Title: Particle Physics Seminar - TBA

Speakers: David Kaplan

Series: Particle Physics

Date: May 28, 2024 - 1:00 PM

URL: https://pirsa.org/24050095

Abstract: Abstract TBA

Zoom link

Non-Linear Quantum Mechanics

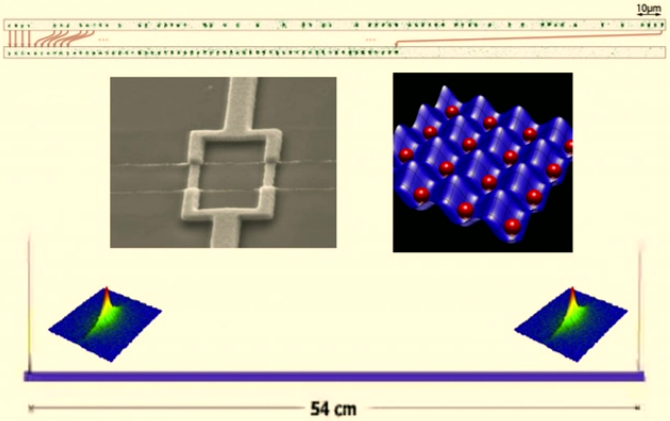
David E. Kaplan

w/ S Rajendran — *PRD* (arxiv:2106.10576)
w/ M Raizen and S Rajendran — *PLB* (arxiv:2203.00195)
w/ M Polkovnikov, et al — *PRL* (arxiv:2204.11875)
w/ Broz, et al — *PRL* (arxiv:2206.12976)

Modify Quantum Mechanics?

Why(nberg-ian) not?

1927



2024

QM is the only known physical theory that is exactly linear

$$i\partial_t|\chi\rangle = \hat{H}|\chi\rangle$$

Quantum Field Theory

The Schrödinger Equation (Schrödinger picture)

$$i\,\partial_t\,|\chi\rangle = \hat{H}\,|\chi\rangle$$

with $\hat{H} = \int d^3x \hat{\mathscr{H}}(\mathbf{x})$

The time evolution operator

$$\hat{U} = e^{-i\hat{H}t}$$

Already non-linear — how do we modify?
Add state-dependent terms.

Quantum Field Theory

'Non-linear' (state-dependent) terms

$$i\,\partial_t|\chi\rangle = \int d^3x \left(\hat{\mathscr{H}}(\mathbf{x}) + \epsilon\langle\chi|\,\hat{\mathcal{O}}_1(\mathbf{x})\,|\chi\rangle\;\hat{\mathcal{O}}_2(\mathbf{x})\right)|\chi\rangle$$

Time evolution includes term that depends on the state itself

If $\mathcal{O}_1$ and $\mathcal{O}_2$ are Hermitian, the norm is constant

$$\partial_t\langle\chi|\chi\rangle = 0$$

Probabilistic interpretation of observables can be maintained

Quantum Field Theory

Path Integral Formulation

Can compute perturbatively — $\langle \chi | \hat{\mathcal{O}}_{1,2} | \chi \rangle$ first at $\epsilon \to 0$

Then insert as a background field

$$S = \int d^4x \, \mathscr{L}(\phi(x), \partial_\mu \phi(x)) + \epsilon \left(\langle \chi | \hat{\mathcal{O}}_1 | \chi \rangle(x) \, \mathcal{O}_2\big(\phi(x)\big) + \langle \chi | \hat{\mathcal{O}}_2 | \chi \rangle(x) \, \mathcal{O}_1\big(\phi(x)\big) \right)$$

$$\mathbb{Z} = \int \mathscr{D}\phi e^{i \int d^4x \, \mathscr{L}(\phi, \partial\phi) + \epsilon\left(\langle \mathcal{O}_1 \rangle \, \mathcal{O}_2(\phi) + \langle \mathcal{O}_2 \rangle \, \mathcal{O}_1(\phi)\right)}$$

QFT Example

YUKAWA COUPLING

$$\hat{\mathscr{H}} \supset y\hat{\phi}\hat{\bar{\Psi}}\hat{\Psi}$$

Add non-linearity

$$\hat{\mathscr{H}} \supset y\hat{\phi}\hat{\bar{\Psi}}\hat{\Psi} + y\epsilon\left(\langle\chi|\hat{\phi}|\chi\rangle\hat{\bar{\Psi}}\hat{\Psi} + \hat{\phi}\langle\chi|\hat{\bar{\Psi}}\hat{\Psi}|\chi\rangle\right)$$

Non-Linear Quantum Mechanics

One-particle, non-relativistic limit

$$\mathscr{L} \supset y(\phi + \epsilon\langle\chi|\phi|\chi\rangle)\bar{\Psi}\Psi$$

To get NR theory for fermions Ψ, compute $\langle\phi\rangle$.

Will depend on initial conditions and sources. At zeroth order, Ψ sources ϕ:

ψ wave function for single fermion Ψ

$$\langle\phi\rangle(x) \supset \int d^4x' |\psi(x')|^2 G_R(x - x')$$

Charge density of Ψ

Causal Green's Function

Non-Linear Quantum Mechanics

One-particle, non-relativistic limit - the non-linear Schrödinger Equation

$$i\,\partial_t \psi(x) = \hat{H}(\mathbf{x})\,\psi(x) + \epsilon \int d^4x' \,|\,\psi(x')\,|^2\, G_R(x' - x)\;\psi(x)$$

Causality guaranteed by the retarded Green's Function:

$$\Box\, G_R(y; x) = \delta^4(x - y)$$

For a massless field:

$$G_R(x - x') = \frac{\delta\left(t' - (t + |\,\mathbf{x}' - \mathbf{x}\,|)\right)}{|\,\mathbf{x}' - \mathbf{x}\,|}$$

Entangled Systems and Causality

One-particle, non-relativistic limit

Wave function for two entangled particles:

$$\psi(\mathbf{x}, \mathbf{y}; t) = \sum_{i,j} c_{ij}(t)\, \psi_i^{I}(\mathbf{x})\, \psi_j^{II}(\mathbf{y})$$

$$\delta \hat{H} \psi = \int d^3x' d^3y' dt' \, |\psi(\mathbf{x}', \mathbf{y}'; t')|^2 \Big(\boxed{G_R(\mathbf{x}, t; \mathbf{x}', t')} + \underbrace{G_R(\mathbf{y}, t; \mathbf{x}', t') + G_R(\mathbf{x}, t; \mathbf{y}', t')}_{\text{causal}} + \boxed{G_R(\mathbf{y}, t; \mathbf{y}', t')} \Big) \; \psi(\mathbf{x}, \mathbf{y}, t)$$

Can a local action at **x** affect the particle at **y**?

$$\delta \hat{H} \subset \int \underbrace{d^3x' d^3y' dt' \, |\psi(\mathbf{x}', \mathbf{y}'; t')|^2}_{\text{e.g., need this } d^3x' \text{ integral unchanged}} G_R(\mathbf{y}, t; \mathbf{y}', t')$$

Entangled Systems and Causality

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$$\delta\hat{H} \subset \int \underbrace{d^3x' d^3y' dt' \,|\,\psi(\mathbf{x}',\mathbf{y}';t')\,|^2}\, G_R(\mathbf{y},t;\mathbf{y}',t')$$

e.g., need this d^3x' integral unchanged

Unchanged if $\psi(\mathbf{x},\mathbf{y},t) \to (\hat{U}_1 \otimes \hat{\mathbf{1}})\,\psi(\mathbf{x},\mathbf{y},t)$. This is true in linear QM (zeroth order in ϵ)

First order correction (time-dep. p.t.) to particle I potential — use zero-order w.f.s

$$V_1(\mathbf{x},t) \subset \int d^3x' d^3y' dt' \,|\,\psi_0(\mathbf{x}',\mathbf{y}';t')\,|^2\, G_R(\mathbf{x},t;\mathbf{x}',t')$$

Similarly factorized — local and unitary. Continue to all orders.

Entangled Systems and Causality

One-particle, non-relativistic limit

Wave function for two entangled particles:

$$\psi(\mathbf{x}, \mathbf{y}; t) = \sum_{i,j} c_{ij}(t)\, \psi_i^I(\mathbf{x})\, \psi_j^{II}(\mathbf{y})$$

$$\delta \hat{H}\psi = \int d^3x' d^3y' dt' \, |\psi(\mathbf{x}', \mathbf{y}'; t')|^2 \left(\boxed{G_R(\mathbf{x}, t; \mathbf{x}', t')} + \underbrace{G_R(\mathbf{y}, t; \mathbf{x}', t') + G_R(\mathbf{x}, t; \mathbf{y}', t')}_{\text{causal}} + \boxed{G_R(\mathbf{y}, t; \mathbf{y}', t')} \right) \psi(\mathbf{x}, \mathbf{y}, t)$$

SM Examples

QED

Add non-linearity to vector: $\mathscr{H} \supset e\left(A_\mu + \epsilon\langle\chi|A_\mu|\chi\rangle\right)J^\mu$

(Defined in a gauge, *e.g.*, Weyl)

Or field strength: $F_{\mu\nu}F^{\mu\nu} \to F_{\mu\nu}F^{\mu\nu} + \epsilon F_{\mu\nu}\langle F^{\mu\nu}\rangle$

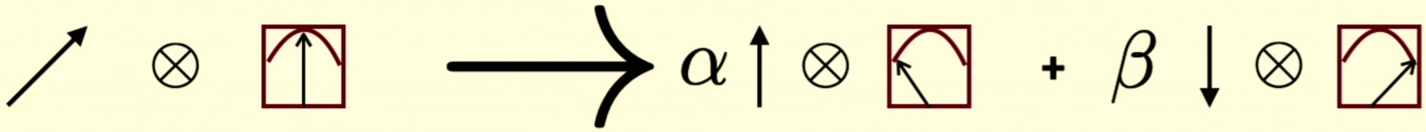
GRAVITY

Replace, *e.g.*, $g_{\mu\nu} \to \dfrac{g_{\mu\nu} + \epsilon\langle g_{\mu\nu}\rangle}{1+\epsilon}$ in interaction terms

Full GCI can be maintained

Measurement in Quantum Mechanics

Time evolution with interaction between the system and measuring device



$|\chi\rangle$ $\quad|A_0\rangle$ $\qquad |i\rangle \otimes |A_i\rangle$

$$|\chi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle$$

Prediction of Quantum Mechanics ("Many Worlds")

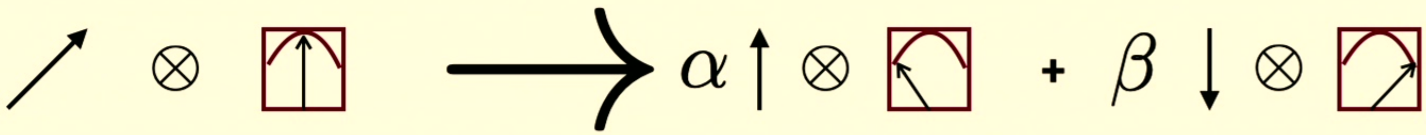
Pick: $\langle A_j | A_i \rangle = \delta_{ij}$

Trace over device $\longrightarrow \rho_{|\chi\rangle} = \sum_i c_i c_i^* |i\rangle\langle i|$

'Probabilistic' outcome

Measurement in Quantum Mechanics

Time evolution with interaction between the system and measuring device



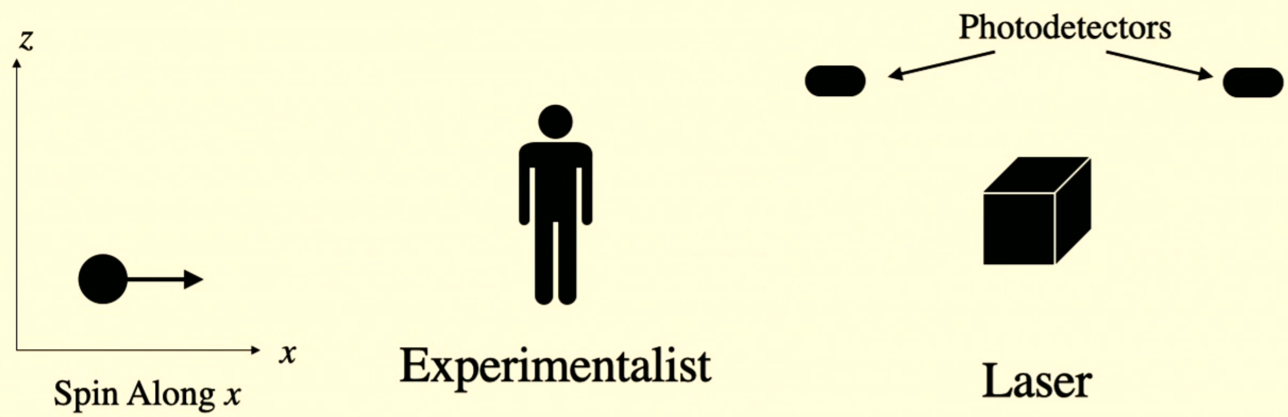
$|\chi\rangle$ $|A_0\rangle$ $|i\rangle \otimes |A_i\rangle$

In NLQM, stationary states generally **not** orthogonal —
the effective Hamiltonian depends on the initial state of the system

No Guarantee: $\langle A_j | A_i \rangle = 0$ for $i \neq j$

$$|\Psi\rangle \otimes |A_0\rangle \to \sum_i c_i |i\rangle \otimes |A_i\rangle + \epsilon \sum_{i,j} d_{i,j} |i\rangle \otimes |A_j\rangle$$ 'Measurement noise'

Linear Quantum Mechanics



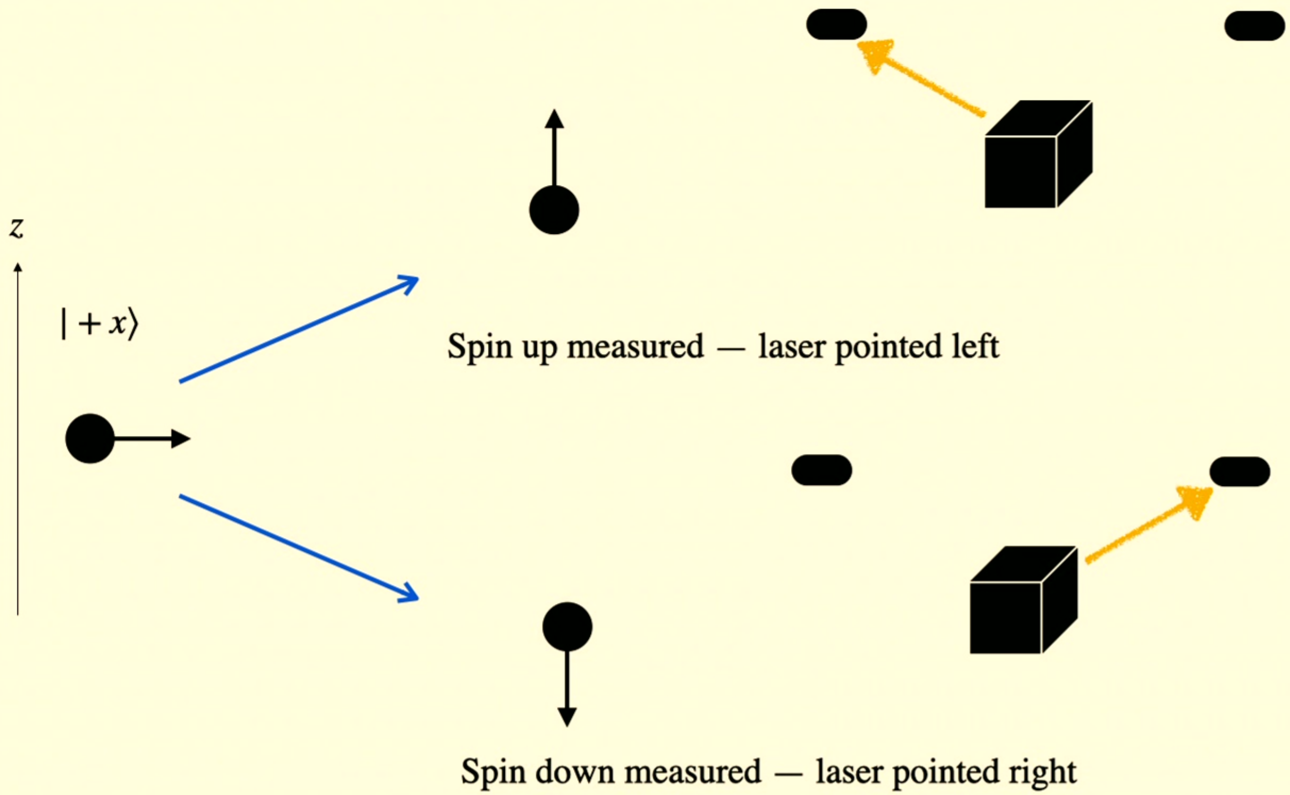
Initial State : $|\chi(0)\rangle$

Represents Full Quantum State (spin, experimentalist…)

Goal: Create Macroscopic Superposition

Method: Measure spin along z.
Depending upon outcome, send laser along different directions

Linear Quantum Mechanics

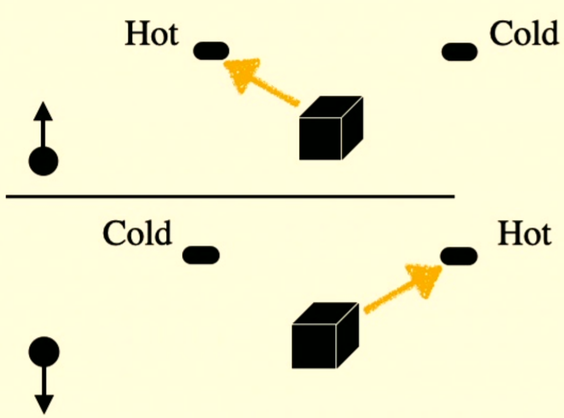


$$|\chi(t)\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle|L\rangle|Env_L\rangle + |-z\rangle|R\rangle|Env_R\rangle\right)$$

Linear Quantum Mechanics

Which photodetectors light up?

$$\mathscr{H} \supset eA_\mu J^\mu$$



Transition Matrix Elements

$$\langle +z | \langle L | \langle Env_L | A_\mu(x_L) J^\mu(x_L) | +z \rangle | L \rangle | Env_L \rangle \neq 0$$

$$\langle +z | \langle L | \langle Env_L | A_\mu(x_R) J^\mu(x_R) | +z \rangle | L \rangle | Env_L \rangle = 0$$

$$\downarrow$$

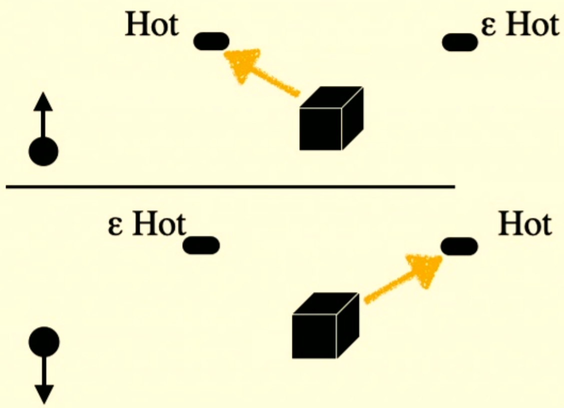
$$\langle L | A_\mu(x_R) | L \rangle = 0$$

$$|\chi(t)\rangle = \frac{1}{\sqrt{2}} \left(|+z\rangle |L\rangle |Env_L\rangle + |-z\rangle |R\rangle |Env_R\rangle \right)$$

Non-Linear Quantum Mechanics

Which photodetectors light up?

$$\mathscr{H} \supset eA_\mu J^\mu + e\varepsilon\langle A_\mu\rangle J^\mu$$



Transition Matrix Elements

$$\langle +z|\langle L|\langle Env_L|\langle A_\mu(x_L)\rangle J^\mu(x_L)|+z\rangle|L\rangle|Env_L\rangle \neq 0$$

$$\langle +z|\langle L|\langle Env_L|\langle A_\mu(x_R)\rangle J^\mu(x_R)|+z\rangle|L\rangle|Env_L\rangle \neq 0$$

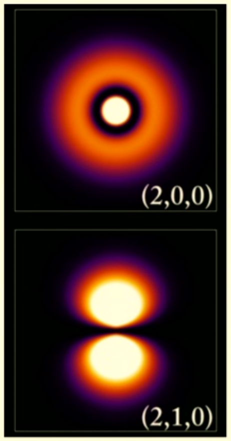
$$\langle\chi|A_\mu(x_L)|\chi\rangle \neq 0,\ \langle\chi|A_\mu(x_R)|\chi\rangle \neq 0$$

Communication between "worlds"

Non-linearity visible despite Environmental De-coherence!
Polchinski: "Everett Phone"

Constraints and Tests: Part I

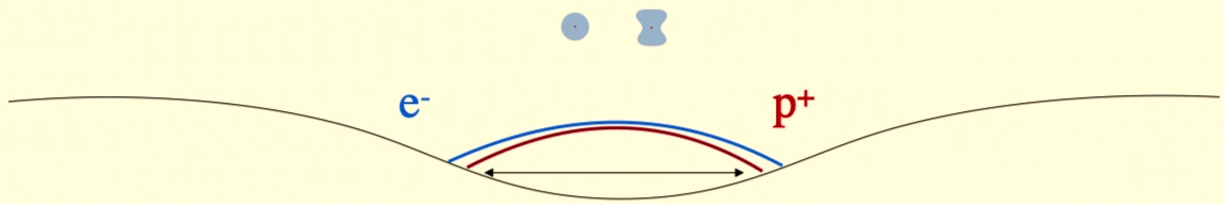
Constraints: Atomic Levels



What does this do to the Lamb Shift?

Say charged particles see their own w.f.:

$$i\,\frac{\partial}{\partial t}\psi(x) = H(\mathbf{x})\,\psi(x) + \epsilon_\gamma\,\alpha \int d^4x'\,|\,\psi(x')\,|^2\,G_R(x'-x)\,\psi(x)$$



Electron spread over the trap (micron) dilutes the electric field and thus the level splitting

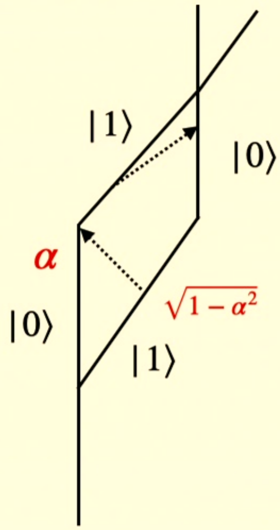
Proton's wave function also produces a field that nearly cancels the electron wave function.

Key — center of mass coordinate cannot be separated from relative coordinate due to locality.

$$\epsilon_\gamma < 10^{-2}$$

Experimental Tests: Local

Interferometry - interaction between paths



Take an ion - split its wave-function

Coulomb Field of one path interacts with the other path

$$|\psi\rangle = \alpha|0\rangle + \sqrt{1-\alpha^2}|1\rangle$$

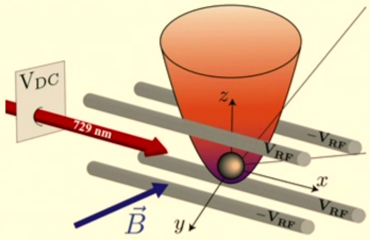
Gives rise to phase shift that depends on the intensity α^2 of the split

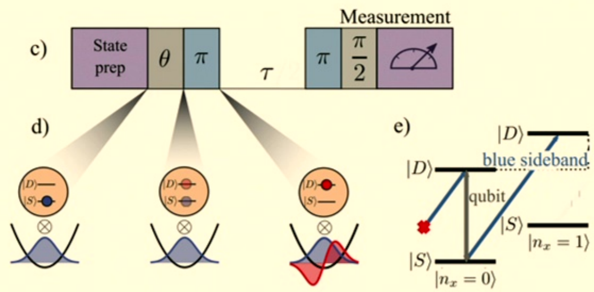
Use intensity dependence to combat systematics

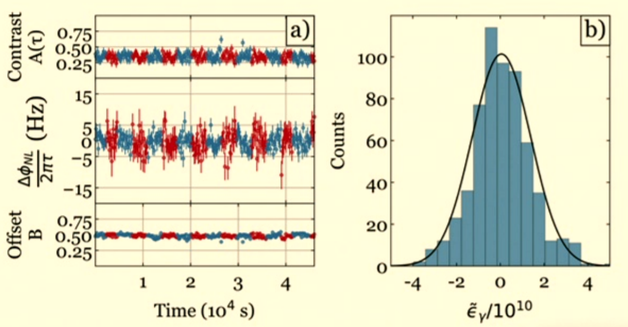
Test … Using a Vibrational Mode of a Trapped Ion

J. Broz, et al. (2022)

State: $|\psi\rangle = a|0\rangle + b|1\rangle$





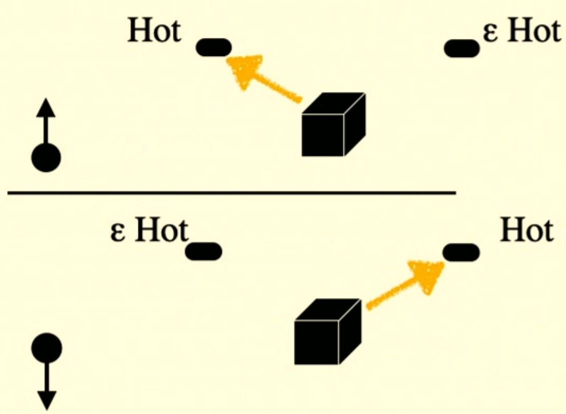


Non-linear phase: $\phi_{NL} = \epsilon_\gamma \alpha \frac{10a^2 + b^2}{30\sqrt{2\pi}\hbar x_0}\tau$

$\epsilon_\gamma = 5 \pm 5.4 \times 10^{-12}$

Experimental Tests: Macroscopic

Create macroscopic superposition
Create Expectation value of EM/Gravity
Search for Expectation value

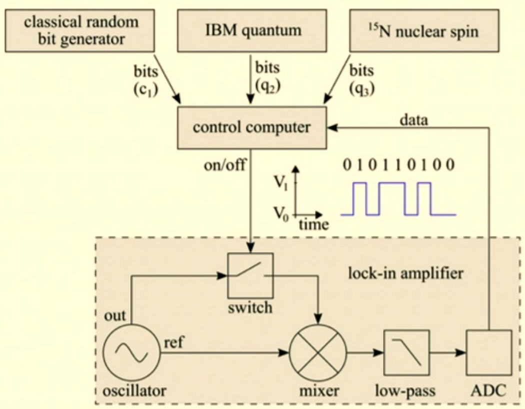


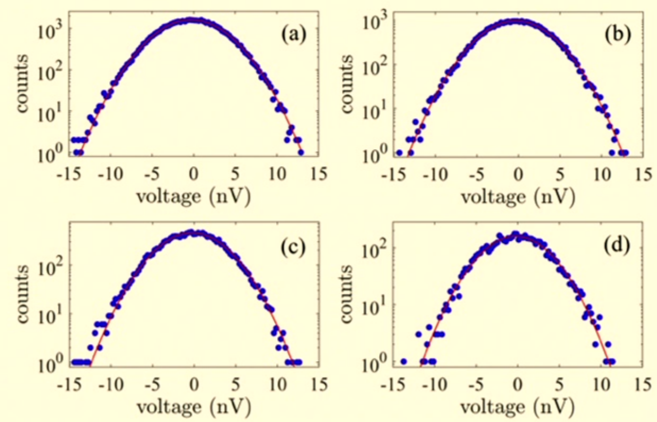
From quantum measurement, turn on outcome directed EM field (laser, voltage, etc)

Similar tests for gravity

Experimental limit on non-linear QM using a voltmeter and quantum bits

M. Polkovnikov, et al (2022)

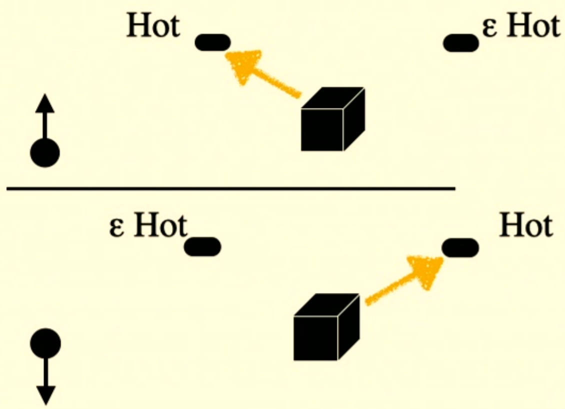




$$|\epsilon_\gamma| \leq 4.7 \times 10^{-11}$$

Delicate Non-Linearity

O performs the laser experiment Today
- discovers non-linear quantum mechanics!



$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle|O_L\rangle + |R\rangle|O_R\rangle\right)$$

Now *O* wants to repeat experiment

Suppose $|O_U\rangle$ decides to run experiment at 9am on October 2
But $|O_D\rangle$ runs experiment on 9am on October 12

State at 9am on October 2

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle|O_L\rangle\frac{|L'\rangle|O_L'\rangle + |R'\rangle|O_D'\rangle}{\sqrt{2}} + |R\rangle|O_R\rangle\right)$$

Delicate Non-Linearity

State on 9am on October 2

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle\,|O_L\rangle \frac{|L'\rangle\,|O_L'\rangle + |R'\rangle\,|O_D'\rangle}{\sqrt{2}} + |R\rangle\,|O_R\rangle \right)$$

Compare with State today

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle\,|O_L\rangle + |R\rangle\,|O_R\rangle \right)$$

$$\langle L|\,\langle O_L|\,\langle L'|\,\langle O_L'|\,\langle A_\mu(x_R)\rangle\, J^\mu(x_R)\,|\chi(t = \text{Oct } 2)\rangle = \frac{1}{2}\langle L|\,\langle O_L|\,\langle A_\mu(x_R)\rangle\, J^\mu(x_R)\,|\chi(t = \text{today})\rangle$$

Effect is 1/2 of prior effect!

But, full effect if O_U and O_D perform experiment at same time!

Quantum Pollution: Without adequate care, superpositions may diverge wildly, preventing exploitability. Not automatic - but need careful protocols!

Delicate Non-Linearity

State on 9am on October 2

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle\,|O_L\rangle \frac{|L'\rangle\,|O'_L\rangle + |R'\rangle\,|O'_D\rangle}{\sqrt{2}} + |R\rangle\,|O_R\rangle \right)$$

Compare with State today

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle\,|O_L\rangle + |R\rangle\,|O_R\rangle \right)$$

$$\langle L|\,\langle O_L|\,\langle L'|\,\langle O'_L|\,\langle A_\mu(x_R)\rangle\, J^\mu(x_R)\,|\chi(t = \text{Oct } 2)\rangle = \frac{1}{2}\langle L|\,\langle O_L|\,\langle A_\mu(x_R)\rangle\, J^\mu(x_R)\,|\chi(t = \text{today})\rangle$$

Effect is 1/2 of prior effect!

But, full effect if O_U and O_D perform experiment at same time!

Quantum Pollution: Without adequate care, superpositions may diverge wildly, preventing exploitability. Not automatic - but need careful protocols!

Particles have been scattering for 13 billion years. Cosmological dilution?

What part of the wave function…

$$|\chi\rangle = \alpha\,|\mathrm{Us}\rangle + \beta\,|\mathrm{Them}\rangle$$

$$\mathscr{H} \supset eA_\mu J^\mu + e\varepsilon\langle A_\mu\rangle J^\mu$$

$$|\chi\rangle = \alpha\,|\mathrm{Us}\rangle + \beta\,|\mathrm{Them}\rangle \to \langle\chi|A_\mu|\chi\rangle = |\alpha|^2\langle \mathrm{U}|A_\mu|\mathrm{U}\rangle + |\beta|^2\langle \mathrm{T}|A_\mu|\mathrm{T}\rangle$$

$$\langle\chi|A_\mu|\chi\rangle \to |\alpha|^2\langle \mathrm{U}|A_\mu|\mathrm{U}\rangle$$

$$\epsilon \to \epsilon_{\mathrm{eff}} \equiv \epsilon\,|\alpha|^2$$

For $\alpha \ll \beta$, the wave function is dominated by something we are not a part of.
Can't turn on coherent fields over there.

Local exploitability completely determined by unchangeable initial conditions dramatic difference from linear QM

Classical World?

$|U(t)\rangle = |$ 🌎 $\rangle + \delta|$ ■ $\rangle$ **Or** $|U(t)\rangle = \delta|$ 🌎 $\rangle + |$ ■ $\rangle$

Are there natural quantum amplifiers, for e.g. in chaotic systems?

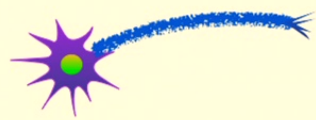
$\Delta x \sim 100\text{nm}$

Changing classical evolution of a system requires coherent motion of N atoms

Probability that N atoms coherently move in some way: p^N

With $p \sim O(1)$ scattering probability

What about the weather?
What about my brain??



100's + ions to get one neuron to fire

$|U(t)\rangle = |$ 🌎 $\rangle + \delta|$ ■ $\rangle$ Reasonable

Quantum Amplifiers are Hard!

Natural Quantum Dilution

The '750 GeV' resonance!

Search for resonances decaying to photon pairs in 3.2 fb^{-1} of pp collisions at $\sqrt{s}$ = 13 TeV with the ATLAS detector

Dec 15, 2015

NF-2015-081

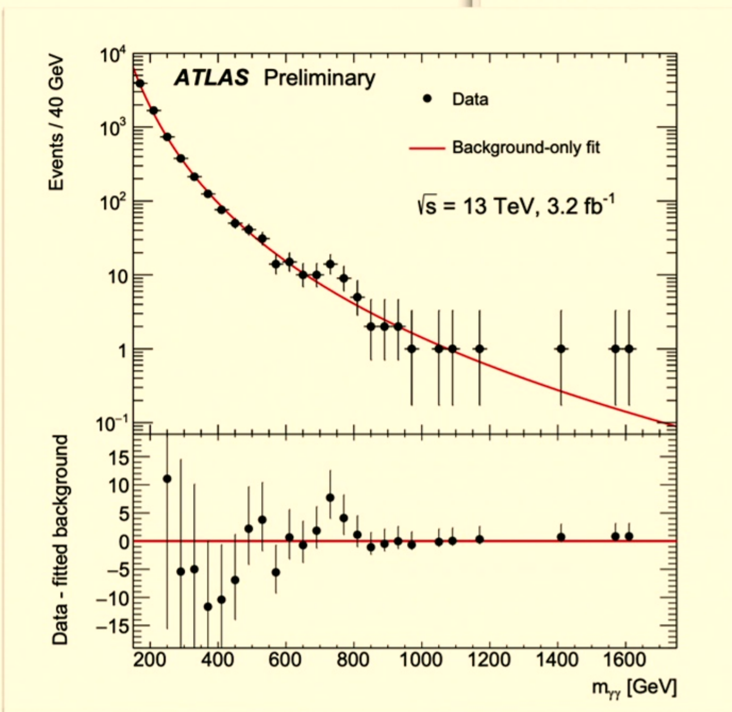
ATLAS

Server

claim

reference search

562 citations



Have we been diluting our wave function on Earth for the past 100 years?

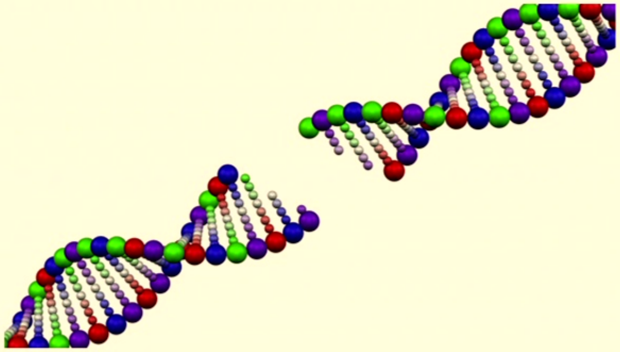
Evolutionary Dilution?

Is N ~ O(few) for evolution?

Maybe for RNA/DNA?

RNA formation?

Is Evolution a quantum amplifier?!?



$|U(t=0)\rangle =$ 

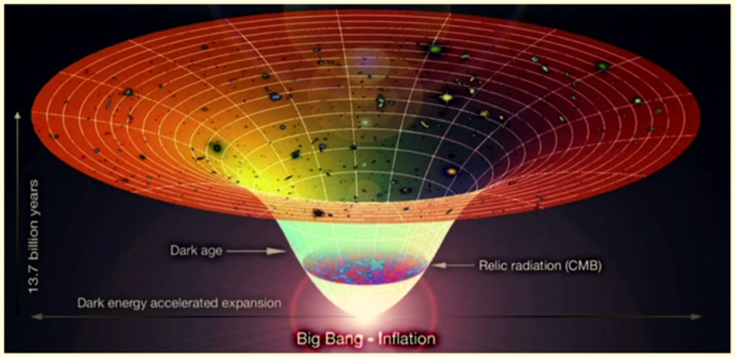
$|U(t)\rangle = |$  $\rangle(|$  $\rangle + |\text{No life}\rangle + \ldots)$

$|U(t=0)\rangle =$ 

$|U(t)\rangle = |$  $\rangle(|$  $\rangle + |$  $\rangle + \ldots)$

Cosmological Quantum Amplifier: Inflation

Standard cosmic inflation:
rapidly places quantum state in a homogenous and isotropic state
(Bunch-Davies Vacuum)



Homogeneous state becomes massive superposition of statistically similar Universes!

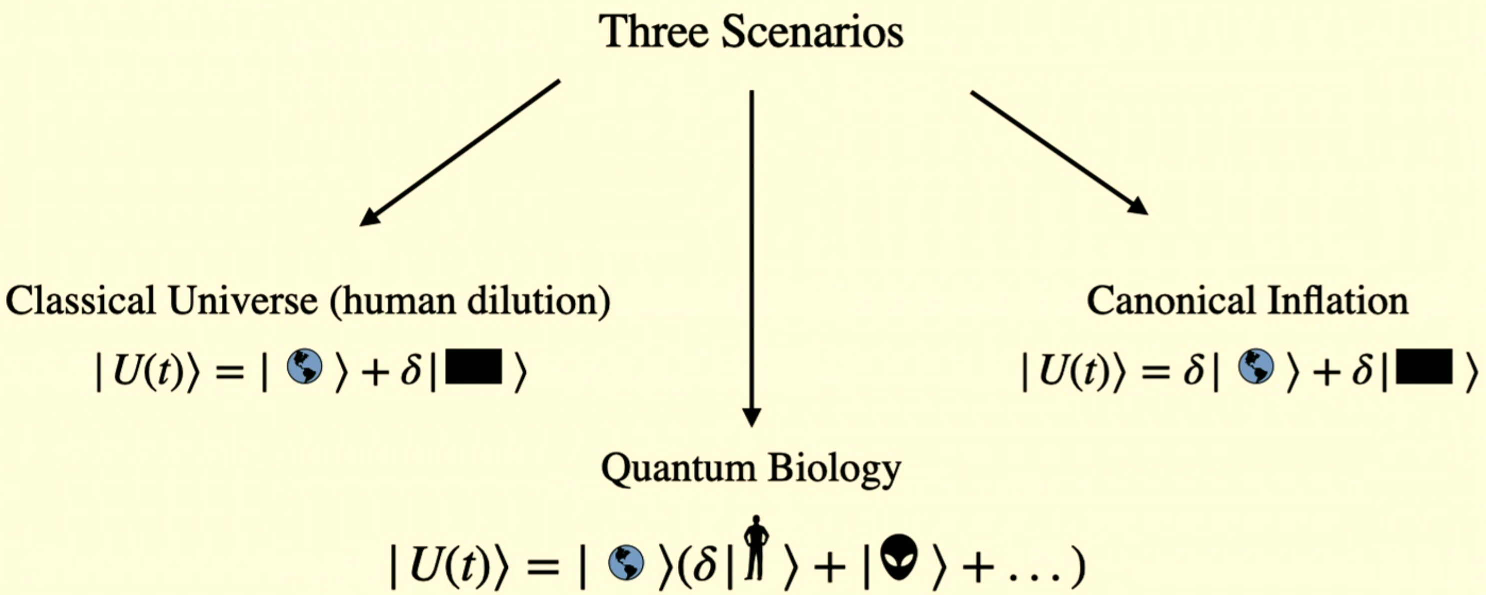
$$|\chi\rangle = \sum_i c_i |U_i\rangle, \;\; c_i \sim e^{-N}$$

Most of the Universe: The space-time point where the Earth is located is in intergalactic space

Probes of Non-Linear Quantum Mechanics

Non-Linear effects deeply tied to unchangeable initial and evolution of total quantum state

Probe not just non-linear quantum mechanics but full evolutionary history of quantum state!

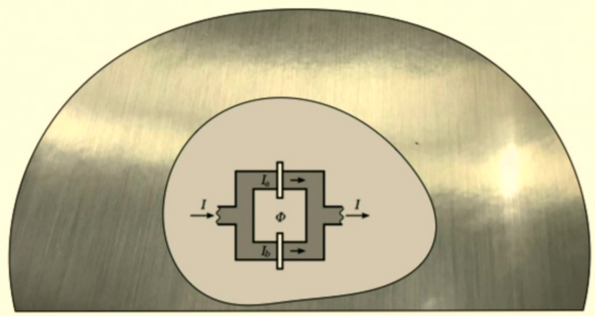


Tests for a Quantum-Diluted Earth

Look for coherent fields turned on in all parts of the wavefunction

$$eJ^{\mu}(A_{\mu} + \epsilon_{\gamma}\langle A_{\mu}\rangle)$$

Remove an ancient magnetic source in our part of the wave function and measure the field in that region.



$|U(t)\rangle = |$ 🌎 $\rangle(\alpha\,|\,\text{missing magnetic material}\,\rangle + \beta\,|\,\text{magnetic material}\rangle)$

Could do the same with gravitational sources on Earth ($e.g.$, Bikini Atoll)

Future Directions

1. Gravity for non-classical states
2. Black Hole horizon with interfering metrics
3. How does non-linear QM affect inflation?
4. Model building with $\langle h \rangle, \langle R \rangle, \ldots$

Conclusions

Remarkably easy to modify QM in a causal consistent way

Locality makes many past tests insensitive — **new probes required**

NL effects can be experimentally tested by amplifying quantum measurements

Quantum amplification in the history of the universe could suppress local non-linearities

If locally diluted, non-zero fields across the wave function could be detected (ancient magnetic/gravitational fields, cosmological metric)

If NLQM is locally accessible, it will radically change what we can do technologically

Thank you!