Title: Quantum Spin Liquid Oasis in Desert States of Unfrustrated Spin Models: Mirage ?

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Abstract: Hilbert spaces are incomprehensibly vast and rich. Model Hamiltonians are space ships. They could take us to new worlds, such as cold \textit{spin liquid oasis} in hot regions in Hilbert space deserts. Exact decomposition of isotropic Heisenberg Hamiltonian on a Honeycomb lattice into a sum of 3 non-commuting (permuted) Kitaev Hamiltonians, helps us build a degenerate \textit{manifold of metastable flux free Kitaev spin liquid vacua} and vector Fermionic (Goldstone like) collective modes. Our method, \textit{symmetric decomposition of Hamiltonians}, will help design exotic metastable quantum scars and exotic quasi particles, in nonexotic real systems.
G. Baskaran, arXiv:2309.07119

Zoom link

Quantum Spin Liquid Oasis in Desert States of Unfrustrated Isotropic Spin Models: Mirage ?

Exploring the Landscape of Metastable Quantum Scars

21st May 2024

G Baskaran

THE INSTITUTE OF MATHEMATICAL SCIENCES

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Quantum Spin Liquid Oasis in Desert States of Unfrustrated Isotropic Spin Models: Mirage ?

Hilbert spaces are incomprehensibly vast and rich. Model Hamiltonians are space ships. They could take us to new worlds, such as cold **quantum spin liquid oasis** in hot regions in Hilbert space deserts. Exact decomposition of isotropic Heisenberg Hamiltonian on a Honeycomb lattice into a sum of 3 non-commuting (permuted) Kitaev Hamiltonians, helps us build a degenerate textit{manifold of metastable flux free Kitaev spin liquid vacua} and vector Fermionic (Goldstone like) collective modes. Our method, **symmetric decomposition of Hamiltonians**, will help design exotic metastable quantum scars and exotic quasi particles, in nonexotic real systems.

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Sahara Desert Tunisia

Contemplating on Quantum Spin Liquids Room Temperature Superconductivity

Thanks to Prof Fethi Madouri **former Student from Tunisia**

Eigen States of Model Manybody Hamiltonians
Eigen State Thermalization Hypothesis ETH
And exceptions (Quantum Scars)

Quasi Quantum Scars - Oasis & Mirage in Hot Hilbert Space Deserts

A method to search for Cold Metastable Quantum Phases

Kitaev Spin Liquid - Exactly Solvable Anisotropic S=1/2 model on a Honeycomb Lattice
Where RVB Meanfield theory gives exact Spectrum
Emergent Z_2 Gauge field and Majorana Fermions

Emergence of Metastable Kitaev type Nematic Spin Liquid phases
via Spontaneous Symmetry Breaking in Isotropic Heisenberg Model
Fermion Condensation & Fermionic Goldstone mode

Manybody Localization … Fate of the False Vacua

Experimental Realizabiliy … Transient Quantum Computation

Richness of Two Qubit Hilbert Space

Lorentzian geometry for detecting qubit entanglement

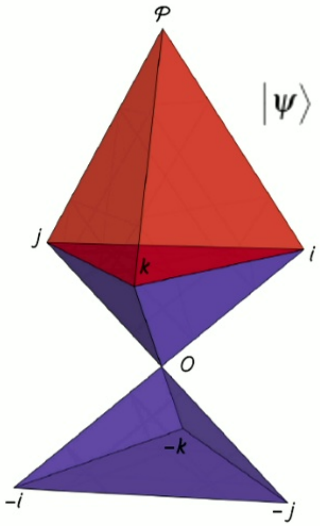
Annals of Physics 396 (2018) 159–172

Joseph Samuel *, Kumar Shivam, Supurna Sinha
Raman Research Institute, Bangalore 560080, India

Dimension of N Qubit Hilbert Space $\mathbb{C}P^{2^N-1}$

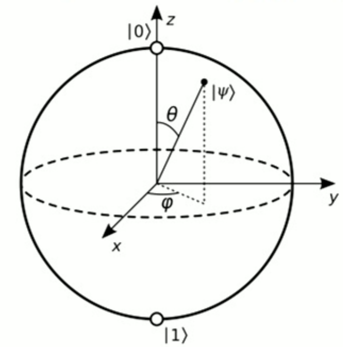
Single Qubit Hilbert Space
Bloch Sphere $\mathbb{C}P^1$,

Two Qubit Hilbert Space



$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$\sum_{ij}|\alpha_{ij}|^2 = 1.$$



.... our analysis uses Energy conditions, which are used in general relativity, ...

Richness of Quantum World
Life of a Quantum Theorist is a Voyage in the Vast Hilbert Space

Reference Quantum Liquids
Fermi sea, Bose gas, Dirac liquid, Weyl liquid, Non-Fermi liquids,
Luttinger liquid, hierarchy of fQH liquids, anyon liquids
Composite Fermi Sea, Majorana Fermi Sea
Superconductor, Superfluid, Vortex liquid ...

Pauling-Anderson RVB liquid, …
Nematic, Kugel-Khomskii, Kitaev Spin Liquid ...

Guided by Experiments we Conceptualize and build
Minimal Quantum Models
Develop Analytical & Numerical Manybody Methods
and the Exploration Continues

Spin Crystals

AFM, Helical, … Landau type Long Range Order,
Bosonic Goldstone modes, Skyrmion (topological) excitations …

Quantum Spin Liquids

Melting of Landau type of Long Range Order
Induced by Frustrating spin-spin interactions and Quantum Fluctuations

…

Isotropic (RVB) and Anisotropic (Nematic) Spin Liquids

Rich Patterns of Quanum Entanglement among Spins

Results in Topological Order, Quantum Order ...

Quasi particles are Extended Topological Objects
Emergent Fermions, Emergent Gauge Fields, Gauge field quanta
and Gauge Field charges and Fluxes, Abelian and Non-Abelian Anyons ...

PW Anderson 1987; GB, Zou, PWA 1987; Kivelson, Rokhsar, Sethna 1987;
GB, PWA 1988, XG Wen 1989 ...

How do we Classify Metastable Phases ?

Glassy Phases ... structural glasses … glassy domain walls in magnets vortex glasses in superconductors … Skyrmion glass … metallurgy

Nonglassy Phases
Metastable structures of ice visible in liquid water ?

We provide a systematic approach

Many body Quantum Scars

Eigen states of many particle Hilbert space obey
Eigen State Thermalization Hypothesis (ETH)

There are Exceptions, however ***Quantum Scars***

Examples

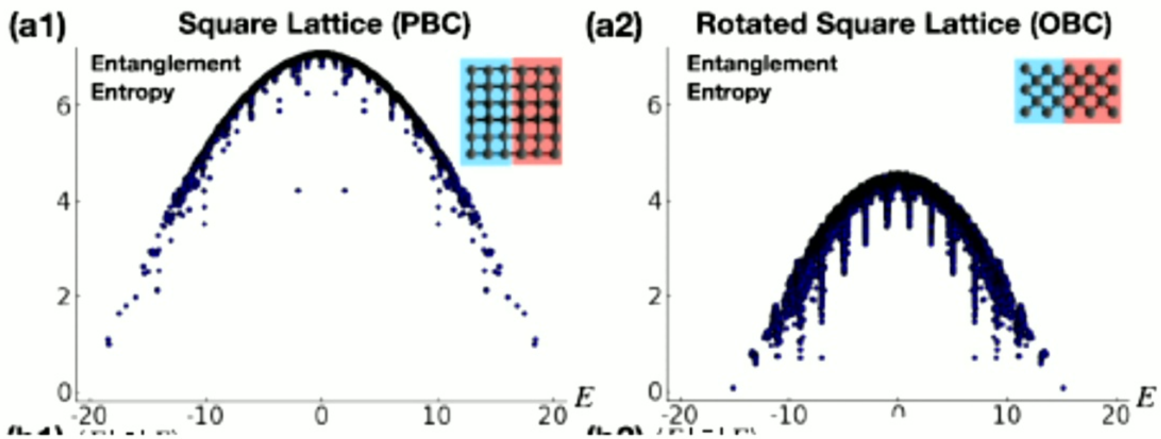
Eta particle condesed states of D-dimensional
Repulsive Hubbard Model in bipartite lattice

Family of states of AKLT model, Spin-1 Kitaev chain, PXP model ..

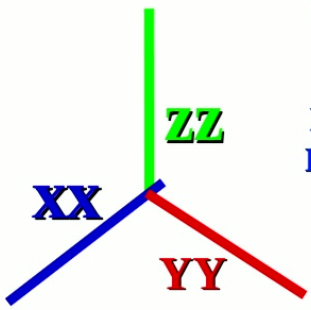
Quantum many-body scar states in two-dimensional Rydberg atom arrays

Cheng-Ju Lin,[1] Vladimir Calvera,[2,1] and Timothy H. Hsieh[1]

PHYSICAL REVIEW B **101**, 220304(R) (2020)



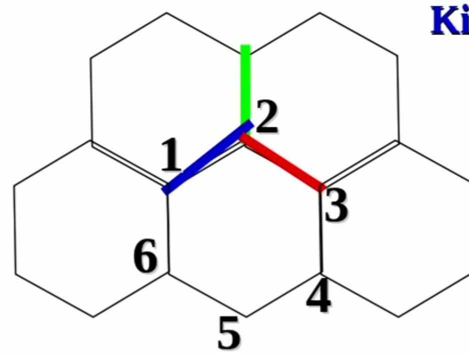
Kitaev Model on a Honeycomb lattice



Frustration in Spin Space via

Bond dependent Ising Interactions

Kitaev 2001, 2003



2N Lattice sites

Periodic Boundary Conditions

$$H = -J_x \sum_{\langle ij \rangle_x} \sigma_i^x \sigma_j^x - J_y \sum_{\langle ij \rangle_y} \sigma_i^y \sigma_j^y - J_z \sum_{\langle ij \rangle_z} \sigma_i^z \sigma_j^z$$

Flux Operator

$$B_p = \sigma_1^y \sigma_2^z \sigma_3^x \sigma_4^y \sigma_5^z \sigma_6^x$$

$B_P^2 = 1$

N conserved local Flux Operators

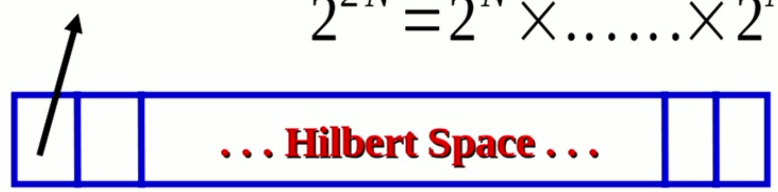
$[B_P, H] = 0$ Eigen values of $B_P = \pm 1$

$[B_P, B_{P'}] = 0$

for any plaquette P and P'

2^N states

$$2^{2N} = 2^N \times \ldots\ldots \times 2^N$$



2^N distinct configurations (sectors) of B_P

2^N different B_P Sectors

Kitaev's method of solution

Similar to Abrikosov Fermions used in RVB Mean Field Theory (GB, Zou, PWA 1987)

But with a Focus on Majorana Components

$$\{\psi,\psi^+\}=1$$

$$\{\xi,\zeta\}=0$$

2 Majorana fermions make one complex or Dirac fermion

$$\psi^+=\xi+i\zeta$$

$$\xi^2=\zeta^2=1$$

$$2=\sqrt{2}\times\sqrt{2}$$

$$c_{i\uparrow}^\dagger = \frac{1}{2}(c_{ix}+ic_{iy}) \text{ and } c_{i\downarrow}^\dagger = \frac{1}{2}(c_{iz}-ic_{i0})$$

Introduce 4 Majorana fermions at each site

$$c^{\alpha},\ \alpha = 0, x, y, z \qquad \{c^{\alpha}, c^{\beta}\} = 2\delta_{\alpha\beta}$$

$$D_i|\Psi\rangle_{\text{phys}} = |\Psi\rangle_{\text{phys}}$$

Dimension of Physical Hilbert Space 2^{2N}

$$D_i \equiv c_i\ c_i^x c_i^y c_i^z$$

Dimension of Enlarged Hilbert Space 4^{2N}

$$\boxed{\sigma_i^a = i c_i c_i^a,} \qquad a = x, y, z \qquad [\sigma_i^a, \sigma_j^b] = i\epsilon_{abc}\sigma^c\delta_{ij}$$

Represent Pauli spin operators
Using 4 Majorana Fermion operators

Kitaev Spin Hamiltonian takes an elegant form:

Majorana Fermions with only two body interactions

$$H = -\sum_{a=x,y,z} J_a \sum_{\langle ij \rangle_a} i c_i \hat{u}_{\langle ij \rangle_a} c_j \qquad \hat{u}_{\langle ij \rangle_a} \equiv i c_i^a c_j^a$$

$$[H, \hat{u}_{\langle ij \rangle_a}] = 0$$

$\hat{u}^2_{\langle ij \rangle_a}$ **= 1** ***eigen value*** $u_{\langle ij \rangle_a} = \pm 1$

$u_{\langle ij \rangle_a}$ (Ising) Z_2 gauge fields on the bonds

Local Z_2 gauge symmetry $u_{\langle ij \rangle a} \longrightarrow \tau_i u_{\langle ij \rangle a} \tau_j$

with $\tau_i \pm 1$

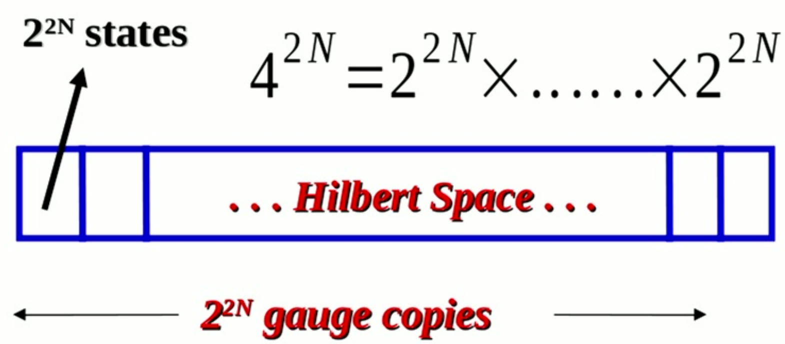
Dimension of Physical Hilbert Space 2^{2N}

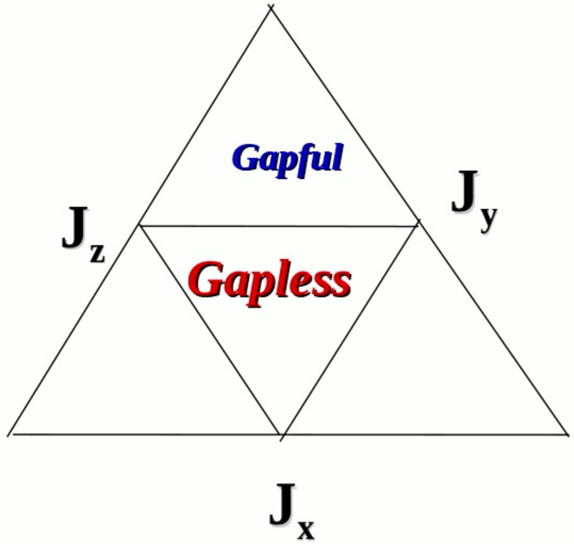
Dimension of Enlarged Hilbert Space 4^{2N}

Spectrum is identical in all gauge copies

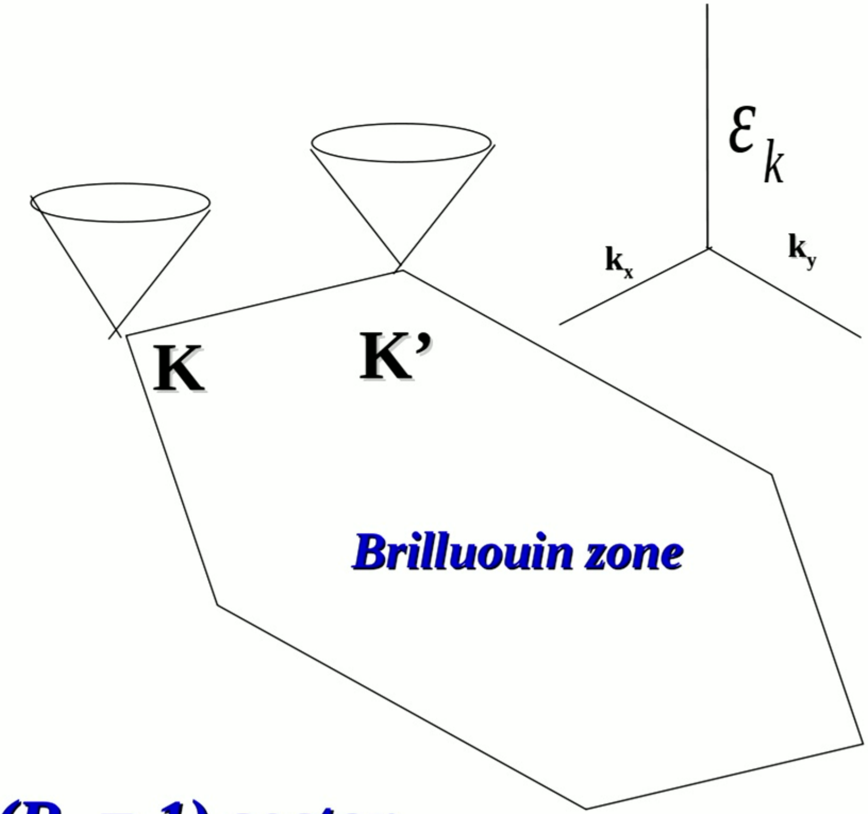
Unphysical Hilbert space has correct energy spectrucm !

$$4^{2N} = 2^{2N} \times \ldots\ldots \times 2^{2N}$$





Only particle like fermionic excitations are present



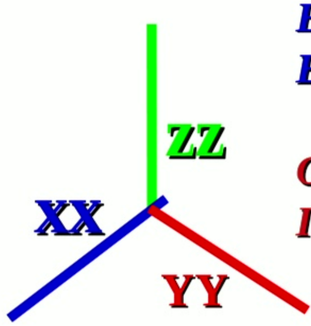
For zero flux (B_p = 1) sector

Exact Spin-Spin Correlation Functions & Quantum Number fractionization

GB, Mondal, Shankar PRL 2007

Kitaev spin liquid is a Nematic Spin Liquid

$$\langle \sigma_i^{\alpha} \sigma_j^{\beta} \rangle = g_{\langle ij \rangle_\alpha} \delta_{\alpha\beta} \quad \text{i, j nearest neighbors}$$
$$= 0 \quad \text{otherwise}$$



Extreme Nematic
Ellipsoid with zero thickness

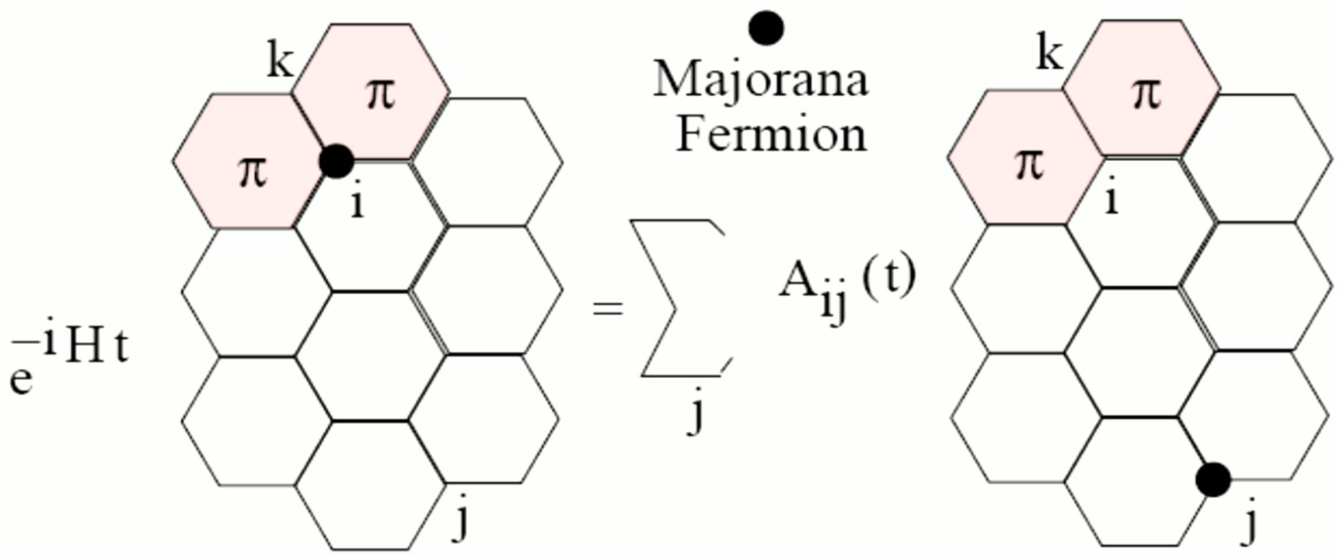
Oriented along X, Y and Z directions
In spin space along xx, yy and zz bonds

Perturbed Kitaev model will
Have general Nematic Ellipsoid

Exact Spin-Spin Correlation Functions & Quantum Number fractionization

GB, Mondal, Shankar PRL 2007

$$e^{iHt}\ \sigma_i^a|\hat{\Psi}\rangle \equiv i c_i(t) T(e^{2u_{\langle ik\rangle a} J_a \int_0^t c_i(\tau) c_k(\tau) d\tau})\hat{\pi}_{\langle ik\rangle a1}\hat{\pi}_{\langle ik\rangle a2}\ |\hat{\Psi}\rangle$$



SU(2) Symmetric S = ½
Heisenberg Hamiltonian on a Honeycomb lattice
can be rewritten as
sum of non-commuting XY and Ising Hamiltonians

$$H_{Heisenberg} = H_{XY} + H_{Ising}$$

Traditionally we break SU(2) symmetry by picking
Ising ground state of exactly solvable Ising part H_{Ising}
and perform spin wave analysis (Kittel)
(Holstein-Primakoff transformation) with H_{XY} as perturbation and obtain

FM or AFM order, Goldstone modes and
fluctuation induced reduction in Sublattice magnetization

G B, arXiv:2309.07119

SU(2) Symmetric S = ½ Heisenberg Hamiltonian can be rewritten as sum of Three non-commuting Kitaev Hamiltonians

$$H_{Heisenberg} = H_{Kitaev}^{xyz} + H_{Kitaev}^{yzx} + H_{Kitaev}^{zxy}$$

Each Kitaev Hamiltonian piece is Exactly solvable

However, energy of this state is high compared to Ising ground state

$$H^K_{xyz} \equiv J \sum_{\langle ij \rangle_x} \sigma^x_i \sigma^x_j + J \sum_{\langle ij \rangle_y} \sigma^y_i \sigma^y_j + J \sum_{\langle ij \rangle_z} \sigma^z_i \sigma^z_j$$

SU(2) Symmetric S = ½
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FM or AFM order, Goldstone modes and
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Residual Interaction among scalar and vector Majorana Fermions

$$H^K_{yzx} + H^K_{zxy} \equiv +J \sum_{\langle ij \rangle_x} c_i c_j (c_i^y c_j^y + c_i^z c_j^z) +$$

$$+J \sum_{\langle ij \rangle_y} c_i c_j (c_i^z c_j^z + c_i^x c_j^x) + J \sum_{\langle ij \rangle_z} c_i c_j (c_i^x c_j^x + c_i^y c_j^y)$$

Hartree Approximation

$$H^H_H(G, xyz) = J_s \sum_{\langle ij \rangle} i c_i c_j \quad + J_v \{ \sum_{\langle ij \rangle_{yz}} i c_i^x c_j^x + \sum_{\langle ij \rangle_{zx}} i c_i^y c_j^y + \sum_{\langle ij \rangle_{xy}} i c_i^z c_j^z \}$$

Hartree Parameters

$$\mathrm{J}_s \equiv J(1+\alpha_s) \qquad \mathrm{J}_v \equiv J(1+\alpha_v)$$

$$\alpha_s \equiv \langle c_i c_j \rangle \qquad \alpha_v \equiv \tfrac{2}{3} \langle \vec{c}_i \cdot \vec{c}_j \rangle$$

are determined self consistantly

$$H_H^H(G, xyz) = J_s \sum_{\langle ij \rangle} i c_i c_j \quad + J_v \{ \sum_{\langle ij \rangle_{yz}} i c_i^x c_j^x + \sum_{\langle ij \rangle_{zx}} i c_i^y c_j^y \; + \; \sum_{\langle ij \rangle_{xy}} i c_i^z c_j^z \}$$

Free Scalar
Majorana Fermion
In zero flux sector

Free vector Majorana Fermion
In three types of decoupled
Zig-zag 1D Chains

X Majorana Fermions hop along
YZ - zig zag chains

Y MF along ZX chain

Z MF along XY chains

In the Heisenberg model in the honeycomb lattice

Ising piece of the Hamiltonian compete
to stablize AFM or FM phase

three Kitaev pieces of the Hamiltonian compete
to stabilize their own Kitaev Spin Liquid
in the corresponding zero flux sector

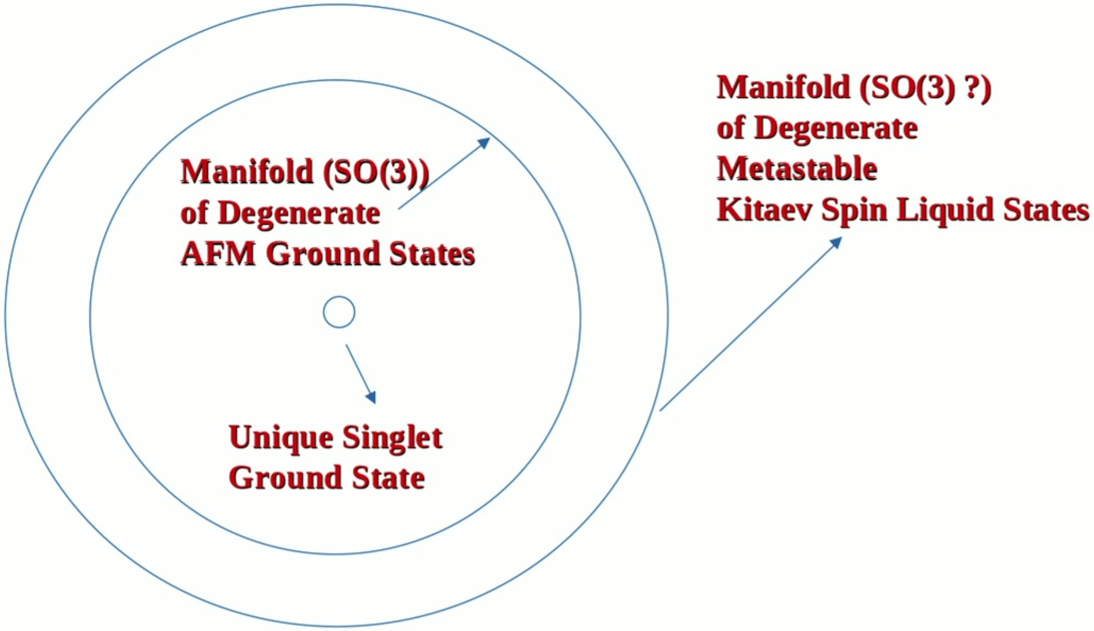
In the competition Kitaev spin liquid looses
and AFM or FM order wins at and close to Ground States

However, we have quantum fluctuating
Zero Flux vectors at higher energy scales

A degenerate manifold of Kitaev spin liquids
become Metastable vacuua and

Goldstone mode emerges

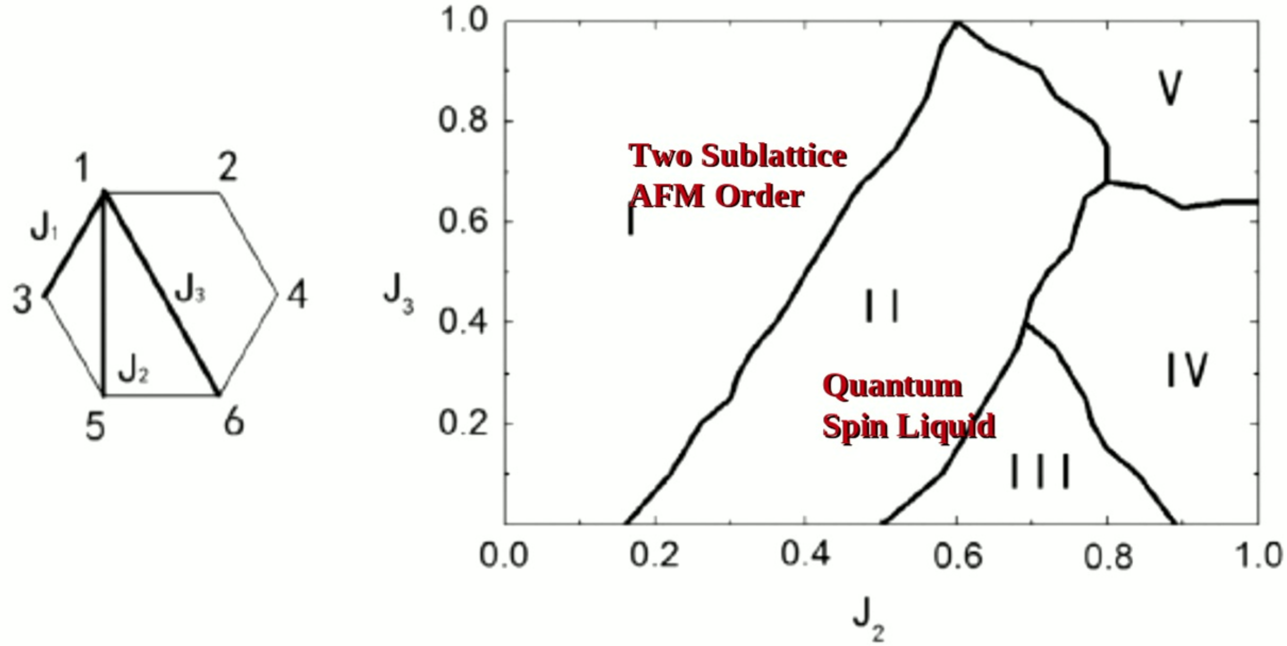
Schematic picture of Hilbert Space and Manifold of Degenerate Metastable Kitaev Spin Liquid States arising from Spontaneous Symmetry Breaking



Frustration could destabilize AFM/FM order and stabilize Kitaev type Nematic Spin Liquid

Francesco Ferrari and GB 2022 (unpublished)

SU(2) Symmetric Heisenberg Spin ½ Model on a Honeycomb Lattce with Frustrating Interactions



Distant Spin Correlations in Quantum Spin Liquid on the Honeycomb Lattice

Hao Geng et al., *Phys. Status Solidi B* **2020**, *257*, 1900659

CRAFTING BY SYMMETRIC DECOMPOSITION OF HAMILTONIANS

$$H^H \equiv H^K_{xyz} + H^{cK}_{xyz}$$

$$H^{cK}_{xyz} \equiv H^K_{yzx} + H^K_{zxy}$$

Similarly, spin-$\frac{1}{2}$ XY model Hamiltonian in a square lattice H^{XY} can be symmetrically decomposed into noncommuting sum of two Kugel-Khomskii [7] pieces H_{xy}^{KK}andH_{yx}^{KK}:

$$H^{XY} = J \sum_{\langle ij \rangle} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \equiv H_{xy}^{KK} + H_{yx}^{KK} \quad (12)$$

Approximate analysis can be performed to find if square lattice XY FM or AFM contains metastable anisotropic spin liquid vacuua corresponding to Kuge-Khomskii compass Hamiltonians.

PREPARATION OF FALSE VACUUM AND WATCHING ITS EVOLUTION

What do we expect ? Our hypothesis is presence of three stages in time evolution. Since there is a finite energy gap for Z_2 flux excitaions, we expect, first, renormalization of the Kitaev spin liquid state (analogue of growth of zero point spin fluctuations in an Ising AFM vacuum in 3 or 2D). After a time interval, the system will nucleate Z_2 fluxes, create a small density of Majorana Fermions as well as topological defects in the Kitaev nematic order, via quantum tunneling. Finally there will be a crossover to nucleation of real (FM/AFM) ground state bubbles and their growth.

Help from Many Body Localization

It is indeed exciting that beyond the comfort zone of equilibrium quantum statistical mechanics, rich new worlds await even in energy eigen states and near eigen (manybody wave packet) states of familiar and well studied Hamiltonians. From the point of view of performing quantum computation and related tasks, present work opens new avenues to *explore and use exotic quasiparticles and exotic metastable states, which are hiding in nonexotic real systems.*

Thank You for Your Attention