

Title: Quantum Machine Learning

Speakers: Alvaro Ballon Bordo

Collection: Navigating Quantum and AI Career Trajectories: A Beginner's Mini-Course on Computational Methods and their Applications

Date: May 23, 2024 - 9:30 AM

URL: <https://pirsa.org/24050085>

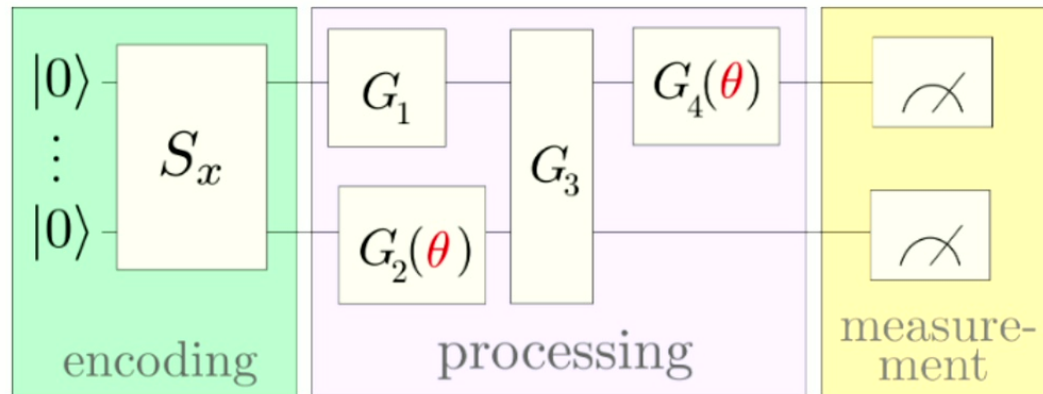
How to embed classical data into a quantum device

Alvaro Ballon



May 2024

Data embedding



Basis Embedding



Used for discrete data, encoded as binary strings

```
N = 3
wires = range(N)
dev = qml.device("default.qubit", wires)

@qml.qnode(dev)
def circuit(b):
    qml.BasisEmbedding(b, wires)
    return qml.state()
>>> circuit([1, 1, 1])
[0.+0.j 0.+0.j 0.+0.j 0.+0.j 0.+0.j 0.+0.j 0.+0.j 1.+0.j]
```


Angle Embedding



Used for real-valued data. Normalize, then embed as rotation angles.

$$x \mapsto R_k(x)|0\rangle = e^{-ix\sigma_k/2}|0\rangle,$$

```
@qml.qnode(dev)
def circuit(val_list):
    qml.AngleEmbedding(val_list, wires)
    return [qml.expval(qml.PauliZ(w)) for w in wires]
>>> circuit([0.0, np.pi / 2, np.pi])
tensor([ 1.0,  0.0, -1.0], requires_grad=True)
```

Amplitude Embedding



Used for complex-valued data.

$$\alpha = (\alpha_0, \dots, \alpha_{2^N-1}) \mapsto \sum_{k=0}^{2^N-1} \alpha_k |k\rangle$$

```
N = 3
wires = range(N)
dev = qml.device("default.qubit", wires)

@qml.qnode(dev)
def circuit(features):
    qml.AmplitudeEmbedding(features, wires)
    return qml.state()
>>> circuit([0.625, 0.0, 0.0, 0.0, 0.625j, 0.375, 0.25, 0.125])
tensor([0.625+0.j , 0.   +0.j , 0.   +0.j , 0.   +0.j ,
        0.   +0.625j, 0.375+0.j , 0.25 +0.j , 0.125+0.j ], requires_grad=True)
```



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```
[1] %%capture
    !pip install pennylane
```

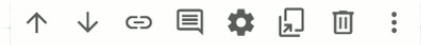


```
[2] import pennylane as qml
    from pennylane import numpy as np
    import matplotlib.pyplot as plt
```



```
X = np.linspace(0,2*np.pi,10)
X.requires_grad = False

tensor([0.          , 0.6981317 , 1.3962634 , 2.0943951 , 2.7925268 ,
        3.4906585 , 4.1887902 , 4.88692191, 5.58505361, 6.28318531], requires_grad=True)
```



```
[ ]
```



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```
[1] %%capture
    !pip install pennylane
```



```
[2] import pennylane as qml
    from pennylane import numpy as np
    import matplotlib.pyplot as plt
```



```
X = np.linspace(0,2*np.pi,10)
X.requires_grad = False
X_test = np.linspace(0.2,2*np.pi+0.2,10)
```



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```
✓ 5s [2] import pennylane as qml
      from pennylane import numpy as np
      import matplotlib.pyplot as plt
```

```
✓ 0s [6] X = np.linspace(0, 2*np.pi, 10)
      X.requires_grad = False
      X_test = np.linspace(0.2, 2*np.pi+0.2, 10)
```

```
▶ dev = qml.device('default.qubit', wires = 1)
```

```
@qml.qnode(dev)
def quantum_model(x, params):
    qml.RX(x, wires = 0)
    qml.Rot(params, wires = 0) I
```



```
+ Code + Text
```

```
[ ] # Step 4 - Classical Pre/Postprocessing
def loss(Y,predictions):
    total_loss=0
    for y,p in zip(Y,predictions):
        total_loss += (y-p)**2
    return total_loss

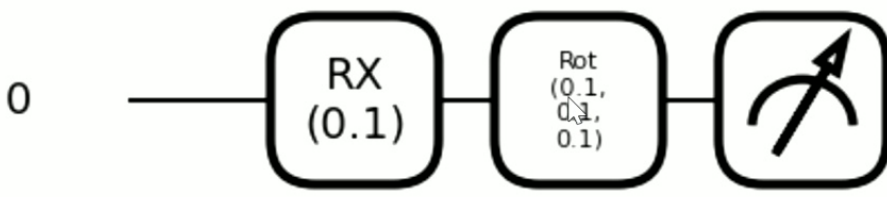
# Step 5 - Define your cost function, including any classical pre/postprocessing
def cost(X,params):
    predictions = [circuit(x,params) for x in X]
    Y = np.sin(X)
    cost = loss(Y,predictions)
    return cost

[ ] # Step 6 - Train your circuit

# Steps 6.1 - Choose an optimizer and a step size
opt = qml.GradientDescentOptimizer(stepsize=0.1)
```

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0s



```
def loss(Y, predictions):  
    total_loss=0  
    for y,p in zip(Y,predictions):  
        total_loss += (y-p)**2  
    return total_loss
```

```
+ Code + Text
```

```
[10] ✓  
0s
```

```
[11] ✓  
0s def loss(Y,predictions):  
    total_loss=0  
    for y,p in zip(Y,predictions):  
        total_loss += (y-p)**2  
    return total_loss
```

```
def cost(X,params):  
    predictions = [quantum_model(x,params) for x in X]  
    Y = np.sin(X)  
    cost = loss(Y,predictions)  
    return cost
```

+ Code + Text

```
opt = qml.GradientDescentOptimizer(stepsize=0.1)

# Step 6.2 Make an initial guess for the parameters and set them as trainable
params = np.array([0.1,0.1,0.1],requires_grad=True)

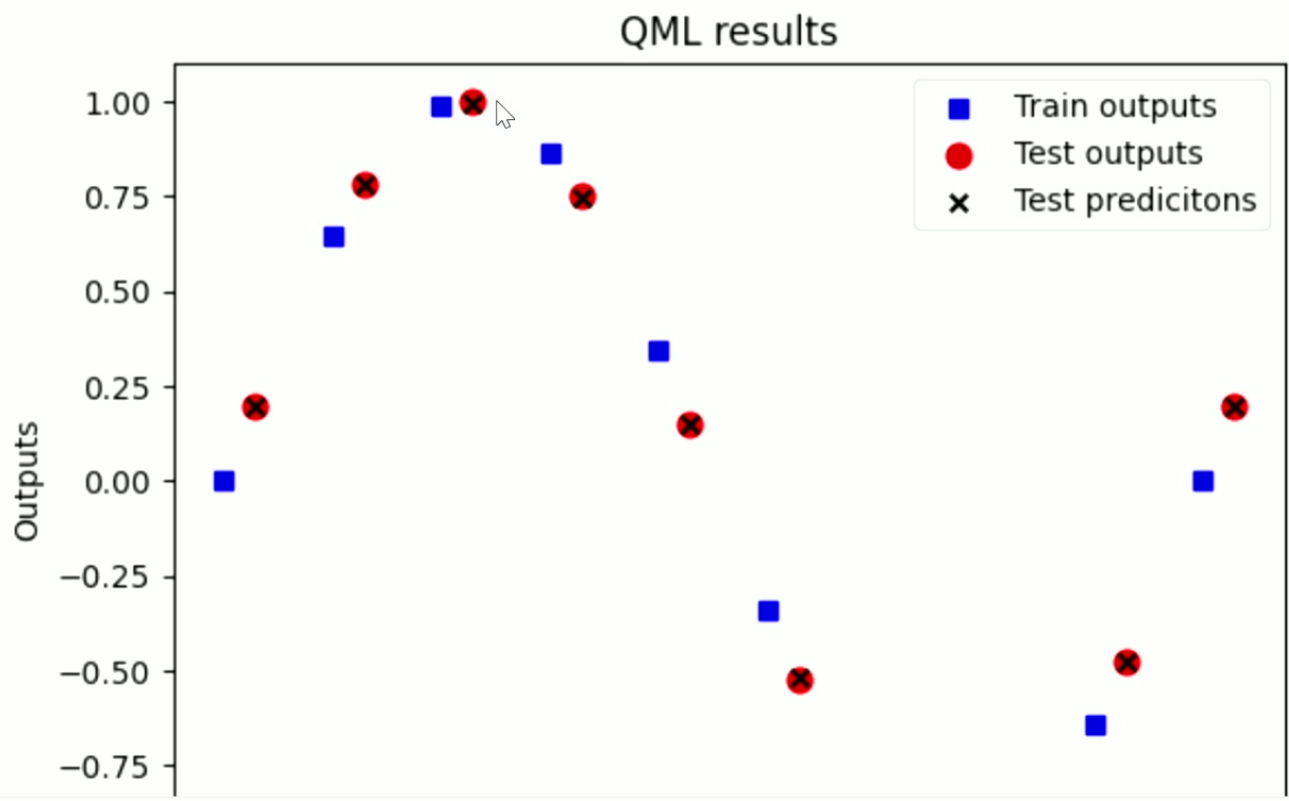
# Step 6.2 - Iterate over a number of defined steps
for i in range(100):
    data_and_params,prev_cost = opt.step_and_cost(cost,X,params)
    params = data_and_params[1]
    if i%10==0:
        print(f'Step: {i},Cost: {cost(X,params)}')
```

[View source](#)

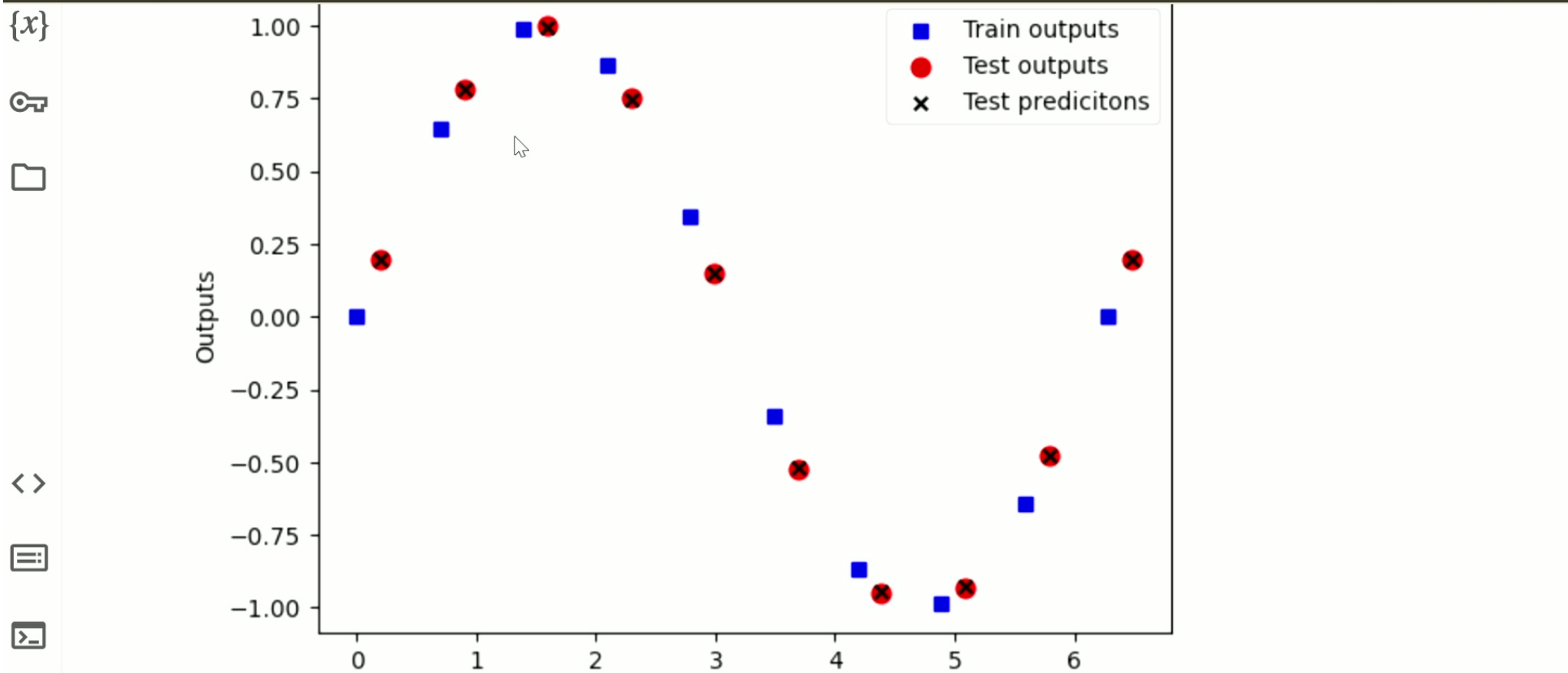
<function cost at 0x7c77e283acb0>

`{x}`

`plt.show()`



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- 4 Angle embedding
- 5 Amplitude Embedding
- 6 A simple QML model
- 7 Variational Quantum Classifier

A simple QML model

Let's do something super simple to warm up

```
graph LR; 0((0)) --> RX[RX (0.1)]; RX --> Rot[Rot (0.1, 0.1, 0.1)]; Rot --> Measure[Measurement];
```

RX here is the angle embedding. Rot is the model

XANADU

Variational Quantum Classifier (VQC)



Aim: Classify **discrete** data.

As per the Machine Learning supervised learning paradigm, we train with labelled data, and then extend to unknown data

In this example, we classify bit strings according to their parity.

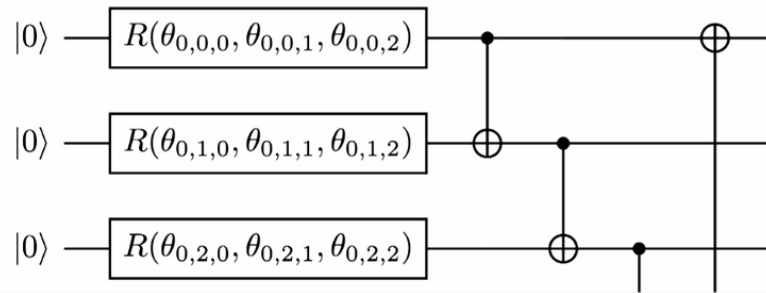
$$f : x \in \{0, 1\}^{\otimes n} \rightarrow y = \begin{cases} 1 & \text{if uneven number of 1's in } x \\ 0 & \text{else.} \end{cases}$$

We then create a quantum device that will run our circuits.

```
[ ] dev = qml.device("default.qubit")
```

Variational classifiers usually define a "layer" or "block", which is an elementary circuit architecture that gets repeated to build the full variational circuit.

Our circuit layer will use four qubits, or wires, and consists of an arbitrary rotation on every qubit, as well as a ring of CNOTs that entangles each qubit with its neighbour. Borrowing from machine learning, we call the parameters of the layer *weights*. Let's take a look on how this quantum circuit looks like:



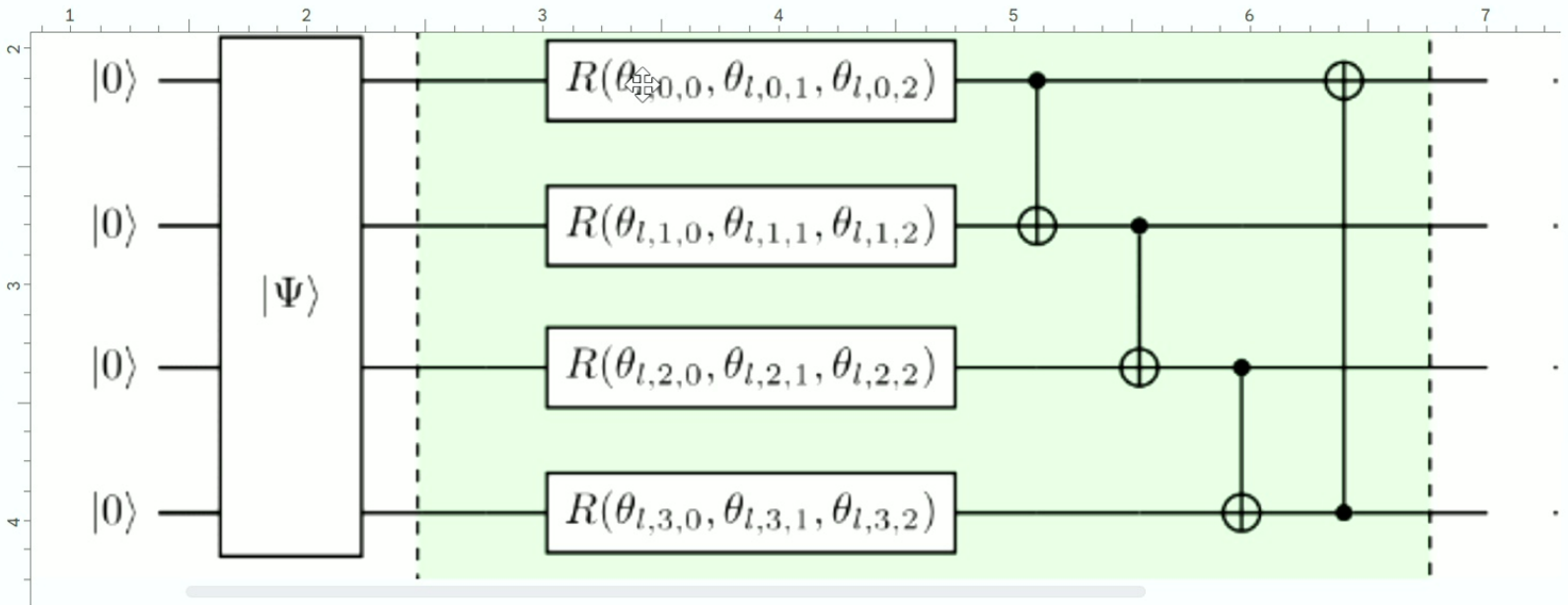
7 Variational Quantum Classifier

8 Variational Quantum Classifier (VQC)

9 The VQC circuit

10 Steps

- 1. Preprocess data if needed
- 2. Encode your data into your model
- 3. Create the model
- 4. Define cost/loss functions



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[]

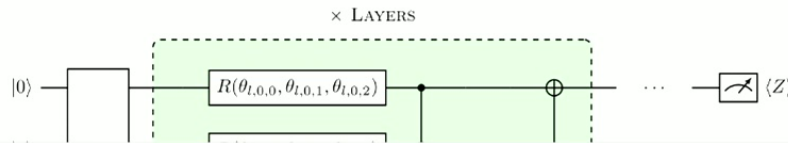
We also need a way to encode data inputs x into the circuit, so that the measured output depends on the inputs. In this first example, the inputs are bitstrings, which we encode into the state of the qubits. The quantum state ψ after state preparation is a computational basis state that has 1s where x has 1s, for example

$$x = 0101 \rightarrow |\psi\rangle = |0101\rangle.$$

The `pennylane.BasisState` function provided by PennyLane is made to do just this. It expects x to be a list of zeros and ones, i.e. `[0,1,0,1]`.

```
def state_preparation(x):  
    ...qml.BasisState(x, wires=[0, 1, 2, 3])
```

Now we define the variational quantum circuit as this state preparation routine, followed by a repetition of the layer structure. This circuit should look like this:





```
for wires in ([0, 1], [1, 2], [2, 3], [3, 0]):  
    qml.CNOT(wires)
```

```
@qml.qnode(dev)  
def circuit(weights, x):  
    qml.BasisState(x, wires=[0, 1, 2, 3])  
  
    for layer_weights in weights:  
        layer(layer_weights)  
  
    return qml.expval(qml.PauliZ(0))
```

```
def square_loss(labels, predictions):  
    # We use a call to qml.math.stack to allow subtracting the arrays directly  
    return np.mean([(labels - qml.math.stack(predictions)) ** 2])
```

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```
{x}
    for layer_weights in weights:
        layer(layer_weights)

    return qml.expval(qml.PauliZ(0))

[20] def variational_classifier(weights, bias, x):
    return circuit(weights, x) + bias

[18] def square_loss(labels, predictions):
    # We use a call to qml.math.stack to allow subtracting the arrays directly
    return np.mean((labels - qml.math.stack(predictions)) ** 2)

[ ] def accuracy(labels, predictions):
    acc = sum(abs(1 - p) < 1e-5 for l, p in zip(labels, predictions))
    acc = acc / len(labels)
    return acc
```

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```
{x}
^ = np.array(data_train[:, :-1])
Y = np.array(data_train[:, -1])
Y = Y * 2 - 1 # shift label from {0, 1} to {-1, 1}

for x,y in zip(X, Y):
    print(f"x = {x}, y = {y}")
    I
⇒ x = [0 0 0 1], y = 1
x = [0 0 1 0], y = 1
x = [0 1 0 0], y = 1
x = [0 1 0 1], y = -1
x = [0 1 1 0], y = -1
x = [0 1 1 1], y = 1
x = [1 0 0 0], y = 1
x = [1 0 0 1], y = -1
x = [1 0 1 1], y = 1
x = [1 1 1 1], y = -1
```



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Connect



```
import pennylane as qml
from pennylane import numpy as np
from pennylane.optimize import NesterovMomentumOptimizer
```

Quantum and classical nodes

We then create a quantum device that will run our circuits.

```
[ ] dev = qml.device("default.qubit")
```

Variational classifiers usually define a "layer" or "block", which is an elementary circuit architecture that gets repeated to build the full variational circuit.

Our circuit layer will use four qubits, or wires, and consists of an arbitrary rotation on every qubit, as well as a ring of CNOTs that entangles each qubit with its neighbour. Borrowing from machine learning, we call the parameters of the layer `weights`. Let's take a look on how this quantum circuit looks like:



```
Iter: 45 | Cost: 1.0785009 | Accuracy: 0.5000000  
Iter: 46 | Cost: 0.9780625 | Accuracy: 0.6000000  
Iter: 47 | Cost: 1.1573709 | Accuracy: 0.6000000  
Iter: 48 | Cost: 1.0235239 | Accuracy: 0.6000000  
Iter: 49 | Cost: 1.2842469 | Accuracy: 0.5000000  
Iter: 50 | Cost: 0.8549226 | Accuracy: 0.6000000  
Iter: 51 | Cost: 0.5136787 | Accuracy: 1.0000000  
Iter: 52 | Cost: 0.2488031 | Accuracy: 1.0000000  
Iter: 53 | Cost: 0.0461277 | Accuracy: 1.0000000  
Iter: 54 | Cost: 0.0293518 | Accuracy: 1.0000000  
Iter: 55 | Cost: 0.0205454 | Accuracy: 1.0000000  
Iter: 56 | Cost: 0.0352514 | Accuracy: 1.0000000  
Iter: 57 | Cost: 0.0576767 | Accuracy: 1.0000000  
Iter: 58 | Cost: 0.0291305 | Accuracy: 1.0000000  
Iter: 59 | Cost: 0.0127137 | Accuracy: 1.0000000  
Iter: 60 | Cost: 0.0058108 | Accuracy: 1.0000000  
Iter: 61 | Cost: 0.0018002 | Accuracy: 1.0000000  
Iter: 62 | Cost: 0.0014089 | Accuracy: 1.0000000
```



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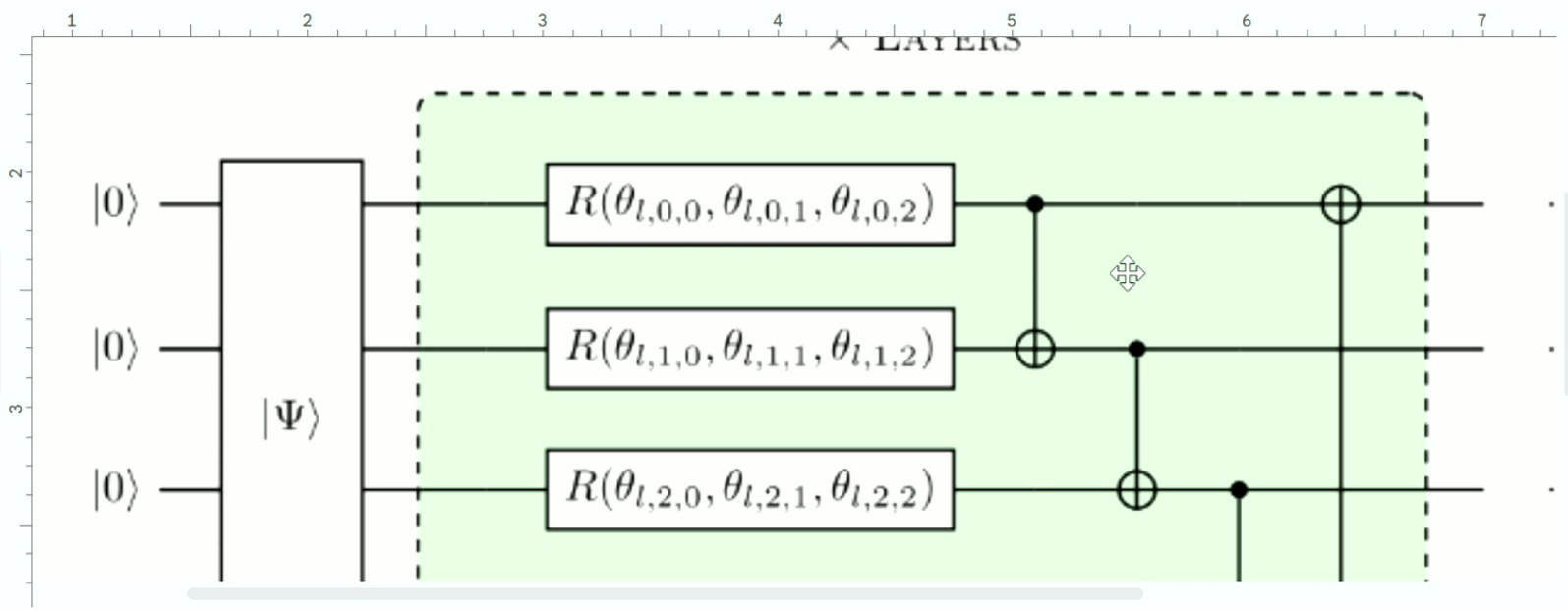
Quantum Machine Learning

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- 8 Variational Quantum Classifier (VQC)
- 9 The VQC circuit
- 10 Steps



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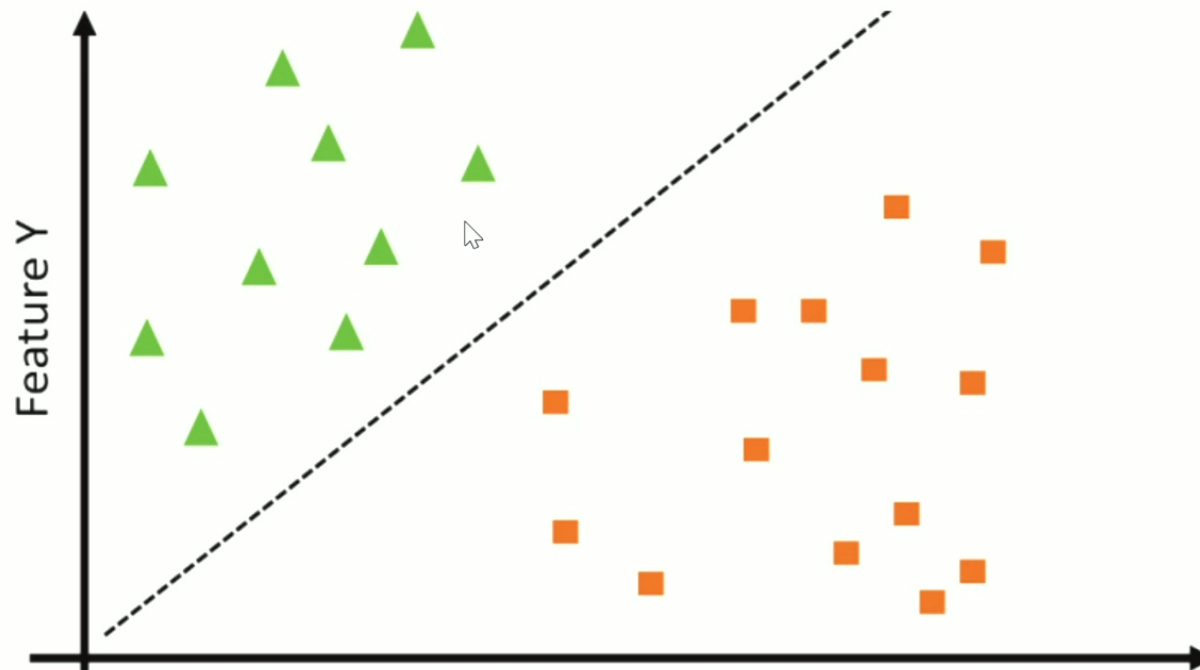
```
0s for x,y,p in zip(X_test, Y_test, predictions_test):  
    print(f"x = {x}, y = {y}, pred={p}")
```

```
acc_test = accuracy(Y_test, predictions_test)  
print("Accuracy on unseen data:", acc_test)
```

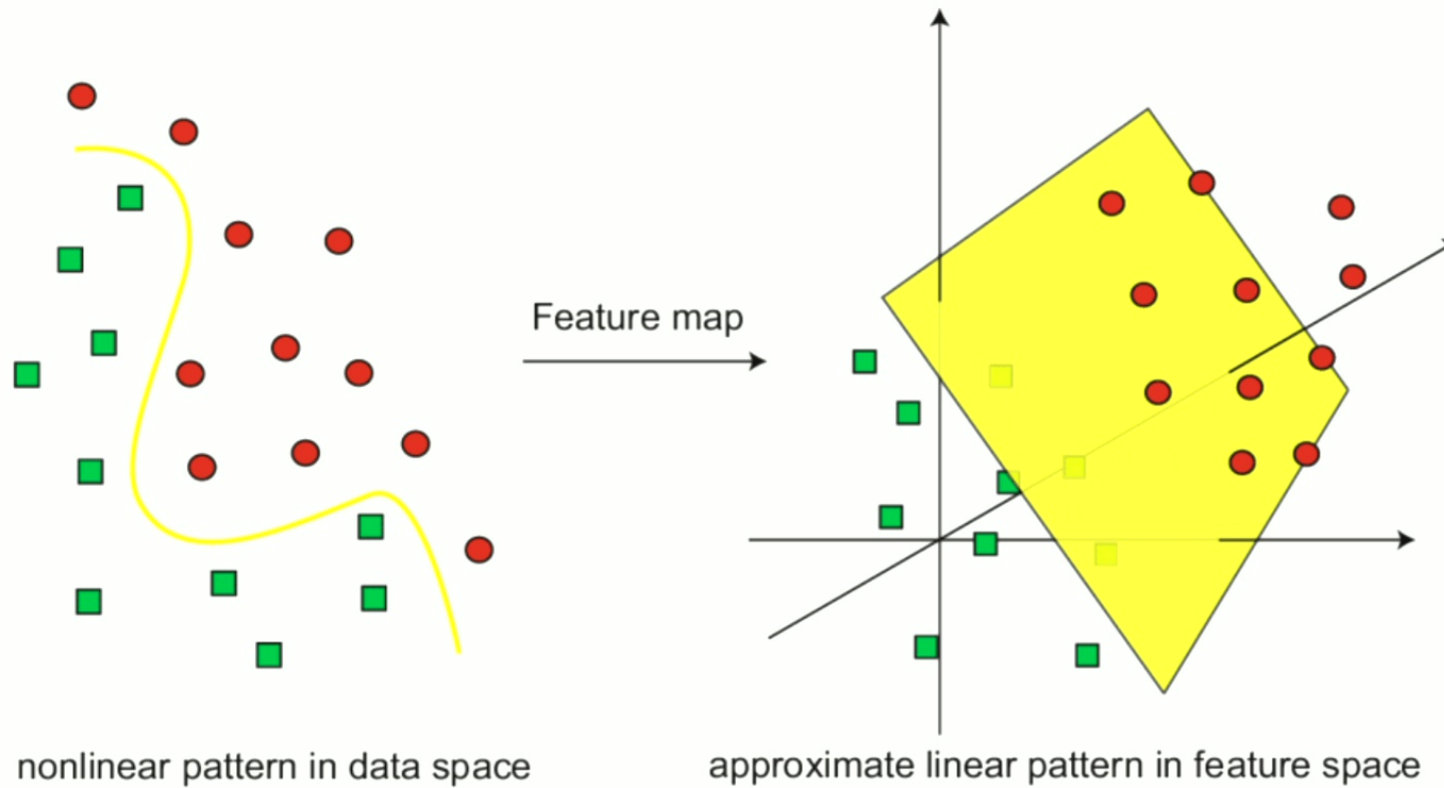
```
x = [0 0 0 0], y = -1, pred=-1.0  
x = [0 0 1 1], y = -1, pred=-1.0  
x = [1 0 1 0], y = -1, pred=-1.0  
x = [1 1 1 0], y = 1, pred=1.0  
x = [1 1 0 0], y = -1, pred=-1.0  
x = [1 1 0 1], y = 1, pred=1.0  
Accuracy on unseen data: 1.0
```

Support vector machine

To exit full screen, press Esc



Feature map





(iv) The way an SVM with a custom kernel is implemented in scikit-learn requires us to pass a function that computes a matrix of kernel evaluations for samples in two different datasets A, B. If $A=B$, this is the [Gram matrix](#). **Complete the kernel_matrix function below to build this matrix of kernel evaluations.**



```
def kernel_matrix(A, B):  
    """Compute the matrix whose entries are the kernel  
    evaluated on pairwise data from sets A and B."""  
    return # Return the kernel matrix
```

(v) Training the SVM optimizes internal parameters that basically weigh kernel functions. It is a breeze in scikit-learn, which is designed as a high-level machine learning library. **Run the cell below to fit the Support Vector Machine.** (I'm aware this is kind of black-boxy, but if you're interested in how this work, do read up on *normal* Machine Learning. You will find that one is very black-boxy too though).



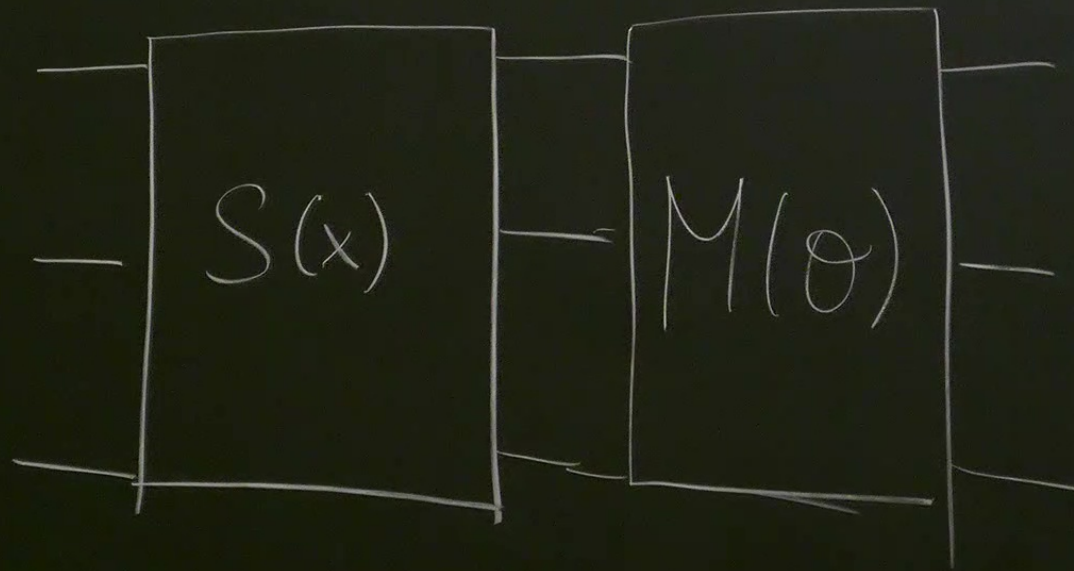
```
[ ] svm = SVC(kernel=kernel_matrix).fit(X_train, y_train)
```



Now run the cell below to compute the accuracy on the test set

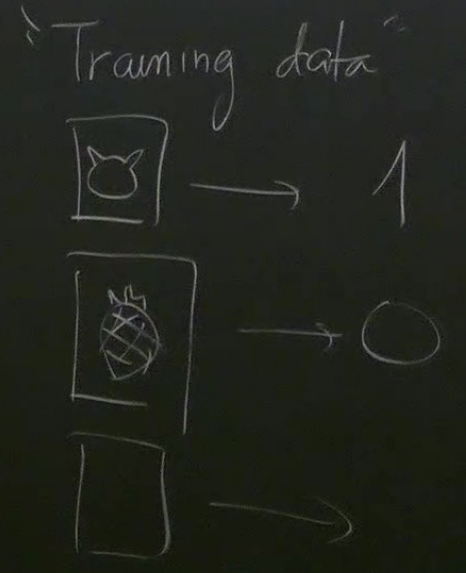
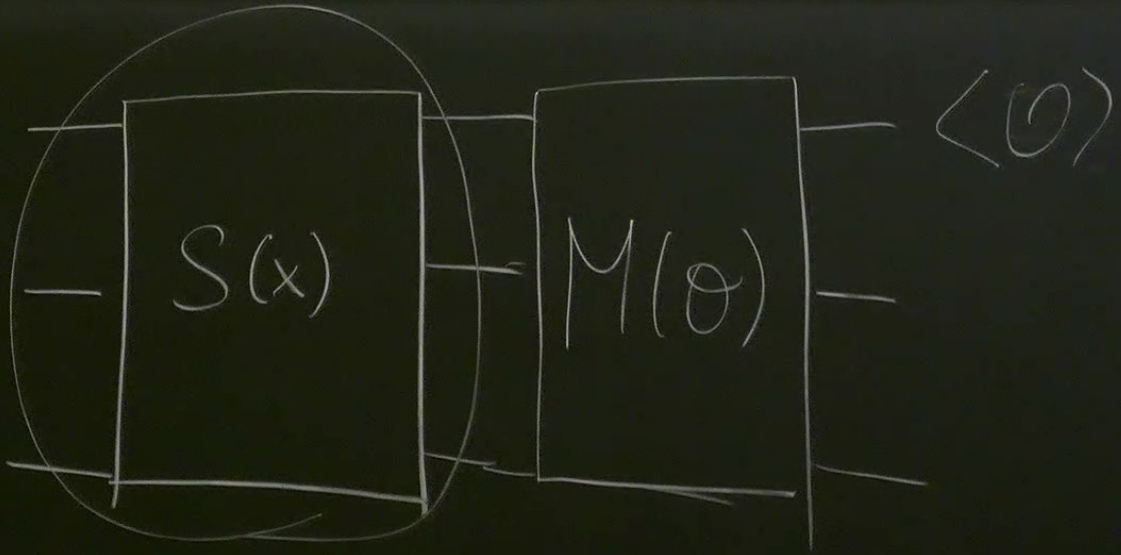


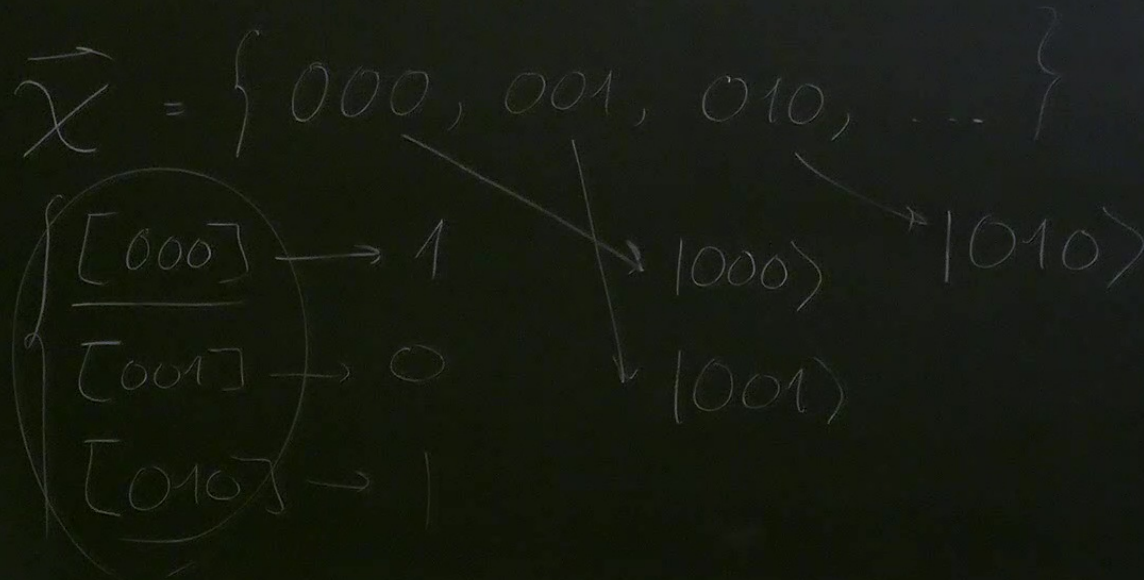
x



$\langle \theta \rangle$

x

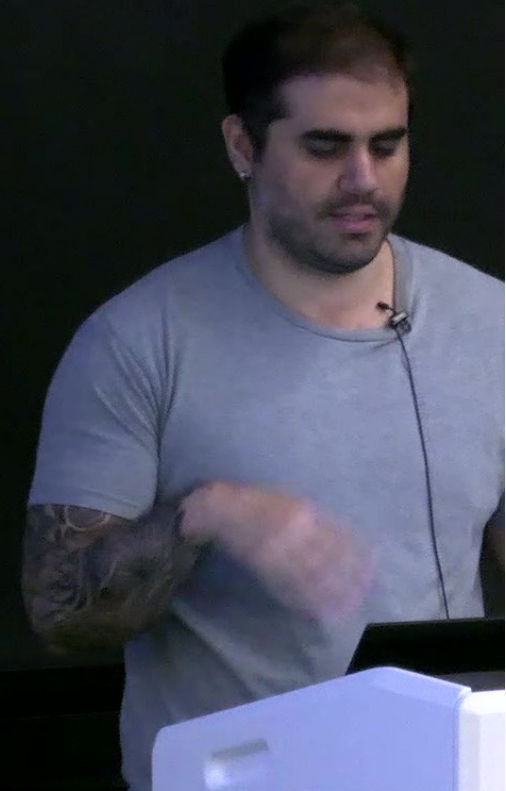


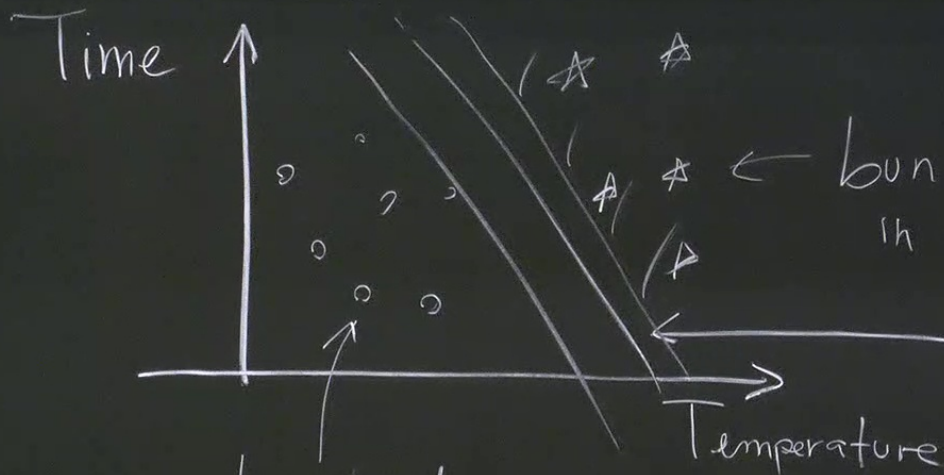


$$[0, \underline{1}, \underline{1}, 0] \rightarrow 0$$

$$[1, 0, 0, 0] \rightarrow 1$$

$$[0, 1, 1, 1] \xrightarrow{?} 1$$





$$\vec{x} = (\vec{w}, b)$$

$$\vec{w} \cdot \vec{x} + b = 0$$

$$\vec{x}_{\text{new}} > 0$$

$$\vec{x}_{\text{new}} < 0$$

$$\vec{\alpha} = \sum_{i=1}^m \alpha_i + \frac{1}{\sum_{i=1}^n \sum_{j=1}^m \alpha_i \alpha_j y_i y_j} \vec{x}_i \cdot \vec{x}_j$$

