Abstract: The axion is one of the most compelling new physics candidates, deriving many of its important properties from an approximate shift symmetry. In this talk, we will consider the general form of the axion coupling to photons in the presence of such a broken shift symmetry. We will show that the axion-photon in general becomes a non-linear monodromic function of the axion. The non-linearity is correlated with the axion mass and singularities in the axion-photon coupling are associated with cusps in the axion potential. We derive the general form of the axion-photon coupling for several examples including the QCD axion and show that there is a uniform general form for this monodromic function. The full non-linear profile of this coupling is phenomenologically relevant to the dynamics induced on axion domain walls/strings and other extended objects involving the axion.
Discovery:

- Non-observation CP
- Cosmo DE or DM
- Test GluTs
- C6 and ST

Motivation:

- Runs
- Slope can dominate
- Corr with m
- Axion Strings

Outline:

- EFT
- Axion-photon
- Examples
**Axion**

\[ g(a) = E_a - \frac{5}{3} N \cdot \text{arctan} \left( \frac{1 - \frac{3}{2}\lambda}{1 + \frac{3}{2}\lambda} \right) \]

\[ a \equiv a + 2\pi \]

\[ \text{shift} \ a \rightarrow a + c \]

\[ \mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{(F_a)^2}{2} (\partial_a)^2 - V(a) + \frac{g(a)}{16\pi^2} F_{\mu\nu} \partial^{\mu} a^{\nu} \]
The monodromic axion-photon coupling

Monodromic:
\[ g(a + 2\pi n) = g(a) + 2\pi in, \quad n \geq 0 \]

Only a coupling \( g(a) = a + a^2 \)

- Preserves \( a \rightarrow a + c \)
- Allows for spontaneous anomalies
- \( e^iS \rightarrow e^iS e^{i\alpha a^2 FF} \)
- Amplitude
\[ g_{\text{eff}} = \frac{a_{\text{em}}}{4\pi F_a} g(a) \]
\[ g(a) = E_0 - \frac{5}{7} \text{Na} \cdot \text{arctan} \left( \frac{1 - \frac{3}{\text{Na}}}{\frac{1}{\text{Na}} \tan \left( \frac{2}{7} \right)} \right) \]

\[ m = 0 \iff g(a) = 2\alpha \]
The mono-chromatic axion-photon coupling

Real parameter $\alpha \in [0, \infty)$

$$g(\alpha) = 2 \alpha \text{ctan} \left( \frac{1 - 2}{1 + 2} \tan \left( \frac{\alpha}{2} \right) \right) + 2 \pi \text{sign}(1 - \alpha) \Theta(\alpha - \pi)$$

$\Theta(\alpha - \pi)$
Coupling

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\[ L = i \Phi \partial_t \Phi - \mu \Phi e^{i \alpha \gamma} \Phi - m \Phi \Phi \]

(Clever trick: \[ \Phi e^{i \alpha \gamma} \gamma_m \Phi = m(a) \Phi e^{i \alpha \gamma} \Phi \]
\[ L = i \Phi \partial_t \Phi - m(a) \Phi \Phi + \frac{g(a)}{4 \pi^2} \frac{\Phi \Phi}{F \tilde{F}} \]

\[ g(a) = \frac{a}{2} - \arctan(\frac{1 - \frac{a}{2}}{1 + \frac{a}{2} \tan(\frac{a}{2})}) + \text{H.s.,} \quad z = \frac{4 \pi^2}{m} \]

\[ M(a)^2 = (m + \Pi)^2 \left(1 - \frac{4 \pi^2}{(1 + \Pi)^2} \sin^2(\frac{a}{2})\right) \]
The monochromatic axion-photon coupling

\[ L \supset \frac{E_a}{16 \pi^2} \vec{F} \cdot \vec{F} + \frac{N_a}{16 \pi^2} \vec{G} \cdot \vec{G} \supset \alpha \vec{F} \cdot \vec{G} \]

\[ \chi = \chi(a, \pi^0) + \left( \frac{e}{\sqrt{3}} \right)^2 \left( \frac{F}{F} + \frac{\pi^0}{F} \right) \]

\[ V(a, \pi^0) = \int \frac{d^2 \pi^0}{2 \pi^0} \left( 1 - \cos \left( \frac{2 \pi^0}{\alpha} \right) \cos \left( \phi \right) + \frac{1 - 2 \gamma}{1 + 2} \sin \left( \frac{2 \pi^0}{\alpha} \right) \sin \left( \phi \right) \right) \]
\[ g(a) = \frac{E_0}{a} - \tan \left( \frac{1}{2} \tan \left( \frac{1}{2} a \right) \right) + \text{H.S.} \]

\[ \Pi^0 = \text{arctan} \left( \frac{1}{1 + e^{-2 |M|}} \right) + \text{H.S.} \]

\[ M^2 = (M + M') \left( 1 - \frac{e^{-2 |M|}}{(1 + e^{-2 |M|})^2} \right) \]