Title: Gyroscopic gravitational memory from binary systems

Speakers: Ali Seraj

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Abstract: I will review the "gyroscopic gravitational memory", the permanent effect of gravitational waves on freely-falling gyroscopes far from the source of gravitational radiation. Then, I will compute the effect created by binary systems in the post-Newtonian approximation. The discussion naturally involves the helicity of gravitational waves and gravitational electric-magnetic duality.

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Zoom link

# Gyroscopic gravitational memory from binary systems

**Ali Seraj**

Physique Théorique et Mathématique, Université Libre de Bruxelles (ULB)

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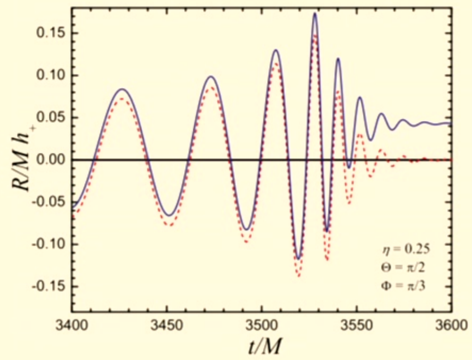
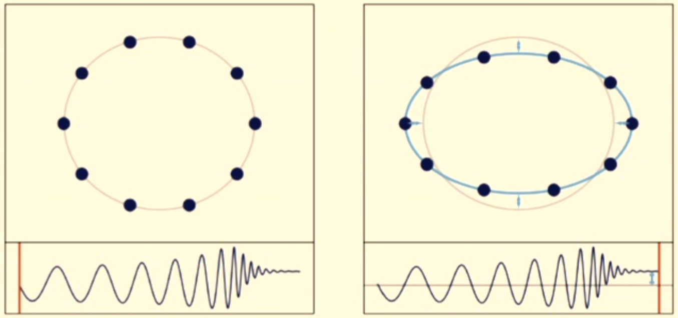
Based on
Work in preparation with **Guillaume Faye**
and PRL 129 (2022) 6, 061101 , 2112.04535 (JHEP) with **Blagoje Oblak**

## Intro: Gravitational memory effects [*Polnarev, Zeldovich, Braginsky, Thorne, Christodoulou, Blanchet, Damour, ...*]

**Gravitational memory effects:** Non-oscillatory part of gravitational waves




[*Credit: M. Favata*]

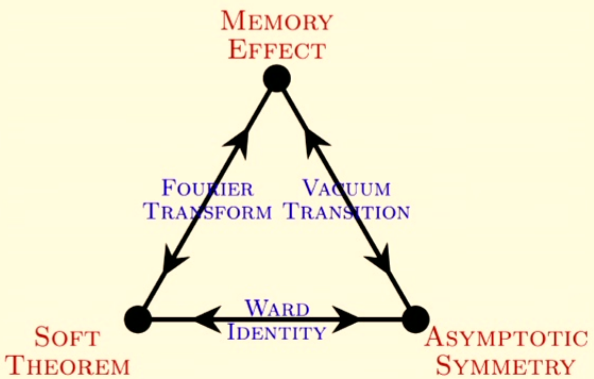


[*Credit: K Mitman*]

**Motivations to study it?**

✓ Observable effect of GR

✓ Low energy structure of quantum gravity



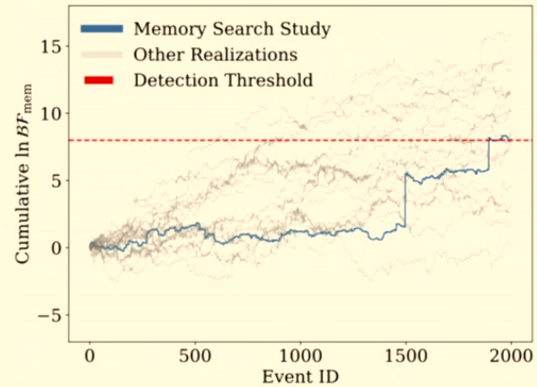

[*Credit: A. Strominger*]

# Prospects for detection: LIGO/Virgo

Not possible to detect in single event

Bayesian model selection: Is a memoryful waveform preferred by data? *[Lasky+ '16]*

Order 2000 signals for detection or 5 years of data collection *[Hubner+'19, Boersma+'20, Grant+'22]*




*[P. Lasky et al.]*

# Prospects for detection: LISA

The Laser Interferometer Space Antenna (LISA) has sufficient sensitivity to observe memory in the GW signal from **supermassive black hole mergers**

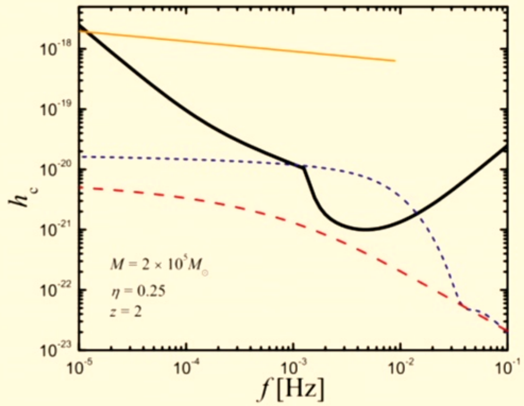



**Figure 1:** GW signal from supermassive mergers. **Black:** Sensitivity curve of LISA, Orange: inspiral signal amplitude, Blue: memory contribution [*Favata '10*]

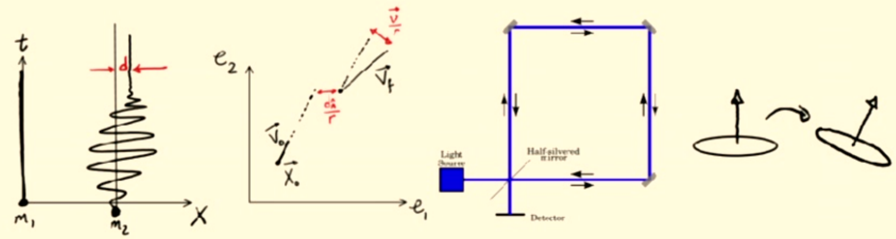
## Subleading memory effects

Low frequency expansion of the waveform

$$h(\omega) = \frac{1}{\omega} A + B \log \omega + C + \cdots, \qquad \omega \to 0$$

Which observables capture subleading effects?

1. Displacement memory [Polnarev, Zel'dovich '74] $\Delta \vec{X} = \vec{X}(t_f) - \vec{X}(t_0)$
2. Kick (velocity) memory [Flanagan+'19] $\Delta \vec{V} = \vec{V}(t_f) - \vec{V}(t_0)$
3. Spin memory [Pasterski+'16] $\Delta T = T^+ - T^-$
4. Gyroscope memory [AS, Oblak '21] $\Delta \vec{S} = \vec{S}(t_f) - \vec{S}(t_0)$



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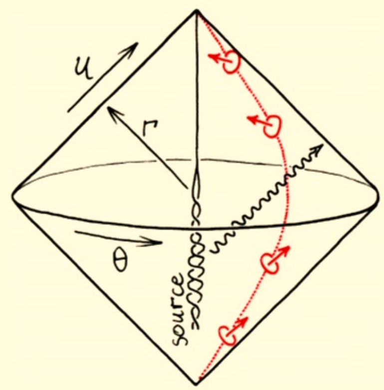
## Gyroscopic Memory effect



## Gyroscope dynamics

A (small) gyroscope with four velocity $\boldsymbol{V} = V^\mu \partial_\mu$ and spin $\boldsymbol{S} = S^\mu \partial_\mu$ obeys **Fermi-Walker transport equation** preserving $\boldsymbol{S} \cdot \boldsymbol{u} = 0$.

$$V^\nu \nabla_\nu S^\mu = (V^\mu a^\nu - V^\nu a^\mu) S_\nu$$

An observer measures $\boldsymbol{S}$ in a **local frame** $\boldsymbol{e}_{\hat{\mu}} = e_{\hat{\mu}}{}^\nu \partial_\nu, \qquad \boldsymbol{e}_{\hat{\mu}} \cdot \boldsymbol{e}_{\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}}$

In a **comoving frame** $\boldsymbol{e}_{\hat{0}} = \boldsymbol{V}$, spin is spatial $\boldsymbol{S} = S^{\hat{i}} \boldsymbol{e}_{\hat{i}}$,

$$\frac{d}{d\tau} S^{\hat{i}} = \Omega^{\hat{i}}{}_{\hat{j}} S^{\hat{j}}\,, \qquad \Omega^{\hat{i}}{}_{\hat{j}} = -V^\mu \big(\omega_\mu{}^{\hat{i}}{}_{\hat{j}} - \overline{\omega}_\mu{}^{\hat{i}}{}_{\hat{j}}\big)$$

where $\frac{d}{d\tau} \equiv V^\mu \overline{D}_\mu$ and $\overline{D}_\mu S^{\hat{i}} = \partial_\mu S^{\hat{i}} + \overline{\omega}_\mu{}^{\hat{i}}{}_{\hat{j}} S^{\hat{j}}$ wrt Minkowski connection

This covariant form requires **background structure**. However, an asymptotic observer has access to this structure, e.g. by referring to **distant stars**.

## GR in Bondi gauge

Asymptotically flat spacetime in Bondi coordinates $(u, r, \theta^A)$ with $g_{rr} = g_{rA} = 0$

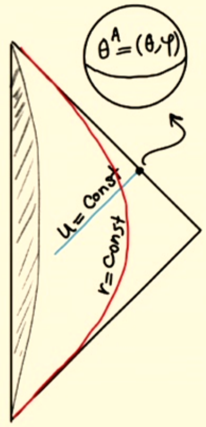
$$ds^2 = = -du^2 - 2du\,dr + r^2 q_{AB}\,d\theta^A d\theta^B + \frac{2m}{r} du^2 + r\,C_{AB}\,d\theta^A\,d\theta^B + D^B C_{AB} du\,d\theta^A + \text{subleading}$$

$q_{AB}(\theta^A)$ round metric on celestial sphere: background structure

$C_{AB}(u, \theta^A)$ Bondi shear (measures GW),

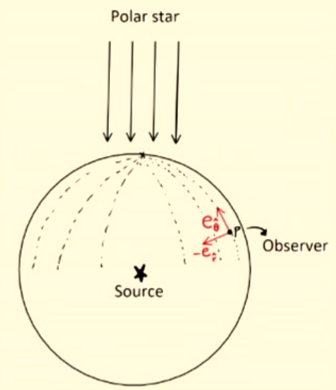
$m(u, \theta^A)$ Bondi mass

$\boldsymbol{\ell} = -du = e^{-2\beta}\partial_r$ tangent to outgoing null geodesics



# Building the frame [AS, Oblak '21]

Build an orthonormal tetrad $(\boldsymbol{e}_{\hat{0}}, \boldsymbol{e}_{\hat{r}}, \boldsymbol{e}_{\hat{a}})$ as follows




**Comoving**: $\boldsymbol{e}_{\hat{0}} = \boldsymbol{V}$ along geodesic.

**Source-oriented**: $\boldsymbol{e}_{\hat{r}}{}^{\mu}$ along outgoing null rays

**Transverse** basis vectors $\boldsymbol{e}_{\hat{a}}$

$$\boldsymbol{e}_{\hat{a}} = \frac{1}{r} E_{\hat{a}}{}^{A} \partial_A + \text{subleading fixed by } E_{\hat{a}}{}^{a} \text{ and metric}$$

where $E_{\hat{a}}{}^{A}(\theta^A)$ orthonormal **dyad on celestial sphere** $q_{AB}\, E_{\hat{a}}{}^{A}\, E_{\hat{b}}{}^{B} = \delta_{\hat{a}\hat{b}}$

## Gyroscopic memory effect

**Precession rate** $\Omega^{\hat{i}}{}_{\hat{j}} = -V^{\mu}(\omega_{\mu}{}^{\hat{i}}{}_{\hat{j}} - \overline{\omega}_{\mu}{}^{\hat{i}}{}_{\hat{j}})$ of distant freely-falling gyroscope

$$\Omega_{\hat{a}\hat{b}} = \frac{\epsilon_{\hat{a}\hat{b}}}{r^2}\hat{\Omega}\,, \qquad \hat{\Omega}(u,\theta^A) = \frac{c}{4}D_A D_B \widetilde{C}^{AB} - \frac{1}{8}\dot{C}_{AB}\widetilde{C}^{AB}$$

where $\widetilde{C}_{AB} \equiv \varepsilon_{CA} C^C{}_B$ is the dual shear. $\hat{\Omega} =$ *dual covariant mass*$= \mathrm{Im}\psi_2^0$ in NP formulation [*Freidel, Pranzetti*]

**Gyroscopic memory** is a net transverse rotation after the passage of the wave

$$\Delta S^{\hat{a}} = \Delta\Phi\, \epsilon^{\hat{a}\hat{b}} S_{\hat{b}}^{\text{initial}} + \mathcal{O}(r^{-3}),$$
$$\Delta\Phi(\theta^A) \equiv \frac{1}{4r^2}\Big( D_A D_B \int du\, \widetilde{C}^{AB} - \frac{1}{2}\int du\, \dot{C}_{AB}\widetilde{C}^{AB} \Big).$$

First term is the spin memory [*Pasterski et al. '15*] related to superrotations

Second term is the helicity of radiation $N_+ - N_-$ at a given point on celestial sphere.

## Complex basis on the sphere

**Null dyad** $(m, \overline{m})$ on the celestial sphere $m^A = E_{\hat{1}}{}^A + \mathrm{i}\, E_{\hat{2}}{}^A$

$$m \cdot \overline{m} = 1\,, \qquad m_A \overline{m}_B = q_{AB} - \mathrm{i}\,\varepsilon_{AB}$$

$m_{A_1 \cdots A_s} = m_{A_1} \cdots m_{A_s}$ complete basis for symmetric trace-free (STF) tensors

$$T_{A_1 \cdots A_s} = m_{A_1 \cdots A_s} T + \overline{m}_{A_1 \cdots A_s} \overline{T}\,, \quad \text{where} \quad T = \overline{m}^{A_1 \cdots A_s} T_{A_1 \cdots A_s}\,.$$

$(T, \overline{T})$ have **spin-weight** $-s, +s$, as they transform under $m \to e^{i\eta} m$ as

$$T \to e^{-is\eta} T, \qquad \overline{T} \to e^{is\eta} \overline{T}$$

**Eth ð operator** [Newman, Penrose '66]

$$\eth T = \overline{m}^{A_1 \cdots A_s} m^{A_{s+1}} D_{A_{s+1}} T_{A_1 \cdots A_s}\,, \qquad \bar{\eth} T = \overline{m}^{A_1 \cdots A_{s+1}} D_{A_{s+1}} T_{A_1 \cdots A_s}$$

$\eth, \bar{\eth}$ change the spin-weight by $+1$,$-1$.

## Precession in complex basis

Precession rate

$$\hat{\Omega}(u,\theta^A) = \underbrace{\frac{c}{4} D_A D_B \widetilde{C}^{AB}}_{\hat{\Omega}_{(\mathrm{S})}} \underbrace{-\frac{1}{8}\dot{C}_{AB}\widetilde{C}^{AB}}_{\hat{\Omega}_{(\mathrm{H})}}$$

In the complex basis, the total precession rate is given by

$$\boxed{\hat{\Omega} = \frac{1}{4}\,\mathrm{Im}(2c\,\eth^2 C + \dot{C}\overline{C})\,, \qquad C = \overline{m}^{AB} C_{AB}}$$

The result is of spin-weight 0, and therefore frame-independent.

# Spin-weighted spherical harmonics

**Spin-weighted spherical harmonics** $(0 \leq s \leq \ell)$

$$ {}_sY^{\ell m} = (-\sqrt{2})^s \sqrt{\tfrac{(\ell-s)!}{(\ell+s)!}}\, \eth^s Y^{\ell m}\,, \quad {}_{-s}Y^{\ell m} = (\sqrt{2})^s \sqrt{\tfrac{(\ell-s)!}{(\ell+s)!}}\, \bar{\eth}^s Y^{\ell m}\,, $$

**Properties**

Transform nicely under $\eth$ derivative

$$ \eth\, {}_sY^{\ell m} = -\sqrt{\tfrac{(\ell-s)(\ell+s+1)}{2}}\, {}_{s+1}Y^{\ell m}\,, \quad \bar{\eth}\, {}_sY^{\ell m} = \sqrt{\tfrac{(\ell+s)(\ell-s+1)}{2}}\, {}_{s-1}Y^{\ell m}\,. $$

Complete basis

$$ \int_S {}_sY^{\ell_1 m_1}\, {}_s\overline{Y}^{\ell_2 m_2} = \delta_{\ell_1 \ell_2}\delta_{m_1 m_2} $$

$$ \int_S {}_{s_1}Y^{\ell_1 m_1}\, {}_{s_2}Y^{\ell_2 m_2}\, {}_sY^{\ell m} = \sqrt{\tfrac{(2\ell_1+1)(2\ell_2+1)(2\ell+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & m \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ -s_1 & -s_2 & -s \end{pmatrix} $$

# Multipole expansion

Bondi shear can be expanded into parity definite scalars $U(u,\theta^A), V(u,\theta^A)$

$$C_{AB} = D_{\langle A}D_{B\rangle}\,U + \varepsilon^C{}_{(A}D_{B)C}\,V\,.$$

Therefore

$$C = \bar{\eth}^2\overline{Z}\,, \qquad Z = U + \mathrm{i}\,V\,.$$

Multipole expansion (normalized to match [*Thorne '80*])

$$C = \sum_{\ell m} C_{\ell m}\,{}_{-2}Y^{\ell m} \qquad\qquad C_{\ell m} = \frac{G}{\sqrt{2}c^{\ell+2}}\left(U_{\ell m} - \mathrm{i}\frac{V_{\ell m}}{c}\right)\,,$$

$$\overline{C} = \sum_{\ell m} C^*_{\ell m}\,{}_{2}Y^{\ell m} \qquad\qquad C^*_{\ell m} = \frac{G}{\sqrt{2}c^{\ell+2}}\left(U_{\ell m} + \mathrm{i}\frac{V_{\ell m}}{c}\right)\,,$$

$U_{\ell m}, V_{\ell m}$ mass, spin radiative multipoles.

# Multipole expansion of precession rates

Soft term $\hat{\Omega}_{(\mathsf{S})} = \frac{c}{2}\,\mathrm{Im}\,\eth^2 C$

$$\hat{\Omega}_{(\mathsf{S})} = -\frac{G}{4c^2}\sum_{\ell m}\sqrt{\frac{(\ell+2)!}{2(\ell-2)!}}\,\frac{V_{\ell m}}{c^\ell}Y^{\ell m}$$

Hard term

$$\hat{\Omega}_{(\mathsf{H})} = \frac{G^2}{16\mathrm{i}\,c^4}\sum_{\ell_1,\,m_1}\sum_{\ell_2,\,m_2}\sum_{\ell=|\ell_1-\ell_2|}^{\ell_1+\ell_2}\sum_{|m|\leqslant\ell}\frac{1}{c^{\ell_1+\ell_2}}\mathcal{G}_{\ell m}^{\ell_1 m_1,\,\ell_2 m_2}Y^{\ell m}$$
$$\times\Big[\big(\dot{U}_{\ell_1 m_1}U_{\ell_2 m_2}+\frac{1}{c^2}\dot{V}_{\ell_1 m_1}V_{\ell_2 m_2}\big)\big(1-(-1)^{\ell_1+\ell_2+\ell}\big)$$
$$+\frac{\mathrm{i}}{c}\big(\dot{U}_{\ell_1 m_1}V_{\ell_2 m_2}-\dot{V}_{\ell_1 m_1}U_{\ell_2 m_2}\big)\big(1+(-1)^{\ell_1+\ell_2+\ell}\big)\Big].$$

where $\mathcal{G}_{\ell m}^{\ell_1 m_1,\,\ell_2 m_2} = (-1)^m\sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell+1)}{4\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell\\ m_1 & m_2 & -m\end{pmatrix}\begin{pmatrix}\ell_1 & \ell_2 & \ell\\ 2 & -2 & 0\end{pmatrix}$

## Total helicity flux

$$\text{Total helicity flux} = \int_{S^2} \hat{\Omega}_{(\mathsf{H})}$$

Integrating over the sphere kills all harmonics except $\ell = 0$.

The triangular property of $3j$-symbols imply that $\ell_1 = \ell_2$. As a result $\ell_1 + \ell_2 + \ell$ is even, which kills $UU, VV$ terms.

$$\int_{S^2} \hat{\Omega}_{(\mathsf{H})} = \frac{G^2}{8c^5} \sum_{\ell,m} \frac{(-1)^m}{c^{2\ell}} \left( \dot{U}_{\ell m} V_{\ell\, -m} - \dot{V}_{\ell m} U_{\ell\, -m} \right) .$$

For planar orbits, it can be proven in general that [Faye et al. '12]

$$U_{\ell m} = 0\,, \qquad \ell + m = \text{odd}\,,$$
$$V_{\ell m} = 0\,, \qquad \ell + m = \text{even}\,.$$

Total helicity flux vanishes for planar orbits (bound or unbound).

## Post-Newtonian expansion

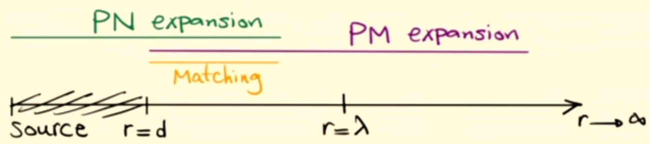
Make a nonrelativistic expansion (in powers of $1/c$)

At leading order

$$\hat{\Omega}_{(\mathsf{H})} = \frac{G^2}{8\mathrm{i}} \frac{1}{c^8} \sum_{\ell=1,3} Y^{\ell m} \sum_{|s| \leq \lfloor \ell/2 \rfloor} \mathcal{G}_{\ell,m}^{2-s,m_1,\,2+s,m_2} \dot{U}_{2-s,m_1} U_{2+s,m_2} + O(c^{-9})$$

# Blanchet-Damour formalism [Blanchet, Damour '86]

**Compact nonrelativistic** source has two length scales: size $d$ and typical wavelength $\lambda$ with $d/\lambda \sim v/c \ll 1$,




Einstein equations in harmonic gauge $\partial_\mu h^{\mu\nu} = 0$ where $h^{\mu\nu} = \sqrt{g} g^{\mu\nu} - \eta^{\mu\nu}$

$$\Box_\eta h^{\mu\nu} = \frac{16\pi G}{c^4} \tau^{\alpha\beta}, \qquad \tau^{\alpha\beta} = (-g)T^{\mu\nu} + \frac{c^4}{16\pi G} \Lambda^{\mu\nu}(h^2, h^3, \cdots)$$

Near zone $r \ll \lambda$: post-Newtonian expansion $\Box_\eta = -\frac{1}{c^2}\frac{\partial}{\partial t^2} + \Delta$

$$h^{\mu\nu} = \sum_{n=2}^{\infty} \frac{1}{c^n} {}^{(n)}\bar{h}^{\mu\nu}, \qquad \Delta^{(n)}\bar{h}^{\mu\nu} = 16\pi G^{(n-4)}\tau^{\mu\nu} + \partial_t^{2\,(n-2)}\bar{h}^{\mu\nu}$$

Outside source $r > d$: post-Minkowskian expansion

$$h^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h_n^{\alpha\beta}, \qquad \Box h_n^{\alpha\beta} = \Lambda_n^{\alpha\beta}\left[h_1, h_2, \ldots, h_{n-1}\right]$$

## Radiation in PN formalism

Radiative vs. source multipole moments

$$U_{\ell m} = \frac{\partial^\ell}{\partial u^\ell} I_{\ell m} + O(c^{-3})\,, \qquad V_{\ell m} = \frac{\partial^\ell}{\partial u^\ell} J_{\ell m} + O(c^{-3})$$

Source multipoles are determined in terms of the stress tensor [Blanchet '06]

$$\begin{aligned}
\mathrm{I}_L(u) =& \mathcal{FP} \int d^3x \int_{-1}^{1} dz \left\{ \delta_l(z) \hat{x}_L \Sigma - \frac{4(2l+1)\delta_{l+1}(z)}{c^2(l+1)(2l+3)} \hat{x}_{iL} \Sigma_i^{(1)} \right. \\
& \left. + \frac{2(2l+1)\delta_{l+2}(z)}{c^4(l+1)(l+2)(2l+5)} \hat{x}_{ijL} \Sigma_{ij}^{(2)} \right\} (u + z|\mathbf{x}|/c, \mathbf{x}) \\
\mathrm{J}_L(u) =& \mathcal{FP} \int d^3x \int_{-1}^{1} dz \epsilon_{ab\langle i_l} \left\{ \delta_l(z) \hat{x}_{L-1\rangle a} \Sigma_b \right. \\
& \left. - \frac{(2l+1)\delta_{l+1}(z)}{c^2(l+2)(2l+3)} \hat{x}_{L-1\rangle ac} \Sigma_{bc}^{(1)} \right\} (u + z|\mathbf{x}|/c, \mathbf{x})
\end{aligned}$$

where $\Sigma = \frac{\bar{\tau}^{00} + \bar{\tau}^{ii}}{c^2}\,, \quad \Sigma_i = \frac{\bar{\tau}^{0i}}{c}\,, \quad \Sigma_{ij} = \bar{\tau}^{ij}.$

# Quasi-circular binary systems

For a quasi-circular binary system [Blanchet et al. '08]

$$\left\{ U_{20} \to -\frac{4}{7} c^2 M \sqrt{\frac{5\pi}{3}}\, x\, \nu,\ U_{21} \to 0,\ U_{22} \to \frac{4}{21} c^2 e^{-2 i \psi} M \sqrt{\frac{2\pi}{5}}\, x\, \nu \left(42 + 84 \pi x^{3/2} + x(-107 + 55\nu)\right),\right.$$
$$\left. U_{30} \to 0,\ U_{31} \to 0,\ U_{32} \to -\frac{8}{3} c^3 e^{-2 i \psi} M \sqrt{\frac{2\pi}{7}}\, x^2 \nu (-1 + 3\nu),\ U_{33} \to 0 \right\}$$

$$\left\{ V_{20} \to 0,\ V_{21} \to -\frac{2}{21} c^3 e^{-i \psi} M \sqrt{\frac{2\pi}{5}}\, x^{3/2} \Delta \nu (28 + x(-17 + 20\nu)),\ V_{22} \to 0,\ V_{30} \to \frac{32}{5} c^4 M \sqrt{\frac{3\pi}{35}}\, x^{7/2} \nu^2,\right.$$
$$\left. V_{31} \to \frac{2}{9} c^4 e^{-i \psi} M \sqrt{\frac{\pi}{35}}\, x^{3/2} \Delta \nu (-3 + 2x(4 + \nu)),\ V_{32} \to 0,\ V_{33} \to 6 c^4 e^{-3 i \psi} M \sqrt{\frac{3\pi}{7}}\, x^{3/2} \Delta (1 + 2x(-2 + \nu))\, \nu \right\}$$

where $M = m_1 + m_2\,, \quad \Delta = \frac{m_1 - m_2}{M}\,, \quad \nu = \frac{m_1 m_2}{M^2}$

## Quasi-circular binary systems

Quasi-circular planar binary described by $(R(t), \psi(t))$, slow and fast variables respectively.

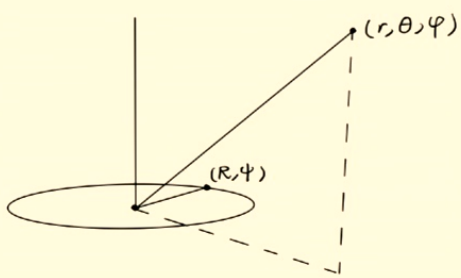
Due to Kepler's law $\omega = \dot{\psi}$ satisfies $\omega^2 R^3 = GM$

More convenient variable is

$$x = \left(\frac{GM\omega}{c^3}\right)^{2/3} = (v/c)^2 = \frac{r_s}{2R}\,, \qquad r_s = \frac{2GM}{c^2}$$

Its evolution is determined by the Energy flux-balance equation

$$\dot{x} = \frac{64}{5}\frac{c^3}{GM^2}\mu\, x^5 \left[1 + O(c^{-2})\right]$$



## Precession at leading PN order

## Precession rates: Non-axisymmetric modes

The precession rate is dominated by the soft term

$$\hat{\Omega}_{(\mathsf{S})} = \frac{GM\Delta\nu}{c} x^{3/2} \Big( \tfrac{1}{4} \cos(\phi - \psi) \sin\theta (65 \cos 2\theta - 16 \cos\theta - 69) + 45 \cos^3(\phi - \psi) \sin^3\theta \Big) + O(c^{-6}) \,,$$

This is 2PN effect $= O(c^{-4})$

$\hat{\Omega}_{(\mathsf{H})}$ starts at 4PN $= O(c^{-8})$

Not axisymmetric: contains $m = 1, 3$

## Precession rates: axisymmetric modes

Precession from quasi-circular binary at leading order

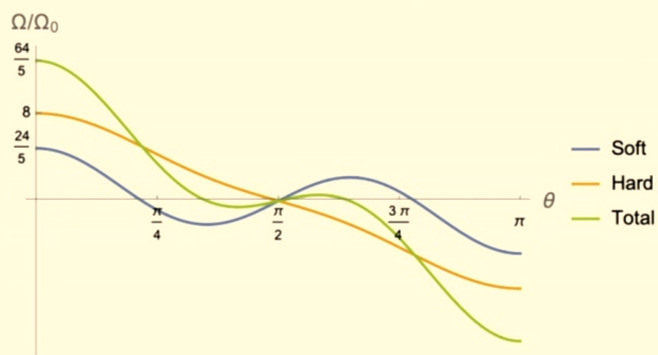
$$\hat{\Omega}_{(\mathsf{S})} = \frac{12}{5}(-3\cos\theta + 5\cos^3\theta)\Omega_0\,, \qquad \hat{\Omega}_{(\mathsf{H})} = 4(\cos\theta + \cos^3\theta)\Omega_0$$

$$\hat{\Omega}_{\mathsf{tot}} = \hat{\Omega}_{(\mathsf{S})} + \hat{\Omega}_{(\mathsf{H})} = \frac{16}{5}(-\cos\theta + 5\cos^3\theta)\Omega_0$$

which contains $\ell = 1, 3$ modes and the size is given by

$$\Omega_0 \equiv -\frac{GM\nu^2}{c}x^{7/2} + O(c^{-9})$$

This is a 4PN effect



# Gyroscopic memory

**Accumulation of DC effects**: As a result of time integration

$$\int du\, x^n = \int dx \frac{x^n}{\dot{x}} = \frac{5}{64}\frac{GM}{c^3\nu}\frac{1}{n-4}x^{n-4}\left(1+\mathcal{O}\left(\frac{1}{c^2}\right)\right), \qquad n \neq 4\,.$$

$$\int du\, x^n e^{-\mathrm{i}\, m\, \psi(t)} = \frac{GM\mathrm{i}}{c^3}\frac{\operatorname{sgn} m}{|m|}x^{\alpha-\frac{3}{2}}e^{-\mathrm{i}\, m\, \psi(t)}$$

No enhancement in non-axisymmetric modes

$\frac{1}{c^3 x^4} \sim c^5$ enhancement in axisymmetric modes

**Gyroscopic memory**

$$\Delta\Phi = \frac{1}{r^2}\int du\, \hat{\Omega} = \left(\frac{r_s}{r}\right)^2 \frac{\nu}{8\sqrt{x}}(5\cos^3\theta - \cos\theta)$$

Gyroscopoc memoy is a 1.5 PN sourced by the accumulation of subleading effects

IR divergence from past infinity: $u \to -\infty \implies x \to 0$

We assume an IR cutoff and integrate from $x_0$ to the ISCO $x = r_s/R = 2/3$

## Summary and outlook

How big is it $\Delta\Phi \sim \left(\frac{r_s}{r}\right)^2 \frac{\nu}{8\sqrt{x}}$ ?

SGR J1745-2900 closest known pulsar (magnetar) to Sgr A* ($M \sim 10^6 M_\odot$)

$$\left(\frac{r_s}{r}\right)^2 \sim 10^{-11}$$

Memory after 10 years $1/\sqrt{x} \sim 100$. Therefore, $\Delta\Phi \sim 10^{-9}\nu$

Elliptic or hyperbolic binaries? Spinning binaries? (in progress)

Contribution from merger and ring down?

Correlation effect on a cluster of pulsars ? Relevant for PTA?

Sagnac interferometer and gyroscopic memory

**Thank you for you attention**