Title: Randomly Monitored Quantum Codes

Speakers: Dongjin Lee

Series: Perimeter Institute Quantum Discussions

Date: May 07, 2024 - 11:00 AM

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Abstract: Quantum measurement has conventionally been regarded as the final step in quantum information processing, which is essential for reading out the processed information but collapses the quantum state into a classical state. However, recent studies have shown that quantum measurement itself can induce novel quantum phenomena. One seminal example is a monitored random circuit, which can generate long-range entanglement faster than a random unitary circuit. Inspired by these results, in this talk, we address the following question: When quantum information is encoded in a quantum error-correcting code, how many physical qubits should be randomly measured to destroy the encoded information? We investigate this question for various quantum error-correcting codes and derive the necessary and sufficient conditions for destroying the information through measurements. In particular, we demonstrate that for a large class of quantum error-correcting codes, it is impossible to destroy the encoded information through random single-qubit Pauli measurements when a tiny portion of physical qubits is still unmeasured. Our results not only reveal the extraordinary robustness of quantum codes under measurement decoherence, but also suggest potential applications in quantum information processing tasks.

Zoom link

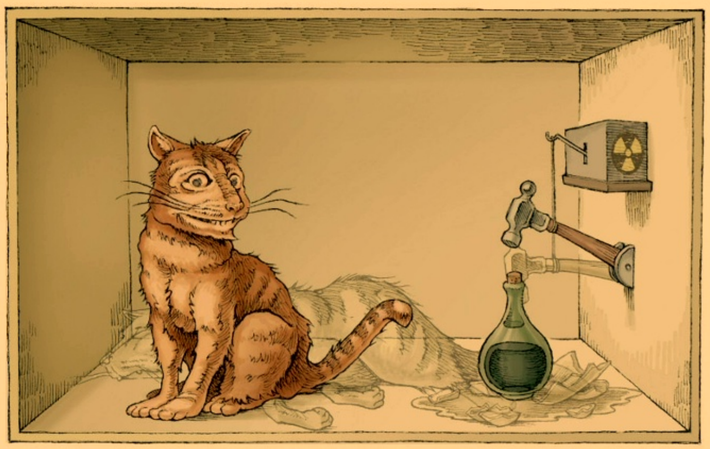
Perimeter QI Group Seminar
May. 7. 2024

Randomly Monitored Quantum Codes

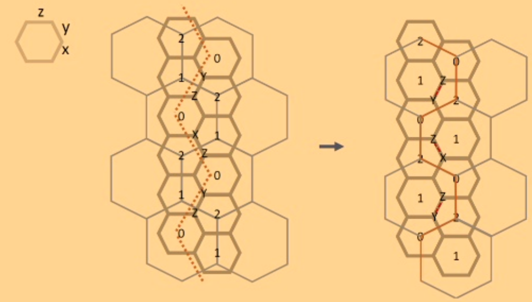
DONGJIN LEE

Based on arXiv: 2402:00145 [DL, Beni Yoshida]

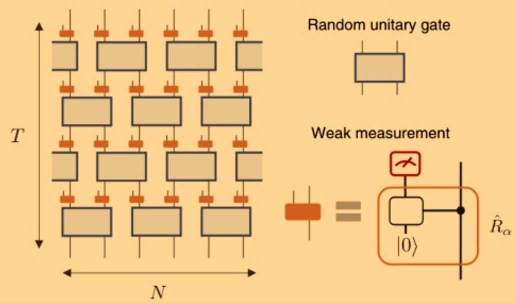
Quantum measurement: old vs. new story



Schrodinger's cat

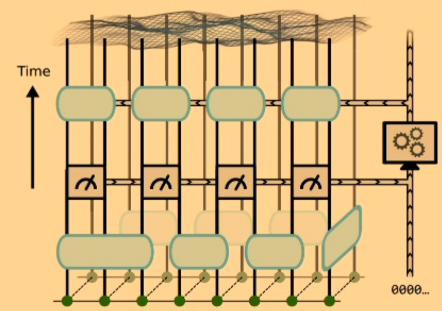


Floquet codes [Hastings,Haah'21]



Monitored random circuits

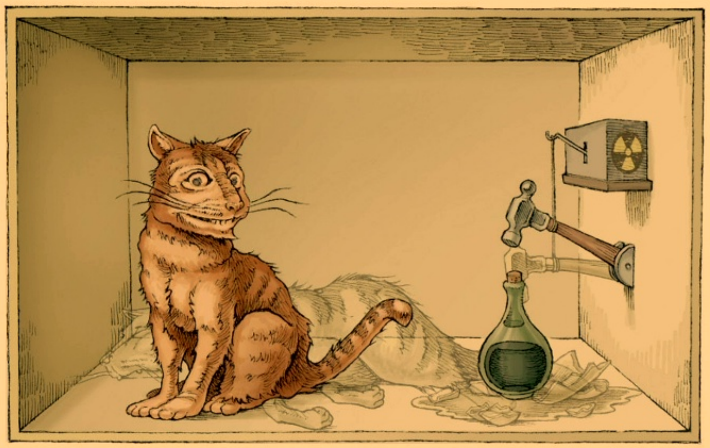
[Li,Chen,Fisher'19] [Skinner,Ruhman,Nahum'19]



Efficient state preparation

[Lu,Lessa,Kim,Hiseh'22]

Quantum measurement: old vs. new story



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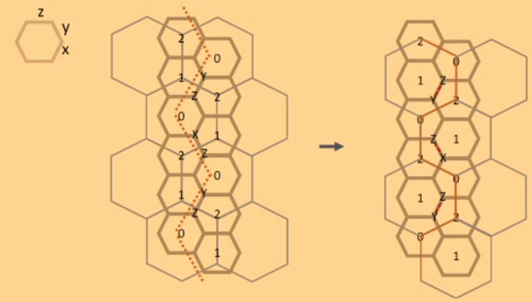

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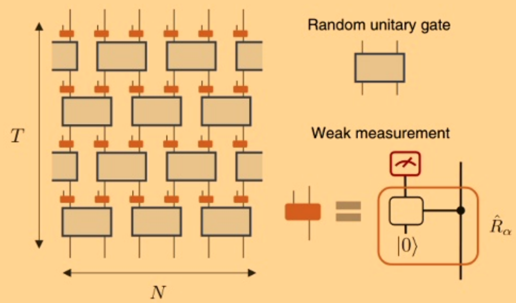

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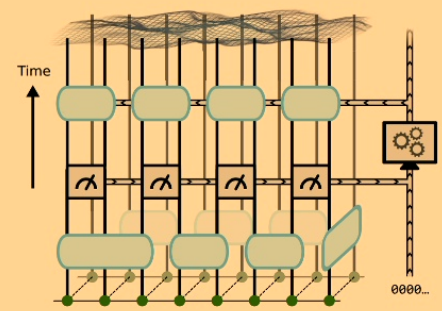


Efficient state preparation

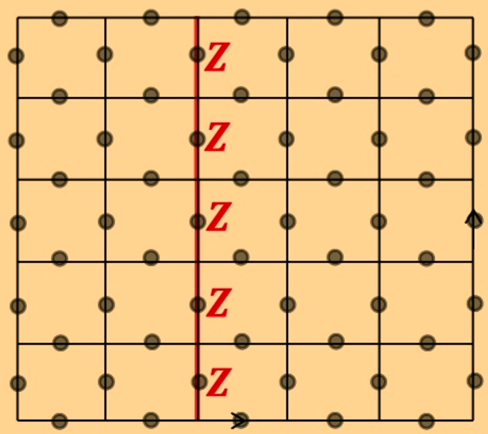
[Lu,Lessa,Kim,Hiseh'22]

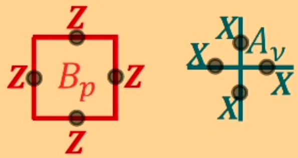
How robust is quantum information under measurements?

Quantum code and measurement

Q1. Given $[[n, k, d]]$ code, **how many qubits** should we **measure** to destroy the encoded information?

A1: d (at least)



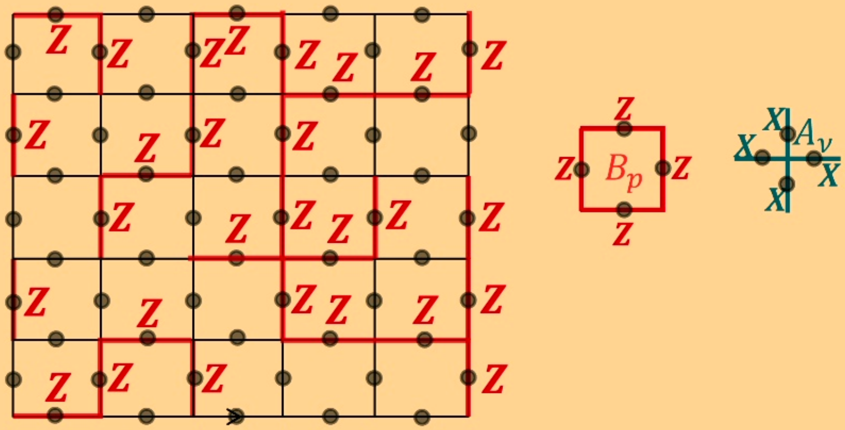


Toric code

Quantum Code and Measurement

Q2. What if we **cannot** choose qubits? (random qubits, but still controlled basis)

A2: $p_e^{th} n$ (p_e^{th}: erasure threshold)



Toric code

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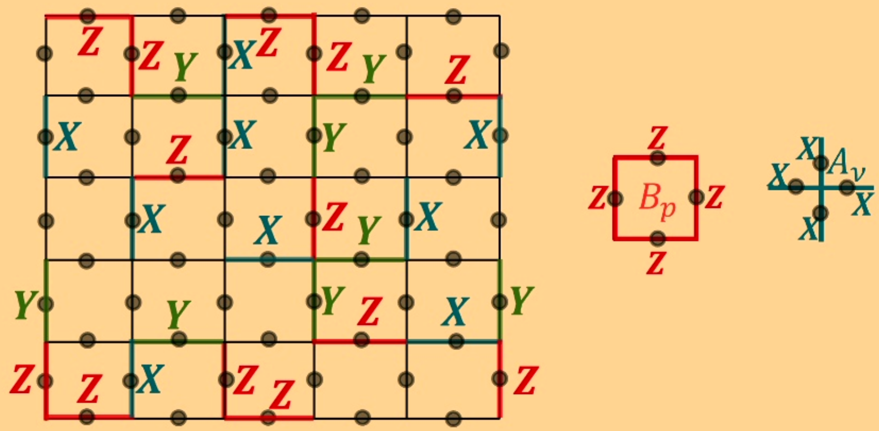
Quantum Code and Measurement

Q3. What if we **cannot choose measurement basis as well**? (random monitoring of system)

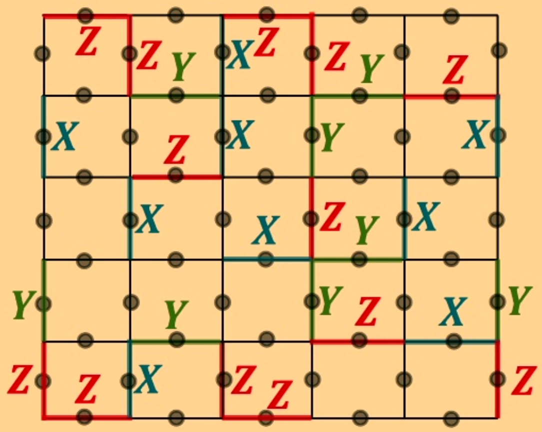
A3: ...?



Toric code

Quantum Code and Measurement

→ In this talk, we introduce **measurement threshold** p_m^{th}, which captures robustness of quantum codes under **random monitoring**.



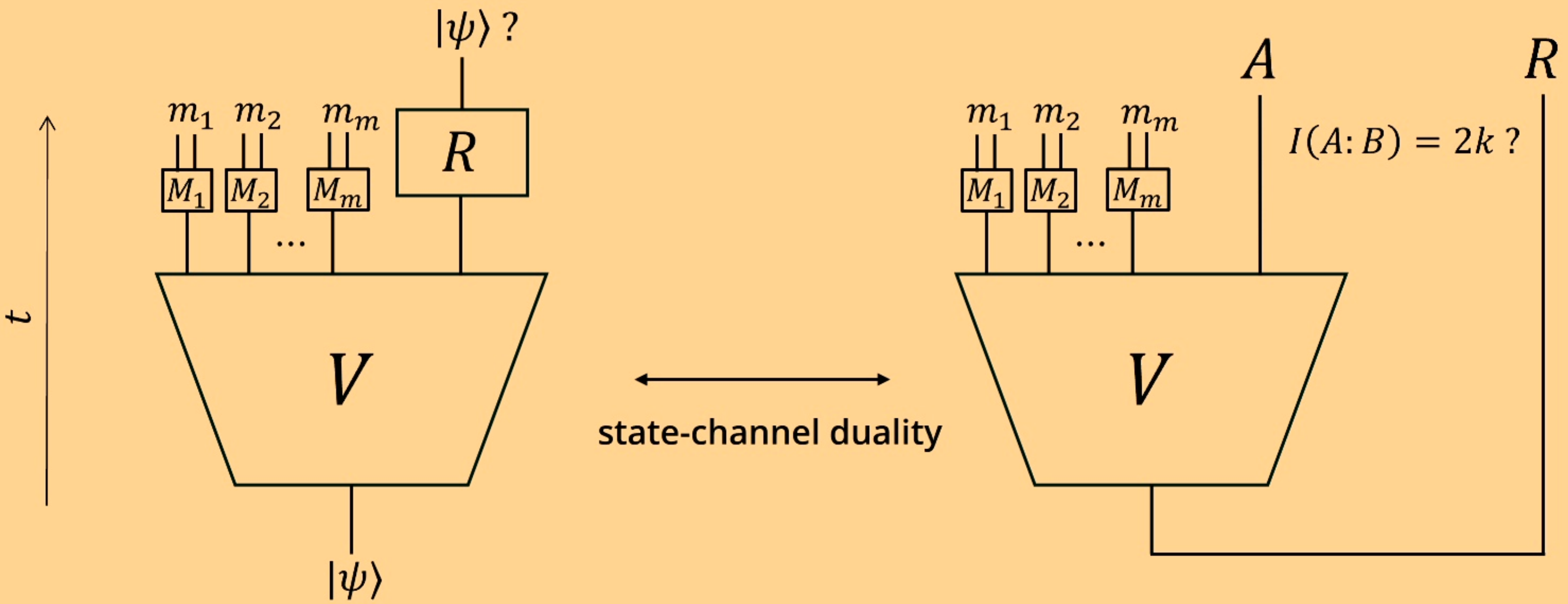
A3: p_m^{th} n

→ We show that large class of quantum codes are **extraordinarily robust** against random monitoring. Namely, $\boldsymbol{p_m^{th} = 1}$!

Outline

1. Setup
2. Concatenated codes
3. 2D Topological codes
4. Holographic codes
5. Summary and open problems

Setup: information preservation condition



Let $M = \langle M_1, M_2, \cdots, M_m \rangle$

→ For stabilizer codes, $I(A:R) = 2k \Leftrightarrow L(S) \cap M \subseteq S$.

→ For subsystem codes, $I(A:R) = 2k \Leftrightarrow L_{\text{dressed}}(G) \cap M \subseteq G$.

→ Information is destroyed if and only if logical operators are measured.

Setup: measurement threshold

→ **We quantify the randomness of Pauli measurements by probability (p_X, p_Y, p_Z).**

$p_m = p_X + p_Y + p_Z \leq 1.$ (if $p_m < 1$, we do not measure all qubits)

Setup: measurement threshold

→ We quantify the randomness of Pauli measurements by probability (p_X, p_Y, p_Z).

$$p_m = p_X + p_Y + p_Z \leq 1.$$ (if $p_m < 1$, we do not measure all qubits)

→ We normalize (p_X, p_Y, p_Z) by introducing relative measurement frequency $(\alpha_X, \alpha_Y, \alpha_Z)$.

$$(p_X, p_Y, p_Z) = p_m(\alpha_X, \alpha_Y, \alpha_Z)\,,\ \ \alpha_X + \alpha_Y + \alpha_Z = 1$$

→ Given $(\alpha_X, \alpha_Y, \alpha_Z)$, p_m^{th} is defined to be **the smallest** p_m such that the **information will be destroyed**.

One simple but fundamental relation: $p_e^{th} \leq p_m^{th}$

Simply because to measure a logical operator, it should be first supported on randomly selected qubits.

Setup: measurement threshold

→ We quantify the randomness of Pauli measurements by probability (p_X, p_Y, p_Z).

$$p_m = p_X + p_Y + p_Z \leq 1. \quad \text{(if } p_m < 1\text{, we do not measure all qubits)}$$

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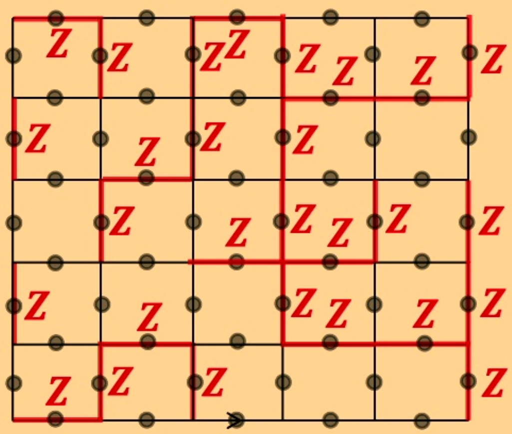
Simply because to measure a logical operator, it should be first supported on randomly selected qubits.

In this talk, we show that p_m^{th} is much larger than p_e^{th} in general.

Setup: examples

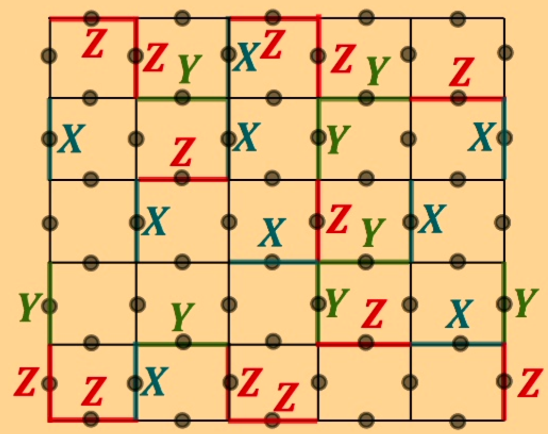
$(p_X, p_Y, p_Z) = (0,0,1/2)$



$p_m^{th}(0,0,1) = 1/2$

$\hookrightarrow (\alpha_X, \alpha_Y, \alpha_Z)$

$(p_X, p_Y, p_Z) = (1/6,1/6,1/6)$



$p_m^{th}(1/3,1/3,1/3) = ?$

$\hookrightarrow (\alpha_X, \alpha_Y, \alpha_Z)$

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Concatenated code: 5 qubit code

Code concatenation: one way to take thermodynamic limit.

Stabilizers: $S = \langle I_1 X_2 Z_3 Z_4 X_5, X_1 Z_2 Z_3 X_4 I_5, Z_1 Z_2 X_3 I_4 X_5, Z_1 X_2 I_3 X_4 Z_5 \rangle$

Logical operators: $\bar{X} = X_1 X_2 X_3 X_4 X_5, \ \bar{Y} = Y_1 Y_2 Y_3 Y_4 Y_5, \ \bar{Z} = Z_1 Z_2 Z_3 Z_4 Z_5$

A hyperbolic tiling of codes is often used as a toy model of AdS/CFT correspondence. [Pastawski,Yoshida,Harlow,Preskill'15]

Concatenated code: 5 qubit code

Code concatenation: one way to take thermodynamic limit.

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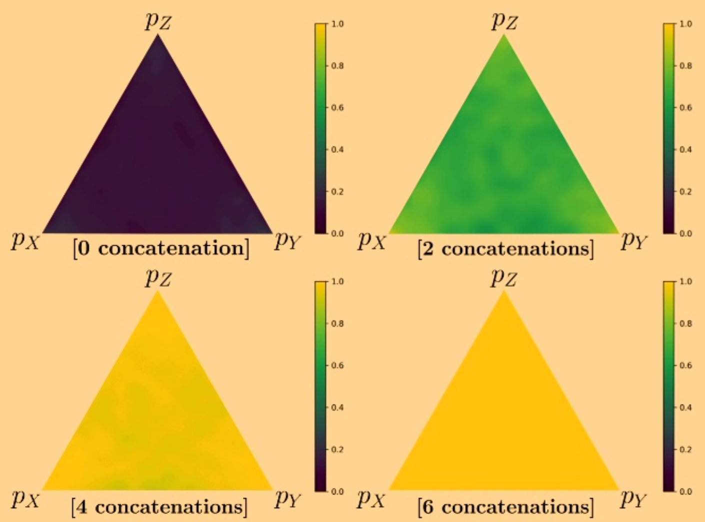
A hyperbolic tiling of codes is often used as a toy model of AdS/CFT correspondence. [Pastawski,Yoshida,Harlow,Preskill'15]



[Information preservation diagram at $p_m = 0.95$]

$p_m^{th} = 1$ for all $(\alpha_X, \alpha_Y, \alpha_Z)$. $p_e^{th} = 1/2$

→ At $p_m = 1$, which logical operator will be measured?

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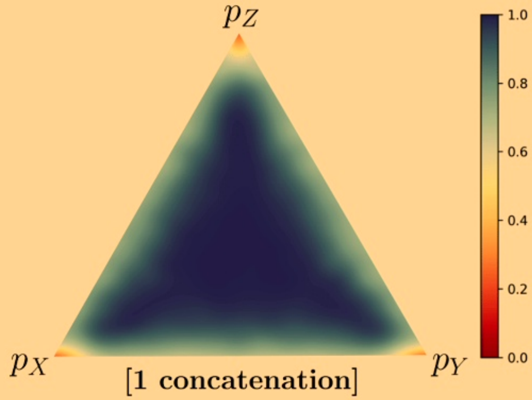
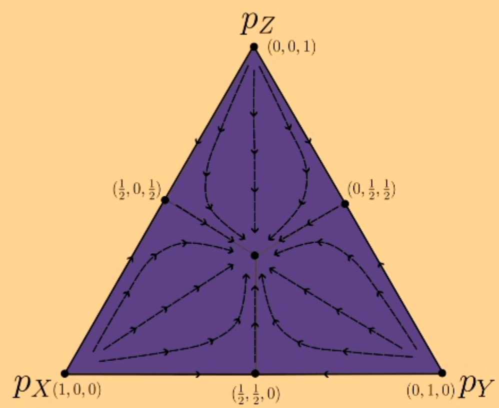
Concatenated code: 5 qubit code, $p_m = 1$

- At $p_m = 1$, one of logical operators $\bar{X}$, $\bar{Y}$, $\bar{Z}$ must be measured.
- We quantify uncertainty of measured logical operator by Renyi-2 entropy.

$$S^{(2)}(p_{\bar{X}}, p_{\bar{Y}}, p_{\bar{Z}}) = -\log_3(p_{\bar{X}}^2 + p_{\bar{Y}}^2 + p_{\bar{Z}}^2)$$



[Uncertainty of measured logical operator]



[Flow of logical measurement probability]

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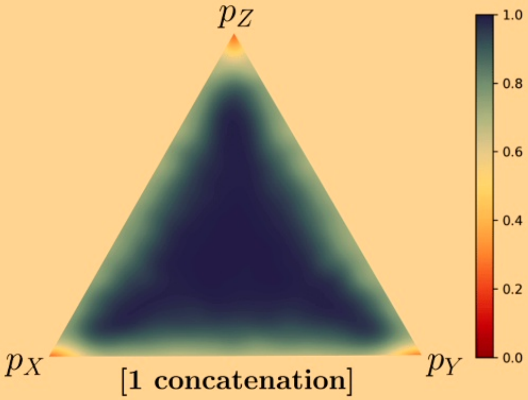

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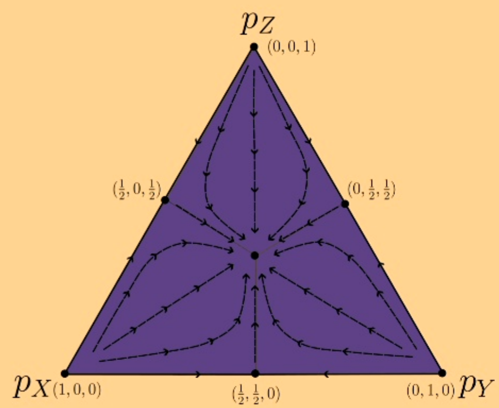

[Flow of logical measurement probability]

- We conjecture if measured logical operator is uncertain, measurement threshold must be 1!

Uncertainty in $(p_{\bar{X}}, p_{\bar{Y}}, p_{\bar{Z}}) \Longrightarrow p_m^{th} = 1$ for corresponding $(\alpha_X, \alpha_Y, \alpha_Z)$

Concatenated code: 7 qubit code

Stabilizers: $S = \langle X_1X_2X_3X_4, X_2X_3X_5X_6, X_3X_4X_6X_7, Z_1Z_2Z_3Z_4, Z_2Z_3Z_5Z_5, Z_3Z_4Z_6Z_7 \rangle$

Logical operators: $\bar{X} = X_1X_2 \cdots X_7, \ \ \bar{Y} = Y_1Y_2 \cdots Y_7, \ \ \bar{Z} = Z_1Z_2 \cdots Z_7$

Can be viewed as the smallest 2D color code with triangular boundary.

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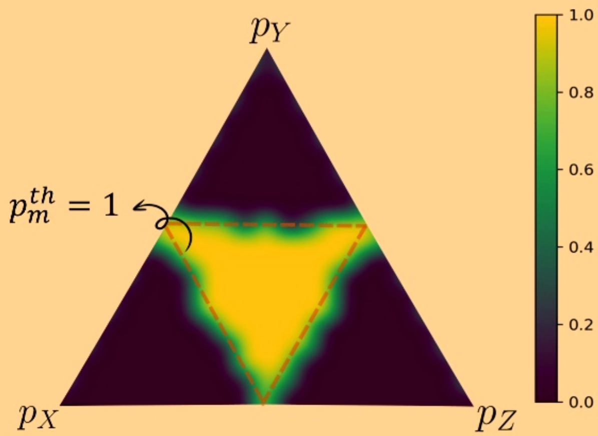
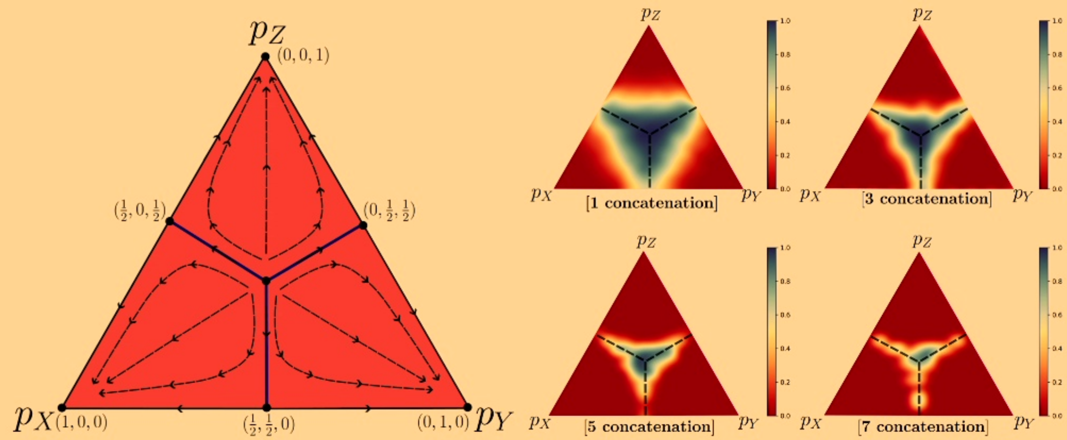


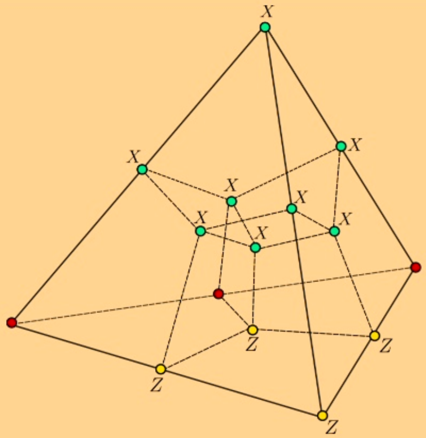
[Information preservation diagram at $p_m = 0.95$]



[Uncertainty of measured logical operator]

$p_m^{th} = 1$ if and only if $\alpha_X, \alpha_Y, \alpha_Z < \frac{1}{2}$

Concatenated code: 15 qubit Reed-Muller code



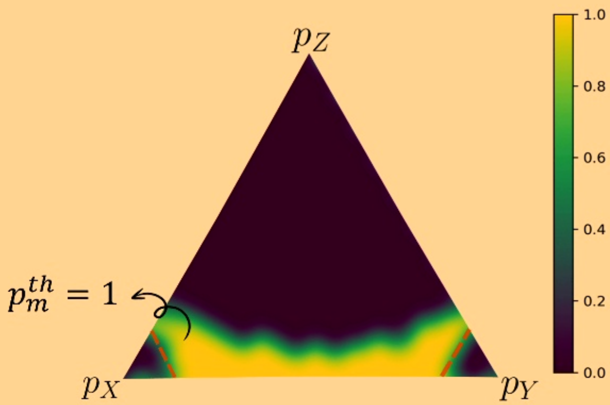
X stabilizers: each body cell

Z stabilizers: each face cell

Can be viewed as 3D color code with boundaries.

→ The minimum weight of logical Z is **three** while logical X and Y operators are **seven**.

→ Because of this asymmetry, **logical Z operator** will be measured at uniform measurement probability $p_X = p_Y = p_Z = \frac{1}{3}$.



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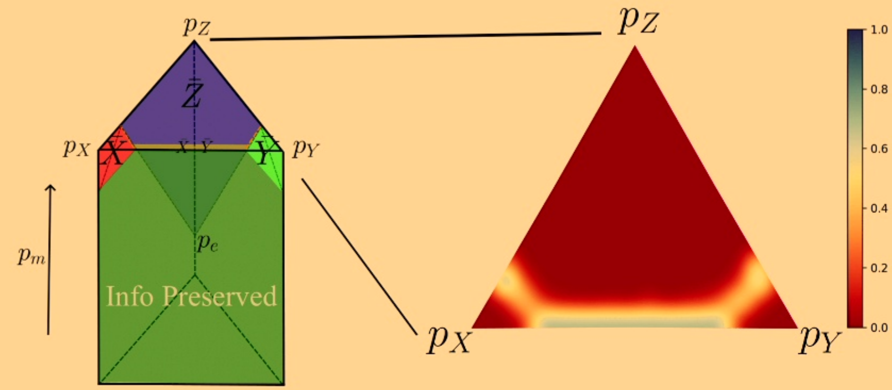

[Uncertainty of measured logical operators]

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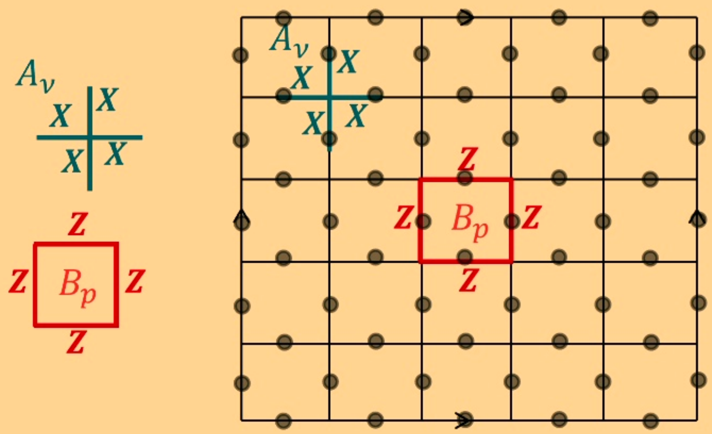
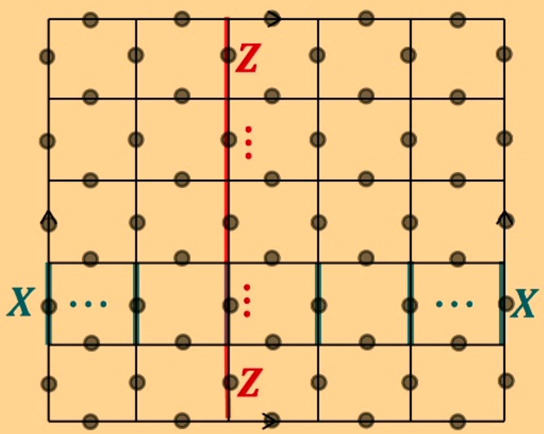
2D toric code

2D toric code review [Kitaev'97]



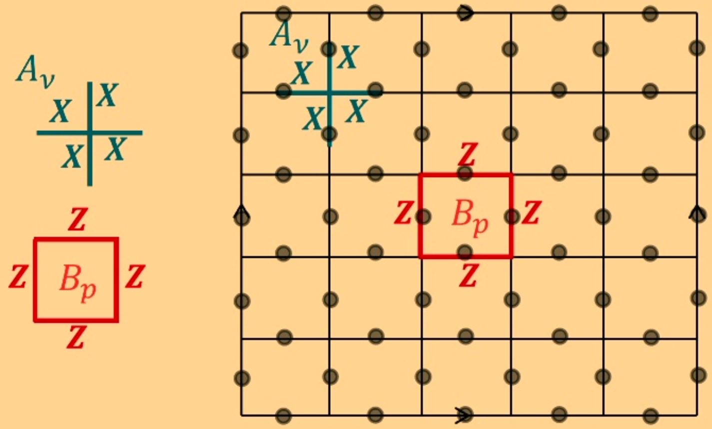


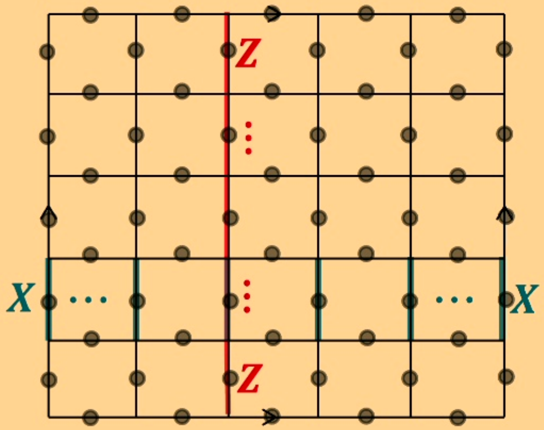
Code space: *Ground space of* $H = -\sum_v A_v - \sum_p B_p$

Logical operators: X, Z *incontractible loop operators*

2D toric code

2D toric code review [Kitaev'97]





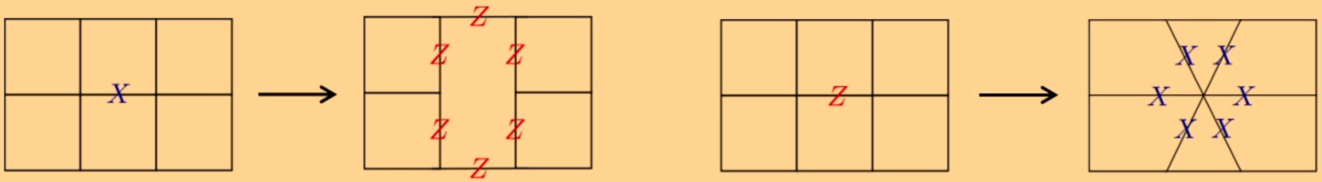
Code space: *Ground space of* $H = -\sum_v A_v - \sum_p B_p$

Logical operators: X, Z *incontractible loop operators*

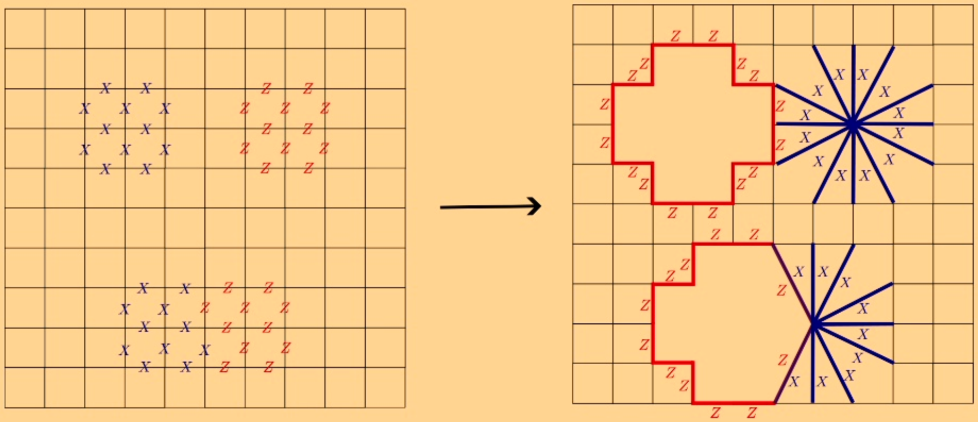
Claim: Information is preserved if and only if $p_X < \frac{1}{2}, p_Z < \frac{1}{2}$ and $p_m < 1$.

2D toric code

Measurement in toric code: X and Z



If measurement operators do not percolate ($p_X, p_Z < \frac{1}{2}$), the information is **still preserved** after random measurements, but in **the form of toric code defined on a deformed lattice.**

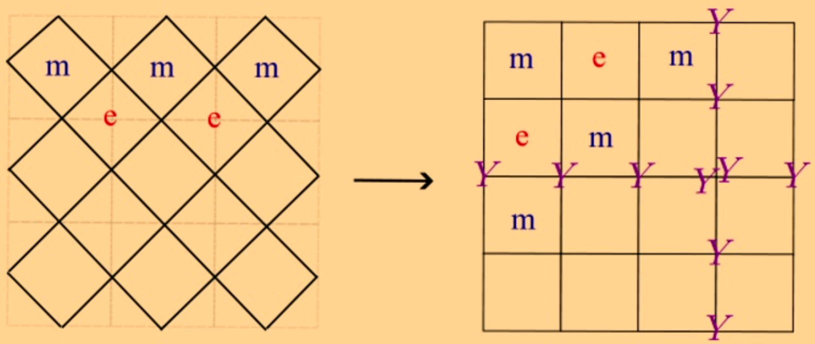


2D toric code

Measurement in toric code: Y

Logical operators consisting of only Y operators **are very rare.** Mainly because they are **not deformable.**

→ All Y-type logical operators are determined by the initial row and column data.



Furthermore, non-deformability makes Y-type logical operators **heavy.**

→ Initial a row, b column operators: $W(a,b) = (L-a)b + (L-b)a$

$$P_{\text{destroy}} \leq \sum_{0\leq a,b\leq 0} \binom{L}{a}\binom{L}{b} p_Y^{W(a,b)} \to 0 \quad (L \to \infty)$$

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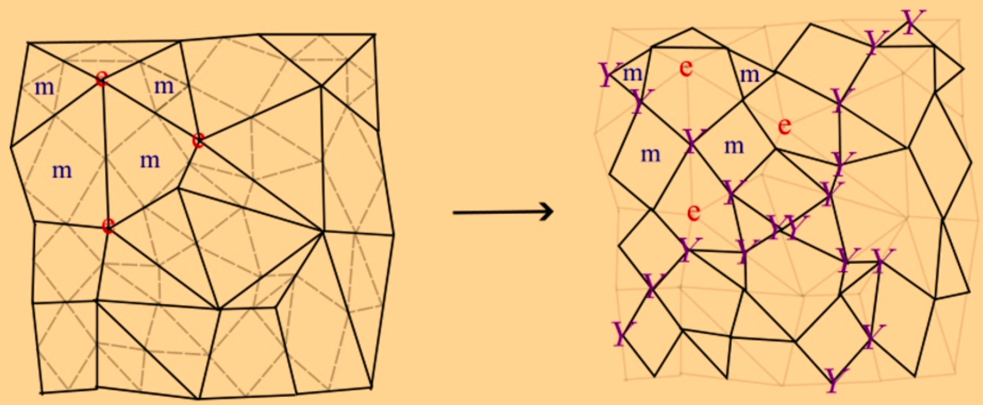
2D toric code

Measurement in toric code: X , Y , Z

After measuring toric code with $p_X, p_Z < \frac{1}{2}$, we get a **deformed toric code**.

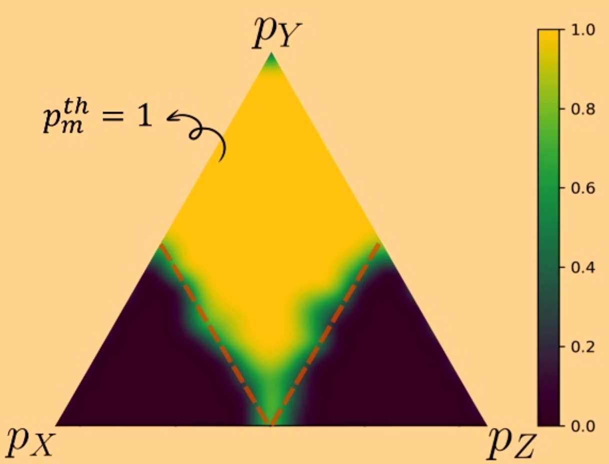


At deformed lattice, it is **harder** to form closed Y loop, and it should have **larger** weight.

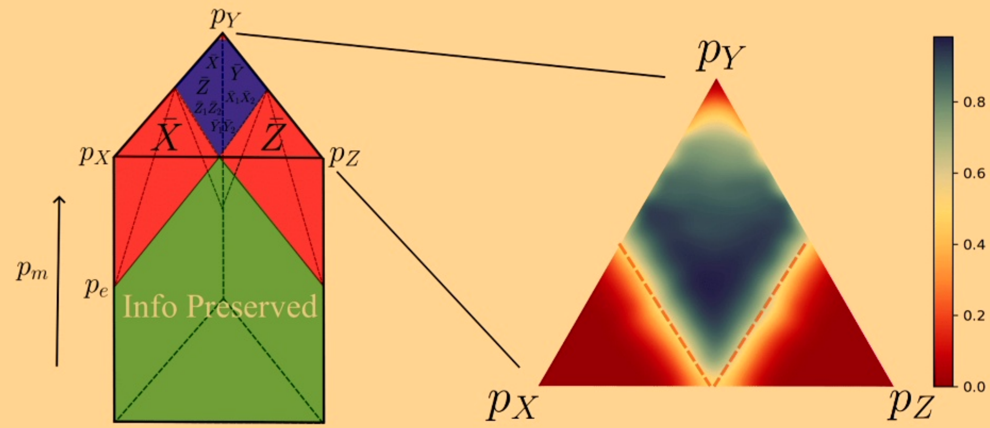
⟶ Information must be preserved if $p_Y < 1$.

2D toric code

Numerical results



[Information preservation diagram at $p_m = 0.95$]



[Uncertainty of measured logical operators]

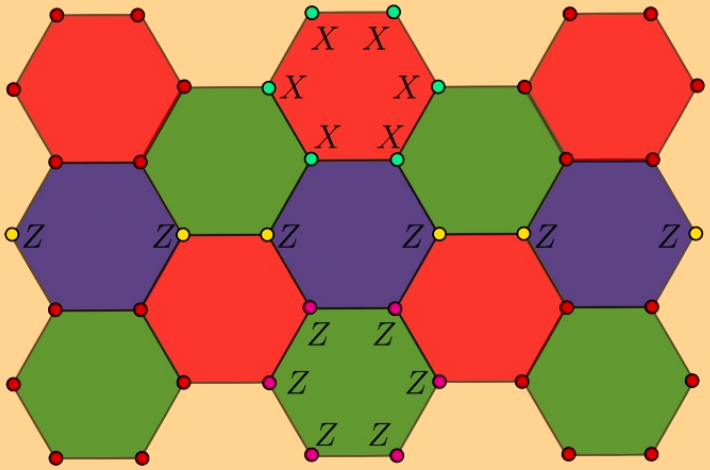
- ✓ Information is preserved if and only if $p_X < \frac{1}{2}, p_Z < \frac{1}{2}$ and $p_m < 1$
- ✓ The relation $S^{(2)}(\alpha_X, \alpha_Y, \alpha_Z) > 0 \implies p_m^{th}(\alpha_X, \alpha_Y, \alpha_Z) = 1$ holds.

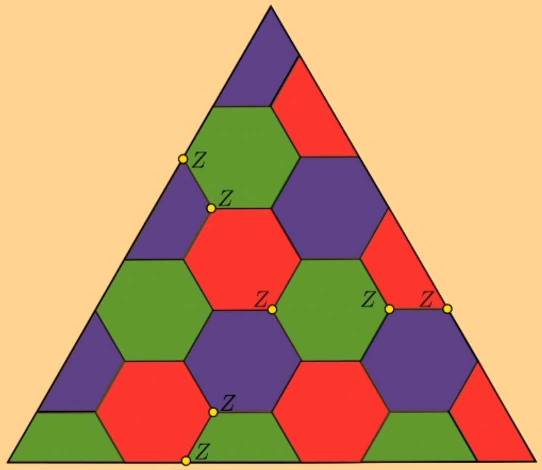
2D color code

2D color code review [Bombin,Martin-Delgado'06]

↳ 2D topological code that has symmetry under exchange of X, Y, Z operators.

Defined on any lattice with three-valent and three-colorable condition.



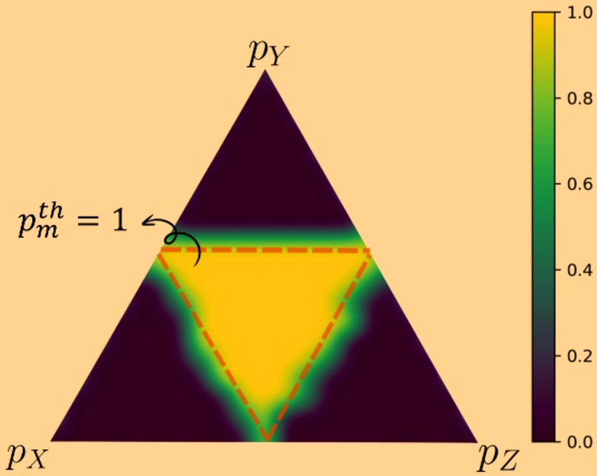


→ **Stabilizer generators**: face operators consisting of only X and Z.

→ **Logical operators**: string operators connecting same-colored faces.

2D color code

Numerical results



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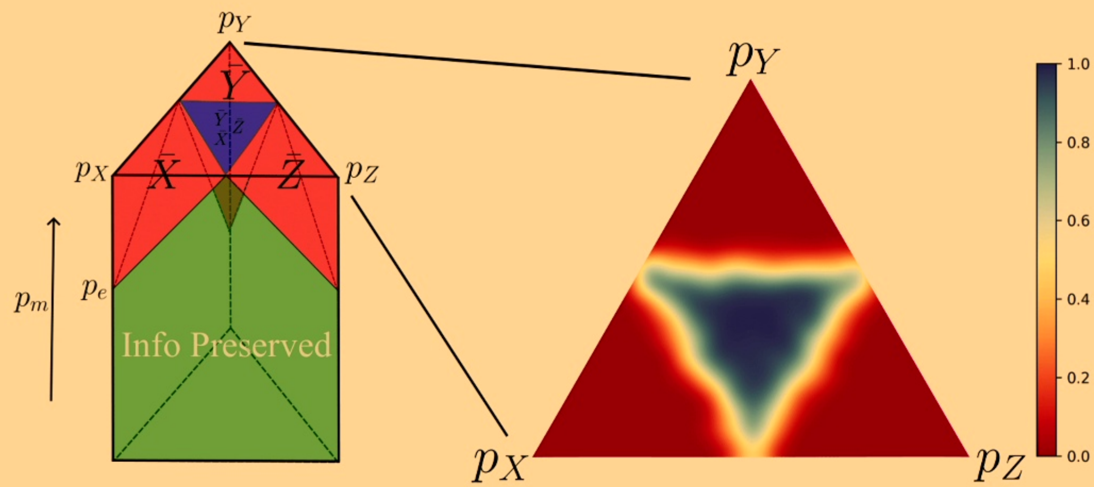


[Uncertainty of measured logical operators]

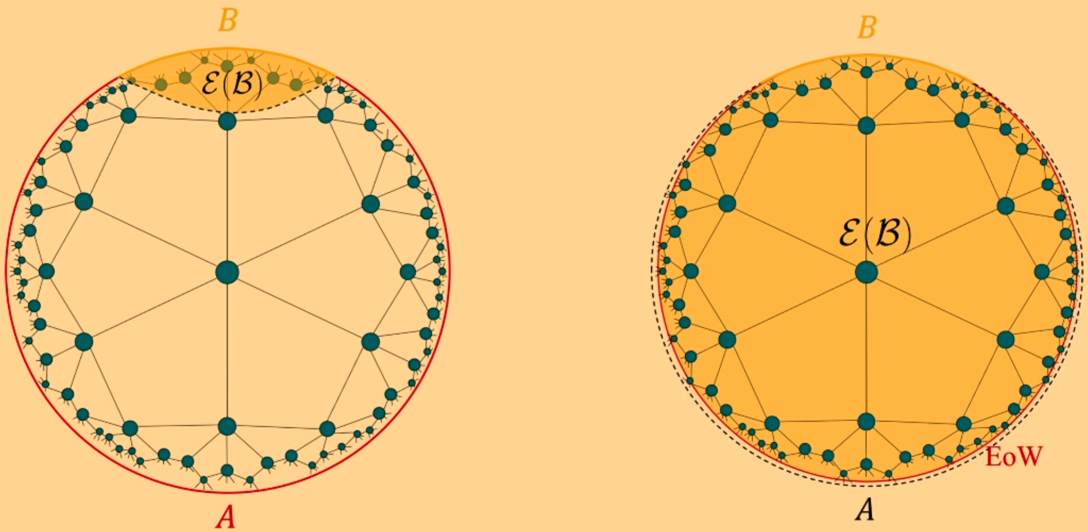
- Information is preserved if and only if $p_X, p_Y, p_Z \leq \frac{1}{2}$, and $p_m < 1$.
- The relation $S^{(2)}(\alpha_X, \alpha_Y, \alpha_Z) > 0 \implies p_m^{th}(\alpha_X, \alpha_Y, \alpha_Z) = 1$ holds.

Outline

1. Setup
2. Concatenated codes
3. 2D Topological codes
4. **Holographic codes**
5. Summary and open problems

Holographic codes

→ AdS/CFT correspondence can be interpreted as QECC where bulk degrees of freedom is holographically encoded into boundary degrees of freedom.



[Before measuring subregion A] [After measuring subregion A]

- ✓ Erasure threshold for holographic QECC : $p_e^{th} = 1/2$.
- ✓ Measurement threshold for holographic QECC: $p_m^{th} = 1$
  - Tensor network models: HaPPY code, RTN [Patawaski,Yoshida,Harlow,Preskill'15] [Hayden, et.al.'16]
  - Measurement induced EoW brane. [Antonini, et.al.'22]

Summary and open problems

Take home message

We demonstrated that QECCs have **extraordinary robustness** against random local monitoring, achieving the maximal measurement threshold $p_m^{th} = 1$ in many cases.

⟶ **A bit of quantum encoding** helps the system to retain the memory of its initial states under projective measurements.

Open problems

1. Effective field theory description of the relation $S^{(2)}(\alpha_X, \alpha_Y, \alpha_Z) \Rightarrow p_m^{th}(\alpha_X, \alpha_Y, \alpha_Z) = 1$ for topological codes?

2. Shadow tomography for QECC? Namely, efficient shadow tomography inside the logical Hilbert space?

3. Relation between transversal logical gates and the measurement threshold?
(7 qubit code vs 15 qubit code)

Thank you