Title: Nonclassicality in correlations without causal order

Speakers: Ravi Kunjwal

Series: Quantum Foundations

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Abstract: A Bell scenario can be conceptualized as a "communication" scenario with zero rounds of communication between parties, i.e., although each party can receive a system from its environment on which it can implement a measurement, it cannot send out any system to another party. Under this constraint, there is a strict hierarchy of correlation sets, namely, classical, quantum, and non-signalling. However, without any constraints on the number of communication rounds between the parties, they can realize arbitrary correlations by exchanging only classical systems. We consider a multipartite scenario where the parties can engage in at most a single round of communication, i.e., each party is allowed to receive a system once, implement any local intervention on it, and send out the resulting system once. Taking our cue from Bell nonlocality in the "zero rounds" scenario, we propose a notion of nonclassicality---termed antinomicity---for correlations in scenarios with a single round of communication. Similar to the zero rounds case, we establish a strict hierarchy of correlation sets classified by their antinomicity in single-round communication scenarios. Since we do not assume a global causal order between the parties, antinomicity serves as a notion of nonclassicality in the presence of indefinite causal order (as witnessed by causal inequality violations). A key contribution of this work is an explicit antinomicity witness that goes beyond causal inequalities, inspired by a modification of the Guess Your Neighbour's Input (GYNI) game that we term the Guess Your Neighbour's Input or NOT (GYNIN) game. Time permitting, I will speculate on why antinomicity is a strong notion of nonclassicality by interpreting it as an example of fine-tuning in classical models of indefinite causality.This is based on joint work with Ognyan Oreshkov, arXiv:2307.02565.

Zoom link

Nonclassicality in correlations without causal order

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Quantum Foundations seminar, Perimeter Institute
May 7, 2024

Based on arXiv:2307.02565 with Ognyan Oreshkov

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What is nonclassical about indefinite causal order?

What is nonclassical about indefinite causal order?

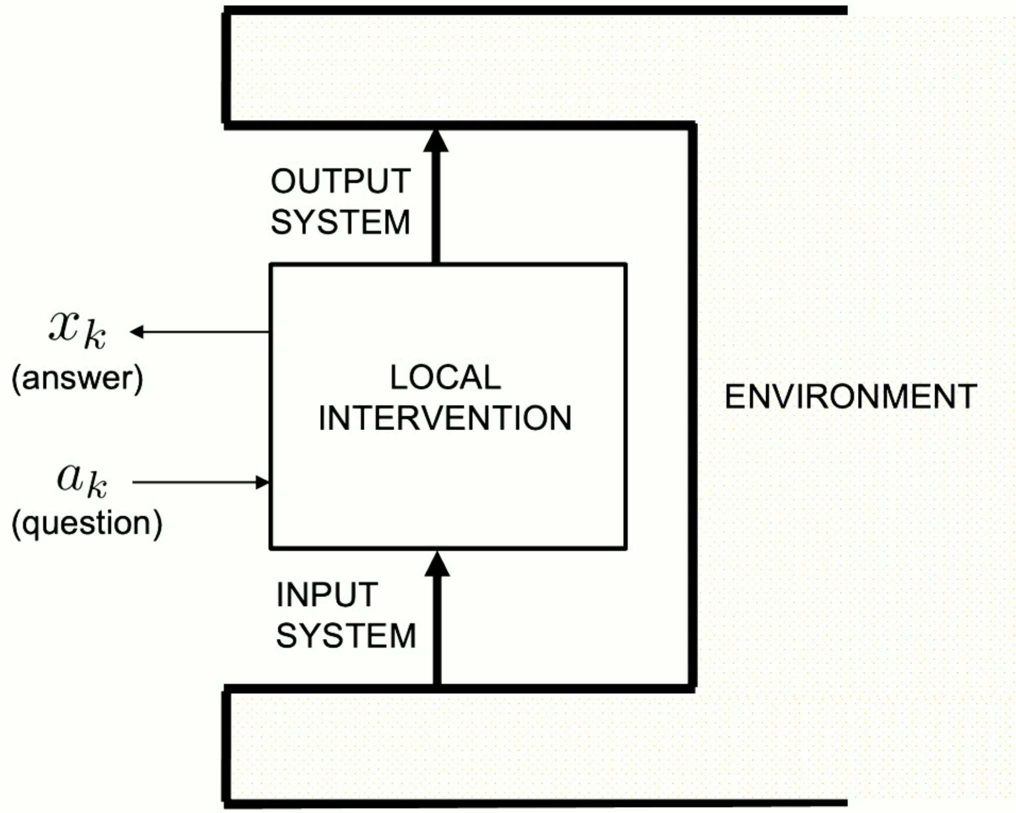
Operational answers only!

Outline

The set-up ⟶ **Causal ineq. viol.** ⟶ **Not always nonclassical**

Antinomicity

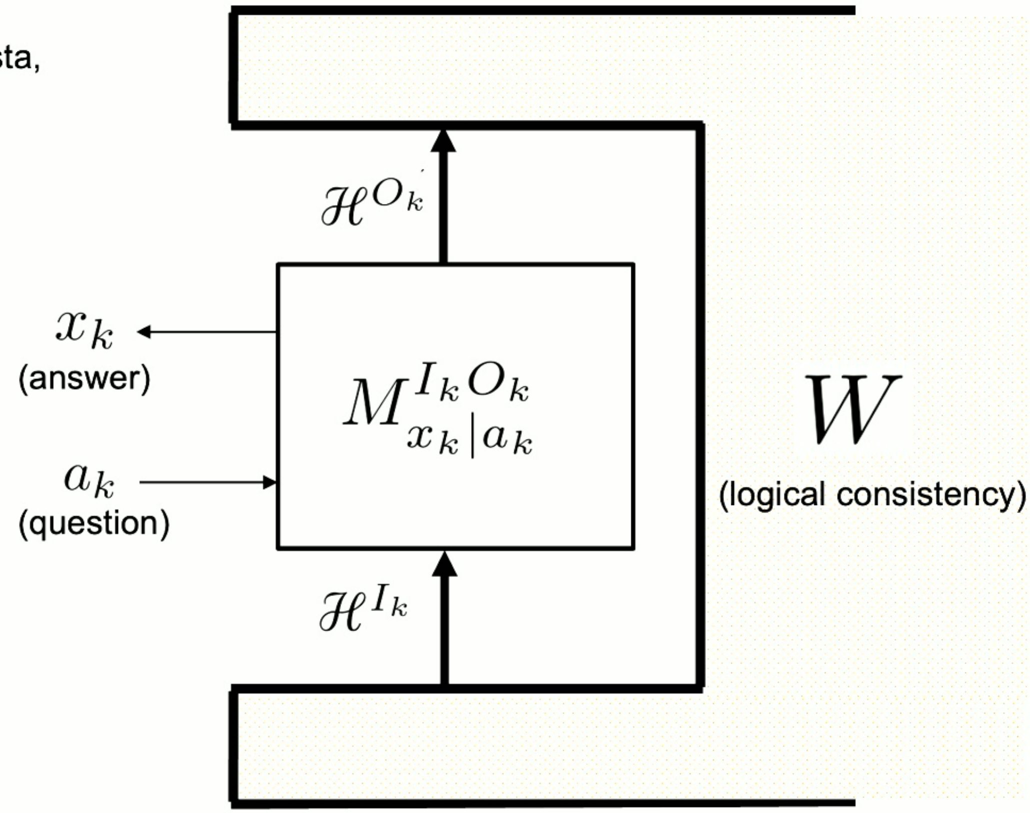
General operational paradigm



$$p(x_1, x_2, \ldots, x_N | a_1, a_2, \ldots, a_N)$$

Process-matrix framework

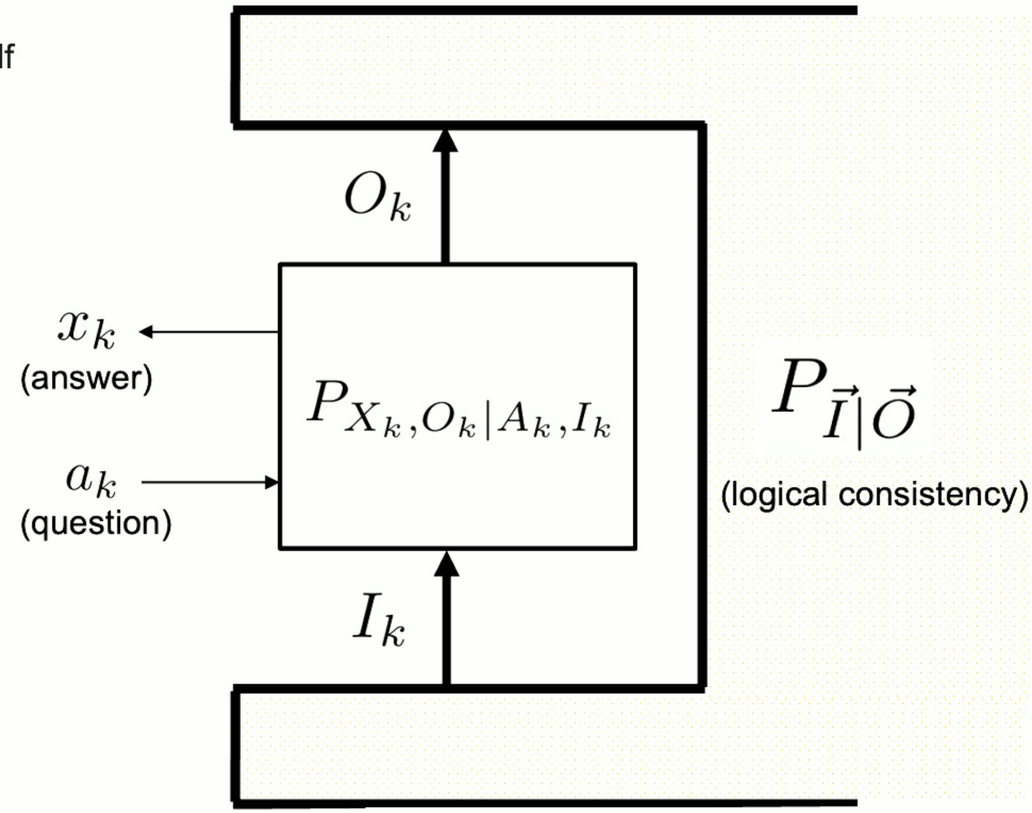
O. Oreshkov, F. Costa,
Č. Brukner (OCB)
arXiv:1105.4464



$$p(\vec{x}|\vec{a}) = \mathrm{Tr}(W M^{I_1 O_1}_{x_1|a_1} \otimes M^{I_2 O_2}_{x_2|a_2} \otimes \cdots \otimes M^{I_N O_N}_{x_N|a_N})$$

Classical process framework

Ä. Baumeler, S. Wolf
arXiv:1511.05444



$$p(\vec{x}|\vec{a}) = \sum_{\vec{i},\vec{o}} \left(\prod_{k=1}^{N} p(x_k, o_k|a_k, i_k) \right) p(\vec{i}|\vec{o})$$

Process function framework

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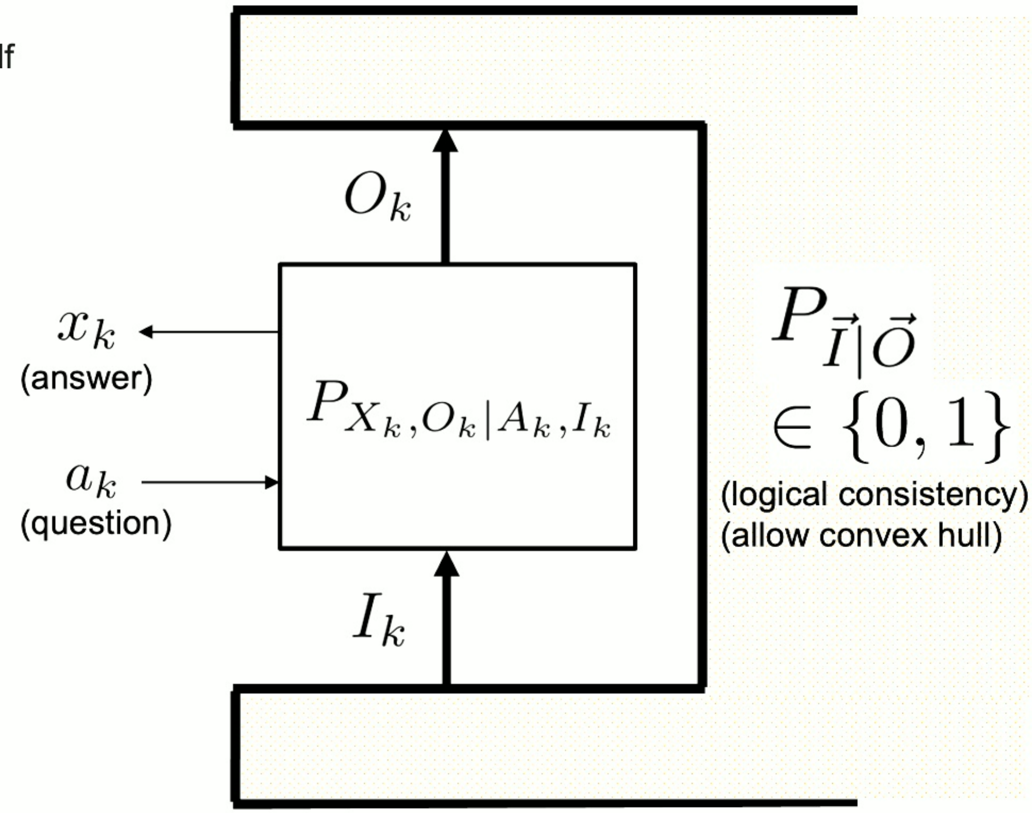

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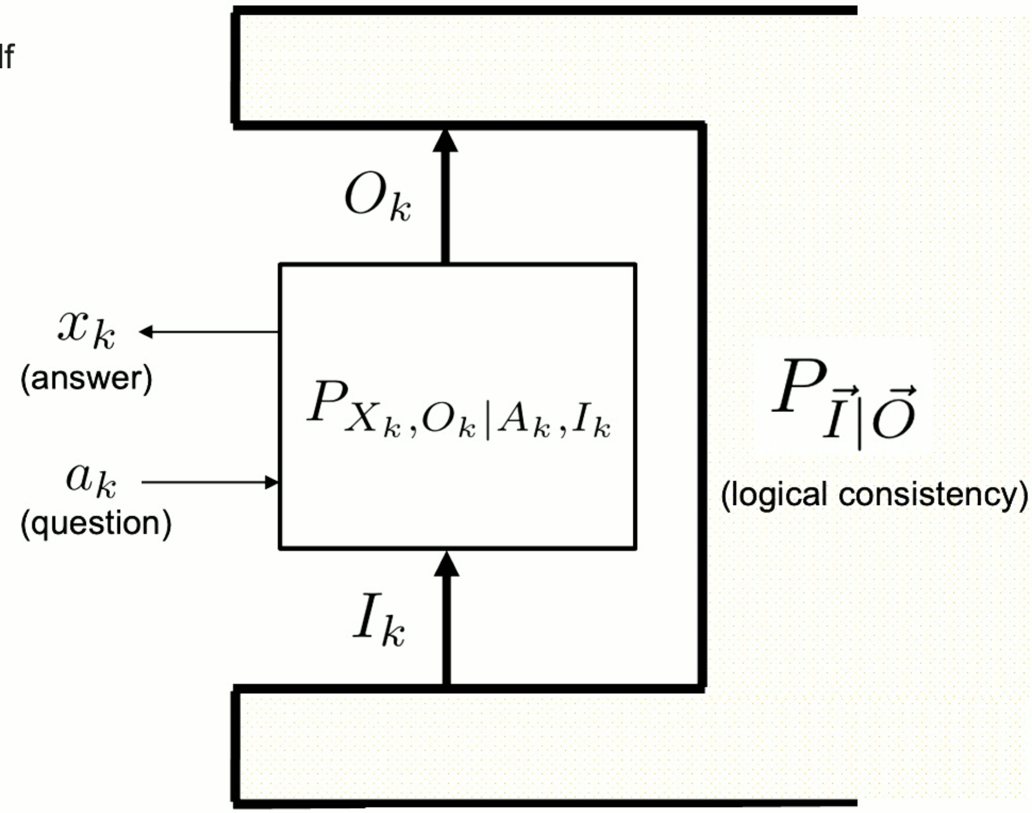

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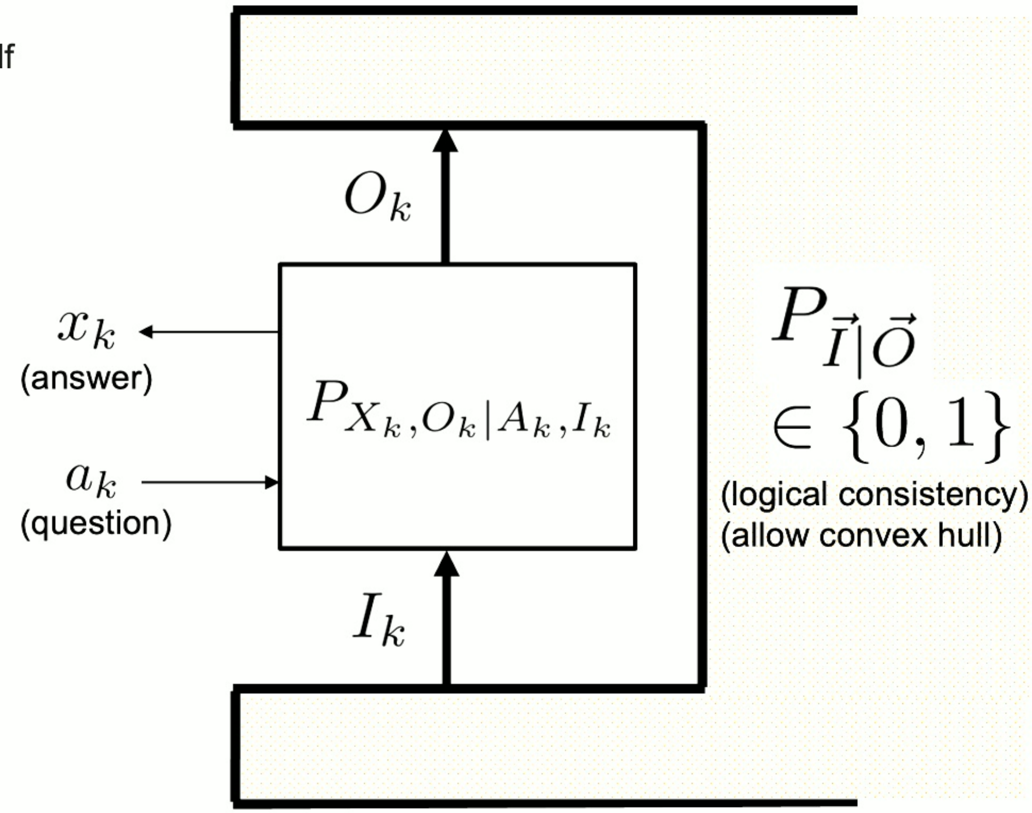

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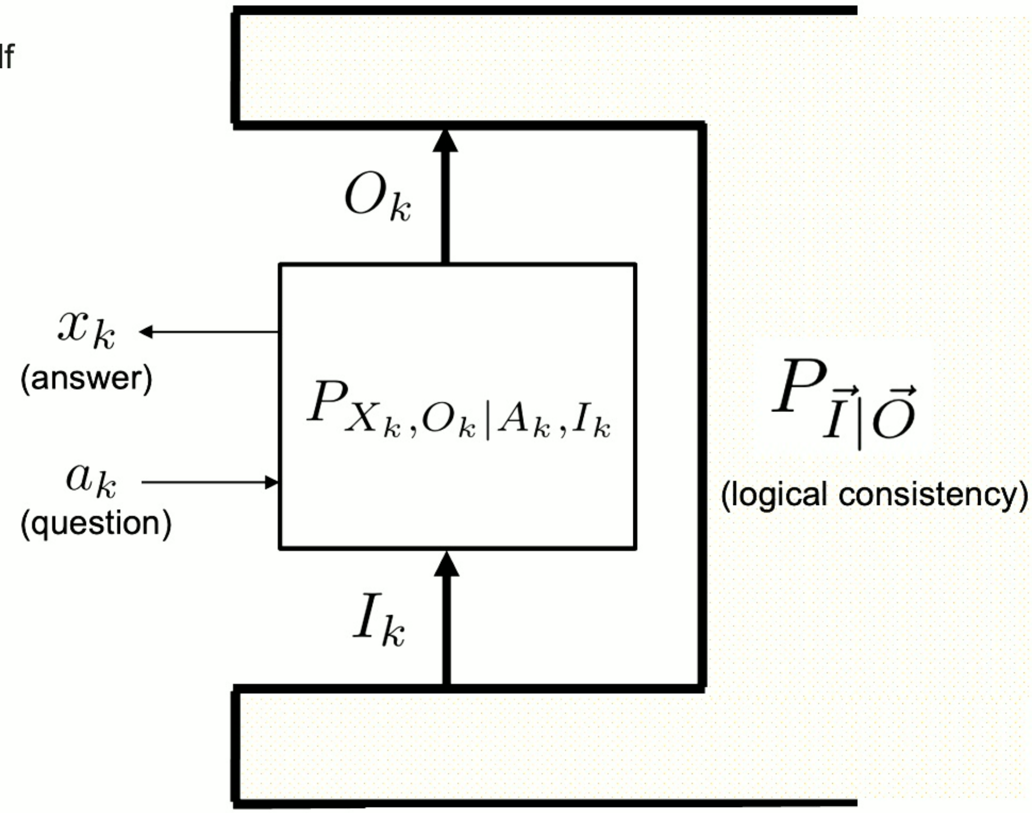

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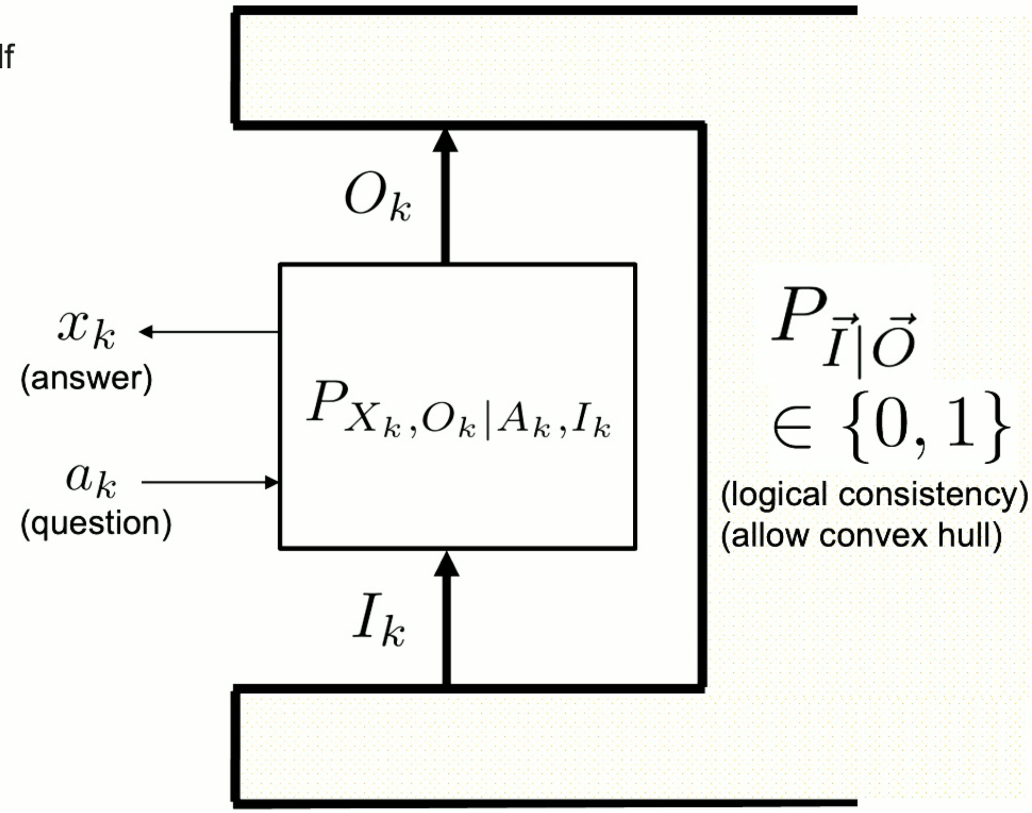

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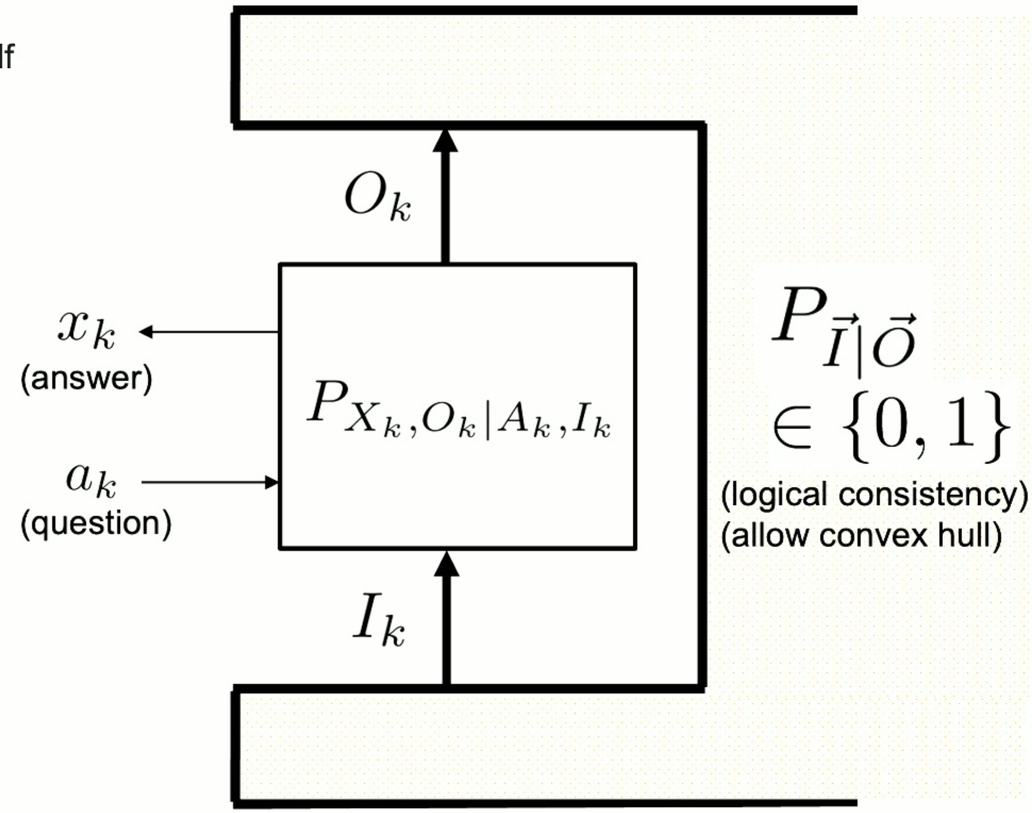
Causal inequalities

Process function framework

Ä. Baumeler, S. Wolf
arXiv:1511.05444



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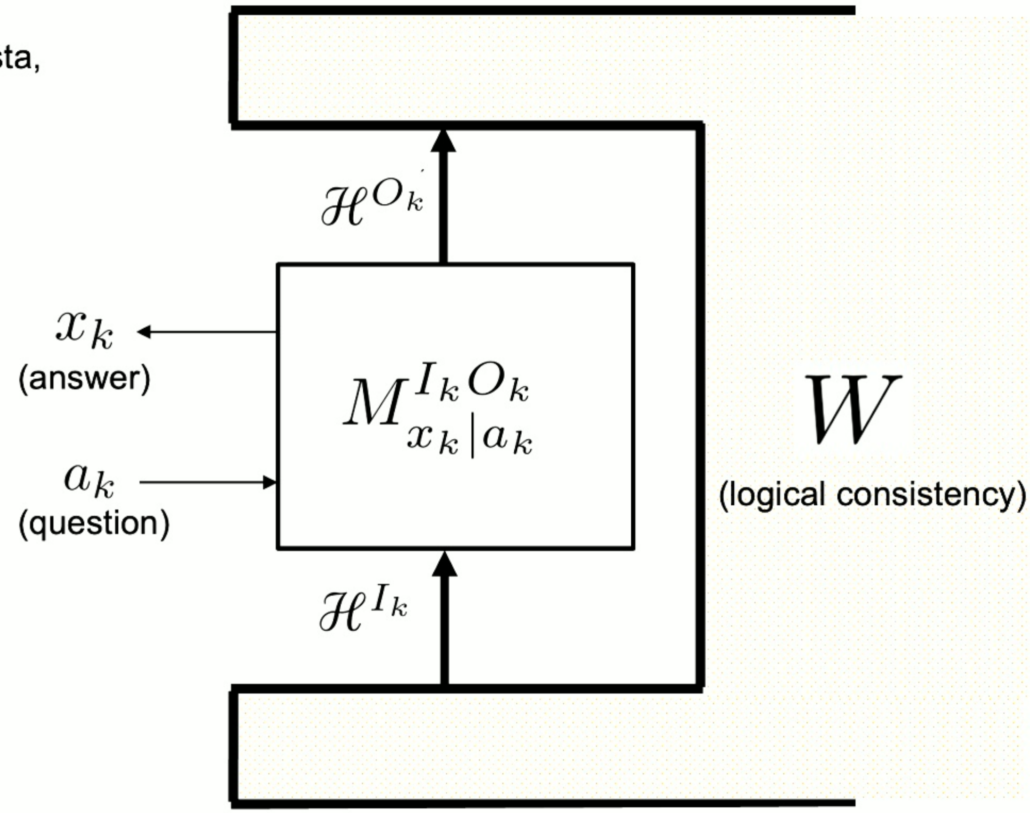

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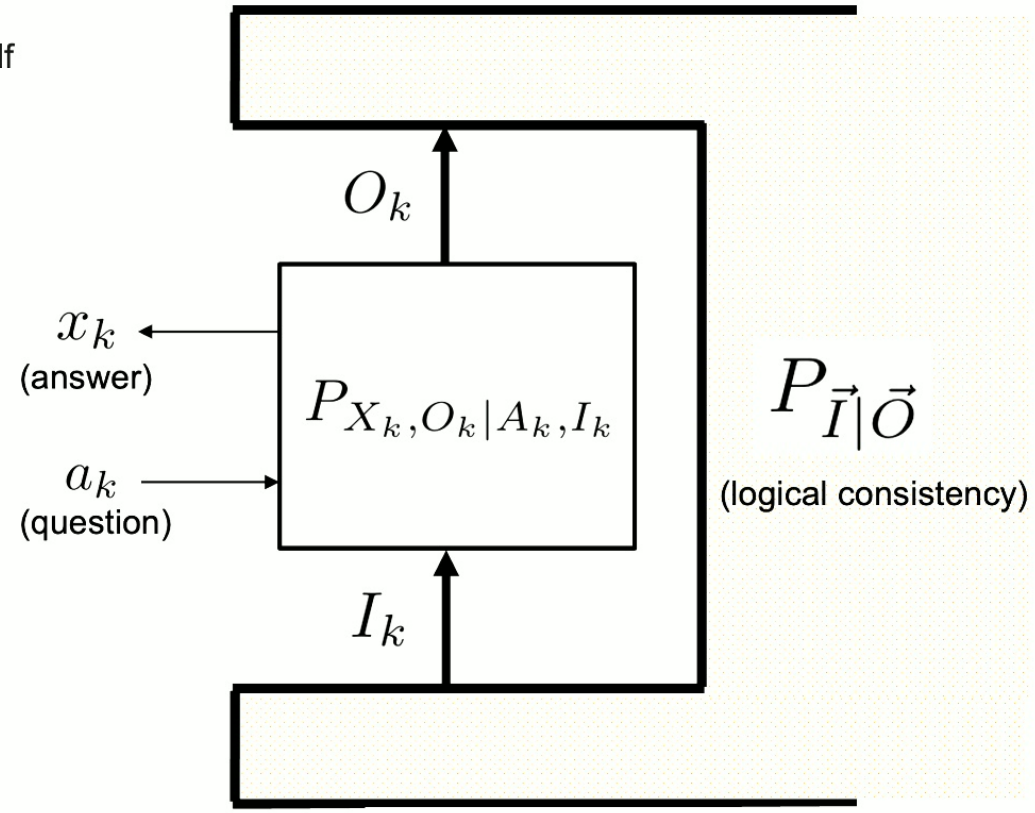

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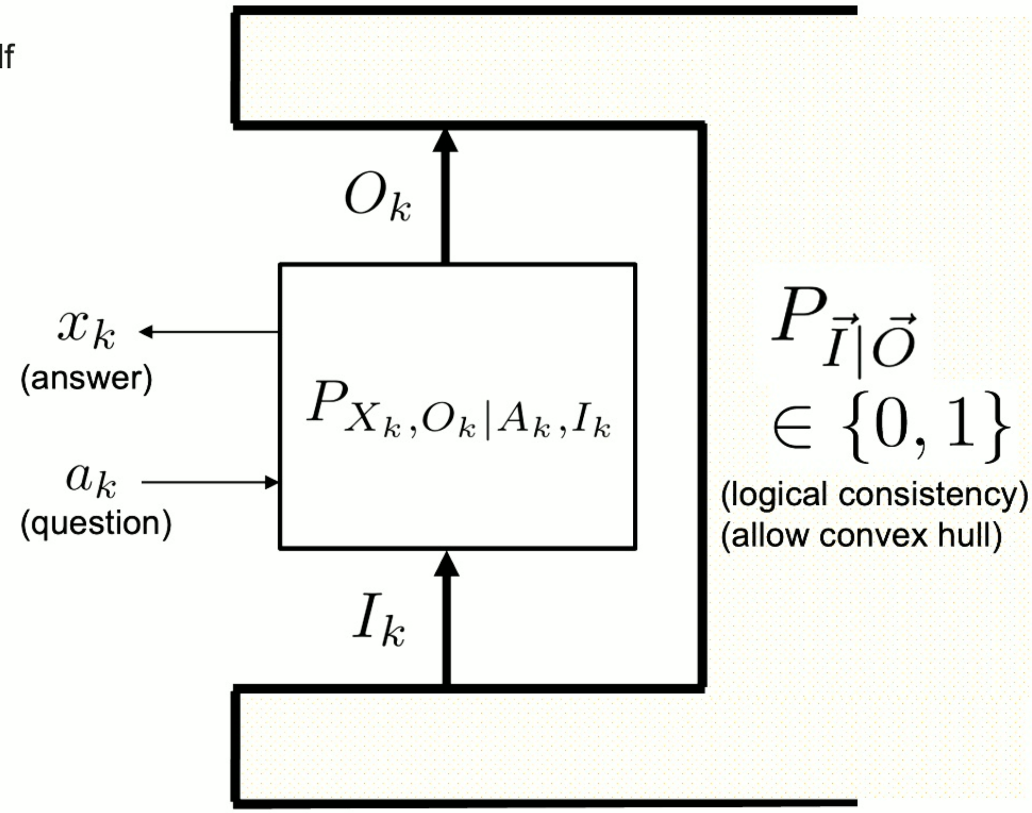

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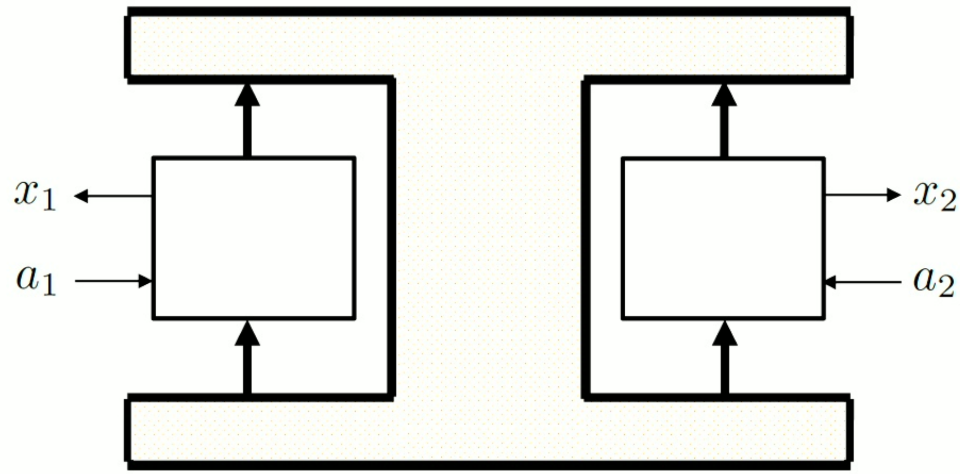
Causal inequalities

Operational constraints from a definite causal order

Example: Guess Your Neighbour's Input (GYNI) inequality

$$x_1 = a_2 \text{ and } x_2 = a_1$$



$$\frac{1}{4} \sum_{a_1, a_2, x_1, x_2} \delta_{x_1, a_2} \delta_{x_2, a_1} P(x_1, x_2 | a_1, a_2) \leq \frac{1}{2}$$

Example: Guess Your Neighbour's Input (GYNI) inequality

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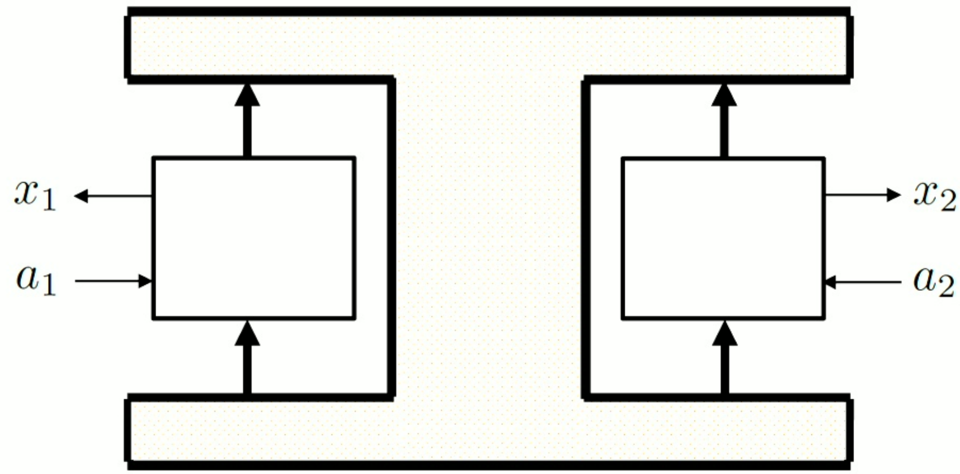


$$\frac{1}{4}\sum_{a_1,a_2,x_1,x_2} \delta_{x_1,a_2}\delta_{x_2,a_1} P(x_1,x_2|a_1,a_2) \leq \frac{1}{2}$$

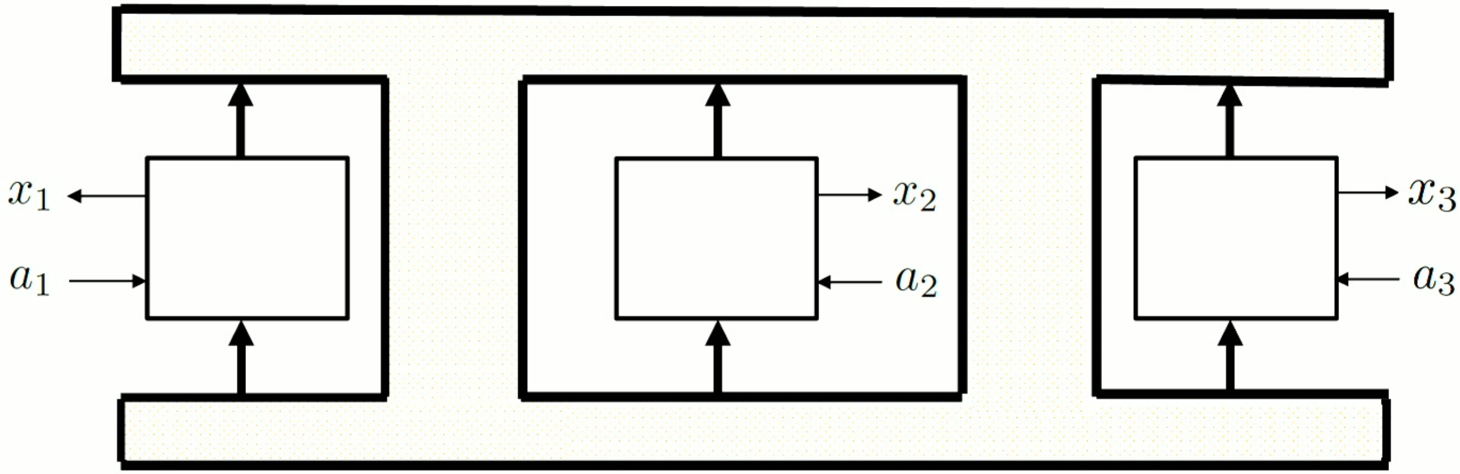
Violated by process-matrix correlations! arXiv:1508.01704

Does the diagonal limit of the process-matrix framework imply causality?

Bipartite: Yes! (OCB)
In general: No! (BFW, AF/BW)

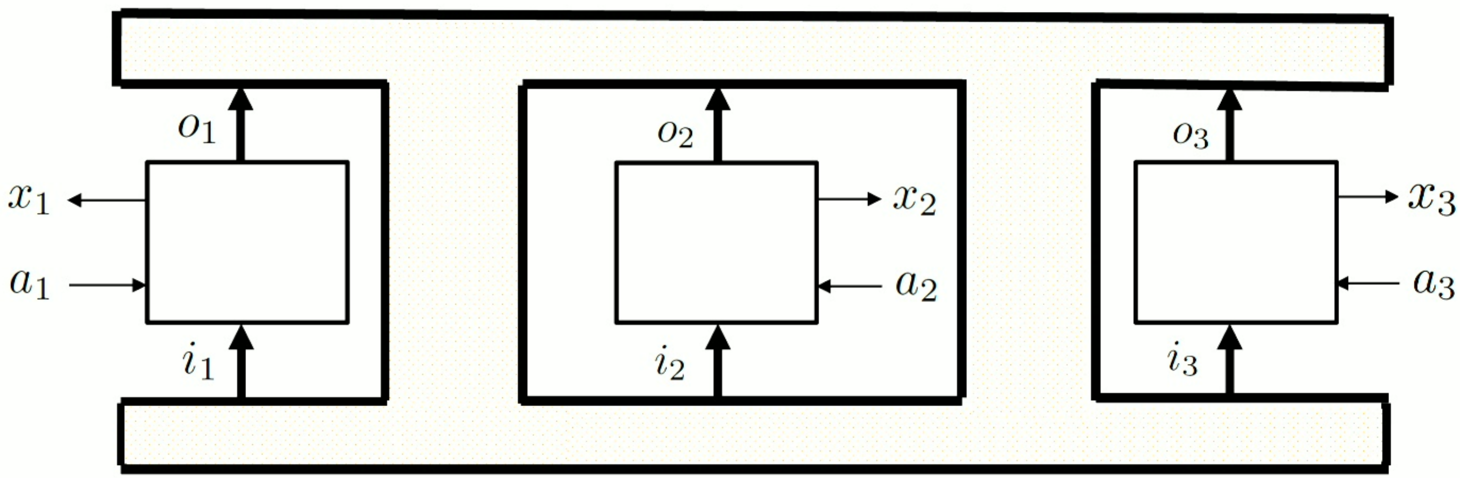
OCB: arXiv:1105.4464
BFW: arXiv:1403.7333
AF/BW: arXiv:1507.01714

Example: a tripartite causal inequality



$$\frac{1}{2}\sum_{\vec{x},\vec{a}} p(x_1,x_2,x_3|a_1,a_2,a_3)\Big(\delta_{x_1,a_3}\delta_{x_2,a_1}\delta_{x_3,a_2}\delta_{\text{maj}(a_1,a_2,a_3),0} + \delta_{x_1,\bar{a}_2}\delta_{x_2,\bar{a}_3}\delta_{x_3,\bar{a}_1}\delta_{\text{maj}(a_1,a_2,a_3),1}\Big) \le \frac{3}{4}.$$

AF/BW or "Lugano" process function



$$i_1 = \bar{o}_2 o_3, i_2 = \bar{o}_3 o_1, i_3 = \bar{o}_1 o_2$$

Causal inequality violations do not require nonclassical resources

Antinomicity

A notion of classicality: Deterministic Consistency (or non-antinomicity)

A multipartite correlation satisfies deterministic consistency if and only if it can be achieved by a classical process in the convex hull of classical deterministic processes, *i.e.*,

$$p(\vec{x}|\vec{a}) = \sum_{\vec{i},\vec{o}} \prod_{k=1}^{N} p(x_k, o_k|a_k, i_k)p(\vec{i}|\vec{o})$$

where $p(\vec{i}|\vec{o}) = \sum_{\lambda} p(\lambda)\delta_{\vec{i},\omega^{\lambda}(\vec{o})}$

For a non-signalling environment, this describes a Bell-local model for the correlation!

Baumeler-Wolf: arXiv:1507.01714

Antinomicity is the failure of deterministic consistency for a correlation

intuitively, it's the property that a classical environment must admit "hidden logical contradictions" to reproduce the correlation

Correlation sets

Correlational scenario (N, M, D)

Settings: $\vec{a} := (a_1, a_2, \ldots, a_N)$

Outcomes: $\vec{x} := (x_1, x_2, \ldots, x_N)$

$$p(\vec{x}|\vec{a}) \geq 0 \quad \forall \vec{x}, \vec{a},$$

$$\sum_{\vec{x}} p(\vec{x}|\vec{a}) = 1 \quad \forall \vec{a}.$$

Four sets of correlations

- $\mathscr{DC}$: deterministically consistent correlations
  (achievable by convex mixtures of process functions)

- $\mathscr{PC}$: probabilistically consistent correlations
  (achievable via diagonal process matrices)

- $\mathscr{QP}$: quantum process correlations
  (achievable via process matrices)

- $q\mathscr{C}$: quasi-consistent correlations
  (the full set of correlations, achievable via arbitrary classical channels)

$$\mathscr{DC} \subsetneq \mathscr{PC} \subsetneq \mathscr{QP} \subsetneq q\mathscr{C}$$

Key theorem

A deterministic correlation can be realized by a process matrix if and only if it can also be realized by a process function

Theorem 4 in arXiv:2307.02565

Key theorem

Hence: any deterministic correlation unachievable by a process function is also unachievable by a process matrix!

Theorem 4 in arXiv:2307.02565

Key theorem

A deterministic correlation can be realized by a process matrix if and only if it can also be realized by a process function

Theorem 4 in arXiv:2307.02565

Logic of the strict inclusions

$$\mathcal{QP} \subsetneq q\mathcal{C}$$

- Every deterministic correlation achievable by a process matrix is achievable by a process function
- Bipartite case: perfect GYNI correlation unachievable by any process function (bipartite diagonal limit => no causal inequality violation)
- Hence, perfect GYNI correlation unachievable by any process matrix

Logic of the strict inclusions

$$\mathscr{PC} \subsetneq \mathscr{QP}$$

Follows from the bipartite case where $\mathscr{DC}$ and $\mathscr{PC}$ coincide and causal inequalities are violated by process matrices

arXiv: 1105.4464
arXiv: 1508.01704

Logic of the strict inclusions

$$\mathscr{DC} \subsetneq \mathscr{PC}$$

Guess Your Neighbour's Input or NOT (GYNIN) game

$$(x_1, x_2, x_3) = (a_3, a_1, a_2) \text{ OR } (x_1, x_2, x_3) = (\bar{a}_3, \bar{a}_1, \bar{a}_2)$$

$$p_{\text{gynin}} := \frac{1}{8} \sum_{\vec{x},\vec{a}} p(\vec{x}|\vec{a}) \Big(\delta_{x_1,a_3} \delta_{x_2,a_1} \delta_{x_3,a_2} + \delta_{x_1,\bar{a}_3} \delta_{x_2,\bar{a}_1} \delta_{x_3,\bar{a}_2} \Big)$$

Logic of the strict inclusions

$$p_{\text{gynin}} \overset{\text{causal}}{\leq} \frac{1}{2} \overset{\text{classical}}{\leq} \frac{5}{8} \overset{\text{antinomic}}{\leq} 1$$

AF/BW: arXiv:1507.01714 $\mathscr{DC} \subsetneq \mathscr{PC}$ BFW: arXiv:1403.7333

Logic of the strict inclusions

$$\mathscr{DC} \subsetneq \mathscr{PC}$$

Guess Your Neighbour's Input or NOT (GYNIN) game

$$(x_1, x_2, x_3) = (a_3, a_1, a_2) \text{ OR } (x_1, x_2, x_3) = (\bar{a}_3, \bar{a}_1, \bar{a}_2)$$

$$p_{\text{gynin}} := \frac{1}{8} \sum_{\vec{x},\vec{a}} p(\vec{x}|\vec{a}) \left(\delta_{x_1,a_3} \delta_{x_2,a_1} \delta_{x_3,a_2} + \delta_{x_1,\bar{a}_3} \delta_{x_2,\bar{a}_1} \delta_{x_3,\bar{a}_2} \right)$$

Logic of the strict inclusions

$$p_{\text{gynin}} \overset{\text{causal}}{\leq} \frac{1}{2} \overset{\text{classical}}{\leq} \frac{5}{8} \overset{\text{antinomic}}{\leq} 1$$

AF/BW: arXiv:1507.01714 $\mathscr{D}\mathcal{C} \subsetneq \mathscr{P}\mathcal{C}$ BFW: arXiv:1403.7333

Takeaway

- Not every causal inequality violation is a witness of nonclassicality
- Antinomicity as nonclassicality in correlations without causal order
- Hierarchy of correlation sets: strict inclusions
- No assumption about network structure beyond logical consistency

Open questions

- Fully characterize the classical polytope in the simplest non-trivial scenario, i.e., (3,2,2)
- Can one witness antinomicity with unitary proces?
- Tsirelson-type bounds on process-matrix correlations? [See arXiv:2403.02749]
- Infinite-dimensional surprises?