Title: Classical Black Hole Scattering from a World-Line Quantum Field Theory - VIRTUAL

Speakers: Jan Plefka

Series: Colloquium

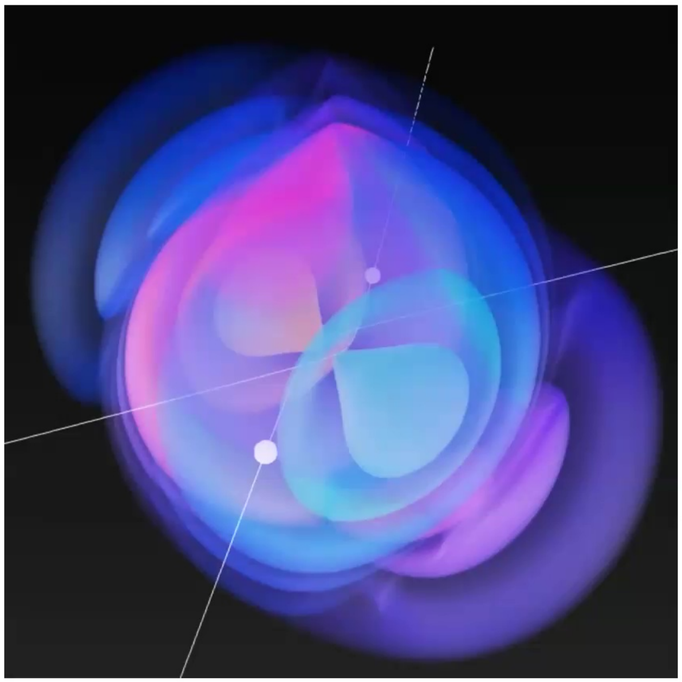
Date: May 29, 2024 - 2:00 PM

URL: https://pirsa.org/24050061

Abstract: Predicting the outcome of scattering processes of elementary particles in colliders is the central achievement of relativistic quantum field theory applied to the fundamental (non-gravitational) interactions of nature. While the gravitational interactions are too minuscule to be observed in the microcosm, they dominate the interactions at large scales. As such the inspiral and merger of black holes and neutron stars in our universe are now routinely observed by gravitational wave detectors. The need for high precision theory predictions of the emitted gravitational waveforms has opened a new window for the application of perturbative quantum field theory techniques to the domain of gravity. In this talk I will show how observables in the classical scattering of black holes and neutron stars can be efficiently computed in a perturbative expansion using a world-line quantum field theory; thereby combining state-of-the-art Feynman integration technology with perturbative quantum gravity. Here, the black holes or neutron stars are modelled as point particles in an effective field theory sense. Fascinatingly, the intrinsic spin of the black holes may be captured by a supersymmetric extension of the world-line theory, enabling the computation of the far field wave-form including spin and tidal effects to highest precision. I will review our most recent results at the fifth order in the post-Minkowskian expansion amounting to the computations of hundreds of thousands of four loop Feynman integrals.

Zoom link

HIGH PRECISION GRAVITATIONAL WAVE PHYSICS FROM A WORLDLINE QUANTUM FIELD THEORY



Jan Plefka

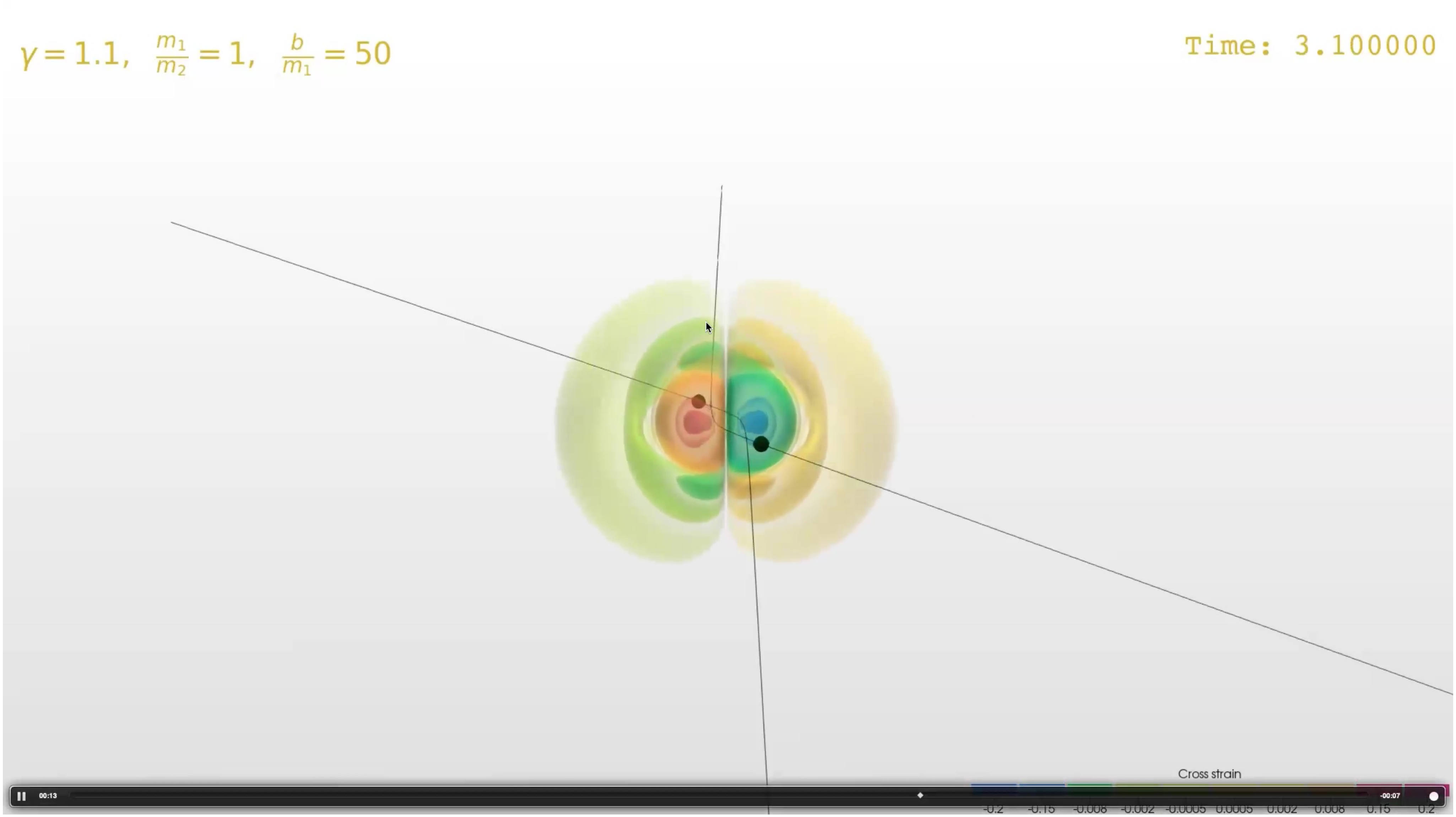
Humboldt Universität zu Berlin

Based on joint work with

Mathias Driesse, Gustav Uhre Jakobsen, Gustav Mogull, Benjamin Sauer, Jan Steinhoff (AEI), Johann Usovitsch (CERN)

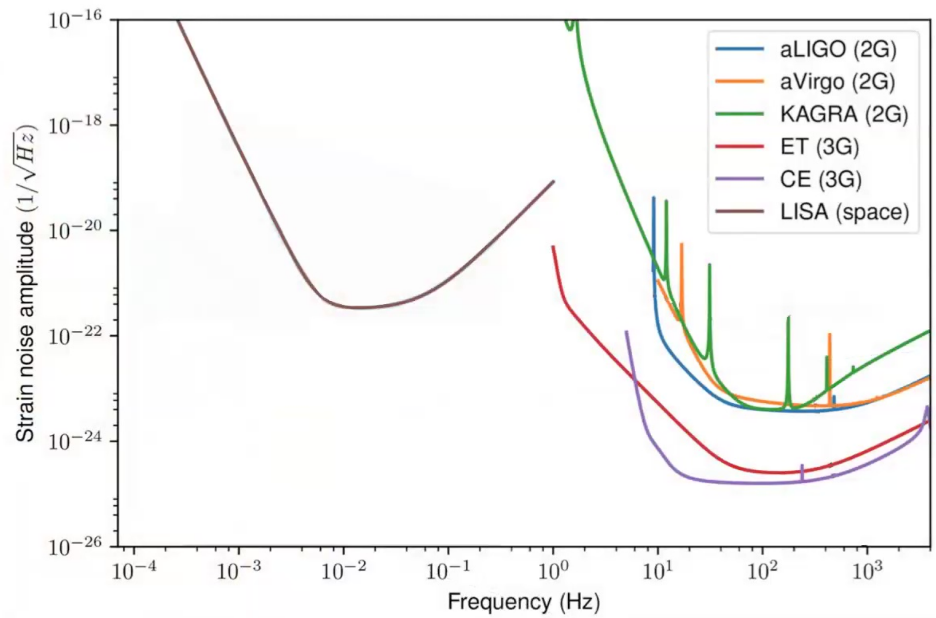
European Research Council
Established by the European Commission

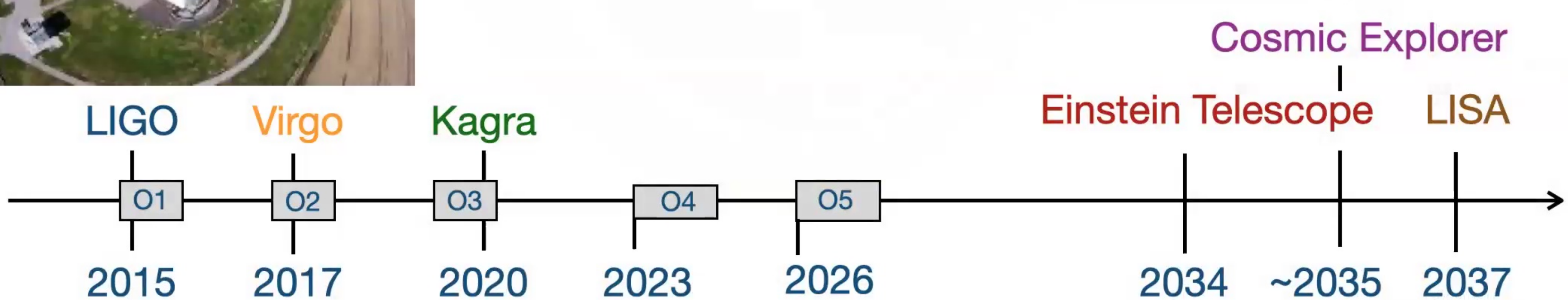
RTG 2575:
Rethinking Quantum Field Theory


$\gamma = 1.1, \ \frac{m_1}{m_2} = 1, \ \frac{b}{m_1} = 50$
Time: 3.100000
Cross strain
-0.2 -0.15 -0.008 -0.002 -0.0005 0.0005 0.002 0.008 0.15 0.2

ERA OF GRAVITATIONAL WAVE PHYSICS: NEED FOR HIGH-PRECISION PREDICTIONS

- Upcoming 3rd generation of gravitational wave observatories with 10^2 sensitivity increase
- Need for accurate waveform predictions well beyond state-of-the art



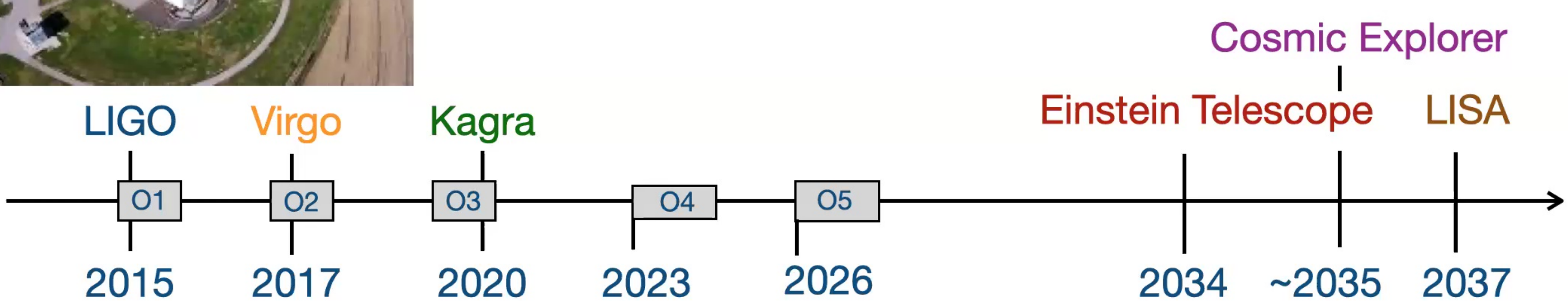


ERA OF GRAVITATIONAL WAVE PHYSICS: NEED FOR HIGH-PRECISION PREDICTIONS

- Upcoming 3rd generation of gravitational wave observatories with 10^2 sensitivity increase
- Need for accurate waveform predictions well beyond state-of-the art

High-precision predictions necessary basis to study fundamental questions in physics:

- Is Einstein's theory correct?
- Black hole formation & population?
- Neutron star properties?
- Physics beyond the standard model?

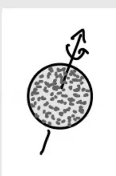


GRAVITATIONAL TWO-BODY PROBLEM

Black Hole

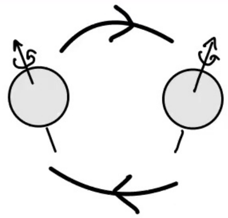
mass, spin

Neutron Star

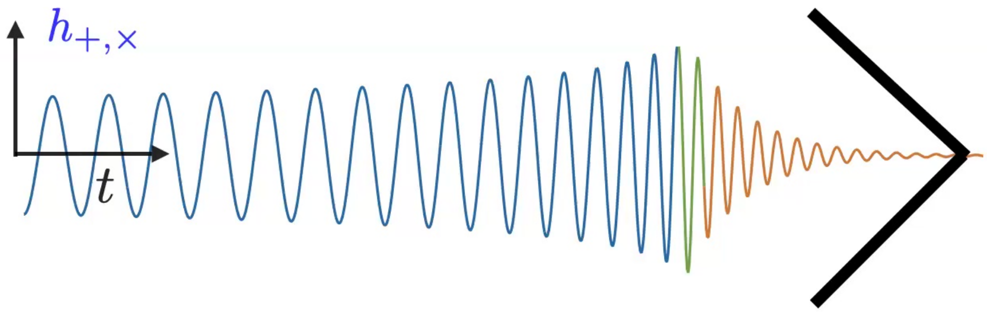


mass, spin, radius, tidal deformability

Black Hole/Neutron Star Binaries:



Bound state



inspiral

merger

- During **inspiral**: weak gravitational fields $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$

- **Quantum** field theory formalism for **classical** two-body problem:

WORLDLINE QUANTUM FIELD THEORY

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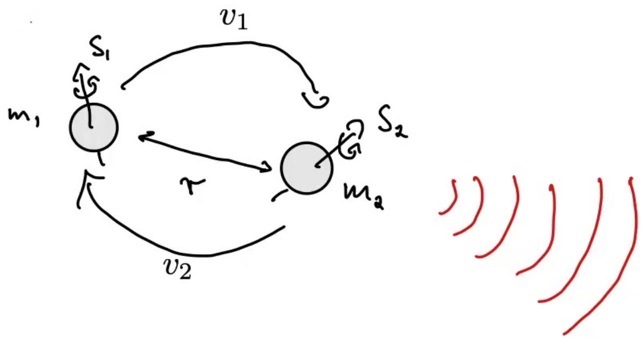
THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



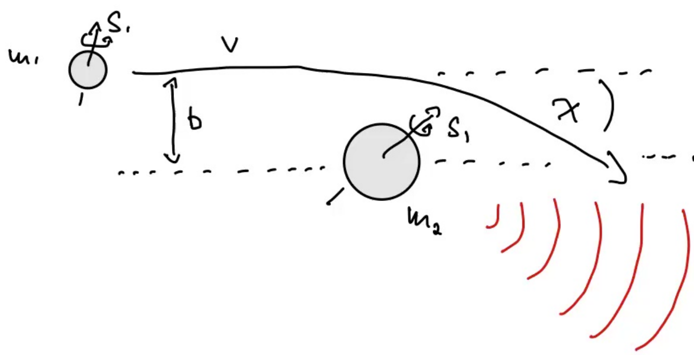
Inspiral of 2 black holes or neutron stars:

Virial-theorem: $\frac{GM}{r} \sim v^2$ $(c = 1)$

post-Newtonian (PN) expansion in G & v^2

Weak field expansion: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$

$\kappa = \sqrt{32\pi G}$ ← Newton's constant



Scattering of 2 black holes or neutron stars:

Weak field (G), but exact in v^2

post-Minkowskian (PM) expansion

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POST-NEWTONIAN VS POST-MINKOWSKIAN EXPANSIONS

Conservative non-spinning 2-body dynamics: Form of potential

PN state-of-the-art ↓ (5PN)

V(G,v,r)	0PN [Newton]	1PN [Einstein,Infeld, Hofmann, 1938]	2PN [Ohta,Okamura,Hiida, Kimura (1974)]	3PN [Damour, Jaranowski,Schaefer; Blanchet,Bohe,Faye]	4PN	(5PN)		Integration complexity
1PM [Westpfahl, 1979]	G/r	$G\,v^2/r$	$G\,v^4/r$	$G\,v^6/r$	$G\,v^8/r$	$G\,v^{10}/r$	...	~ tree-level
2PM [Damour, 2017]		$G^2\,1/r^2$	$G^2\,v^2/r^2$	$G^2\,v^4/r^2$	$G^2\,v^6/r^2$	$G^2\,v^8/r^2$	...	~ 1-loop
3PM			$G^3\,1/r^3$	$G^3\,v^2/r^3$	$G^3\,v^4/r^3$	$G^3\,v^6/r^3$	...	~ 2-loop
4PM				$G^4\,1/r^4$	$G^4\,v^2/r^4$	$G^4\,v^4/r^4$	...	~ 3-loop
(5PM) [Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]					$G^5\,1/r^5$	$G^5\,v^2/r^5$	...	~ 4-loop

4PN/(5PN): [Bini,Damour,Geralico] [Foffa;Porto, Rothstein, Sturani] [Blümlein,Marquardt,Maier]

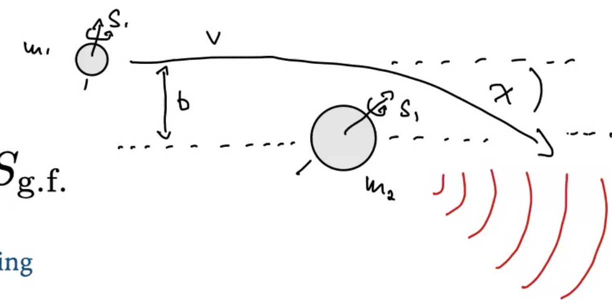
3PM: [Bern,Cheung,Roiban,Shen, Solon,Zeng] [Källin, Liu, Porto] [Di Vecchia, Heissenberg, Russo,Veneziano] [Bjerrum-Bohr,Vanhove,Damgaard] [Brandhuber,Chen,Travaglini,Wen] [Jakobsen,Mogull,JP,Sauer]

4PM: [Bern,Parra-Martinez,Roiban,Ruf,Shen,Solon,Zeng] [Dlapa,Källin,Liu,Porto] [Vanhove,Damgaard,Plante] [Jakobsen,Mogull,JP,Sauer]

(5PM) ← PM state-of-the-art

RELATIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH

Point-particle approximation for BHs (or NSs) $|b| \gg Gm_*$



$$S = -\sum_{i=1}^{2} \int d\tau_i \sqrt{g_{\mu\nu}\dot{x}_i^\mu(\tau_i)\dot{x}_i^\nu(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\mathrm{g.f.}}$$

Point particle approximation — Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu}$$

Einstein's eqs.

$$\ddot{x}_i^\mu + \Gamma^\mu{}_{\nu\rho}\dot{x}_i^\nu\dot{x}_i^\rho = 0$$

Geodesic eqs.

2) Solve iteratively in G

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\sum_{n=0}^{\infty} G^n h_{\mu\nu}^{(n)}(x)$$

emitted radiation

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu\,\tau + \sum_{n=1}^{\infty} G^n z_i^{(n)\,\mu}(\tau)$$

straight line: „in" state — deflections

3) Construct observables

Far field waveform:

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}(\frac{1}{r^2})$$

„Impulse" (change in momentum):

$$\Delta p_i^\mu = m_i\dot{x}_i^\mu\Big|_{\tau=-\infty}^{\tau=+\infty} = m_i\int d\tau\,\ddot{x}_i^\mu(\tau)$$

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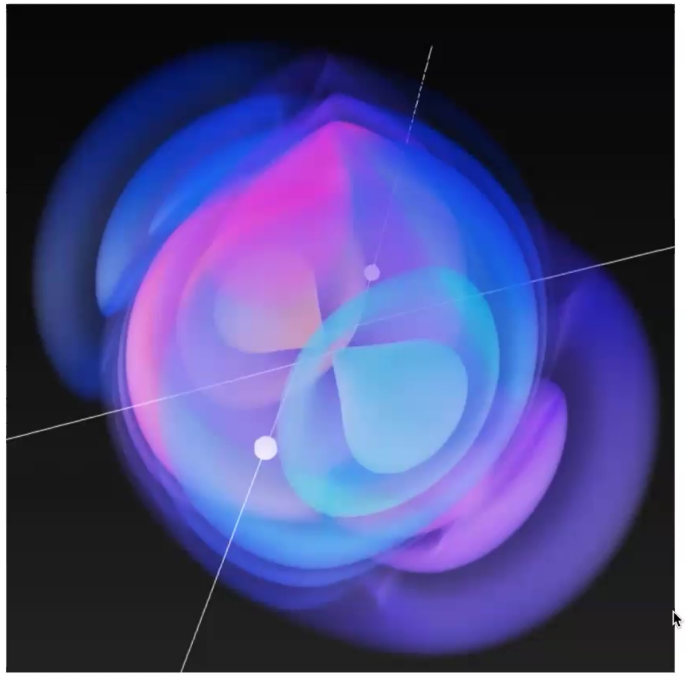
WORLDLINE QUANTUM FIELD THEORY



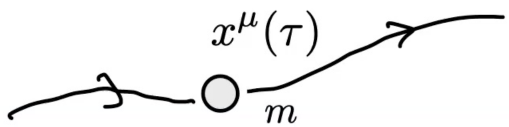
[Jakobsen,Mogull,JP,Sauer]

WORLDLINE QUANTUM FIELD THEORY

Mogull, **JP**, Steinhoff *JHEP* 02 (2021) 048

- Model Black Holes/Neutron Stars as point particles

$$S_{\mathrm{BH/NS}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\, \dot{x}^{\mu}(\tau)\, \dot{x}^{\nu}(\tau) \quad + \text{[spin \& tidal effects]}$$



They interact through Einstein's gravity:

$$S = S_{\mathrm{BH/NS}} + \frac{1}{16\pi\, G}\int d^4x \sqrt{-g}\, R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$

- Path integral quantisation perturbative in Newton's constant G but exact in velocity

$$\langle \mathcal{O} \rangle_{\mathrm{WQFT}} = \int D[h, z]\, \mathcal{O}\, e^{-\frac{i}{\hbar} S[z,h]} \quad \xrightarrow{\hbar \to 0}$$

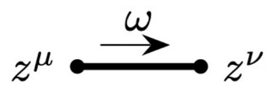
Tree-level one-point functions $\langle h_{\mu\nu} \rangle$ and $\langle z^{\mu} \rangle$ solve classical equations of motion

$\Rightarrow$ Advanced quantum field theory technology for classical gravitational wave physics

WORLDLINE QUANTUM FIELD THEORY: PROPAGATORS

$$S_{\mathrm{WQFT}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot{x}^{\mu}(\tau)\,\dot{x}^{\nu}(\tau) + \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$
- Worldline propagators:



$$\langle z^{\mu}(\omega) z^{\nu}(-\omega)\rangle = -\frac{i}{m}\frac{\eta_{\mu\nu}}{(\omega + i0)^2}$$

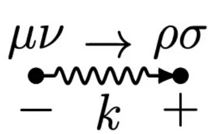
- Perturbative (quantum) gravity (in the bulk):

$$\sqrt{-g}\,R(g) = -\frac{1}{2}h_{\mu\nu}(P^{-1})^{\mu\nu;\rho\sigma}\Box h_{\rho\sigma} + \sqrt{G}[\partial^2 h^3] + \sqrt{G}^2[\partial^2 h^4] + \sqrt{G}^3[\partial^2 h^5] + \dots$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

⇒ Graviton propagator:



$$= i\frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i0)^2 - \mathbf{k}^2}$$

N.B. need to take retarded propagator (in-in formalism)

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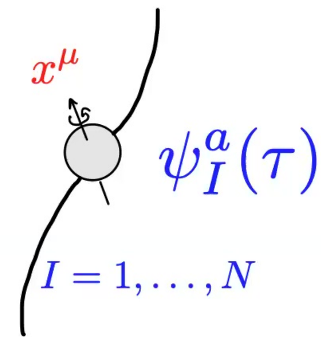
PUTTING SPIN ON THE WORLD-LINE

[Jakobsen,Mogull,JP,Steinhoff]



- Hidden **supersymmetry** of spinning-black holes!

  Add anti-commuting fields ψ^a: Captures (Spin)^N interactions

- Spin-orbit & spin-spin interactions via $N = 2$ superparticle action

$$S_{\mathrm{BH/NS}} = -m \int d\tau \left[\tfrac{1}{2} g_{\mu\nu} \dot{x}^\mu \, \dot{x}^\nu + i\bar{\psi} D_\tau \psi + \tfrac{1}{2} R_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d + C_E R_{a\mu b\nu} \dot{x}^\mu \dot{x}^\nu \bar{\psi}^a \psi^b \, \bar{\psi} \cdot \psi \right]$$

spin degrees of freedom — neutron star term

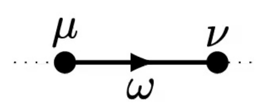
Scattering scenario:

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \, \tau + z_i^\mu(\tau)$$
$$\psi_i^a(\tau) = \Psi_i^a + \psi_i'^a(\tau)$$

Spin tensor of BHs/NSs

$$S_i^{ab} = -2im \bar{\psi}_i^{[a} \psi_i^{b]}$$

Quantize $z_i^\mu, \psi_i'^a, \bar{\psi}_i'^a$



$$\langle \psi^a(\omega) \bar{\psi}^b(-\omega) \rangle = \frac{-i\eta^{ab}}{m(\omega + i0)}$$

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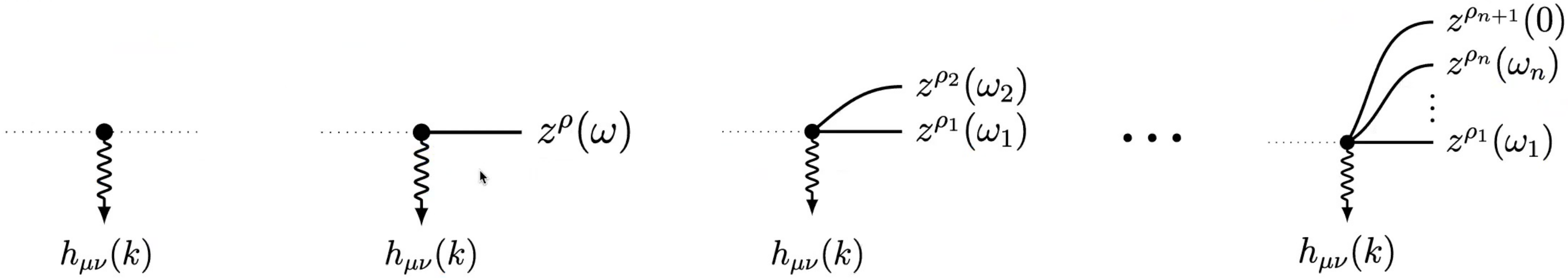
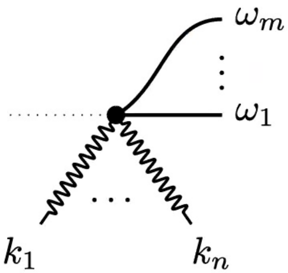
WORLD LINE VERTICES

$$-\frac{m\,G}{2}\int d\tau\, h_{\mu\nu}[x(\tau)]\,\dot{x}^{\mu}(\tau)\,\dot{x}^{\nu}(\tau)$$



- Worldline vertices with spin & tidal effects: n-gravitons & m world-line fluctuations

$V_{n|m} =$



$$= m\sqrt{G}^{n}\, e^{ib\cdot\sum_j k_j}\,\delta\!\left(v\cdot\sum_{j=1}^{n}k_j+\sum_{i=1}^{m}\omega_i\right)\times\left(\begin{array}{c}\text{polynomial in }\omega_i, k_j\\ \text{of degree }2n+m\\ \text{depending on }v^{\mu},\ S^{\mu\nu}\end{array}\right)$$

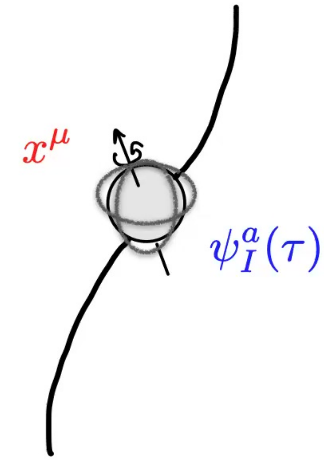
+ C_E, c_{E^2}, c_{B^2} for neutron stars

Energy conservation on worldline

TIDAL INTERACTIONS

- First layer of tidal & finite size effects:

$$S_{\text{tidal}} = m \int d\tau \left[c_{E^2} E_{\mu\nu} E^{\mu\nu} + c_{B^2} B_{\mu\nu} B^{\mu\nu} \right]$$



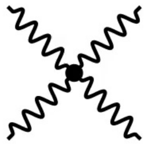
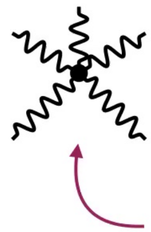
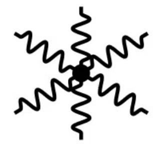
Electric and magnetic curvature:

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta \qquad B_{\mu\nu} := R^*_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta$$

Wilson coefficients (or „Love numbers“): c_{E^2} & c_{B^2} (vanish for black holes)

BULK GRAVITON VERTICES

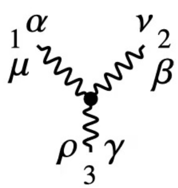
$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g)$$

 $\sim \sqrt{G}\,k^2$  $\sim \sqrt{G}^2\,k^2,$  $\sim \sqrt{G}^3\,k^2$  $\sim \sqrt{G}^4\,k^2, \ldots$

Four-momentum conservation in bulk $\delta^4(\sum p)$

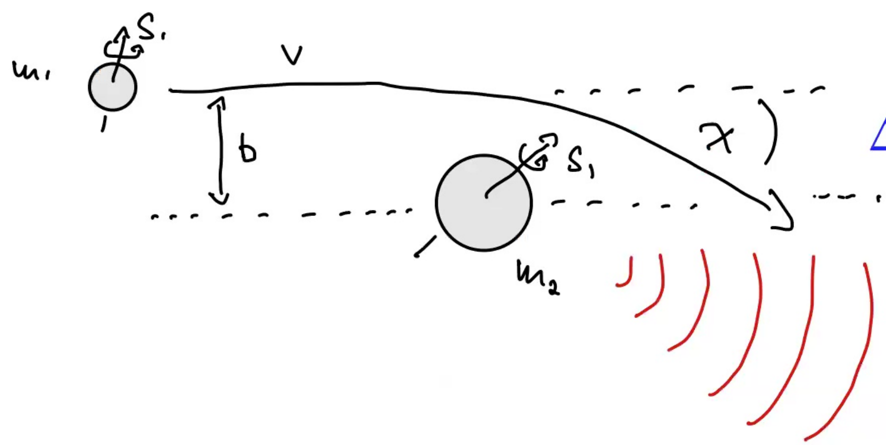
Highly involved structures! Emerge from Einstein-Hilbert action.

- E.g. three-point vertices:



$$= i\sqrt{G}\,\mathrm{sym}[$$
$$-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\rho\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\rho\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\rho\gamma})$$
$$+ P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\rho}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\rho}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\rho\gamma})$$
$$+ P_3(k_{1\rho}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\rho}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) - 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\rho})$$
$$+ 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\rho}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\nu\alpha}\eta_{\rho\beta}\eta_{\mu\gamma})],$$

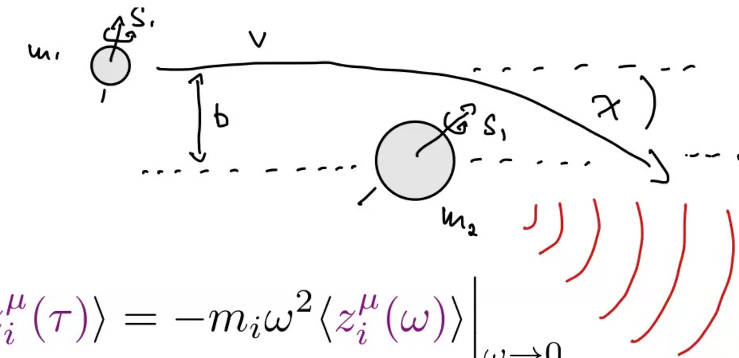
WQFT OBSERVABLES



$$\Delta p_1^\mu = \sum_{n=1}^{\infty} G^n \, \Delta p_1^{(n)\,\mu}$$

$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

OBSERVABLES OF WQFT: ONE POINT FUNCTIONS



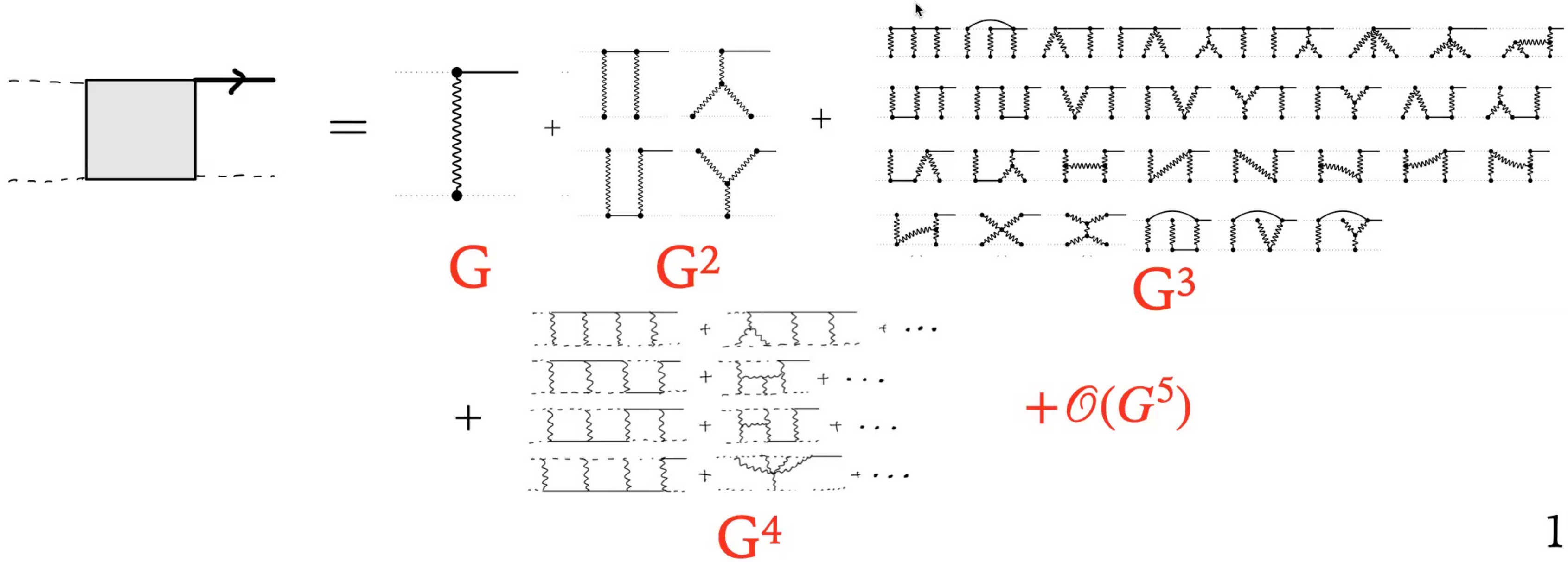
1) Impulse (change of momentum)

$$\Delta p_i^\mu = m_i \langle \dot{x}_i^\mu \rangle \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau \langle \ddot{x}_i^\mu(\tau) \rangle = m_i \int d\tau \frac{d^2}{d\tau^2} \langle z_i^\mu(\tau) \rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega \to 0}$$

Fourier trans. ↑

Needs sum of all graphs with outgoing z-line:

$$\langle \Delta p_1^\mu \rangle =$$



$+\mathcal{O}(G^5)$

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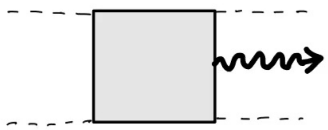
OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

2) **Emitted Waveform** (Gravitational Bremsstrahlung)

$\langle h_{\mu\nu} \rangle =$



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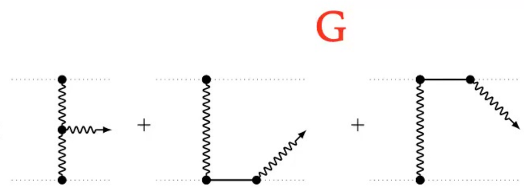
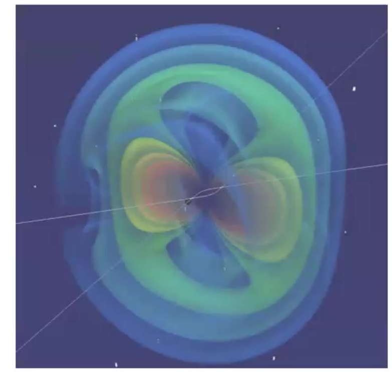


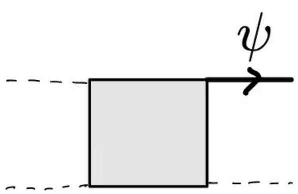
$+ \dots$



3) **Spin-kick:** Change in spin-tensor from $\langle \Delta\psi_1^\mu \rangle$

$$S_1^{\mu\nu} = -2i\bar{\psi}_1^{[\mu}\psi_1^{\nu]} \quad \Longrightarrow \quad \Delta S_1^{\mu\nu} = -2i(\Delta\bar{\psi}_1^{[\mu}\psi_1^{\nu]} + \bar{\psi}_1^{[\mu}\Delta\psi_1^{\nu]} + \Delta\bar{\psi}_1^{[\mu}\Delta\psi_1^{\nu]})$$

$\langle \Delta\psi_1^\mu \rangle =$



Spin-vector: $S_1^\mu = \frac{1}{2}\epsilon^\mu{}_{\nu\rho\sigma}p_1^\nu S_1^{\rho\sigma}$

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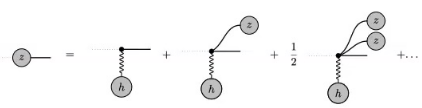
HIGH PRECISION WQFT COMPUTATIONS: WORKFLOW

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Generate graphs & Insert Feynman rules

Berends-Giele recursion



(5PM: 6g vertex)

↓

Reduction to scalar integral family

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

↓ $i\epsilon$

Integration by parts: Master Integrals

Kira

Reduced symmetries due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

→

Solve masters by Diff. Eq. technique

Single scale integrals

$$x = \gamma - \sqrt{\gamma^2 - 1}$$

$$\gamma = v_1 \cdot v_2$$

↑

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

↑ $i\epsilon$

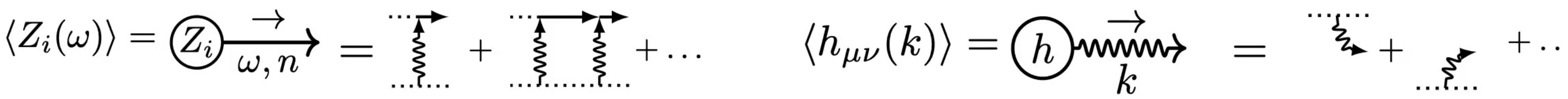
$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

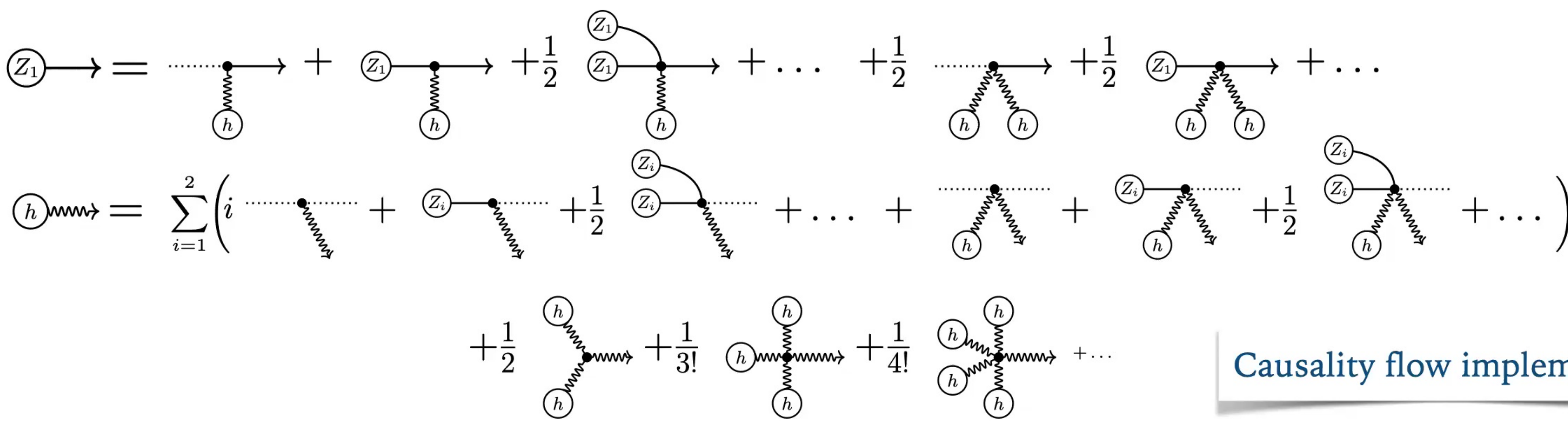
Find same set of Master integrals for spin and tidal effects

Generate graphs &
Insert Feynman rules

[Jakobsen,Mogull,JP,Sauer]



- All graphs are **trees**: Use **Berends-Giele recursion** to efficiently generate all contributions.



Causality flow implemented

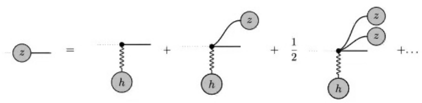
- Insertion of Feynman rules with **FORM** - fast on tensor algebra & convenient for anti-commuting variables.

HIGH PRECISION WQFT COMPUTATIONS: WORKFLOW

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Generate graphs & Insert Feynman rules

Berends-Giele recursion



(5PM: 6g vertex)

↓

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↗ $i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

Reduction to scalar integral family

STRUCTURE OF WQFT INTEGRALS: IMPULSE & SPIN KICK

[Jakobsen,Mogull,JP,Sauer]

- Order **N-PM** : Single scale **(N-1)-loop** integrals:

$$I_{\mathrm{nPM}} = \int_q e^{-q\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{l_1,l_2\ldots l_{n-1}} \frac{\mathrm{num}[l_i]}{D_1^{a_1}\ldots D_j^{a_j}}\delta(l_1\cdot v_*)\delta(l_2\cdot v_*)\ldots\delta(l_{n-1}\cdot v_*)$$

Propagators $D_i(l_i, q, v_*)$ are linear or quadratic $v_* \in \{v_1, v_2\}$ (*'s determined by self-force order)

$$D_i(l_i, q, v_*) = \begin{cases} l_i\cdot v_* + i0 \\ (l_i^0 + i0)^2 - \vec{l}_i^2 \end{cases}$$

← For conservative sector use Feynman

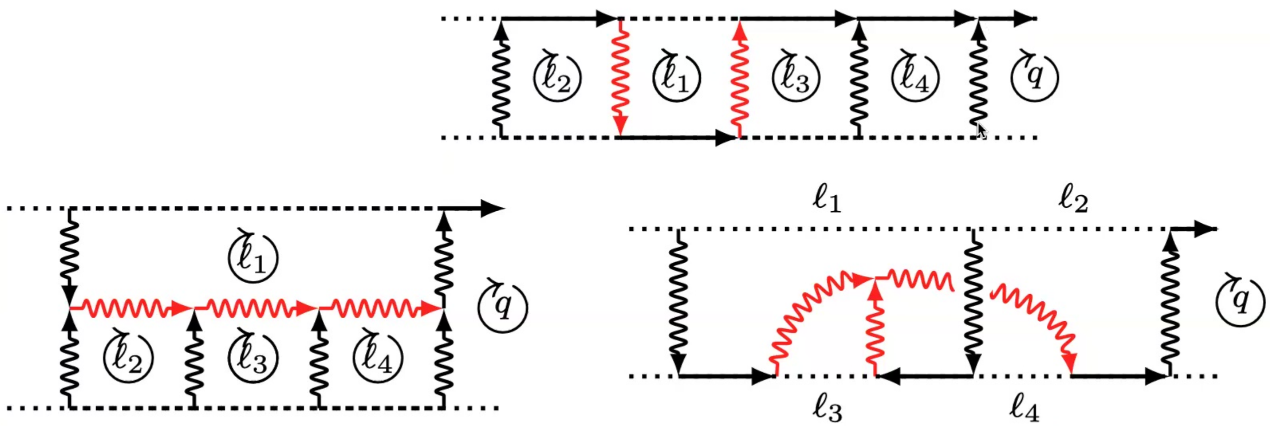
Depend on single dimensionful scale q —> left with **single parameter** integral $\gamma = v_1 \cdot v_2$

- Passarino-Veltman tensor reduction yields scalar integrals (spin-less case)

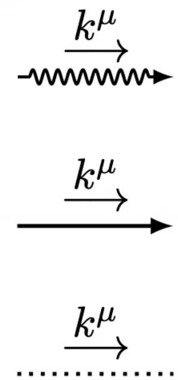
$$\ell_i^\mu \to \ell_i\cdot v_1\,\hat{v}_1^\mu + \ell_i\cdot v_2\,\hat{v}_2^\mu + \frac{\ell_i\cdot q}{q^2}q^\mu \qquad v_i\cdot\hat{v}_j = \delta_{ij}$$

WQFT LOOP INTEGRALS: IMPULSE & SPIN KICK

Four loop example diagrams:



(Memory Effect)



$$= \frac{1}{(k_0 + i\epsilon)^2 - \boldsymbol{k}^2}$$

$$= \frac{1}{k \cdot v_j + i\epsilon}$$

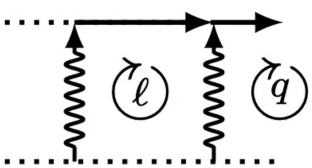
$$= \delta(k \cdot v_j)$$

External Kinematics:

$$v_1 \cdot v_1 = v_2 \cdot v_2 = 1$$

$$v_1 \cdot q = v_2 \cdot q = 0$$

$$v_1 \cdot v_2 = \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$



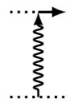
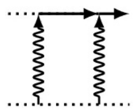
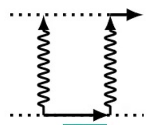
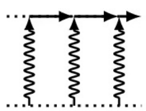
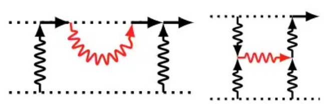
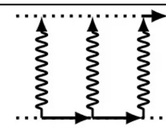
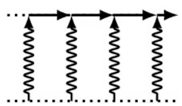
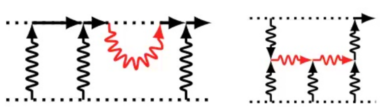
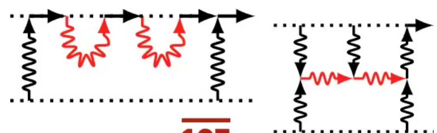
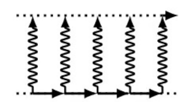
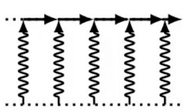
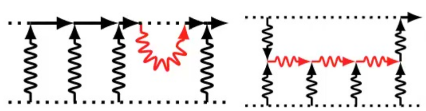
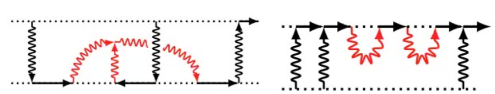
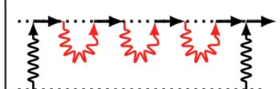
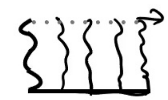
$$= \int d^D\ell \frac{\delta(\ell \cdot v_2)}{\ell^2 (q - \ell)^2 \ell \cdot v_1}$$

$$= \int d^D\ell \frac{\delta(\ell \cdot v_2)}{((\ell_0 + i\epsilon)^2 - \boldsymbol{\ell}^2)((q_0 - \ell_0 + i\epsilon) - (\boldsymbol{q} - \boldsymbol{\ell})^2)(\ell \cdot v_1 + i\epsilon)}$$

(Regulator: Dimensional Regularization)

PM EXPANSION: LOOP AND SELF-FORCE (SF) ORDERS

$\nu = m_1/m_2$

	$m_1 m_2^{L+1}$	$m_1^2 m_2^{L}$	$m_1^3 m_2^{L-1}$	$m_1^4 m_2^{L-2}$	$m_1^5 m_2^{L-3}$
1PM tree-level	 0SF				
2PM 1-loop	 0SF	 $\overline{\text{0SF}}$			
3PM 2-loop	 0SF	 1SF	 $\overline{\text{0SF}}$		
4PM 3-loop	 0SF	 1SF	 $\overline{\text{1SF}}$	 $\overline{\text{0SF}}$	
5PM 4-loop	 0SF	 1SF	 2SF	 $\overline{\text{1SF}}$	 $\overline{\text{0SF}}$

SCATTERING OF SPINNING BLACK-HOLES UP TO 5PM ACCURACY

0SF	S^0	S^1	S^2	S^3	S^4
tree level	1PM	2PM	3PM	4PM	5PM
1 loop	2PM	3PM	4PM	5PM	6PM
2 loops	3PM	4PM	5PM	6PM	7PM
3 loops	4PM	5PM	6PM	7PM	8PM
4 loops	5PM	6PM	7PM	8PM	9PM

$\ldots$

Physical PM counting for Kerr black holes!

$$\ldots \quad a_i^\mu = G m_i \chi_i^\mu \,, \qquad |\chi_i| < 1$$

Vines [1709:06016]
Chen, Chung, Huang, Kim [2111.13639]
Damgaard, Hoogeveen, Luna, Vines [2208.11028]
...

1SF	S^0	S^1	S^2
2 loops	3PM	4PM	5PM
3 loops	4PM	5PM	6PM
4 loops	5PM	6PM	7PM

$\ldots$

Jakobsen, Mogull [2201.07778] Cons Cons
Jakobsen, Mogull [2210:06451] Diss Diss

Jakobsen, Mogull, JP, Sauer, Xu [2306:01714]: Cons
Jakobsen, Mogull,JP, Sauer [2308.11514]: Diss
Driesse, Jakobsen, Mogull, JP, Sauer, Usovitsch [2403.07781]: Cons
Preparations for Diss : Klemm, Nega, Sauer, JP: [2401.07899]

2SF	S^0
4 loops	5PM

$\ldots$

UNKNOWN

$\ldots$

Work on spinless 3 loops: Bern, Parra-Martinez, Robin, Ruf, Shen [2112:10750]
Dlapa, Kälin, Neef, Porto [2210:05541]
Damgaard, Hansen, Planté, Vanhove [2307.04746]

* Cons=Conservative, Diss=Dissipative

PLANARIZATION AT FIRST SELF-FORCE ORDER

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

- **All loop** 1SF integral family:

$$I_{\{n_A\}} = \int_{\ell_1 \dots \ell_L} \frac{\delta(\ell_1 \cdot v_1) \prod_{i=2}^{L} \delta(\ell_i \cdot v_2)}{\prod_A D_A^{n_A}}$$

$$A \in \{iv, ij, iq, i0\}$$

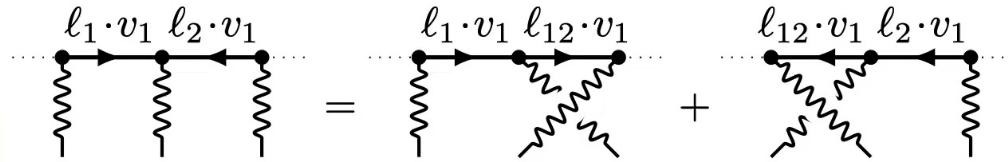
$$D_{iv} = \begin{cases} \ell_1 \cdot v_2 & i = 1 \\ \ell_i \cdot v_1 & i = 2, 3, \dots, L \end{cases}$$

$$D_{ij} = (\ell_i - \ell_j)^2$$

$$D_{iq} = (\ell_i - q)^2$$

$$D_{i0} = \ell_i^2$$

- Reachable via **planarization:**

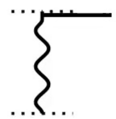


$$\frac{1}{(\ell_1 \cdot v_1 + i0^+)(\ell_2 \cdot v_1 + i0^+)} = \frac{1}{(\ell_1 \cdot v_1 + i0^+)(\ell_{12} \cdot v_1 + i0^+)} + \frac{1}{(\ell_2 \cdot v_1 + i0^+)(\ell_{12} \cdot v_1 + i0^+)}$$

At L-loop order have L linear and L(L+3)/2 quadratic propagators & L delta-functions.

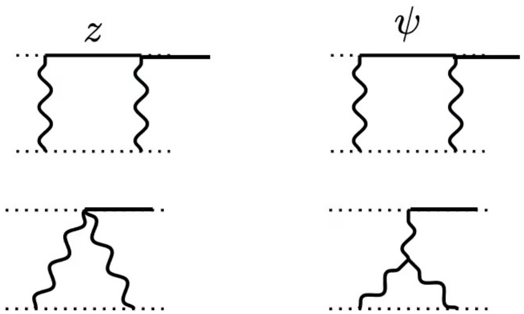
1PM AND 2PM: IMPULSE

- **1PM: Trivial - pure Fourier transform**

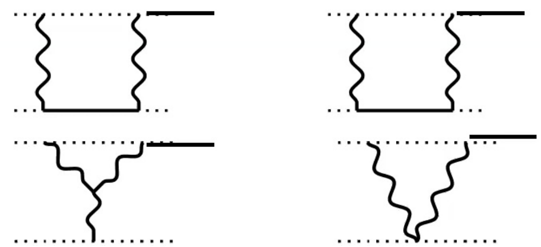


$\sim G m_1 m_2$

- **2PM: 1-loop:** 8 graphs, 3 propagators



$\sim G^2 m_1 m_2^2$



$\sim G^2 m_1^2 m_2$

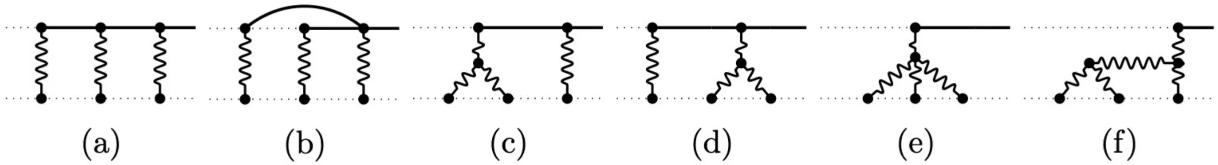
$$D_1 = l \cdot v_1 \pm i\epsilon \ ,$$
$$D_2 = l^2 \ ,$$
$$D_3 = (l+q)^2 \ .$$

$$\int_l \frac{\text{num}[l]}{D_1 D_2 D_3} \delta(l \cdot v_*)$$

3PM: 2 LOOPS

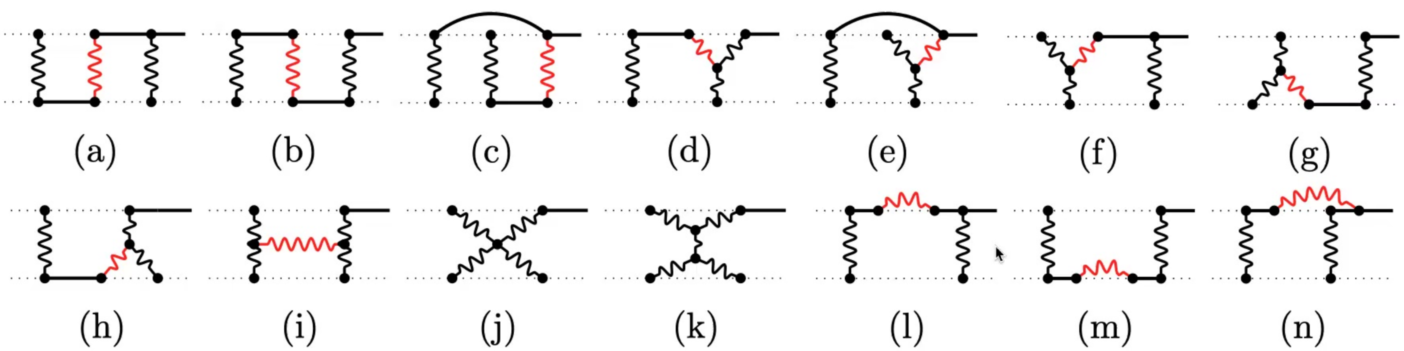
[Jakobsen,Mogull]

- 6 test body diagrams (geodesic motion in Schwarzschild background):



$\sim G^3 m_1 m_2^3$ **0SF**

- 14 comparable mass diagrams (i0 prescription relevant for red propagators):



$\sim G^3 m_1^2 m_2^2$ **1SF**

1 active graviton propagator

- Integral family: 7 propagators + 2 delta functions

$$\int_{l_1 l_2} \frac{\delta(l_1 \cdot v_{\bar{\imath}_1})\delta(l_2 \cdot v_{\bar{\imath}_2})}{\prod_{j=1}^{7} D_j^{n_j}}$$

$$D_1 = l_1 \cdot v_{i_1} + \sigma_1 i0 \,, \qquad D_2 = l_2 \cdot v_{i_2} + \sigma_2 i0 \,,$$

$$D_4 = l_1^2 \,, \qquad D_5 = l_2^2 \,,$$

$$D_6 = (l_1 + q)^2 \,, \qquad D_7 = (l_2 + q)^2 \,.$$

$$D_3 = (k^0 + \sigma_3 i0)^2 - \boldsymbol{k}^2$$

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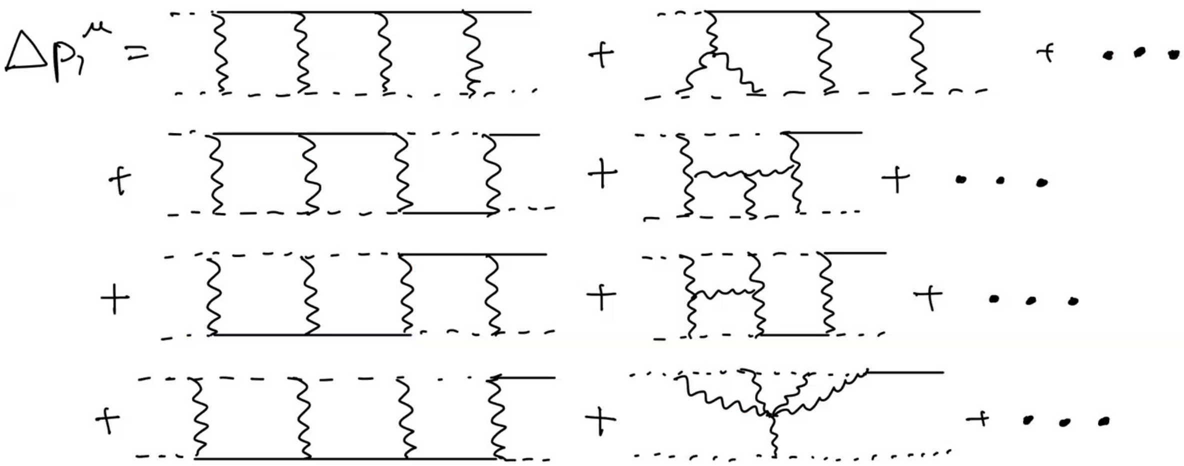
4PM MOMENTUM IMPULSE

201 non-spinning diagrams, 529 linear in spin diagrams

[Jakobsen,Mogull,JP,Sauer]



$m_1 m_2^4$ 0SF

$m_1^2 m_2^3$ 1SF

$m_1^3 m_2^2$ $\overline{1}$SF

$m_1^4 m_2$ $\overline{0}$SF

- Integral family: 3 linear and 9 quadratic propagators

$$\int_q e^{-iq\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{\ell_1,\ell_2,\ell_3}\frac{\text{num}[\ell_i]}{D_1\cdots D_{12}}\delta(\ell_1\cdot v_{i_1})\delta(\ell_2\cdot v_{i_2})\delta(\ell_3\cdot v_{i_3})$$

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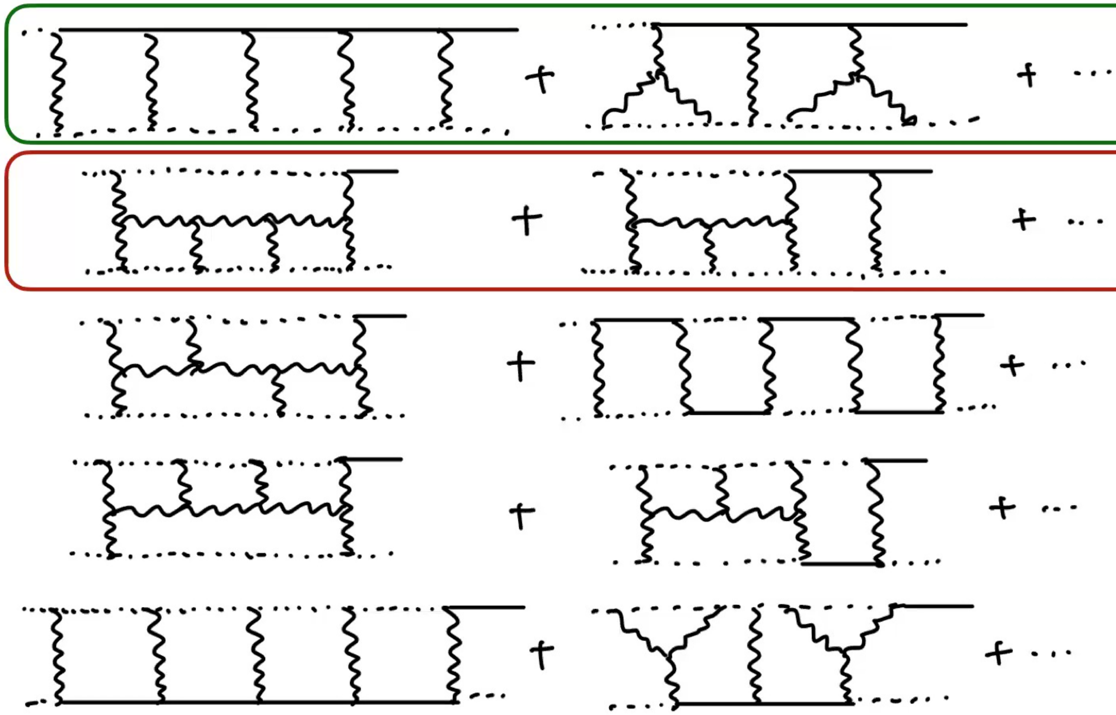
5PM MOMENTUM IMPULSE

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

- Focus on conservative sector: $\Delta p_1 = -\Delta p_2$ 0SF: 63 diagrams 1SF: 363 diagrams

$\Delta p_1^{(5)\mu} =$

Our calculation:



$m_1 m_2^5$ 0SF

$m_1^2 m_2^4$ 1SF

$m_1^3 m_2^3$ 2SF

$m_1^4 m_2^2$ $\overline{1}$SF

$m_1^5 m_2^1$ $\overline{0}$SF

- Integral family: 4 linear and 14 quadratic propagators + 4 delta functions

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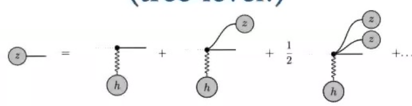
HIGH PRECISION WQFT COMPUTATIONS: WORKFLOW

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Generate graphs

Berends-Giele recursion
(tree-level!)



↓

Insert Feynman rules

Form script
(5PM: 6g vertex)

↓

Reduction to
scalar integral family

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

↓ $i\epsilon$

Integration by parts:
Master Integrals

Kira
Reduced symmetries
due to retarded prop.
$l \to -l \quad \epsilon \to -\epsilon$

→

Solve masters by Diff.
Eq. technique

Single scale
integrals
$x = \gamma - \sqrt{\gamma^2 - 1}$

↑

Fix boundary conditions
in static limit (PN) $\gamma \to 1$

Method of regions
(potential & radiation
gravitons)

↑ $i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

31

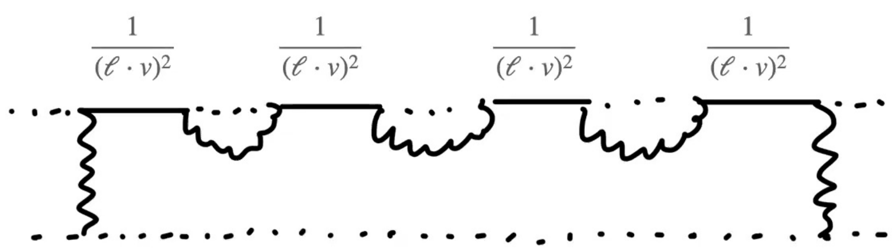
Integration by parts: Master Integrals

5PM-1SF: INTEGRATION-BY-PARTS (IBP) IDENTITIES

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Reduce integral family to basis of **Master Integrals:**

- Reduced 10^6 conservative scalar integrals to basis of 470 Master Integrals
- Used **KIRA 3.0** (pre-release) in Finite-Field mode (300k core hours on HPC clusters)
- Reconstructed rational functions from finite-field samples with **FireFly**
- Key improvement: generate fewer equations (Laporta algorithm) by tightly controlling powers of propagators



- Maximum of 8 allowed powers on worldline propagators (7 dots)
- Maximum of 9 scalar products overall

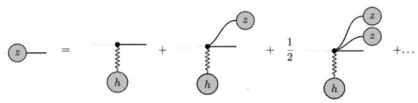
Johann Usovitsch

HIGH PRECISION WQFT COMPUTATIONS: WORKFLOW

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Generate graphs & Insert Feynman rules

Berends-Giele recursion



(5PM: 6g vertex)

↓

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$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \ldots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

↓ $i\epsilon$

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Reduced symmetries due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

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Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

↑

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

↗ $i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

33

Solve masters by Diff. Eq. technique

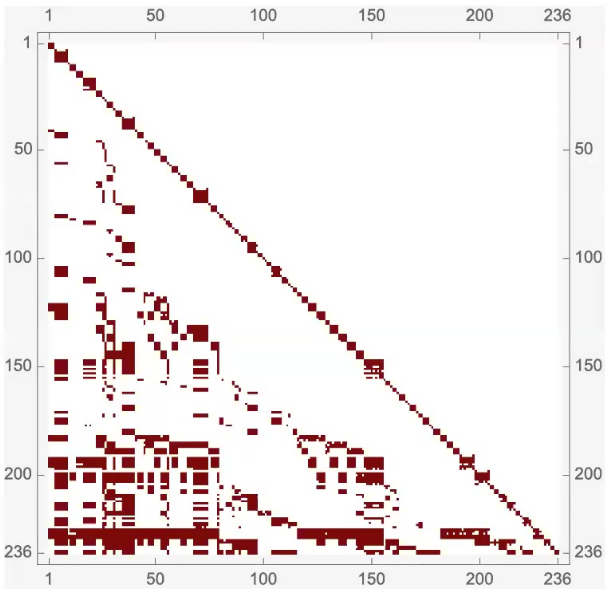
DIFFERENTIAL EQS @ 5PM-1SF

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

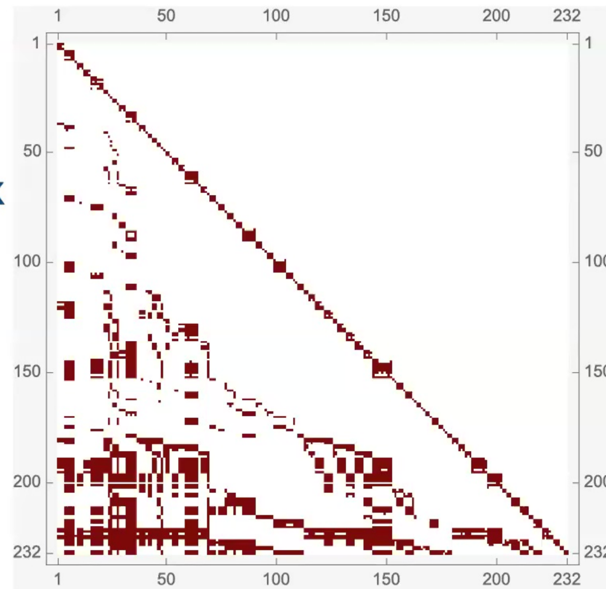
- After IBP reduction: Basis of Master Integrals $\vec{I}(x,\epsilon)$ for 5PM-1SF planar family

$$\frac{d}{dx}\vec{I} = M(x,\epsilon)\,\vec{I} \qquad x = \gamma - \sqrt{\gamma^2-1} \qquad \gamma = \frac{1}{2}(x+x^{-1})$$

- Total of 236 even & 234 odd Master Integrals (MIs)



Heat plots of matrix $M(x,\epsilon)$



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Solve masters by Diff. Eq. technique

DIFFERENTIAL EQS @ 5PM-1SF

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

- **Canonical form** of Diff. eq. through rotation to new basis:

[Gehrmann Remiddi][Henn]

$$\frac{d}{dx}\vec{I}(x,\epsilon) = M(x,\epsilon)\,\vec{I}(x,\epsilon) \xrightarrow{\vec{I}(x,\epsilon) = T(\epsilon,x)\,\vec{I}'(x,\epsilon)} \frac{d}{dx}\vec{I}'(x,\epsilon) = \epsilon\,A(x)\,\vec{I}'(x,\epsilon)$$

Straightforward solution for masters: $\vec{I}'(x,\epsilon) = P\exp[\epsilon\int_C dA(x)]\,\vec{I}'_0(x_0,\epsilon)$

boundary integrals $x_0 = 1 \Leftrightarrow v = 0$

- Tools to find **canonical** form:

  Good choice of Master Integrals: Simple pole structure of $A(x)$

  Most useful algorithms for finding $T(x,\epsilon)$: **CANONICA, INITIAL, FiniteFlow**

Resulting **function basis** up to 5PM (dissipative): Multiple polylogarithms, Complete Elliptic Integrals, Calabi-Yau periods (K3 & new CY 3-fold)

Benjamin Sauer

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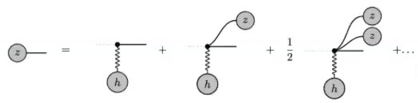
HIGH PRECISION WQFT COMPUTATIONS: WORKFLOW

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Generate graphs & Insert Feynman rules

Berends-Giele recursion



(5PM: 6g vertex)

↓

Reduction to scalar integral family

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

↓ $i\epsilon$

Integration by parts: Master Integrals

Kira

Reduced symmetries due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

→

Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

↑

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

↑ $i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

Fix boundary conditions in static limit (PN)

Compute boundary integrals $\vec{I}_0'(x,\epsilon)$ by **Method of Regions** [Beneke,Smirnov] in static limit $v \to 0$

- Regions determined by scalings graviton momenta:

$$\ell_i^{\mathrm{P}} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, 1)\,, \qquad \ell_i^{\mathrm{R}} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, v)\,.$$

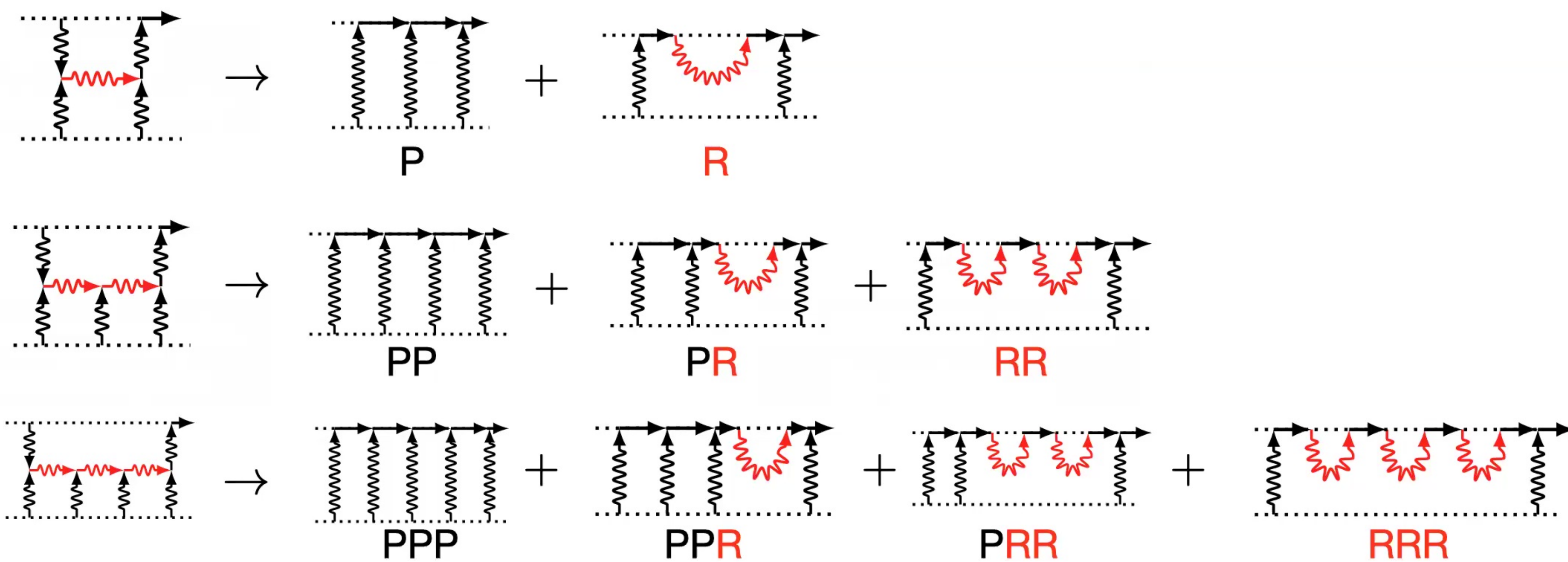
potential (P) radiation (R)

- **Active:** Propagator can go on-shell, $i\epsilon$ prescription matters:

2 loops

3 loops

4 loops



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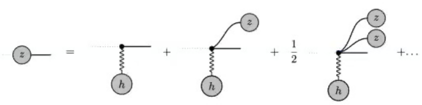
HIGH PRECISION WQFT COMPUTATIONS: WORKFLOW

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

Generate graphs & Insert Feynman rules

Berends-Giele recursion



(5PM: 6g vertex)

↓

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$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

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Single scale integrals
$x = \gamma - \sqrt{\gamma^2 - 1}$

↑

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

↗ $i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$
ΔS_i^μ

Find same set of Master integrals for spin and tidal effects

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RESULTS: SCATTERING ANGLE@ 1PM, 2PM & 3PM PRECISION:

Δp_i^μ

- Scattering angle in center of mass frame: $p_1^\mu = (E_1, \boldsymbol{p})$ $p_2^\mu = (E_2, -\boldsymbol{p})$

$$\cos\theta = \frac{\boldsymbol{p}\cdot(\boldsymbol{p}+\Delta\boldsymbol{p})}{|\boldsymbol{p}||\boldsymbol{p}+\Delta\boldsymbol{p}|}$$

Including recoil: $\Delta\boldsymbol{p} = \Delta\boldsymbol{p}_1 + E_1 \boldsymbol{P}_{\rm rad}/E$

$$\frac{\theta}{\Gamma} = \underbrace{\frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1}}_{\text{1PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^2 \frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)}}_{\text{2PM}}$$

$$\gamma = v_1 \cdot v_2 \qquad \Gamma = E/M = \sqrt{1+2\nu(\gamma-1)} \qquad \nu = \frac{m_1 m_2}{M^2}$$

$$+\left(\frac{GM}{|b|}\right)^3\left(2\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4-12\gamma^2-3)}{(\gamma^2-1)}\frac{\operatorname{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)$$

3PM conservative

[Bern,Cheung,Roiban,Shen, Solon,Zeng] [Kälin, Liu, Porto] [Di Vecchia, Heissenberg, Russo,Veneziano] [Bjerrum-Bohr,Vanhove,Damgaard] [Brandhuber,Chen,Travaglini,Wen] [Jakobsen,Mogull,JP,Sauer]

$$+\left(\frac{GM}{|b|}\right)^3 \frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3}+\frac{1}{v^2}+\frac{(3v^2-1)}{v^3}\operatorname{arccosh}\gamma\right)$$

1SF piece

3PM radiation-reaction

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RESULT: SCATTERING ANGLE@ 4PM PRECISION: CONSERVATIVE + DISSIPATIVE

$$\frac{\theta}{\Gamma} = \quad \cdots \quad + \left(\frac{GM}{|b|}\right)^4 \Bigg(\frac{105\pi \left(33\gamma^4 - 18\gamma^2 + 1\right)}{64\left(\gamma^2-1\right)^2}$$

4PM conservative

[Bern,Roiban,Shen,Parra-Martinez,Ruf,Zeng..]
[Dlapa, Kälin, Liu, Neef,Porto]

$$+\nu\Bigg[-\frac{3h_3 \mathrm{K}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{32\left(\gamma^2-1\right)^2} + \frac{3h_4 \mathrm{E}\left(\frac{\gamma-1}{\gamma+1}\right)\mathrm{K}\left(\frac{\gamma-1}{\gamma+1}\right)}{32\left(\gamma^2-1\right)^2} + \frac{\pi^2 h_5}{16(1-\gamma^2)} + \frac{3h_{27}\log^2\left(\frac{\gamma+1}{2}\right)}{4(1-\gamma^2)} - \frac{h_6 \log\left(\frac{\gamma-1}{2}\right)}{32\left(\gamma^2-1\right)^2} + \frac{3h_{15}\log\left(\frac{\gamma-1}{2}\right)\log\left(\frac{\gamma+1}{2}\right)}{16\left(\gamma^2-1\right)}$$

$$-\frac{h_{22}\log\left(\frac{\gamma+1}{2}\right)}{32\left(\gamma^2-1\right)^2} - \frac{h_{23}\log(\gamma)}{4\left(\gamma^2-1\right)^2} + \frac{3h_{26}\,\mathrm{arccosh}^2(\gamma)}{64\left(\gamma^2-1\right)^4} + \frac{h_{24}\,\mathrm{arccosh}(\gamma)}{32\left(\gamma^2-1\right)^{7/2}} - \frac{3h_{16}\log\left(\frac{\gamma-1}{2}\right)\mathrm{arccosh}(\gamma)}{32\left(\gamma^2-1\right)^{5/2}} - \frac{3h_{28}\log\left(\frac{\gamma+1}{2}\right)\mathrm{arccosh}(\gamma)}{32\left(\gamma^2-1\right)^{5/2}}$$

$$-\frac{h_{62}}{384\gamma^7\left(\gamma^2-1\right)^3} - \frac{21h_2\mathrm{E}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{64(\gamma-1)^2(\gamma+1)} - \frac{3\sqrt{\gamma^2-1}\,h_7\mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{2(\gamma-1)^2(\gamma+1)^3} + \frac{h_{29}\mathrm{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)}{8(1-\gamma^2)} + \left(\frac{3\sqrt{\gamma^2-1}\,h_7}{8(\gamma-1)^2(\gamma+1)^3} + \frac{3h_{30}}{16-16\gamma^2}\right)\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)\Bigg]$$

$$+\left(\frac{GM}{|b|}\right)^4\left(\cdots+\nu\left[\cdots\right]\right)$$

4PM radiation-reaction

[Dlapa, Kälin, Liu, Neef,Porto]
[Damgaard,Plante,Vanhove]
[Jakobsen,Mogull,JP,Sauer]

$$h_3 = 1200\gamma^2 + 2095\gamma + 834$$
$$h_4 = 1200\gamma^3 + 2660\gamma^2 + 2929\gamma + 1183$$
$$h_5 = -25\gamma^6 + 30\gamma^4 + 60\gamma^3 - 129\gamma^2 + 76\gamma - 12$$
$$h_6 = 210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151$$
$$h_7 = -\gamma\left(2\gamma^2-3\right)\left(15\gamma^2-15\gamma+4\right)$$
$$\vdots$$

- Function space: Logs, Dilogs & Elliptic Integrals of 1st & 2nd kind: E(x) & K(x)

OUR RESULT: SCATTERING ANGLE@ 5PM-1SF PRECISION: CONSERVATIVE

[Driesse,Jakobsen,Mogull,JP,Sauer, Usovitsch]

$$\frac{\theta}{\Gamma} = \quad \dots + \left(\frac{GM}{|b|}\right)^5 \Bigg(\frac{2\left(1792\gamma^{10} - 5760\gamma^8 + 6720\gamma^6 - 3360\gamma^4 + 630\gamma^2 - 21\right)}{5\left(\gamma^2-1\right)^5} + \nu \sum_{k=1}^{31} c_k(\gamma)\, f_k(\gamma)$$

$$+\nu^2\theta_{5,2}$$

0SF

1SF

- 5PM-1SF function space: Multiple polylogarithms up to weight 3!

$$G(a_1,\dots,a_n;y) = \int_0^y \frac{\mathrm{d}t}{t-a_1} G(a_2,\dots,a_n;t) \qquad y = 1-x = 1-\gamma+\sqrt{\gamma^2-1} \in \{0,1,2,1\pm i\}$$

- $c_k(\gamma)$ polynomials:

$$c_1(\gamma) = \frac{1880064\gamma^{19} + 1880064\gamma^{18} + 42654086\gamma^{17} + 20978054\gamma^{16} - 305752626\gamma^{15} - 236079666\gamma^{14} + 597683406\gamma^{13} + 516398286\gamma^{12} - 403178675\gamma^{11}}{7560(\gamma^2-1)^4\gamma^7(\gamma+1)}$$

$$+ \frac{-362536115\gamma^{10} + 77856912\gamma^9 + 70236432\gamma^8 + 16701489\gamma^7 + 16955505\gamma^6 - 536235\gamma^5 - 536235\gamma^4 + 393120\gamma^3 + 393120\gamma^2 + 10395\gamma + 10395}{7560(\gamma^2-1)^4\gamma^7(\gamma+1)}$$

$$c_2(\gamma) = -\frac{651264\gamma^{20} - 7809042\gamma^{18} - 23185512\gamma^{16} + 169295016\gamma^{14} - 315460542\gamma^{12} + 277369170\gamma^{10} - 134264214\gamma^8 + 6510035\gamma^6 - 988015\gamma^4 + 240905\gamma^2 - 18585}{2520\gamma^8\left(\gamma^2-1\right)^{9/2}}$$

$$c_3(\gamma) = \frac{6144\gamma^{16} - 587336\gamma^{14} + 4034092\gamma^{12} - 417302\gamma^{10} - 5560073\gamma^8 - 142640\gamma^6 + 35710\gamma^4 - 8250\gamma^2 + 1575}{360(\gamma^2-1)^3\gamma^7}$$

$$c_4(\gamma) = -\frac{\gamma\left(32768\gamma^8 - 90112\gamma^6 + 1564672\gamma^4 - 1872978\gamma^2 - 7817455\right)}{336(\gamma^2-1)^2}$$

$$c_5(\gamma) = -\frac{491520\gamma^{22} - 2482176\gamma^{20} + 10655064\gamma^{18} - 32742084\gamma^{16} + 17085516\gamma^{14} + 61205662\gamma^{12} - 59068870\gamma^{10} - 5433687\gamma^8 + 1352120\gamma^6 - 330890\gamma^4 + 72450\gamma^2}{840(\gamma^2-1)^5\gamma^7}$$

$$\vdots \quad + \frac{11025}{840(\gamma^2-1)^5\gamma^7}$$

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RESULT: SCATTERING ANGLE@ 5PM–1SF PRECISION: CONSERVATIVE

[Driesse,Jakobsen,Mogull,JP,Sauer,Usovitsch]

- 5PM-1SF function space: Multiple polylogarithms up to weight 3!

$$G(a_1,\ldots,a_n;y)=\int_0^y \frac{\mathrm{d}t}{t-a_1}G(a_2,\ldots,a_n;t) \qquad y=1-x=1-\gamma+\sqrt{\gamma^2-1}\in\{0,1,2,1\pm i\}$$

- $f_k(\gamma)$ Basis functions:

$$\theta_{5,1}=\nu\sum_{k=1}^{31}c_k(\gamma)\,f_k(\gamma)$$

5PM-1SF Angle

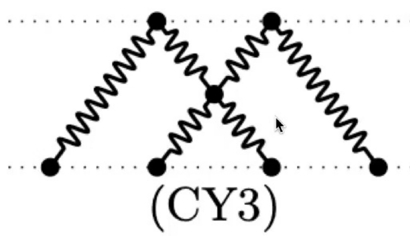
$$f_1(\gamma)=1$$
$$f_2(\gamma)=G(1;y)$$
$$f_3(\gamma)=2(G(0;y)-G(1;y)+G(2;y)+\log(2))$$
$$f_4(\gamma)=\pi^2$$
$$f_5(\gamma)=G(1;y)^2$$
$$f_6(\gamma)=2G(1;y)(G(0;y)-G(1;y)+G(2;y)+\log(2))$$
$$f_7(\gamma)=\frac{1}{2}G(1;y)^2+(-G(1;y)+G(1-\mathrm{i};y)+G(1+\mathrm{i};y))G(1;y)-G(1,1-\mathrm{i};y)-G(1,1+\mathrm{i};y)$$
$$f_8(\gamma)=-\frac{1}{2}G(1;y)^2+G(2;y)G(1;y)+G(0,1;y)-G(1,2;y)$$
$$f_9(\gamma)=-G(1;y)G(2;y)+G(0,1;y)+G(1,2;y)$$
$$f_{10}(\gamma)=\pi^2G(1;y)$$
$$f_{11}(\gamma)=-G(1;y)\left(-G(1;y)^2-2G(0;y)G(1;y)+G(2;y)G(1;y)-\log(4)G(1;y)-G(0,1;y)+4G(1,1-\mathrm{i};y)+4G(1,1+\mathrm{i};y)-G(1,2;y)\right)$$
$$f_{12}(\gamma)=-G(1;y)^3+2G(2;y)G(1;y)^2+(G(1;y)G(2;y)-G(0,1;y)-G(1,2;y))G(1;y)+4\left(-\frac{1}{6}G(1;y)^3+G(1,1,1-\mathrm{i};y)+G(1,1,1+\mathrm{i};y)\right)$$

$\vdots$

No elliptic or CYs!

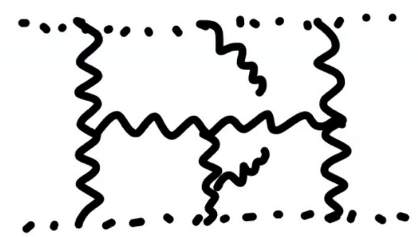
LOOKING AHEAD: FUTURE 5PM CALCULATIONS

- Our next step will be to calculate the **complete momentum impulse**, including **dissipative (radiation) effects**.
- Will yield **full 1SF angle, radiated energy flux**.
- Means upgrading to **retarded graviton propagators,** and including the **(PPR) & (RRR) regions** (odd-in-v)



Klemm, Nega, Sauer, JP '24

- Will we see a **Calabi-Yau 3-Fold**? They appear in the DEs…
- Then, 2SF! M**ore difficult integrals**, including genuine **nonplanars**:



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RADIATION REACTED IMPULSE (CHANGE OF MOMENTUM) @ 4PM & LINEAR IN SPIN

[Jakobsen,Mogull,JP,Sauer]

$$a_i^\mu = S_i^\mu / m_i$$

$$\Delta p_1^{(4)\mu} = \frac{m_1^2 m_2^2}{|b|^4} \sum_{l,\sigma=b,v} \rho_l^{(\sigma)\mu} \left[\left(\frac{m_2^2}{m_1} c_l^{(\sigma)}(\gamma) + \frac{m_1^2}{m_2} \bar{c}_l^{(\sigma)}(\gamma) \right) + \sum_\alpha F_\alpha(\gamma) \left(m_2 d_{\alpha,l}^{(\sigma)}(\gamma) + m_1 \bar{d}_{\alpha,l}^{(\sigma)}(\gamma) \right) \right],$$

$$\rho_l^{(b)\mu} = \left\{ \hat{b}^\mu, \frac{a_i \cdot \hat{L}}{|b|} \hat{b}^\mu, \frac{a_i \cdot \hat{b}}{|b|} \hat{L}^\mu \right\},$$

$$\rho_l^{(v)\mu} = \left\{ v_j^\mu, \frac{a_i \cdot \hat{L}}{|b|} v_j^\mu, \frac{a_i \cdot v_{\bar{\imath}}}{|b|} \hat{L}^\mu \right\}.$$

- **Basis of 19 functions** of weight-0,1,2 functions:

$$F_{1,\dots,14}(\gamma) = \Big\{1, \frac{\log[x]}{\sqrt{\gamma^2-1}}, \log[\gamma], \log\left[\frac{\gamma_\pm}{2}\right], \log^2[x],$$
$$\log^2\left[\frac{\gamma_+}{2}\right], \frac{\log[x]\log[\gamma]}{\sqrt{\gamma^2-1}}, \frac{\log[x]\log\left[\frac{\gamma_\pm}{2}\right]}{\sqrt{\gamma^2-1}},$$
$$\log\left[\frac{\gamma_+}{2}\right]\log\left[\frac{\gamma_-}{2}\right], \mathrm{Li}_2\left[\pm\frac{\gamma_-}{\gamma_+}\right],$$
$$\frac{\mathrm{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right] - 4\mathrm{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right]}{4\sqrt{\gamma^2-1}}\Big\},$$

mixed

$$\gamma_\pm = \gamma \pm 1$$
$$x = \gamma - \sqrt{\gamma^2-1}$$

Rational functions: $c_l^{(\sigma)}(\gamma)$, $d_{\alpha,l}^{(\sigma)}(\gamma)$

$$F_{15,16}(\gamma) = \Big\{\mathrm{Li}_2[-x^2] - 4\mathrm{Li}_2[-x] + \log[4]\log[x] - \frac{\pi^2}{4},$$
$$\frac{\mathrm{Li}_2[-x] - \mathrm{Li}_2\left[-\frac{1}{x}\right] - \log[4]\log[x]}{\sqrt{\gamma^2-1}}\Big\},$$

purely dissipative

$$F_{17\dots19}(\gamma) = \left\{\mathrm{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{K}\left[\frac{\gamma_-}{\gamma_+}\right]\mathrm{E}\left[\frac{\gamma_-}{\gamma_+}\right]\right\}$$

purely conservative

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SCATTERING SPINNING BLACK HOLES: STATE-OF-THE-ART

Higher orders in spin are suppressed by physical PM counting:

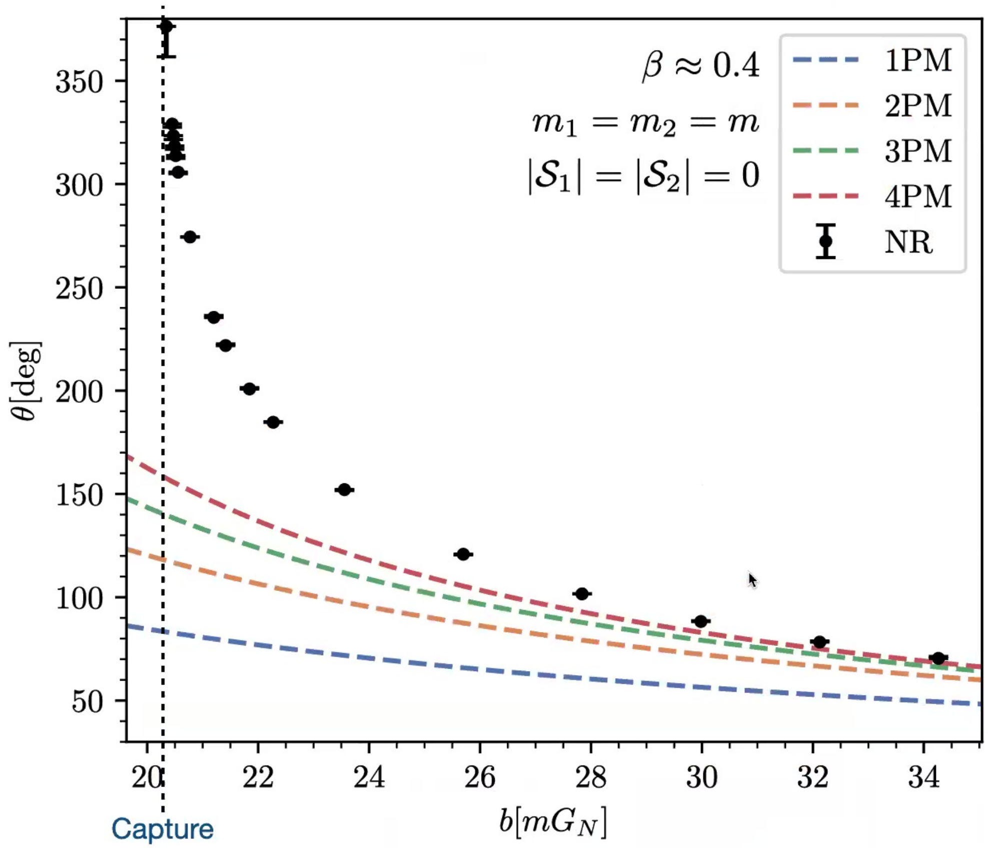
$$a_i^\mu = G m_i \chi_i^\mu \,, \qquad |\chi_i| < 1$$

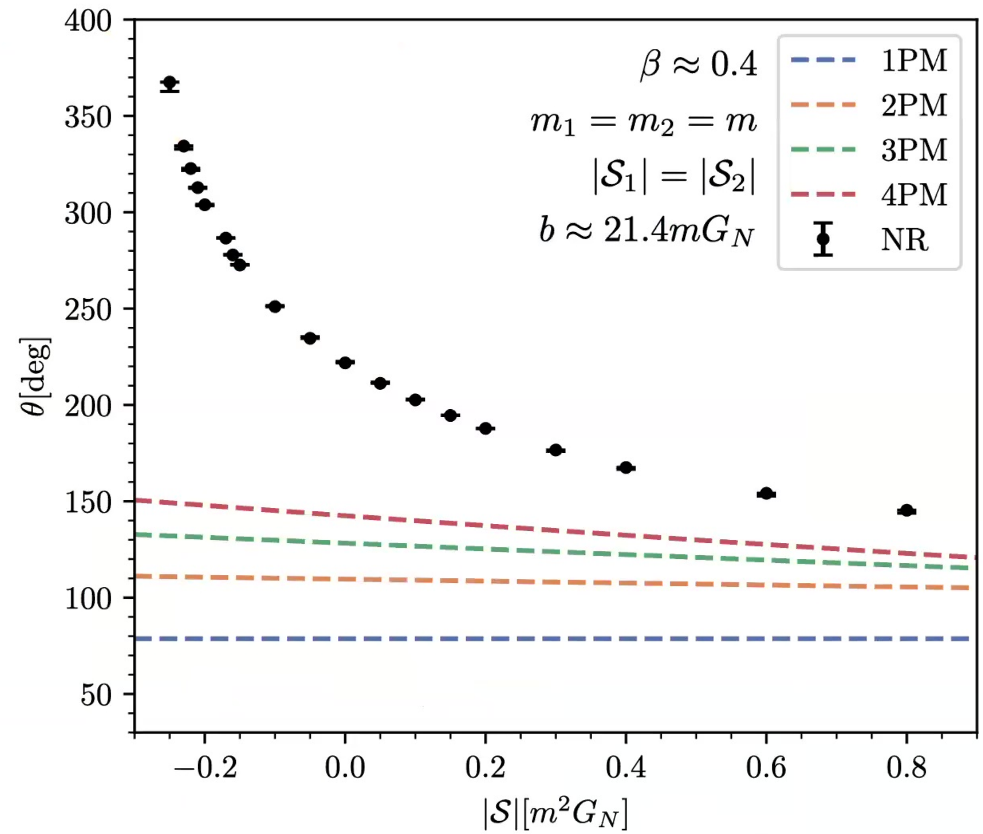
	S^0 (Spin-0)	S^1 (Spin-1/2)	S^2 (Spin-1)	S^3 (Spin-3/2)	S^4 (Spin-2)	S^5 (Spin-5/2)
1PM (tree level)	G	G^2	G^3	G^4	G^5	G^6
2PM (1 loop)	G^2	G^3	G^4	G^5	G^6	G^7
3PM (2 loops)	G^3	Jakobsen, Mogull '22	Jakobsen, Mogull '22	G^6	G^7	G^8
4PM (3 loops)	G^4	Jakobsen, Mogull, JP, Sauer, Xu '23	G^6	G^7	G^8	
5PM (4 loops)	Driesse, Jakobsen, Mogull, JP, Sauer, Usovitsch '24	G^6	G^7	G^8	G^9	G^{10}

Nearing completion of full G^5 perturbative order!

COMPARISON TO NUMERICAL RELATIVITY

Buonanno, GUJ, Mogull: [2402.12342]
Damour, Rettegno: [2211.01399]





NR data: Damour, Guercilena, Hinder, Hopper, Nagar, Rezzolla: [1402.7307]
Rettegno, Pratten, Thomas, Schmidt, Damour: [2307.06999]

EFFECTIVE ONE BODY FORMALISM: RESUMPTION TECHNIQUE

[Buonanno,Jakobsen,Mogull]

From PM data

$$V(r,p_\infty,L) = \frac{G_N\, c_1(p_\infty,L)}{r} + \frac{G_N^2\, c_2(p_\infty,L)}{r^2} + \frac{G_N^3\, c_3(p_\infty,L)}{r^3} + \frac{G_N^4\, c_4(p_\infty,L)}{r^4} + \mathcal{O}(G_N^5)$$

$$p_r^2 = \frac{1}{(1+B_{\rm np}^{\rm Kerr})}\left[\frac{1}{A^{\rm Kerr}}\left(E_{\rm Kerr} - \frac{2MLa}{r^3+a^2(r+2M)}\right)^2 - \left(\mu^2 + \frac{L^2}{r^2} + B_{\rm npa}^{\rm Kerr}\frac{L^2a^2}{r^2}\right)\right]$$

$$= p_\infty^2 - V(r,p_\infty,L)$$

$$u = GM/r$$
$$\chi u = S/r$$

$$A^{\rm Kerr} = \frac{1-2u+\chi^2u^2}{1+\chi^2u^2(2u+1)}$$

$$B_{\rm np}^{\rm Kerr} = \chi^2u^2 - 2u$$

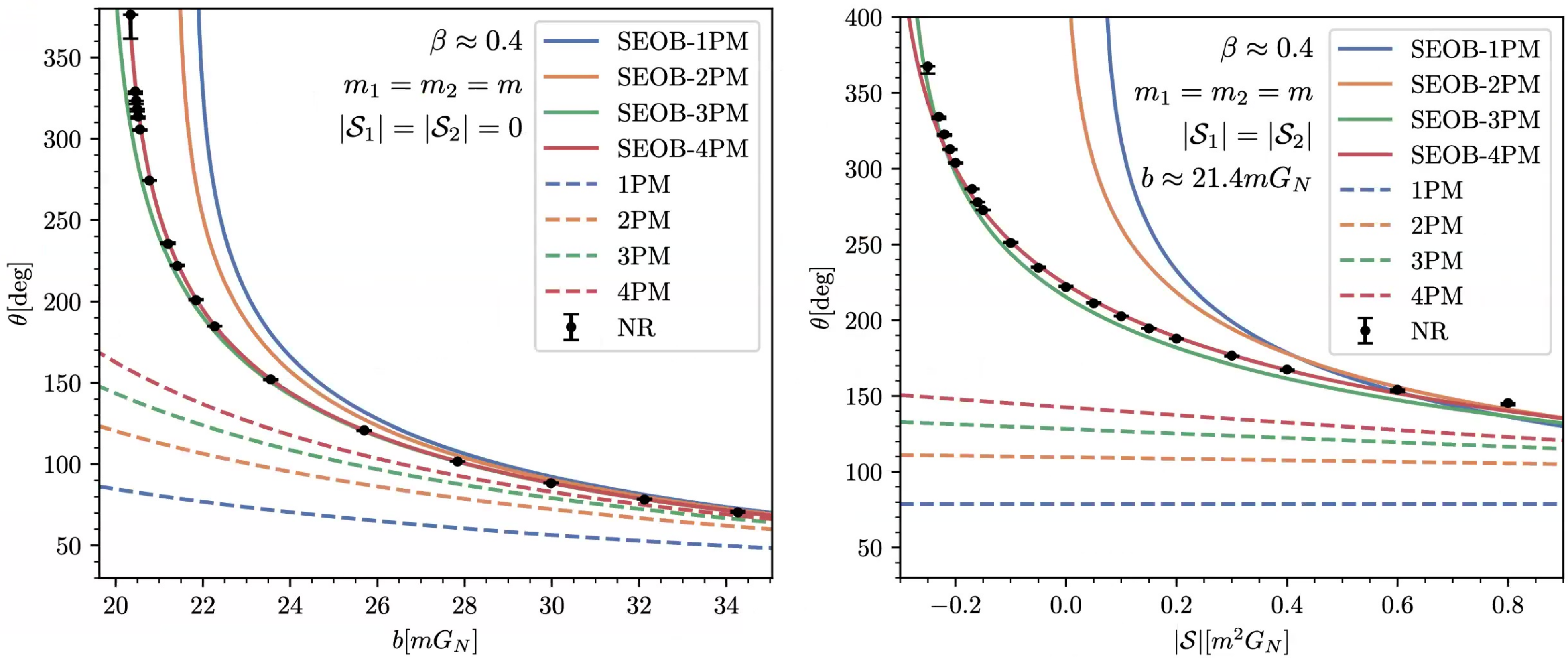
$$B_{\rm npa}^{\rm Kerr} = -\frac{1+2u}{r^2+a^2(1+2u)}$$

From geodesic limit (effective-one-body formalism)

COMPARISON TO NUMERICAL RELATIVITY USING RESUMMATION

[Buonanno,Jakobsen,Mogull]

- SEOB: Spinning **Effective One Body** model: resummation technique



* See also the w_{EOB} model in [2211.01399] by Damour and Rettegno

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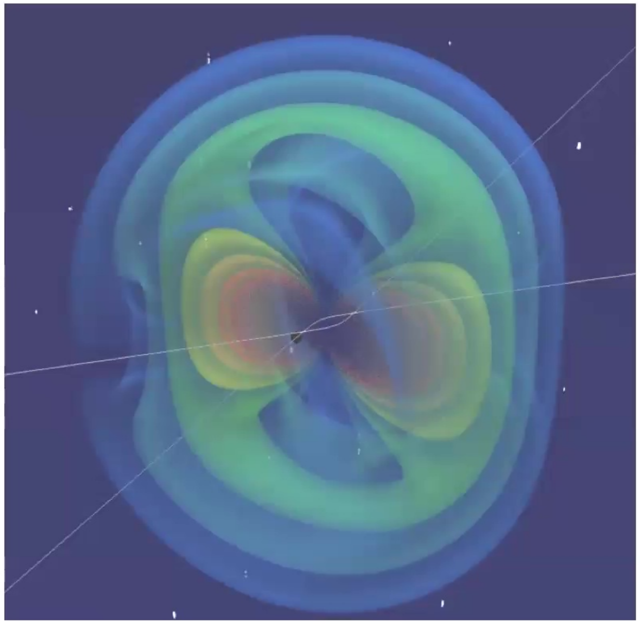
THE FARFIELD GRAVITATIONAL WAVEFORM



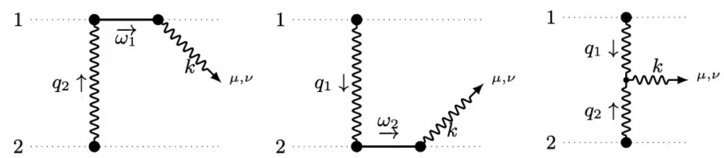
[Jakobsen,Mogull,JP,Sauer]

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum of diagrams with outgoing graviton:

$$\langle h_{\mu\nu}(k)\rangle =$$



For time-domain waveform needs to integrate over outgoing energy : Ω

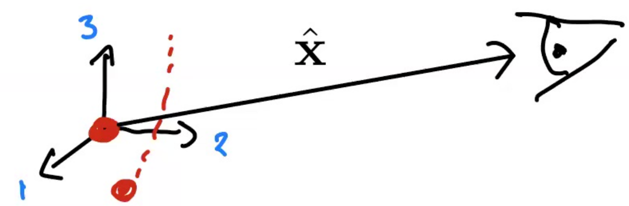
$$\frac{f_{+,\times}(t-r,\hat{\mathbf{x}})}{r} = \frac{4G}{r}\int d\Omega\, e^{-i\Omega(t-r)}\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega(1,\hat{\mathbf{x}}))\rangle$$

where unit vector $\hat{\mathbf{x}}$ points towards the observer

The waveform has two polarizations

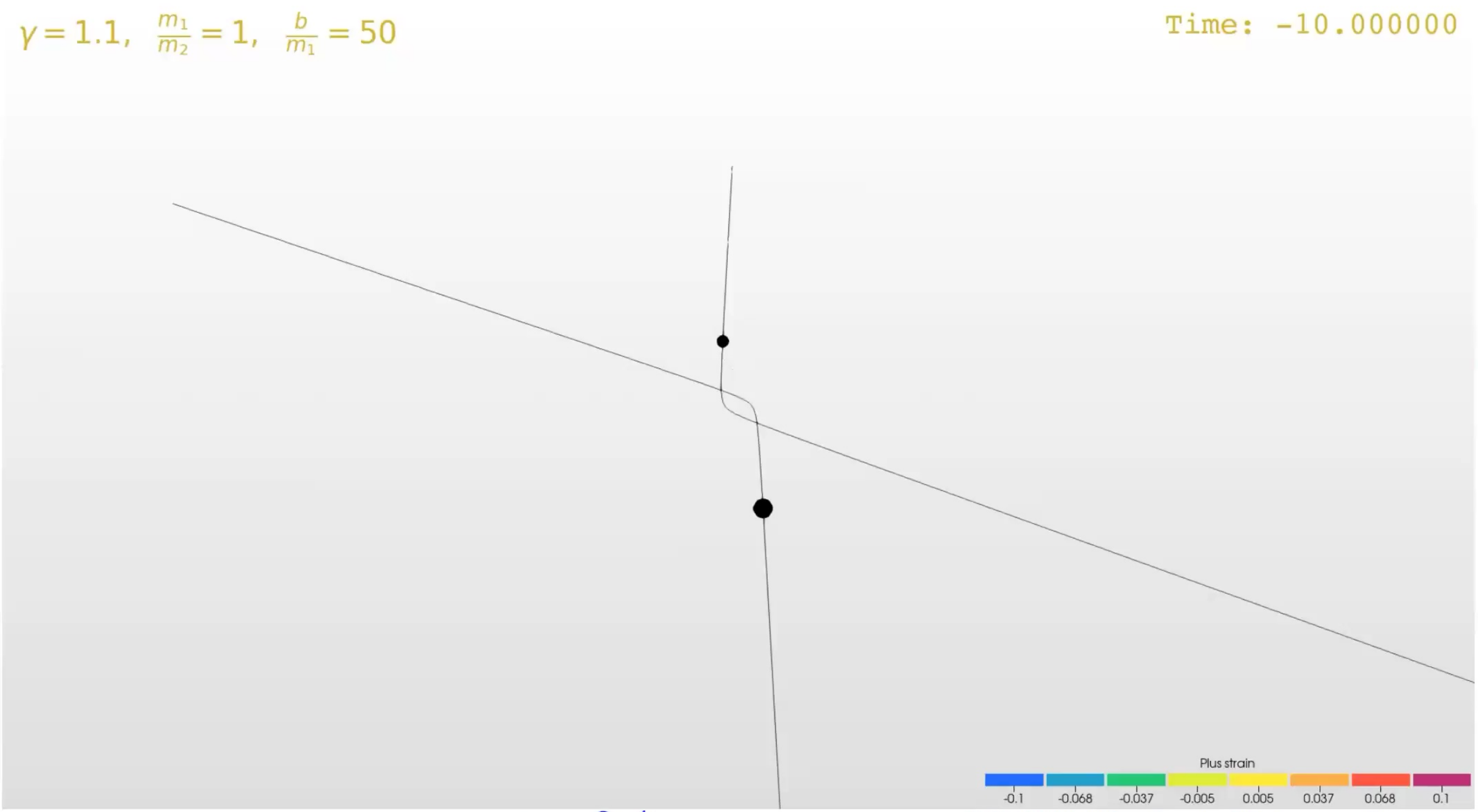
$$f_{+,\times}(\underbrace{t-r}_{u},\underbrace{\theta,\phi}_{\hat{\mathbf{x}}};v,|b|,m_1,m_2)$$

retarded time



50

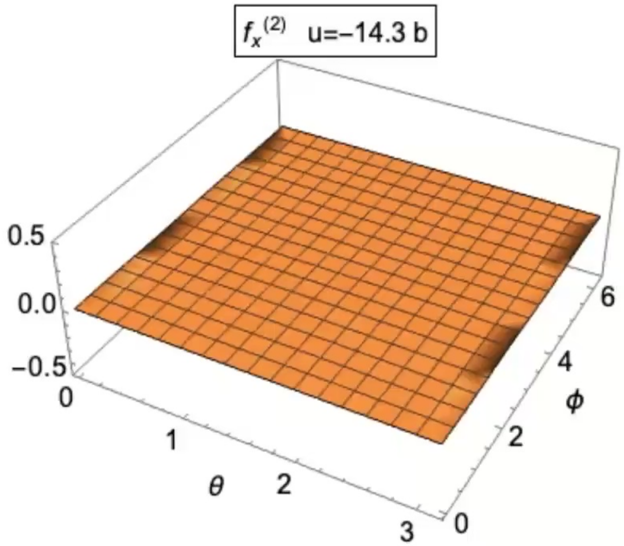
Visualization: Plus-Polarization $f_+^{(2)}$

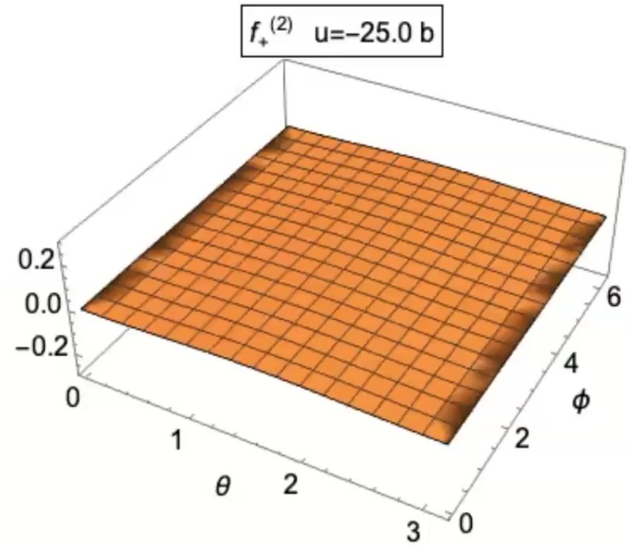


$v = 0.4$

[HU/AEI BSc thesis Kleinmond]

Memory effect



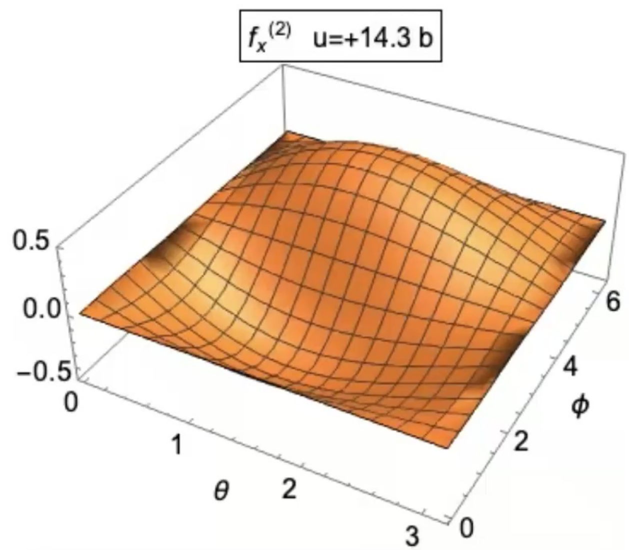


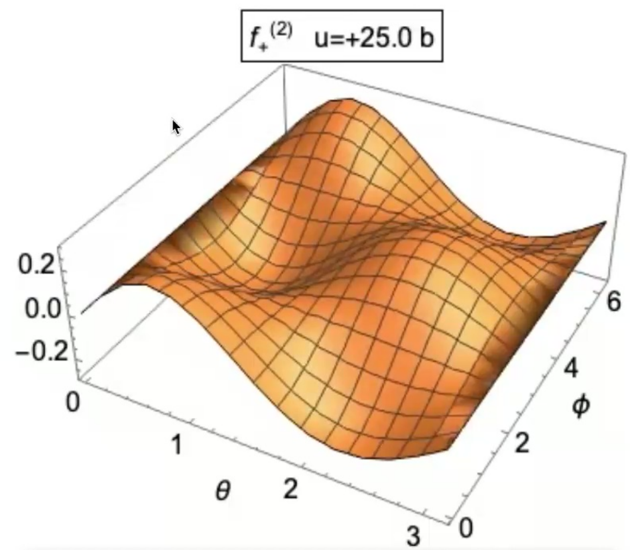
$$\frac{\Delta f^{(2)}_{\mathcal{S}=0}}{m_1 m_2} = \frac{4(2\gamma^2-1)\epsilon\cdot v_1(2b\cdot\epsilon\,\rho\cdot v_1 - b\cdot\rho\,\epsilon\cdot v_1)}{|b|^2\sqrt{\gamma^2-1}(\rho\cdot v_1)^2}$$

$$\rho = (1,\hat{\mathbf{x}})$$

$$\gamma = v_1\cdot v_2$$

Memory effect





$$\frac{\Delta f^{(2)}_{\mathcal{S}=0}}{m_1 m_2} = \frac{4(2\gamma^2-1)\epsilon\cdot v_1(2b\cdot\epsilon\,\rho\cdot v_1 - b\cdot\rho\,\epsilon\cdot v_1)}{|b|^2\sqrt{\gamma^2-1}(\rho\cdot v_1)^2}$$

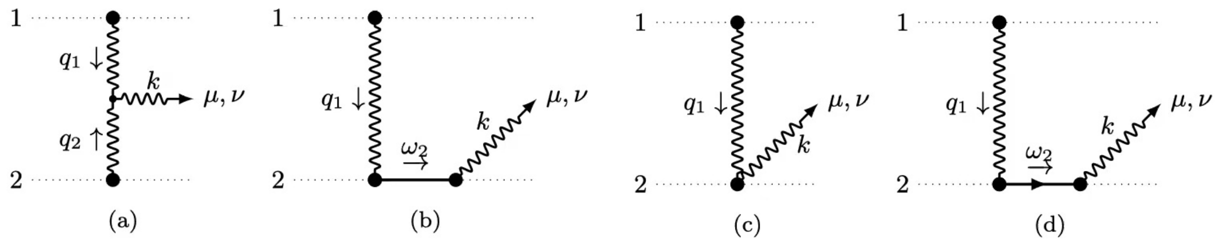
$$\rho = (1, \hat{\mathbf{x}})$$

$$\gamma = v_1\cdot v_2$$

SPINNING WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

- Update Kovacs-Thorne with spin.



$$\frac{f^{(2)}}{m_1 m_2} = \sum_{s=0}^{2} \frac{1}{|\tilde{\mathbf{b}}|_1^{2s+1}} \left[\alpha_1^{(s)} + \frac{\beta_1^{(s)}}{|\tilde{b}|^{2s+2}} \right] + (1 \leftrightarrow 2)$$

- The spinning wave memory:

$$\Delta f^{2)} = f^{(2)}(u = +\infty) - f^{(2)}(u = -\infty)$$

$$\Delta f^{(2)} = \left(1 + \frac{2v|a_3|}{b(1+v^2)} + \frac{|a_3|^2}{|b|^2} - \sum_{i=1}^{2} \frac{C_{E,i}|a_i|^2}{|b|^2}\right) \Delta f^{(2)}_{\mathcal{S}=0}$$

Using Pauli-Lubanski vector: $\mathcal{S}_i^{\mu\nu} = \epsilon^{\mu\nu}{}_{\rho\sigma} v_i^\rho a_i^\sigma$ $\quad a_3^\mu = a_1^\mu + a_2^\mu$

- Radiated angular momentum in COM:

$$\frac{J_{xy}^{\mathrm{rad}} + i J_{zx}^{\mathrm{rad}}}{J_{xy}^{\mathrm{init}}\big|_{\mathcal{S}=0}} = \frac{4G^2 m_1 m_2}{|b|^2} \frac{(2\gamma^2 - 1)}{\sqrt{\gamma^2 - 1}} \mathcal{I}(v)$$
$$\times \left(1 - \frac{2iv\, \mathbf{a}_3 \cdot \mathbf{l}}{|b|(1+v^2)} - \frac{(\mathbf{a}_3 \cdot \mathbf{l})^2}{|b|^2} + \sum_{i=1}^{2} \frac{C_{E,i}}{|b|^2} (\mathbf{a}_i \cdot \mathbf{l})^2\right)$$

PM STATE-OF-THE-ART

WQFT [us]

WEFT **Worldline effective theory** [Källin,Porto,Dlapa,Cho,Liu,..] [Riva,Vernizzi,Mougiakakos..]

HEFT **Heavy BH effective theory** [Aoude,Haddad,Helset,Damgaard] [Brandhuber,Travaglini,Chen]

Amps **Scattering amplitudes** [Bern,Roiban,Shen,Parra-Martinez,Ruf,Zeng..] [Bjerrum-Bohr,Damgaard,Plante,Vanhove,..] [Di Vecchia,Veneziano,Heissenberg,Russo] [Solon,Cheung,..] [Huang,..] [Guevera,Ochirov,Vines,…] [Johansson,Pichini[Kosower,O'Connell,Maybee

order	deflection & spin kick					waveform			
	plain	spin-orbit	spin-spin	spin$^{>2}$	tidal	plain	spin-orbit spin-spin	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	WQFT WEFT Amps	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT	~ 1-loop
3PM cons	WQFT WEFT Amps HEFT	WQFT Amps	WQFT (Amps)		WQFT WEFT	Amps HEFT	Amps		
3PM diss	WQFT WEFT Amps HEFT	WQFT	WQFT		WQFT WEFT				~ 2-loop
4PM cons	WQFT WEFT Amps	WQFT			WQFT				
4PM diss	WQFT WEFT Amps	WQFT			WQFT				~ 3-loop
5PM-1SF cons	WQFT								~ 4-loop

r-r: Radiation-reaction (…) : partial results

SUMMARY

WQFT: Highly efficient quantum field theory technology for classical scattering in GR

- „Quantize“ world-line degrees of freedom
- One-point functions = observables
- Classical theory = tree-level diagrams
- IN-IN Formalism: All propagators retarded.
- Include spin degrees of freedom through world-line supersymmetry

OUTLOOK

WQFT still needs to be extended:

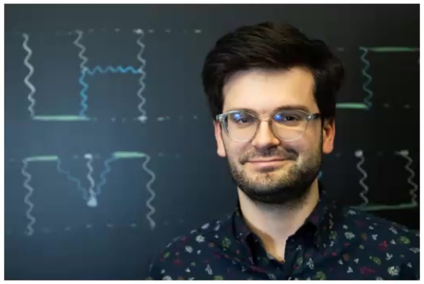
- Higher precision (4PM Spin-Spin, 5PM)
- Higher spin (beyond Spin-Spin)
- Bound orbits? Innovate Effective-one-body Formalism
- Innovate self force expansion

THANKS TO MY COLLABORATORS!

Gustav Uhre Jakobsen

Benjamin Sauer

Mathias Driesse



Gustav Mogull

Jan Steinhoff

Johann Usovitsch