Title: Colloquium - TBA

Speakers: Svitlana Mayboroda

Series: Colloquium

Date: May 15, 2024 - 2:00 PM

URL: https://pirsa.org/24050059

Abstract: Abstract TBA

Zoom link

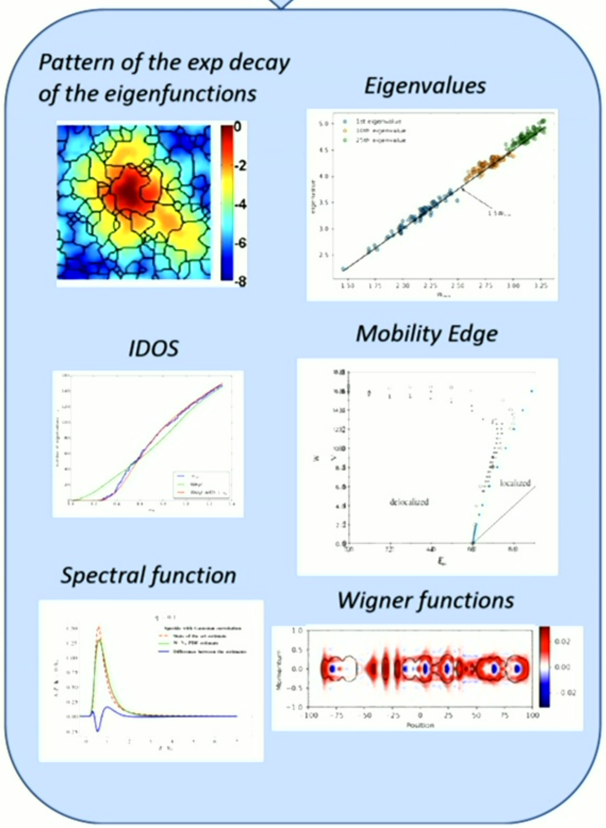
Wave localization

Svitlana Mayboroda
ETH & University of Minnesota

May 2024

SIMONS COLLABORATION ON LOCALIZATION OF WAVES

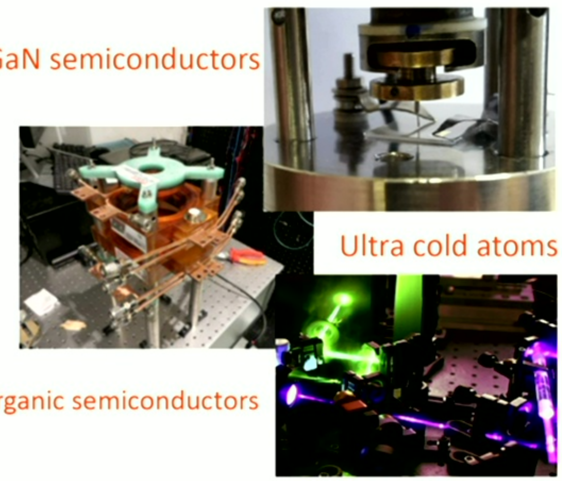
$$\mathbf{Hu = 1}$$



Very few believed [localization] at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. *It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations* to settle even the simplest questions about it.

- Philip W. Anderson, Nobel Lecture, 8 December 1977

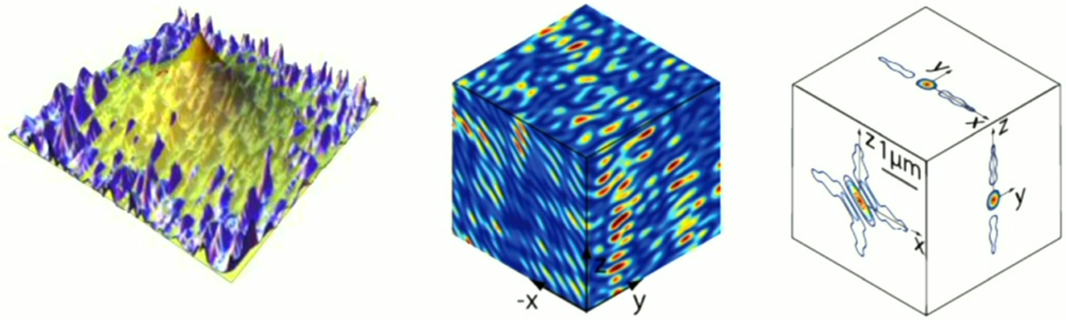
GaN semiconductors



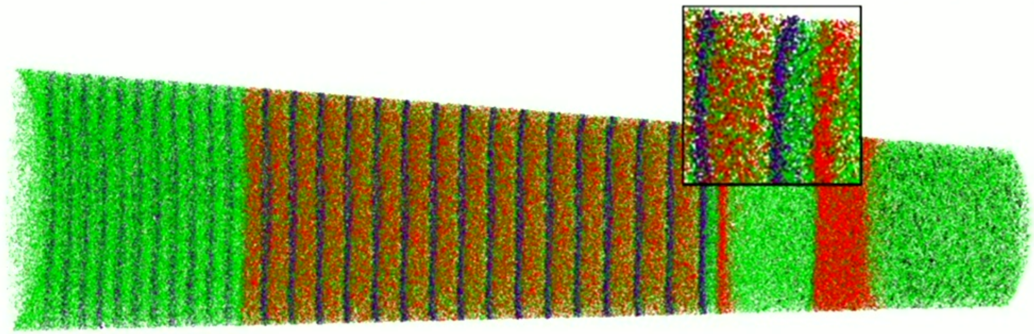
Ultra cold atoms

Organic semiconductors

... technology reveals disorder

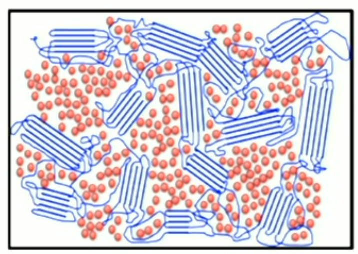


Anderson localization in Bose-Einstein condensate [Aspect lab]

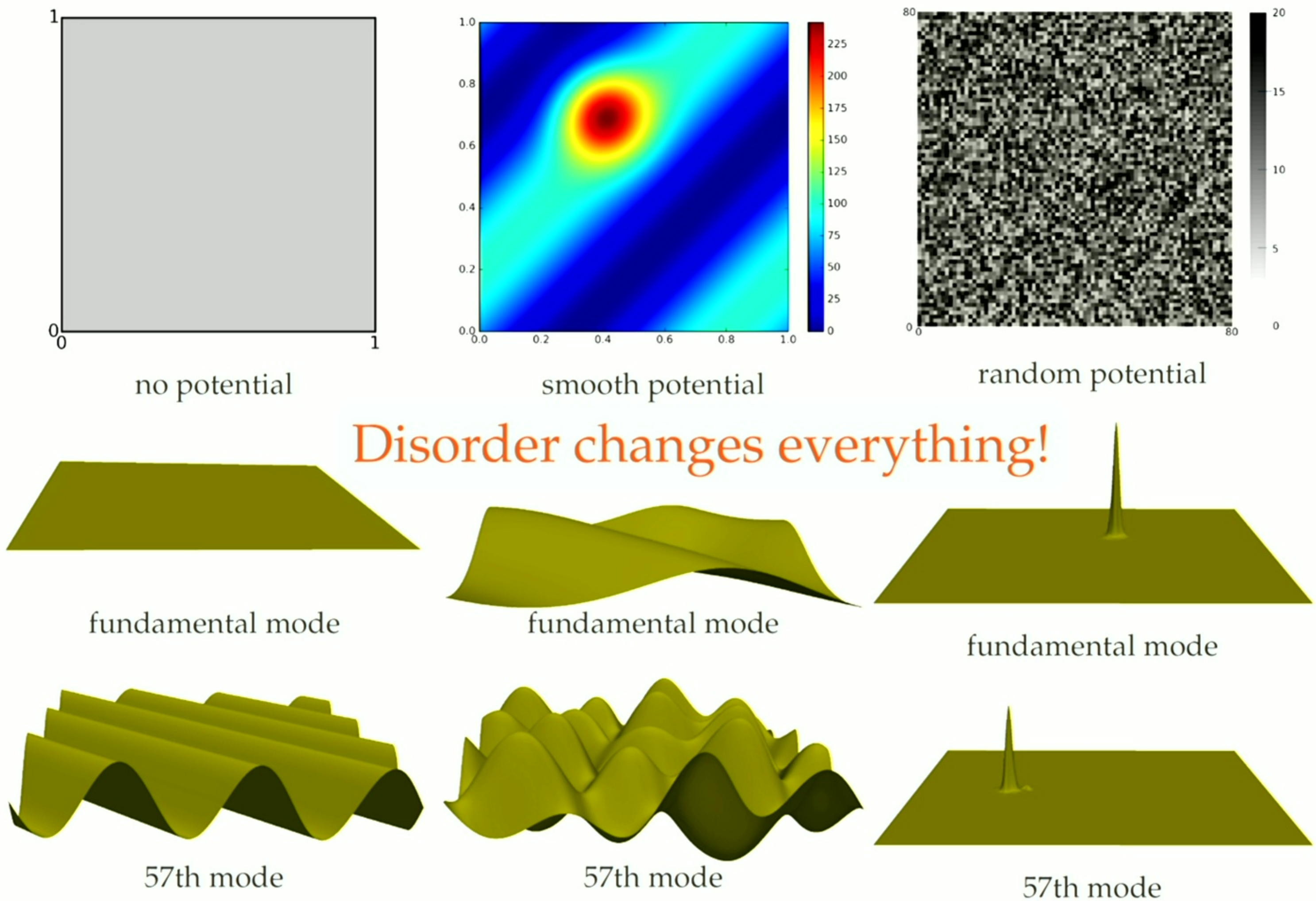


Atomic map of InGaN semiconductor [Speck lab]

Mixed donor-acceptor morphology in an organic solar cell [Friend lab]



Smooth versus disordered potential in Schrödinger equation



Anderson localization

The localization of Schrödinger eigenfunctions with random potential was discovered by Philip Anderson in his Nobel-prize-winning work of 1958.

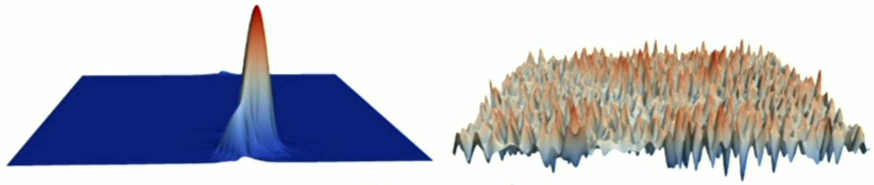
> *Unfortunately, electron localization was devilishly hard to confirm... experimental observations are sparse and covered with disputes and controversies.*
>
> – Lagendijk, van Tiggelen, Wiersma, *50 Years of Anderson Localization*, 2009

> *Most theoretical work [7-9] predicts [the critical exponent]* $\mu = 1$*, but there is also a prediction of* $\mu = 1/2$ *[10]. Numerical simulation [11] gives* $\mu = 2/3\ldots$
>
> – I. Shlimak, *Is Hopping a Science?*, 2015

Mathematical proofs (Frohlich-Spencer, Aizenman-Molchanov) are in extreme regimes (1D, edge of the spectrum, or strong potential).

Waves in disordered media

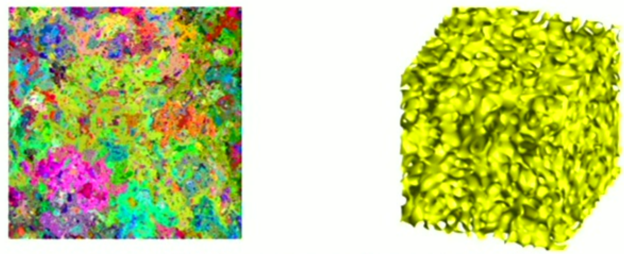
Potential



Eigenmodes

In a disordered environment, **waves localize** ... or not ....

Spherical harmonics (SH)



Nodal domains of random SH

Sarnak and Bogomolny-Schmit **percolate in 3D** ... not in 2D ....

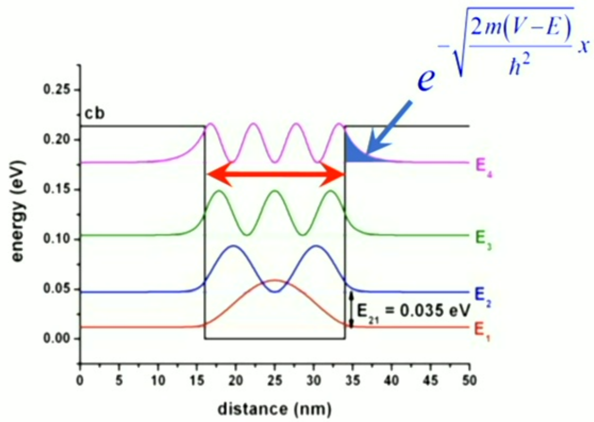
DOE Energy Savings Forecast of Solid State Lighting in the US, 2017–2035:

- Goal: **50% improvement** in LED efficiency
- Energy savings: **more than 92 1GW power plants**
- Cumulative US cost savings: **$890 billion**
- Obstacles: **Green Gap, efficiency droop at high currents, lack of accurate computations/modeling**

Particle vs. wave localization

Waves go where particles don't go

The quantum well

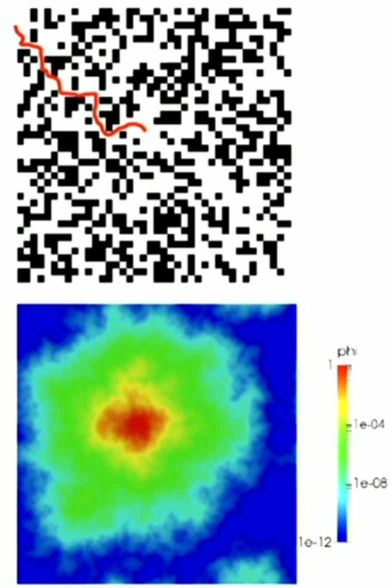


We see the classical potential

Waves see "something different"

Waves don't go where particles go

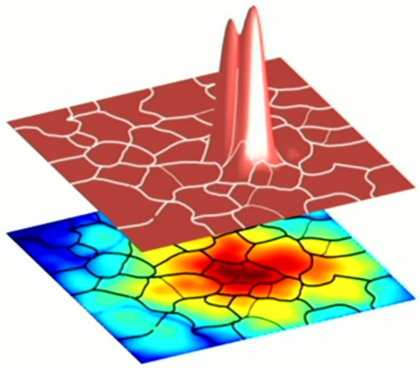
Boolean potential (60% of **0**, 40% of **1**)



Fundamental quantum state ($E>0$)

Take on the perspective of a wave

The main hero:
THE LANDSCAPE



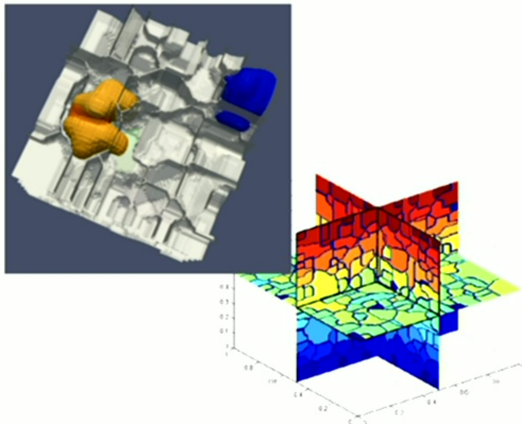
A hidden landscape that waves recognize and obey

- born of the equation but invisible to the naked eye
- contains both spatial and spectral information

The goal is to

Discover and master this landscape in order to

- understand
- predict
- manipulate
- govern
- and, ultimately, design matter waves



Curves/surfaces of the landscape vs. eigenfunctions

Landscape theory

Geometry (spacial info):

- exp decay: $1/u$ as an effective potential
- level sets/free boundary
- random monochromatic waves
- sharp characterizations of the boundary impact (rectifiability)

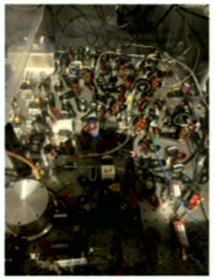
Spectrum (energy info):

- spectrum via min $\frac{1}{u}$
- the new Weyl law
- the new Uncertainty principle
- the Landscape Law: the first non-asymptotic prediction of IDOS

Wigner-Weyl (quantum observables):

- general scheme
- spectral function
- absorption

Cold atoms:



- experimental set-up for Mobility Edge
- landscape percolation vs ME
- Spectral function

Semiconductors:

- 33% improvement of green LEDs (Green Gap)
- 1000x faster computations: from 1D to 3D

Organics:

- transporting energy 10 times further than in photosynthesis
- high efficiency perovskite-based green LEDs

A different perspective: the effective potential

Arnold, David, Filoche, Jerison, Mayboroda, PRL 2016: a new idea

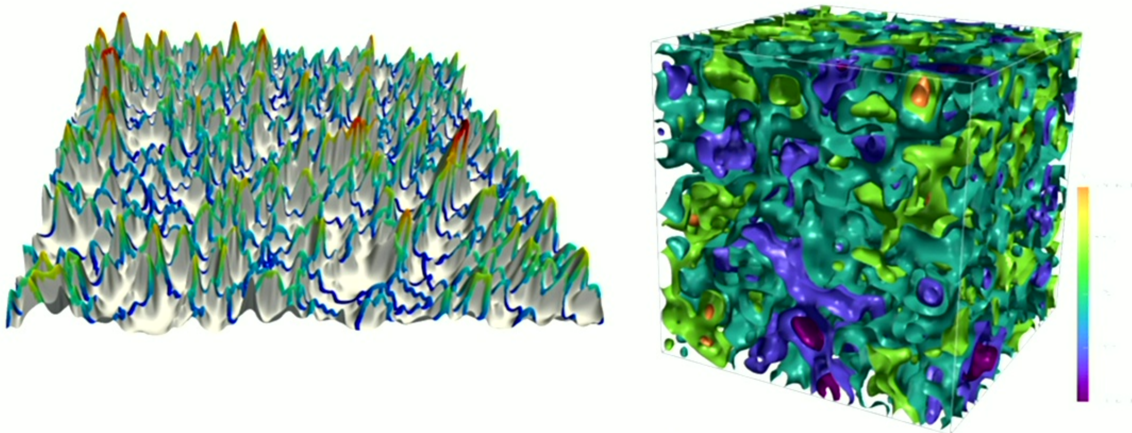
linear equation $\Longrightarrow$ nonlinear control

$\frac{1}{u}$ is an *effective potential* which is often confining.

$$-\Delta\psi + V\psi = E\psi \iff -\frac{1}{u^2}\nabla\cdot(u^2\nabla\phi) + \frac{1}{u}\phi = E\phi$$

– exactly the same eigenvalues! ($\psi = u\phi$)

$Hu = 1 \implies$ enhanced *Agmon-type distance* $\rho_{1/u} \implies$ exp decay



2D and 3D effective potential $\frac{1}{u}$ for Bernoulli V

Exponential decay: $\frac{1}{u}$ is an effective potential

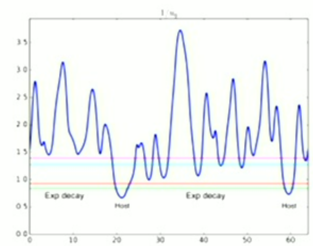
Theorem (Arnold, David, Filoche, Jerison, Mayboroda, 2018)

Let $L = -\mathrm{div} A \nabla + V, 0 \leq V(x) \leq \bar{V}$*, on* M*, a Lipschitz domain on a compact* C^1 *manifold,* $L\psi = \lambda\psi$*, and* $Lu = 1$ *on* M*, Neumann BC. Let*

$$E(\lambda + \delta) = \{x \in M : 1/u(x) \leq \lambda + \delta\}, \quad \delta > 0,$$

(collection of $1/u$*-wells). Define*

$$\rho_{1/u}(x,y) = \inf_{\gamma(x,y)} \int \left((1/u - \lambda)_+ \, b_{ij} \dot{\gamma}_i(t) \dot{\gamma}_j(t) \right)^{1/2} dt$$



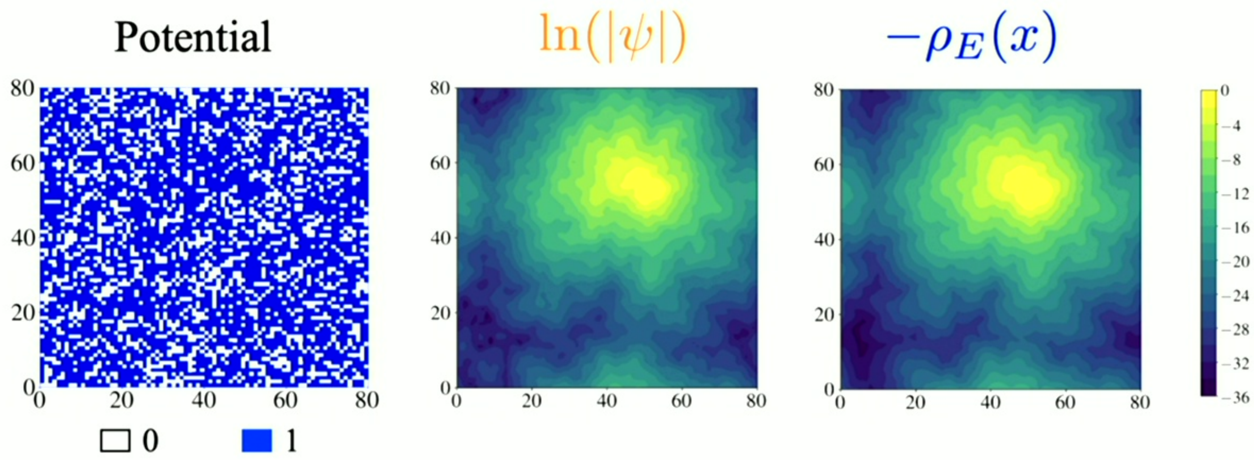
where the infimum is taken over all absolutely continuous paths γ *from* x *to* y*, and* $B = \{b_{ij}\} = A^{-1}$*. Then*

$$\int_{\{\rho_{1/u}(x,E(\lambda+\delta)) \geq 1\}} e^{\rho_{1/u}(x,E(\lambda+\delta))} (|\nabla\psi|^2 + \bar{V}\psi^2) \, dx \leq C \int \bar{V}\psi^2 \, dx$$

Roughly speaking, $\psi(x) \sim e^{-\rho_{1/u}(x,E(\lambda+\delta))}$ away from the $1/u$-wells.

Exponential decay: $\frac{1}{u}$ is an effective potential

$$|\psi(x)| \approx e^{-\rho(x_0,x)} \qquad \rho(x_0,x) := \inf_{\gamma} \int_{\gamma} \left(\frac{1}{u} - \lambda\right)_+ ds$$



Numerical computations: D. Arnold, 2019

Uncertainty principle

The landscape concisely encodes precise spectral and spatial information in one structure, and is easily computable, but it does not lose information.

small and positive

$$E = \langle \nabla\psi, \nabla\psi \rangle + \langle \psi | V | \psi \rangle = \langle u \nabla (\frac{\psi}{u}), u \nabla (\frac{\psi}{u}) \rangle + \langle \psi | \frac{1}{u} | \psi \rangle$$

kinetic energy — potential energy — reduced kinetic energy — effective pot. energy

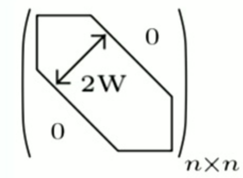
infinite quantum well (Dirichlet problem) lhs 100%+0%; rhs 4%+96%

- at the first approximation, we get a new Uncertainty Principle.
- in some sense, $1/u$ seems to translate interferencial effects into a confinement picture viewed by the eigenmode

Localization for M-matrices

Theorem (M. Filoche, S.M., T. Tao, 2021)

Let A be a symmetric $n \times n$ M-matrix with at most W_c non-zero entries in every row. Let $u = A^{-1}1$, $v_i = \left(\frac{1}{u\,i} - E\right)_+$, *and*

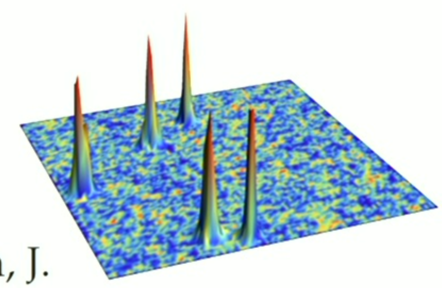


$$\rho(i,j) = \inf_{L \geq 0} \inf_{\substack{i_0,\ldots,i_L: \\ i_0=i, i_L=j}} \sum_{\ell=0}^{L} \ln\left(1 + \sqrt{\frac{\sqrt{v_{i_\ell} v_{i_{\ell+1}}}}{|a_{i_\ell i_{\ell+1}}|}}\right).$$

Then

$$\sum_k \varphi_k^2 \, e^{\frac{2\rho(k,K)}{\sqrt{W_c}}} \left(\frac{1}{u_k} - E\right)_+ \leq W_c \max_{1 \leq i,j \leq n} |a_{ij}|.$$

- Many-body systems and statistical physics:
  S. Balasubramanian, Y. Liao, **V. Galitski**, 2020
  Collaboration with Simons Collaboration on Ultra-Quantum Matter



- localization for Dirac fermions:
  G. Lemut, M. J. Pacholski, O. Ovdat, A. Grabsch, J. Tworzydło, **C. W. J. Beenakker**, 2019

Assessing Anderson localization in BEC of cold atoms

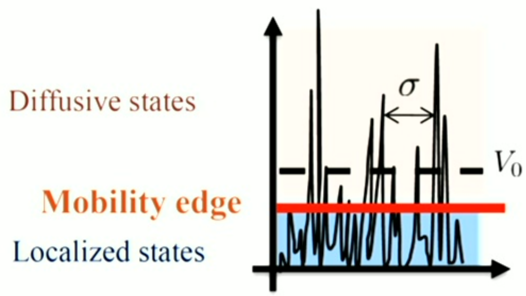
Nobel Prize 2001: Bose-Einstein condensation

Eric A. Cornell | Wolfgang Ketterle | Carl E. Wieman

- Outstandingly clean system
- Pure potentials (no absorption)
- Controllable dimensionality: 1D, 2D, 3D
- Controllable wavelength, 1 nm to 10 μm
- Controllable disorder (laser speckle)
- High temporal resolution

A. Aspect's group achieved the *first direct observation* of a localized wave function in Bose-Einstein condensate of cold atoms in 1D in 2008, 3D in 2012.

Mobility edge:

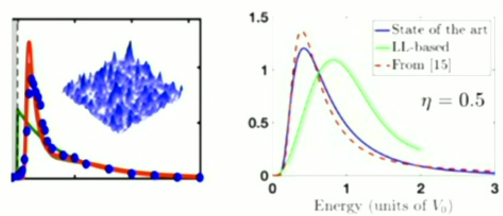


Nobel Prize 2022: testing Bell inequalities

Alain Aspect | John F. Clauser | Anton Zeilinger

Spectral functions:

Spectral functions exhibit a very peculiar behavior in laser speckle potentials. Recent work have shown that the Landscape theory provides an accurate prediction.



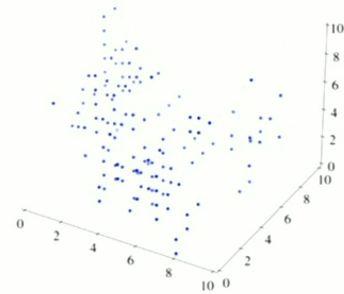
Anderson transition: the mobility edge

The mobility edge in tight binding models is directly related to the percolation threshold of the landscape-based effective potential.

$$(H\psi)_n = -t \sum_{|m-n|=1} (\psi_m - \psi_n) + V_n \psi_n$$

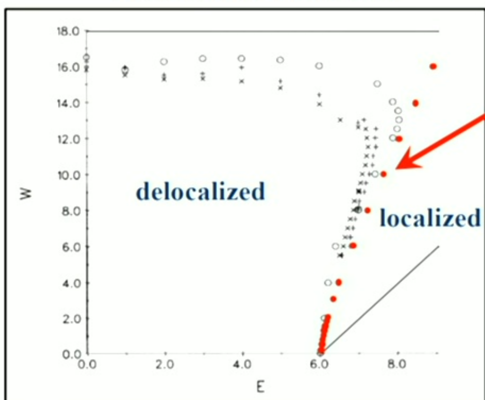
$$\boxed{Hu = 1}$$

Effective potential $\frac{1}{u}$



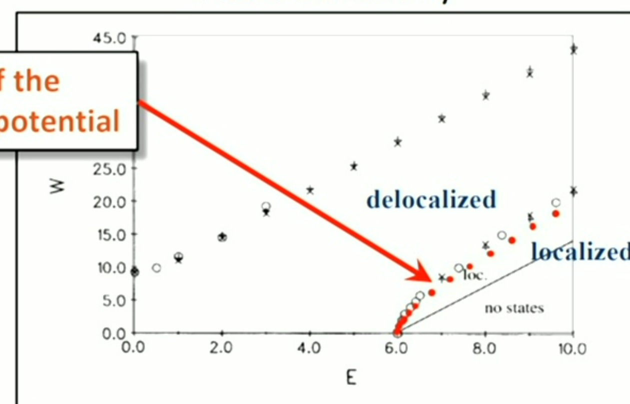
sites s.t. $\frac{1}{u} < 0.45$

Anderson uniform



Percolation of the landscape-based potential

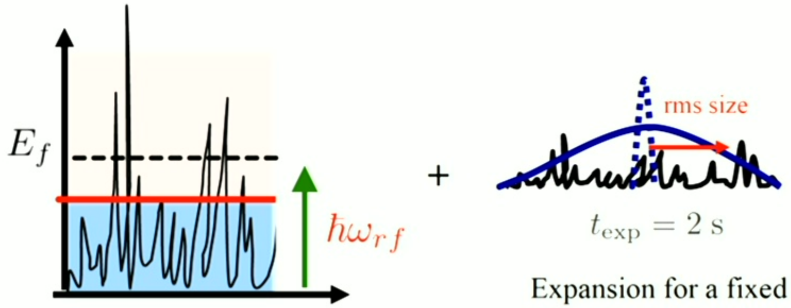
Anderson binary



Comparison with the computed mobility edge from Grussbach & Schreiber, *Phys. Rev. B* (1995)

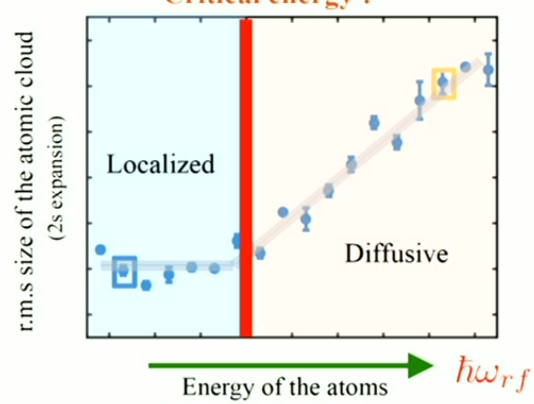
2023: experimental observation of the mobility edge

Direct signature of a critical energy

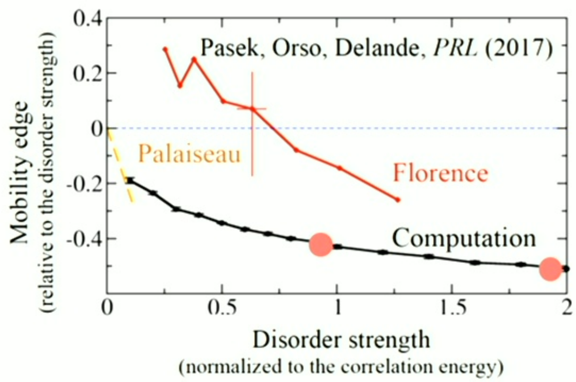


Transfer of the atoms at a well defined energy

Expansion for a fixed duration in disorder



Comparison with numerical prediction of the mobility edge



Excellent agreement without any adjustable parameter !

Working progress to span various disorder strength (from quantum to classical regime)

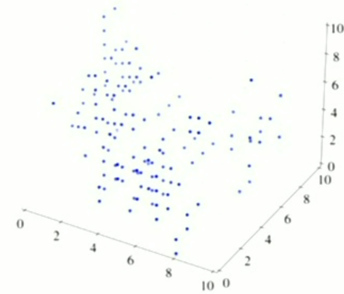
Anderson transition: the mobility edge

The mobility edge in tight binding models is directly related to the percolation threshold of the landscape-based effective potential.

$$(H\psi)_n = -t \sum_{|m-n|=1} (\psi_m - \psi_n) + V_n\psi_n$$

$$\boxed{Hu = 1}$$

Effective potential $\frac{1}{u}$



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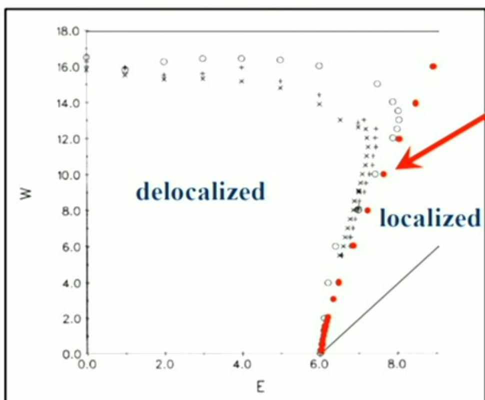
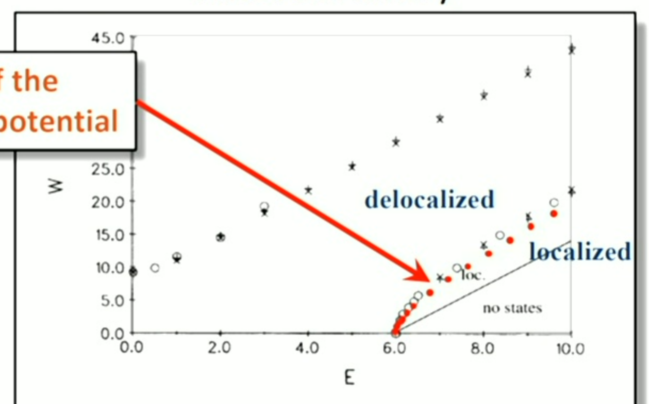


Percolation of the landscape-based potential

Anderson binary

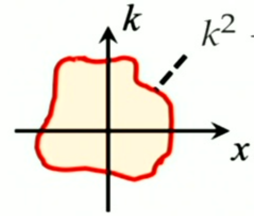


Comparison with the computed mobility edge from Grussbach & Schreiber, *Phys. Rev. B* (1995)

Landscape-based Weyl law

Weyl's asymptotic law

$$\mathrm{IDOS}(E) = \#\{E_i \leq E\} \approx \frac{1}{(2\pi)^d} \iint_{k^2+V(x)\leq E} dx\, dk$$

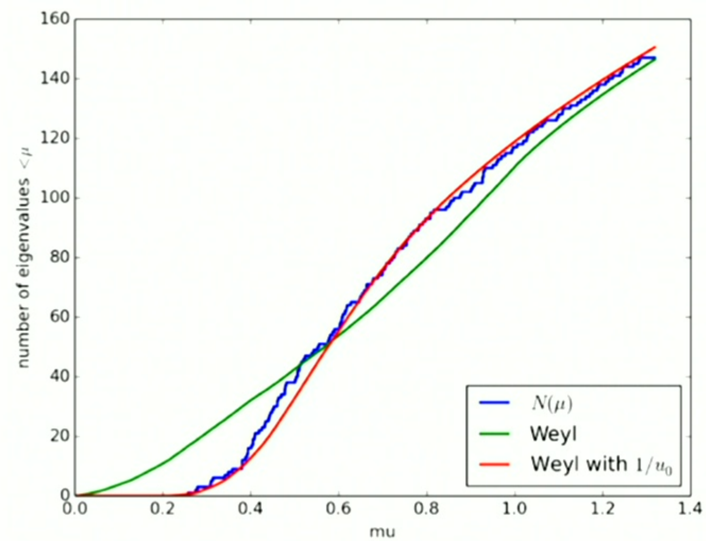


$k^2 + V(x) \leq E$

Counting eigenvalues below E ⟺ Counting volume in phase space

Landscape-based Weyl law

$$\mathrm{IDOS}(E) \approx \frac{1}{(2\pi)^d} \iint_{k^2+\frac{1}{u(x)}\leq E} dx\, dk$$

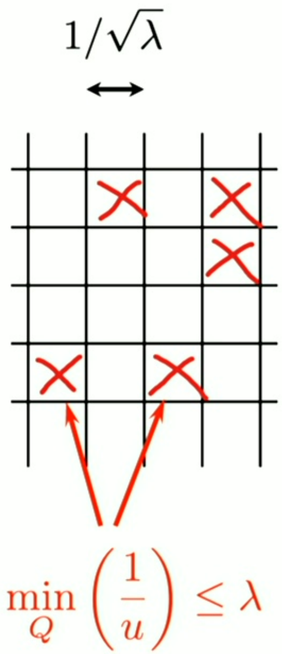


Blue: reality; green: old Weyl; red: new Weyl

Eigenvalue count: the Landscape Law

New counting function: $H = -\Delta + V$, $Hu = 1$

$$N_u(\lambda) := \left\{ \# \text{ of cubes of sidelength } \lambda^{-1/2} : \min_Q \frac{1}{u} \le \lambda \right\}$$



Theorem (G. David, M. Filoche, S. M., 2019 for continuous; D. Arnold, M. Filoche, S.M., W. Wang, S. Zhang 2020 for tight-binding)

There exist constants C_i depending on the dimension only, such that

$$C_1 \alpha^d N_u(C_2 \alpha^{d+2} \mu) - C_3 N_u(C_2 \alpha^{d+4} \mu) \le N(\mu) \le N_u(C_4 \mu)$$

for every $\alpha < 2^{-4}$ and every $\mu > 0$.
If, in addition, u^2 is a doubling weight or V is a disordered potential, then

$$N_u(C_5\, \mu) \le N(\mu) \le N_u(C_4\, \mu) \text{ for every } \mu > 0.$$

Landscape-based Weyl law

Weyl's asymptotic law

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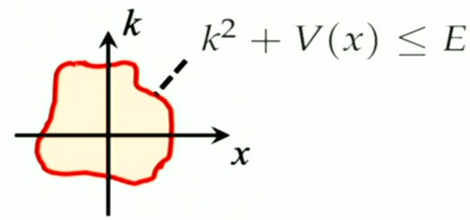

Counting eigenvalues below E ⟷ Counting volume in phase space

Landscape-based Weyl law

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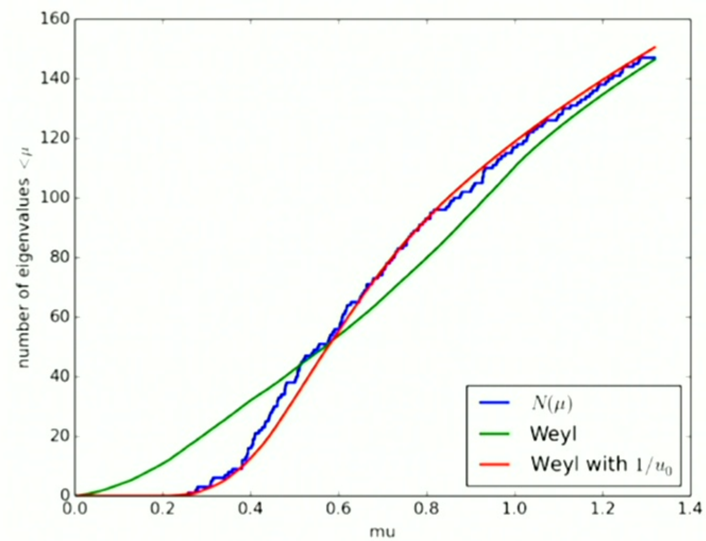

Blue: reality; green: old Weyl; red: new Weyl