Title: Universal aspects of decohered and dissipative quantum many-body systems

Speakers: Tarun Grover

Series: Colloquium

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Abstract: Ground states as well as Gibbs states of many-body quantum Hamiltonians have been studied extensively since the inception of quantum mechanics. In contrast, the landscape of many-body quantum states that are not in thermal equilibrium is relatively less explored. In this talk I will discuss some of the recent progress in understanding decohered or dissipative quantum many-body states. One of the key ideas I will employ is that of "separability", i.e., whether a mixed state can be expressed as an ensemble of short-range entangled pure states. I will discuss several quantum phase transitions in topological phases of matter subjected to Markovian environmental noise from a separability viewpoint, and argue that such a framework also subsumes our understanding of pure quantum states as well as Gibbs states. Time permitting, I will also provide a brief overview of quantum spin-systems subjected to non-Markovian noise originating from an electronic bath, and discuss a new critical phase of matter where quantum coherence coexists with dissipation.

References: 2307.13889, 2309.11879, 2310.07286

Zoom link

Universal aspects of decohered and dissipative quantum many-body systems

Tarun Grover
(UCSD)

Yu-Hsueh Chen (UCSD)

Simon Martin (UCSD)

Chia-Min Chung (Neils Bohr Inst.)

Peter Lu (Perimeter)

Tim Hsieh (Perimeter)

Kai-Hsin Wu (Boston U.)

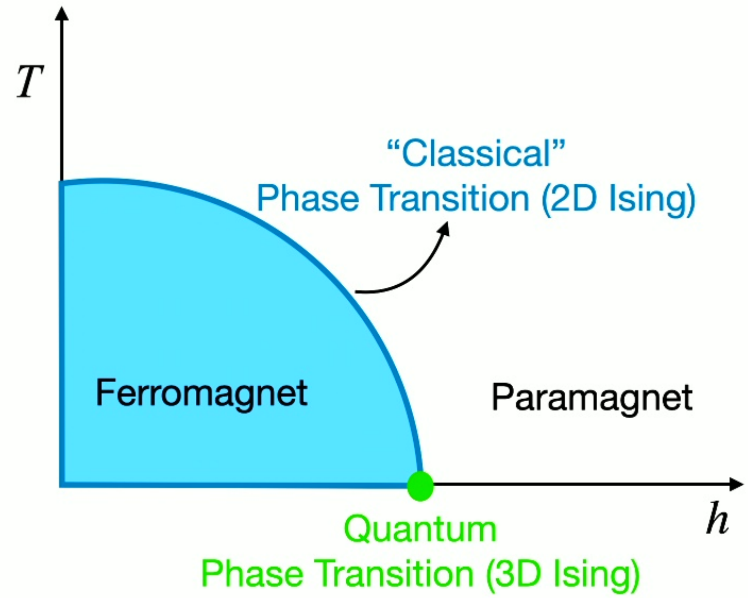
Ying-Jer Kao (National Taiwan U.)

Does quantum mechanics matter in an open quantum system?

Motivation #1:

Quantum Vs Classical Phase Transitions

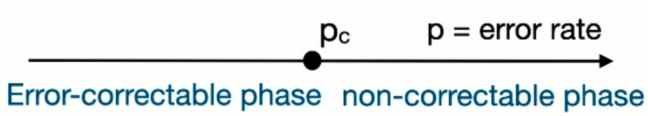
$H = \sum_{<i,j>} - Z_i Z_j - h \sum_i X_i$ on a square lattice:



Can there exist quantum phase transition at non-zero temperature?

Motivation #2:

Stability of “Quantum hard-drive” against Decoherence

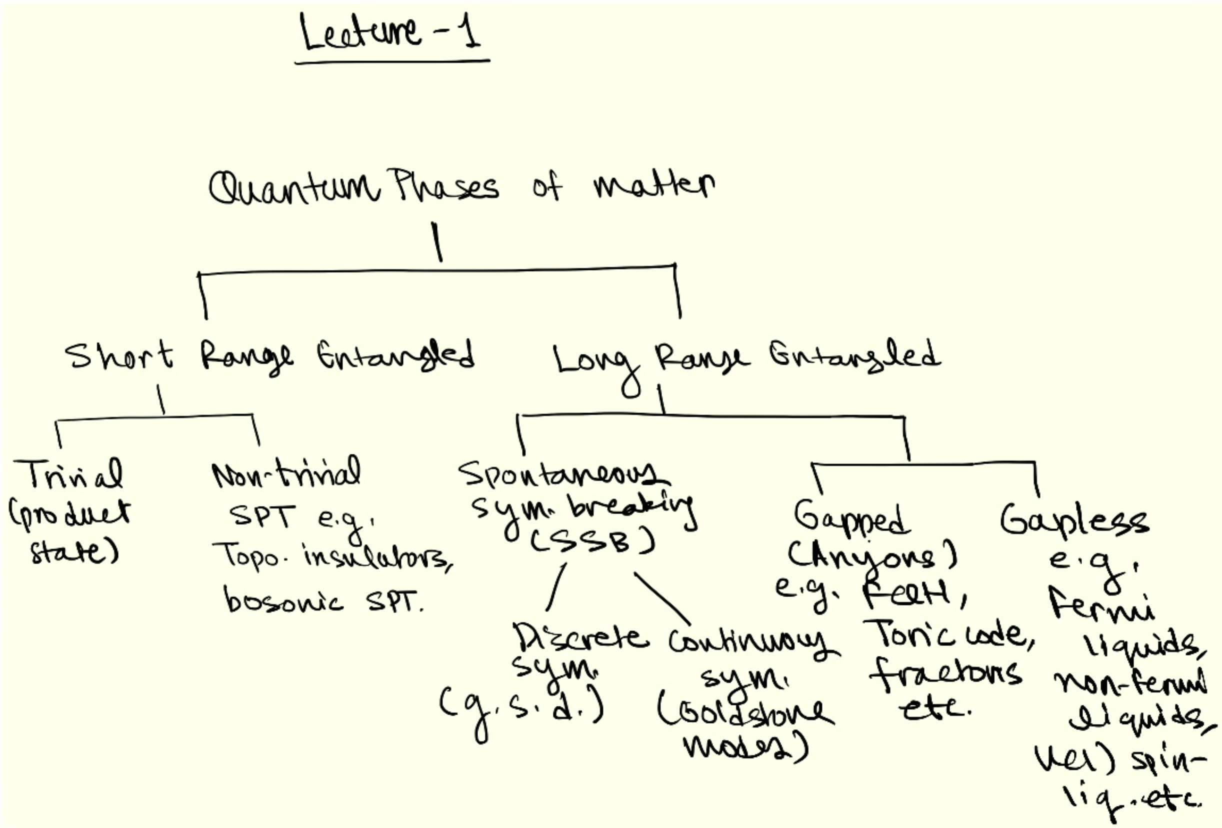


[Dennis, Kitaev, Landahl, Preskill 2001]

What is the nature of phase transition between the quantum coherent and incoherent regimes in a quantum computer?

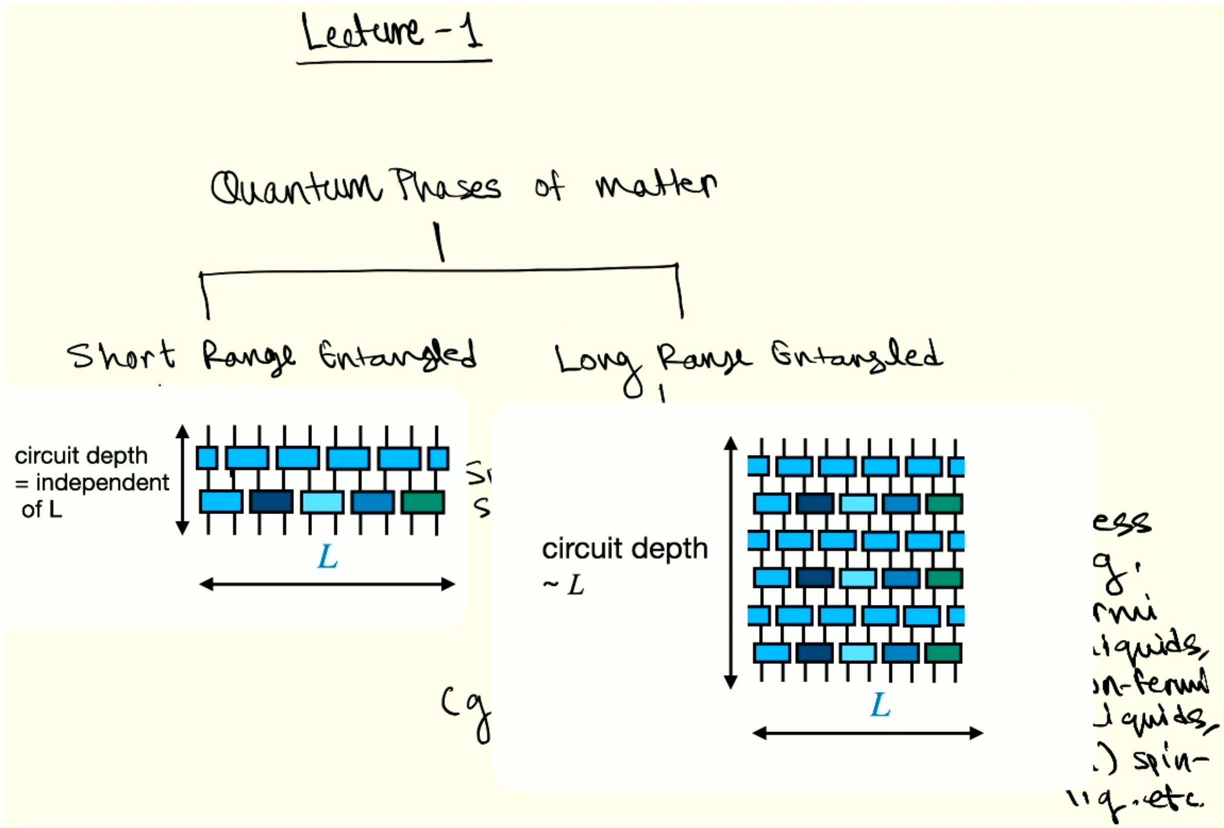
Motivation #3:

Quantum Phases of Matter beyond pure states



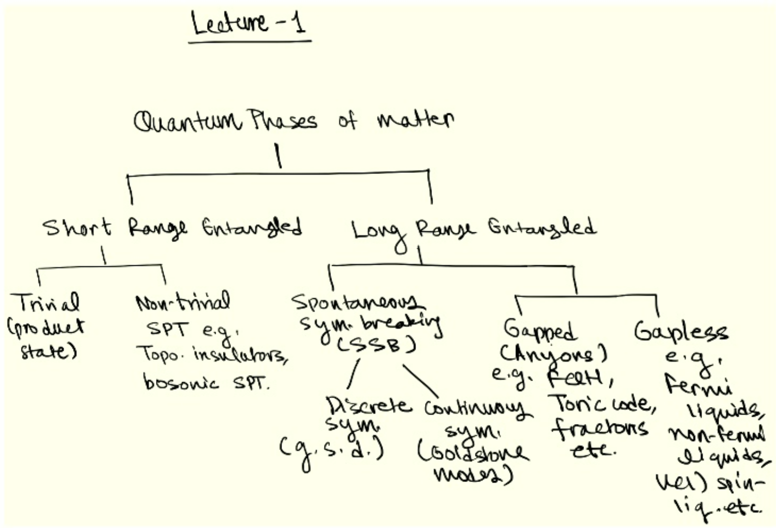
Motivation #3:

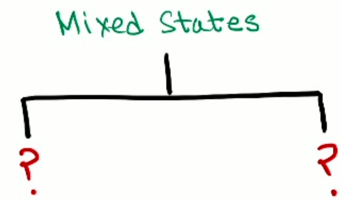
Quantum Phases of Matter beyond pure states



Motivation #3:

Quantum Phases of Matter beyond pure states





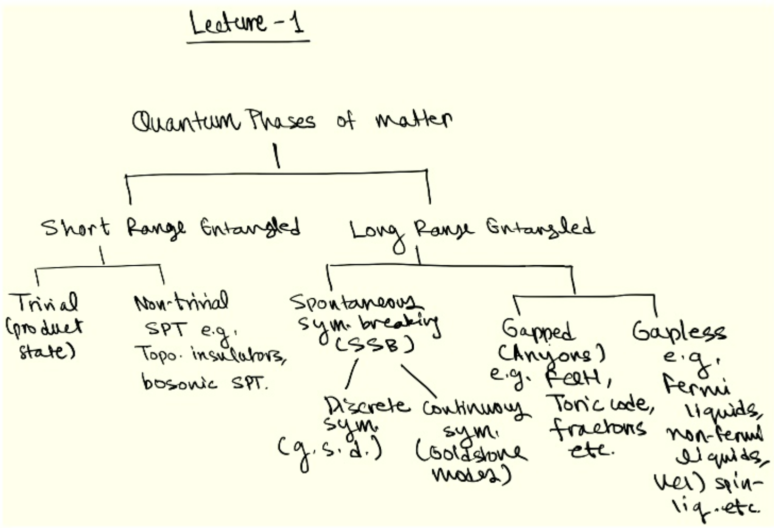
Interesting, non-fine tuned, mixed states?

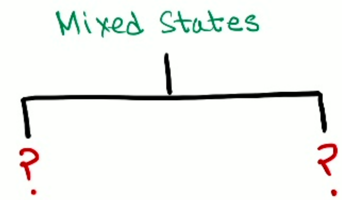
Gibbs states of k-local Hamiltonians, long-range entangled states + local decoherence, non-Markovian noise, engineered dissipation,…

More ideas needed…

Motivation #3:

Quantum Phases of Matter beyond pure states





Interesting, non-fine tuned, mixed states?

Gibbs states of k-local Hamiltonians, long-range entangled states + local decoherence, non-Markovian noise, engineered dissipation,…

More ideas needed…

Zeroth Order question:

When is a mixed state unentangled (“separable”)?

Separable (= Unentangled) Mixed States

[Werner 1989] If $\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i|$, with $p_i > 0$

where each $|\psi_i\rangle$ is unentangled between parties A and B i.e.

$|\psi_i\rangle = |\phi_{i,A}\rangle \otimes |\phi_{i,B}\rangle$, then ρ is bipartite separable (i.e. unentangled).

Consider $\rho = p\,|\psi_{Bell}\rangle\langle\psi_{Bell}| + (1-p)\frac{\mathbb{1}}{4}$

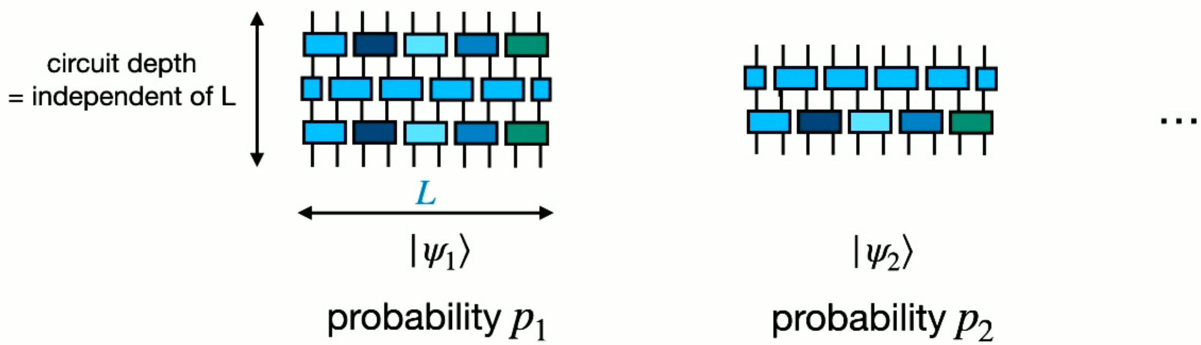
where $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$

Many-body analogs of such transitions?

Short-ranged entangled (SRE) mixed states
= generalization of separability to many-body setup

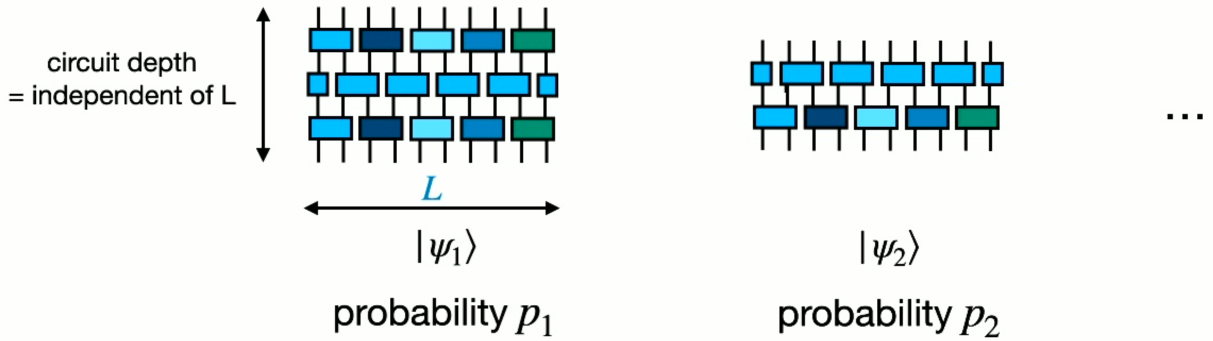
If a density matrix admits a decomposition $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ where each $|\psi_i\rangle$ is short-ranged entangled (i.e. can be prepared via a finite-depth, local, unitary circuit), then we will call ρ a "short-ranged entangled (SRE) mixed-state".

(Motivated from [Hastings 1106.6026])



Symmetric Short-ranged entangled (sym-SRE) mixed states

If a density matrix admits a decomposition $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ where each $|\psi_i\rangle$ is short-ranged entangled, and can be prepared via a finite-depth, local, unitary circuit composed of symmetric gates, then we will call ρ a "sym-SRE mixed-state".



Each local gate ▭ satisfies, [▭, U] = 0, where U is the generator of the symmetry.

Infinitely many decompositions of a density matrix into pure states (!)

$$\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i| = \sum_i p'_i \, |\psi'_i\rangle\langle\psi'_i| = \sum_i p''_i \, |\psi''_i\rangle\langle\psi''_i| = \ldots$$

No general algorithm to determine if it is SRE/LRE.

"All possible decompositions"
(Credit: Google Gemini)

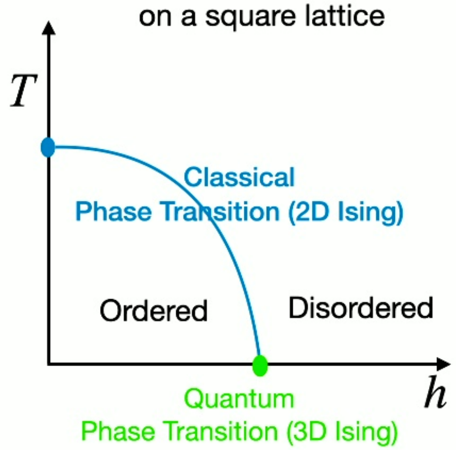
Some useful tools to make progress:

- Explicit demonstration of SRE decomposition.
- Symmetry arguments to show LRE.
- Lieb-Robinson bounds to show LRE.

Spontaneous symmetry breaking as a separability transition

$H = -\sum_{\langle i,j \rangle} Z_i Z_j - h \sum_i X_i$

on a square lattice



Claim:

The Gibbs state $\rho \propto e^{-H/T}$ is sym-LRE for $T < T_c$

Proof by contradiction: Assume ρ is sym-SRE for $T < T_c$.

$$\rho = \rho_+ + \rho_-$$

$$\rho_\pm = \left(\tfrac{1 \pm U}{2}\right)\rho \qquad U = \prod_i X_i$$

(generator of Ising symmetry)

$$U\rho_\pm = \pm \rho_\pm$$

$$\rho_\pm = \sum_\alpha p_{\alpha,\pm} |\psi_{\alpha,\pm}\rangle\langle\psi_{\alpha,\pm}|$$

$$\rho \text{ sym-SRE} \Rightarrow |\psi_{\alpha,\pm}\rangle \text{ SRE}$$

$$\Rightarrow \langle\psi_{\alpha,\pm}|Z_j Z_k|\psi_{\alpha,\pm}\rangle - \underbrace{\langle\psi_{\alpha,\pm}|Z_j|\psi_{\alpha,\pm}\rangle}_{0}\underbrace{\langle\psi_{\alpha,\pm}|Z_k|\psi_{\alpha,\pm}\rangle}_{0} \sim e^{-|i-j|/\xi}$$

$$\Rightarrow \mathrm{tr}\,(\rho Z_j Z_k) = \sum_\pm \sum_\alpha p_{\alpha,\pm}\langle\psi_{\alpha,\pm}|Z_j Z_k|\psi_{\alpha,\pm}\rangle \sim e^{-|i-j|/\xi}$$

Contradiction because of spontaneous long-range order for $T < T_c$

[Yu-Hsueh Chen, TG, 2310.07286; Argument inspired from Lu, Zhang, Hsieh, Vijay 2303.15507]

Spontaneous symmetry breaking as a separability transition

$H = -\sum_{\langle i,j \rangle} Z_i Z_j - h \sum_i X_i$

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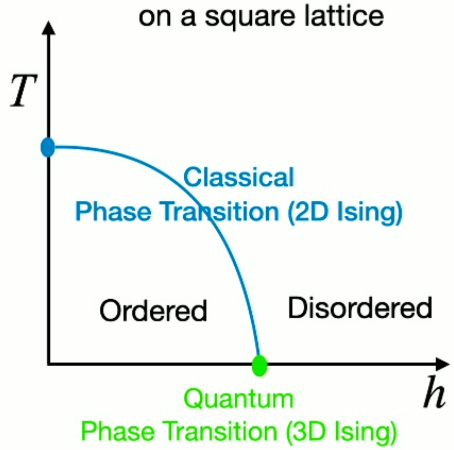


Claim:

The Gibbs state $\rho \propto e^{-H/T}$ is sym-LRE for $T < T_c$

sym-SRE decomposition for $T > T_c$:

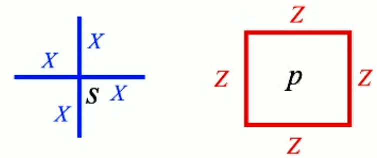
$$\rho = \sum_{x_{\mathbf{v}}} \sqrt{\rho}\, |x_{\mathbf{v}}\rangle\langle x_{\mathbf{v}}| \sqrt{\rho}$$

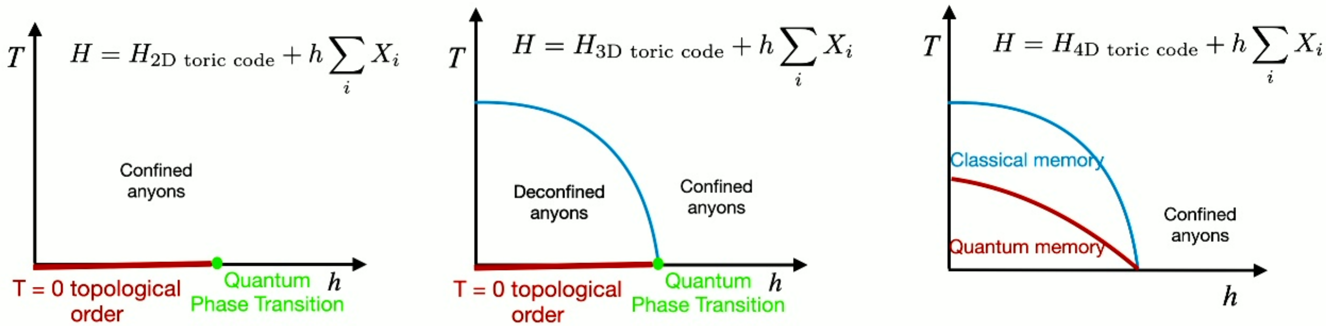
Claim: Pure states $\sqrt{\rho}\,|x_{\mathbf{v}}\rangle$ are SRE for $T > T_c$ and LRE for $T < T_c$.

Heuristic argument: Using field-theory arguments, correlations functions with respect to $\sqrt{\rho}\,|x_{\mathbf{v}}\rangle$ can be mapped to that in the 2d classical Ising model.

Another example: Separability in Gibbs state of toric code

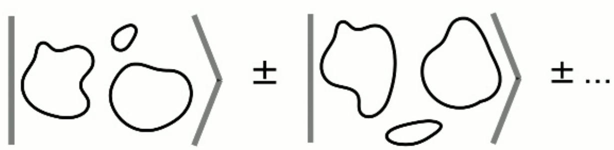
$$H = -\lambda_A \sum_s A_s - \lambda_B \sum_p B_p$$



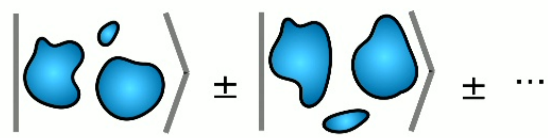


[Dennis, Kitaev, Landahl, Preskill 2001; Yoshida 2011; Hastings 2011]

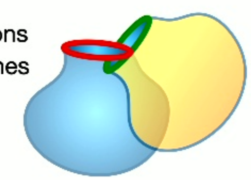
2D, 3D: eigenstates = superposition of closed loops



4D: eigenstates = superposition of closed membranes



anyonic excitations
= open membranes

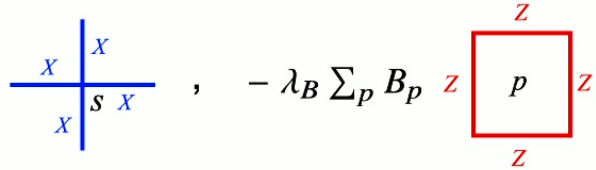


Another example: Separability in Gibbs state of toric code

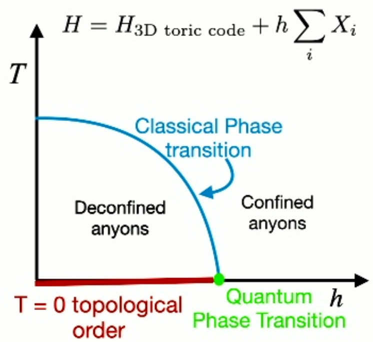
$$\rho = \frac{1}{Z}\sum_m \underbrace{e^{-\beta H/2}|m\rangle}_{=|\phi_m\rangle}\langle m|e^{-\beta H/2} = \frac{1}{Z}\sum_m |\phi_m\rangle\langle\phi_m|$$

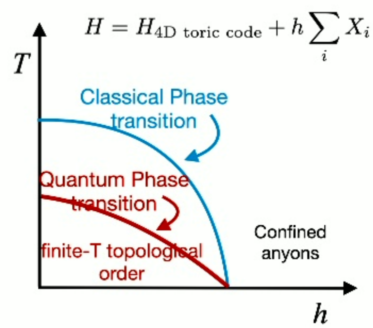
where $\{|m\rangle\}$ = complete set of product states in the X or Z basis.

One can argue that all $|\phi_m\rangle$ are SRE whenever $T > \min(T_A, T_B)$ where T_A, T_B correspond to the critical temperatures of the classical Hamiltonians

$-\lambda_A \sum_s A_s$  , $-\lambda_B \sum_p B_p$

Dimension	T_A	T_B
2D	$\frac{O(\lambda_A)}{\log L}$	$\frac{O(\lambda_B)}{\log L}$
3D	$\frac{O(\lambda_A)}{\log L}$	$O(\lambda_B)$
4D	$O(\lambda_A)$	$O(\lambda_B)$





[Tsung-Cheng Lu, Tim Hsieh, TG 2019]

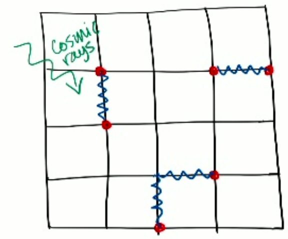
Decoding transition in toric code as an intrinsic mixed-state transition

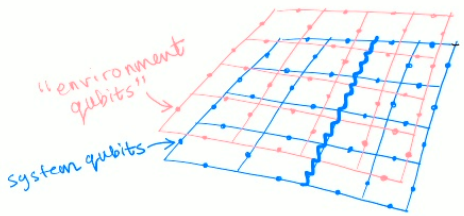
Active quantum error-correction (e.g. 2D toric code)

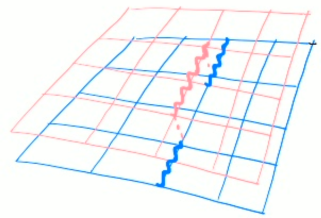
[Dennis, Kitaev, Landahl, Preskill 2001]

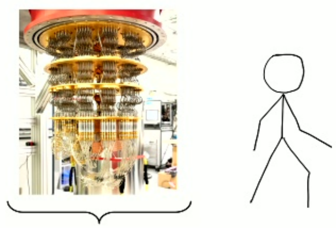
p_c p = error rate

Correctable phase   Non-correctable phase

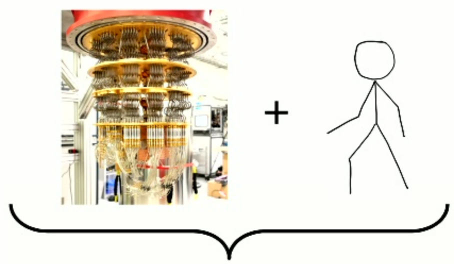








topologically ordered   Environment

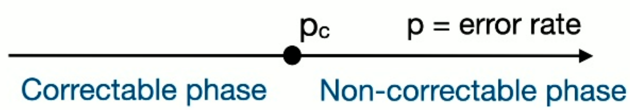


topologically ordered

Decoding transition in toric code as an intrinsic mixed-state transition

Recent works, in particular, Fan, Bao, Altman, Vishwanath [2301.05689; 2301.05687], and Lee, Jian, Xu [2301.05238] have formulated decoding transition as an intrinsic transition for the decohered mixed-state.

- Coherent information jumps across the transition from 2 log(2) to zero at $p = p_c$.
- Renyi negativity also shows a phase transition from log(2) to zero.

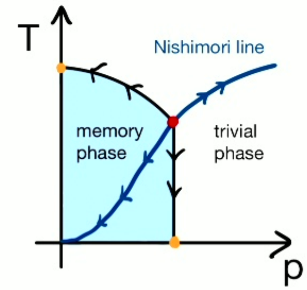


Is the density matrix SRE in the non-correctable phase?

Decoding transition as a separability transition

Key idea: Write decohered ρ as

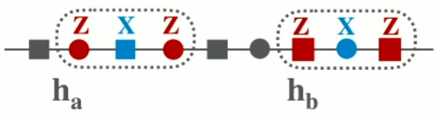
$$\rho = \sum_{z_{\mathbf{e}}} \underbrace{\rho^{1/2}|z_{\mathbf{e}}\rangle}_{=|\psi_m\rangle}\langle z_{\mathbf{e}}|\rho^{1/2} \equiv \sum_m |\psi_m\rangle\langle\psi_m|$$



All $|\psi_m\rangle$ undergo transition from topological to trivial precisely at p_c corresponding to the decoding transition.

[Yu-Hsueh Chen, TG, 2309.11879]

Symmetry enforced separability transitions in cluster states



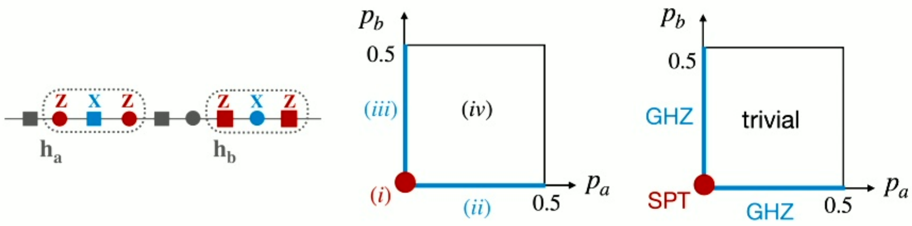
$$H = -\sum_{j=1}^{N} (Z_{b,j-1} X_{a,j} Z_{b,j} + Z_{a,j} X_{b,j} Z_{a,j+1})$$
$$= \sum_{j=1}^{N} h_{a,j} + h_{b,j}$$

Ground state $\rho_0 = \prod_j (1 - h_{a,j})(1 - h_{b,j})$ is a non-trivial SPT phase (i.e. sym-LRE) protected by $Z_2 \times Z_2$ symmetry.

Let's subject ρ_0 to the channel $\mathcal{E}_{a/b,j}[\rho] = (1 - p_{a/b})\rho + p_{a/b} Z_{a/b,j} \rho Z_{a/b,j}$

Is the resulting state sym-SRE at any non-zero p_a and/or p_b ?

Symmetry enforced separability transitions in cluster states



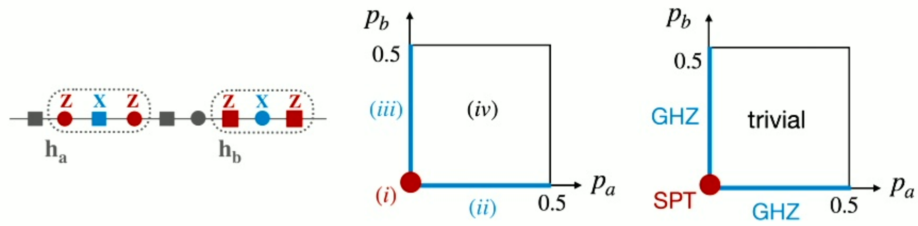
Result: ρ sym-LRE as long as $p_a = 0$ or $p_b = 0$ (regions i, ii, iii). sym-SRE if both p_a, p_b non-zero (region iv). Proof only uses Lieb-Robinson bounds and works in the whole SPT phase.

[Yu-Hsueh Chen, TG, 2310.07286]

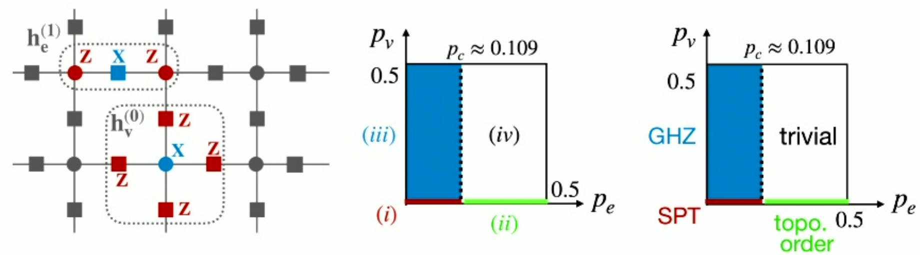
Related results by Ma, Wang [2209.02723], and Ma et al [2305.16399]: in regions i, ii, iii, ρ cannot be purified to an SRE pure state using a symmetric, finite-depth channel.

Symmetry enforced separability transitions in cluster states

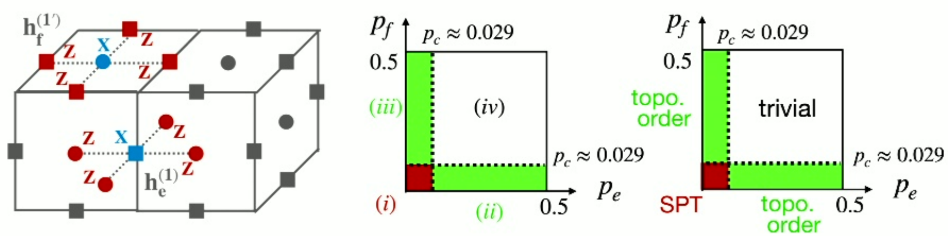
(a)



(b)



(c)



[Yu-Hsueh Chen, TG, 2310.07286]

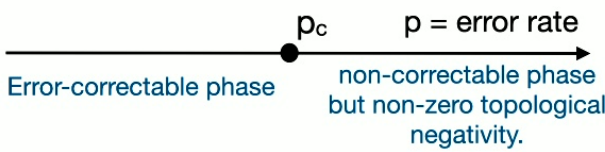
3d result related to finite-T SPT order (Roberts, Yoshida, Kubica, Bartlett, 1611.05450), however the universality for the separability transition is different (3d random plaquette gauge model).

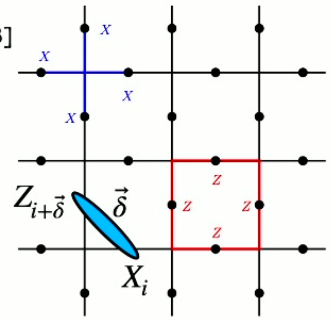
More exotic mixed states

"XZ" channel [Wang, Wu, Wang 2307.13758]

$$\rho \to p X_i Z_{i+\vec{\delta}} \, \rho \, X_i Z_{i+\vec{\delta}} + (1-p)\rho$$

Environment entangled with the fermion of toric code





$$H = -\lambda_A \sum_s A_s - \lambda_B \sum_p B_p$$

$A_s = \prod_{i \in s} X_i \quad B_p = \prod_{i \in p} Z_i$

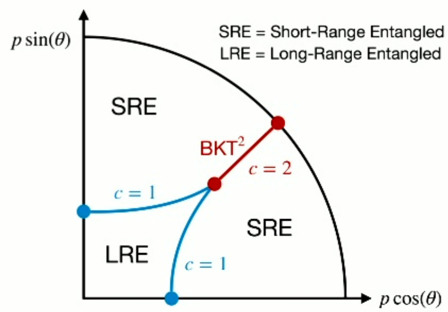
Separability perspective?

X+Z channel:

$$\rho \to p(X_i \cos(\theta) + Z_i \sin(\theta)) \, \rho \, (X_i \cos(\theta) + Z_i \sin(\theta)) + (1-p)\rho$$

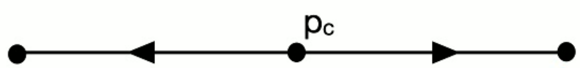
Non-trivial braiding between e and m leads to a critical phase for the double state $|\rho\rangle = \sum_{i,j} \rho_{i,j} |i\rangle |j\rangle$.

[Chen, TG, 2403.06553]

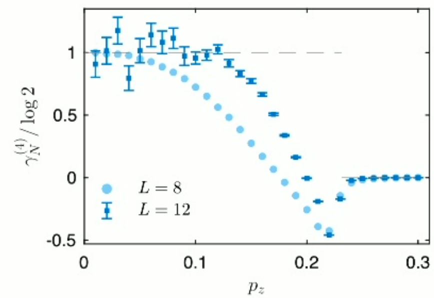


New mixed state phases due to non-Markovian bath?

Markovian (i.e. memoryless), local, baths tend to decrease entanglement.



RG flow for active quantum error-correction (e.g. in 2D toric code)



non-Markovianity can generate entanglement

Mixed-State Long-Range Order and Criticality from Measurement and Feedback

Tsung-Cheng Lu,[1,*] Zhehao Zhang,[2,†] Sagar Vijay,[2,‡] and Timothy H. Hsieh[1,§]
[1] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[2] *Department of Physics, University of California, Santa Barbara, California 93106, USA*

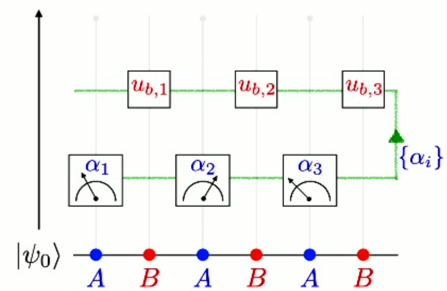


Figure from Lu, Zhang, Hsieh, Vijay 2303.15507.

Non-Markovianity rather common in solid-state physics...

Well-known example: single impurity Kondo problem.
Electrons act as a non-Markovian bath for the impurity spin.

To make the subsystem, i.e., localized spin a many-body system, let's instead consider a *spin-chain* coupled to fermions.

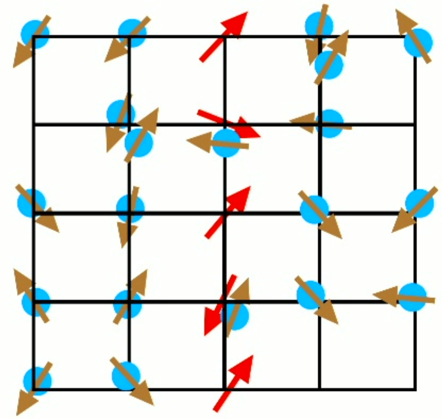
Kinetic energy of fermions with Fermi surface

Kondo interaction

$$\hat{H} = -t \sum_{\langle \boldsymbol{i},\boldsymbol{j} \rangle} (\hat{\boldsymbol{c}}_{\boldsymbol{i}}^{\dagger} \hat{\boldsymbol{c}}_{\boldsymbol{j}} + \mathrm{H.c}) + \frac{J_k}{2} \sum_{\boldsymbol{l}=1}^{L} \hat{\boldsymbol{c}}_{\boldsymbol{l}}^{\dagger} \boldsymbol{\sigma} \hat{\boldsymbol{c}}_{\boldsymbol{l}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{l}}$$

$$+ J_h \sum_{\boldsymbol{l}=1}^{L} \hat{\boldsymbol{S}}_{\boldsymbol{l}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{l}+\Delta \boldsymbol{l}}.$$

1d spin-system



Integrating out fermions generates dissipation for the spins ("Landau damping")

Field theory for d=1 Spin Chain coupled to d > 1 Free fermions

Weak-coupling approach: $SU(N)_k$ WZW CFT perturbed by dissipation

Kinetic energy $\quad S_{\text{Grad}} = \frac{1}{\lambda}\int d\tau dx \, \operatorname{tr}\left(\frac{1}{c^2}\partial_\tau g \partial_\tau g^{-1} + \partial_x g \partial_x g^{-1}\right)$ (g = SU(N) matrix)

Wess-Zumino-Witten term ("Berry phase") $\quad S_{\text{WZ}} = -\frac{\mathrm{i}\,k}{12\pi}\int d^3y\, \epsilon^{ijk}\, \operatorname{Tr}[g^{-1}\partial_i g g^{-1}\partial_j g g^{-1}\partial_k g]$

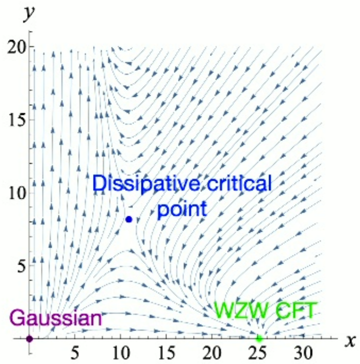
Dissipation $\quad S_{\text{Dis}} = k^2\gamma \int d\tau d\tau' dx\, K(\tau-\tau')\, \operatorname{tr}[\mathbb{1} - g(\tau,x)g^{-1}(\tau',x)]$

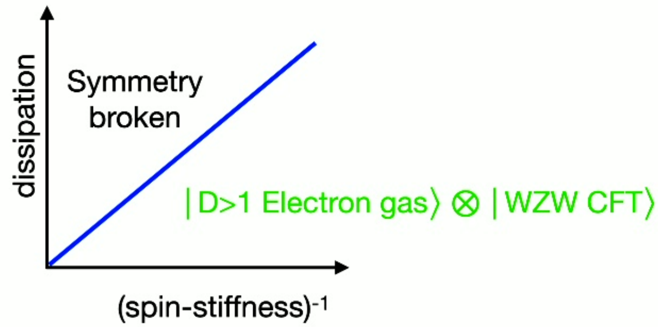
$$K(\tau-\tau') = \frac{A}{|\tau-\tau'|^{3-\delta}} \quad \delta = \tilde{\delta}/k \;,\; \tilde{\delta} = O(1)$$

Controlled large-k limit, analogous to Ed Witten, Comm. Math. Phys. 92, 455 (1984).

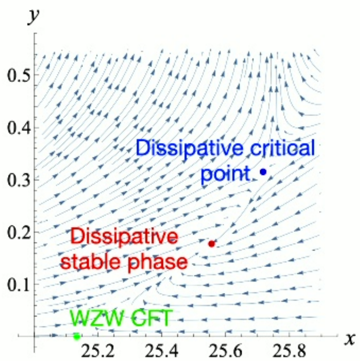
[Simon Martin, TG, 2307.13889]

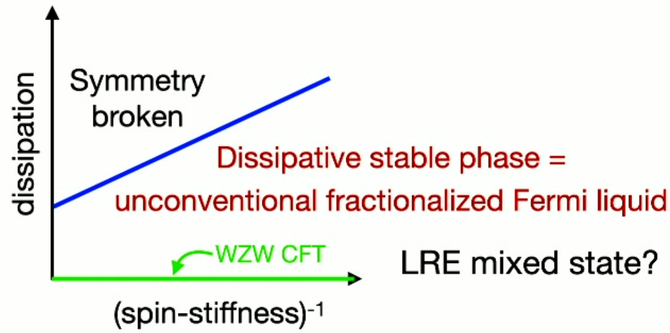
Weak dissipation



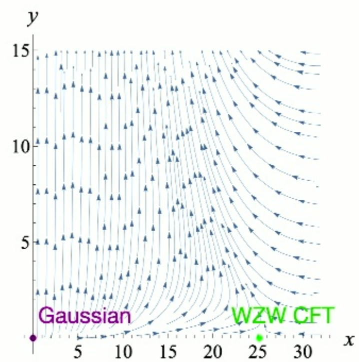


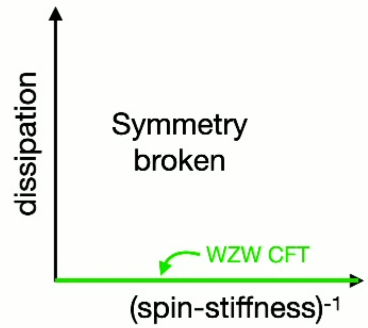
Intermediate dissipation





Strong dissipation





Summary and some future directions

- Separability provides an organizing principle to characterize mixed states as long range or short range entangled, with or without imposing symmetry. Seems to subsume partition-function based definition of equilibrium phases while also giving insights into non-equilibrium settings.

- Finer classification of LRE mixed states? Renormalization group perspective [Sang, Zou, Hsieh, 2310.08639] an important step. Relation to separability?

- Tempting to define complexity of a mixed state as $C(\rho) = \min \sum_i p_i C(\psi_i)$ where the minimum is taken over all possible decompositions $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ of the mixed state ρ, and $C(\psi_i)$ is the complexity of the pure state $|\psi_i\rangle$.

  By definition, $C(\rho)$ is constant for SRE mixed states and scales with system size for LRE mixed states. What is the scaling of $C(\rho)$ at phase transitions from an LRE to an SRE mixed state?