Title: Reducing the overhead of quantum error correction

Speakers: Aleksander Kubica

Series: Perimeter Institute Quantum Discussions

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Abstract: Fault-tolerant protocols and quantum error correction (QEC) are essential to building reliable quantum computers from imperfect components that are vulnerable to errors. Optimizing the resource and time overheads needed to implement QEC is one of the most pressing challenges that will facilitate a transition from NISQ to the fault tolerance era. In this talk, I will discuss two intriguing ideas that can significantly reduce these overheads. The first idea, erasure qubits, relies on an efficient conversion of the dominant noise into erasure errors at known locations, greatly enhancing the performance of QEC protocols. The second idea, single-shot QEC, guarantees that even in the presence of measurement errors one can perform reliable QEC without repeating measurements, incurring only constant time overhead.

Zoom link

Reducing the overhead of quantum error correction

Aleksander Kubica

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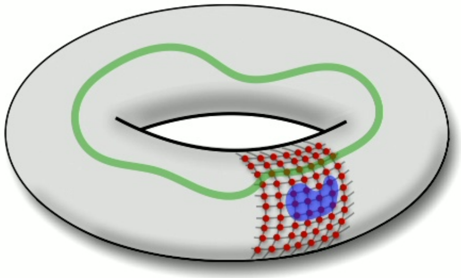
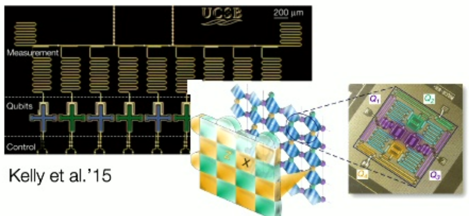


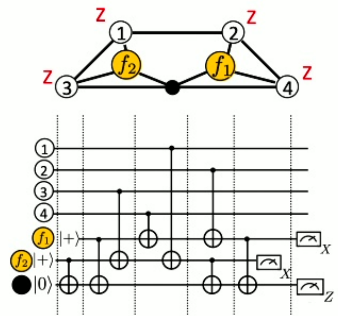
Perimeter Institute

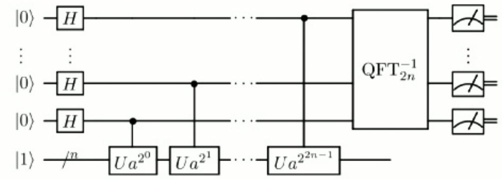
May 1, 2024

A path to fault tolerance

physical system & operations → quantum error correction (QEC) → quantum algorithms

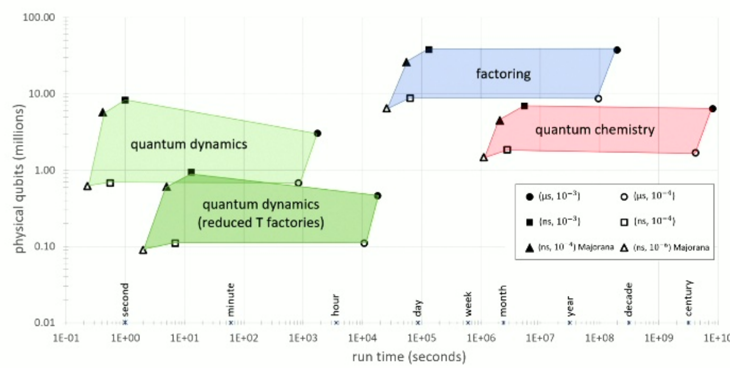

Kelly et al.'15
Córcoles et al.'15





- Quantum algorithms require:
  — error rates $\sim 10^{-10}$–10^{-15},
  — fast logical clock speed.

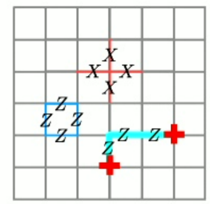
- The space and time overheads of QEC are a major roadblock!



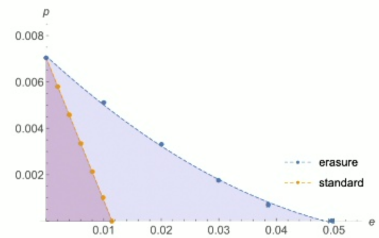
Beverland et al.'22

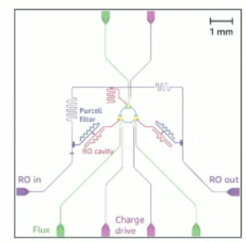
Motivation & outline

- In this talk:
  — reducing the qubit and time overheads of QEC.

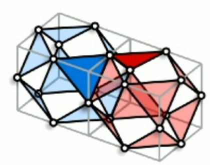


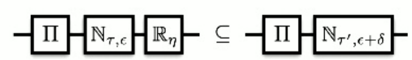
1. Decoding problem:
   — toric/surface code & biased noise.





2. Erasure qubits:
   — **AK**+22, arXiv:2208.05461,
   — Gu,**AK**+, arXiv:2312.14060,
   — Levine,**AK**+23, arXiv:2307.08737.

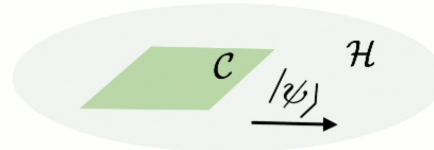




3. Single-shot QEC:
   — **AK**,Vasmer21, arXiv:2106.02621,
   — Gu,**AK**+23, arXiv:2306.12470.

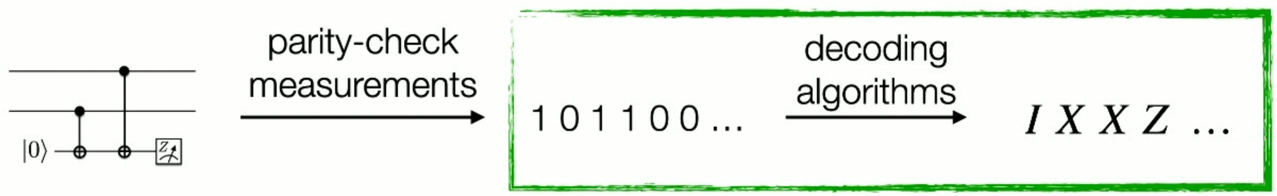
Detecting & correcting errors

- Quantum code = a subspace of the Hilbert space.



- Errors take the encoded state $|\psi\rangle$ outside $\mathscr{C}$.

- Gather information about errors w/o revealing the encoded information:



- Decoding problem: given the syndrome, find a recovery operator.

- Processing of classical information needs to be fast:
  — the backlog problem [Terhal'15],
  — computationally-hard problem [Iyer,Poulin'15].

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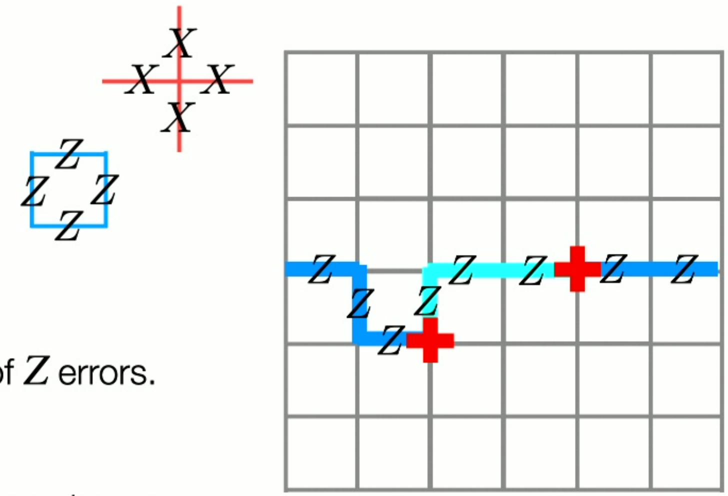
Toric/surface code & decoding problem

- Toric/surface code [Kitaev97]:
  — qubits on edges of a lattice,
  — +1 eigenspace of parity checks.

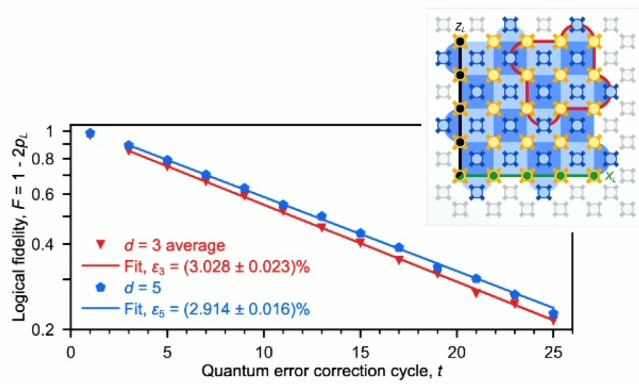


- Measuring X parity checks:
  — syndrome = endpoints of strings of Z errors.

- Different Z errors can have the same syndrome.

- Decoding problem = matching problem.

- Min-weight perfect matching [Dennis+02]:
  — good performance & practically useful,
  — further improvements [Higgott,**AK**+23].



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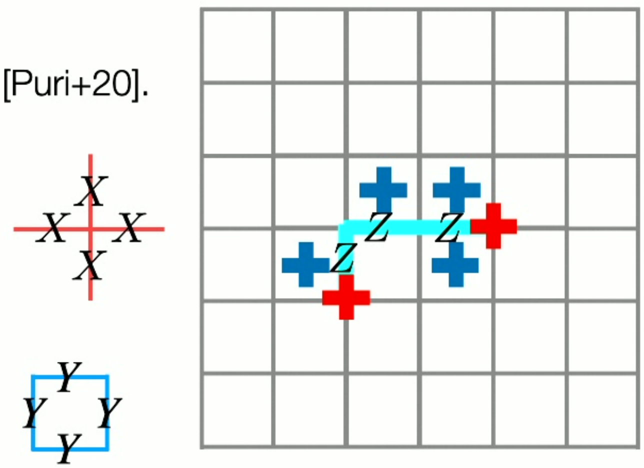
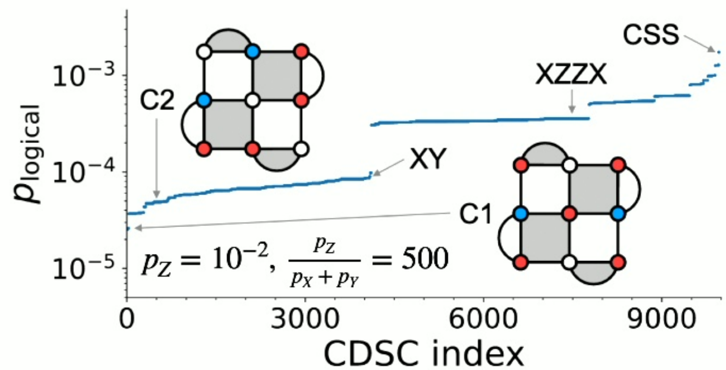
Exploiting bias of Pauli noise

- Biased Pauli noise $p_Z \geq p_X = p_Y$:
  — bosonic qubits & bias-preserving gates [Puri+20].
- Can we leverage the noise bias?
  — Yes [Tuckett+18;Bonilla+21;…].
- Apply Clifford operators to modify parity checks [Dua,**AK**+22;Vasmer,**AK**22].
- We can lower the logical error rate & reduce the qubit overhead!





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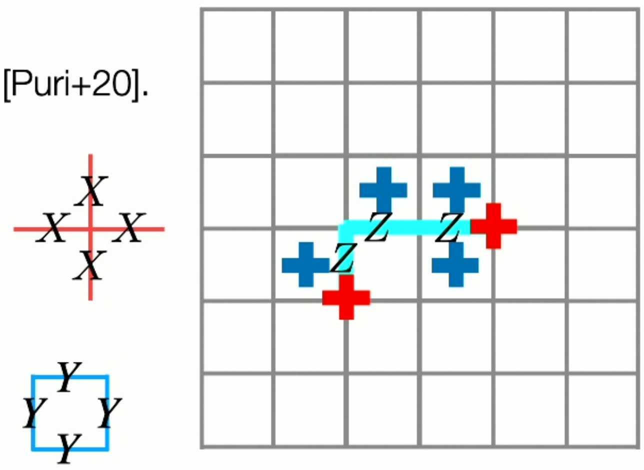
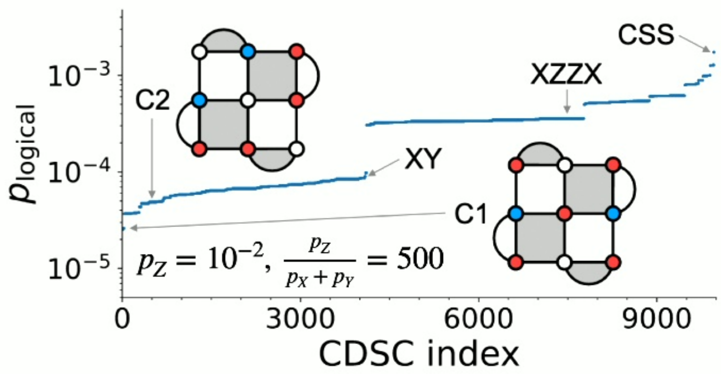
  — Yes [Tuckett+18;Bonilla+21;…].
- Apply Clifford operators to modify parity checks [Dua,**AK**+22;Vasmer,**AK**22].
- We can lower the logical error rate & reduce the qubit overhead!
- Can there be other noise bias?





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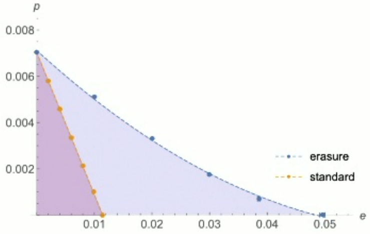

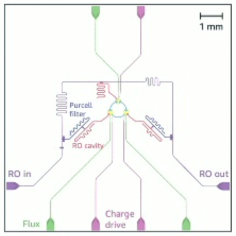

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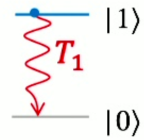
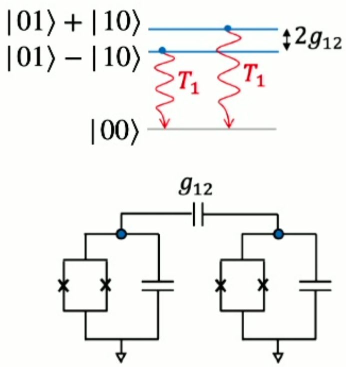
Exploiting other bias: $T_1 \leq T_\phi$

- Amplitude damping noise $\mathscr{A}$:
  — energy relaxation from $|1\rangle$ to $|0\rangle$ w/ probability γ.



- Correcting amplitude damping noise:
  — 4-qubit code [Leung+98],
  — surface-code threshold of 39% [Darmawan,Poulin16].

- Dual-rail encoding [Duan+10]:
  — qubit subspace $\mathrm{span}\{|01\rangle, |10\rangle\}$,
  — $\mathscr{A}^{\otimes 2}(\rho) = (1-\gamma)\rho + \gamma |00\rangle\langle 00|$.



- Erasure qubits [Wu+22;**AK**+22;Kang+22]:
  the effective noise is erasure-biased.

Correcting erasures is simple

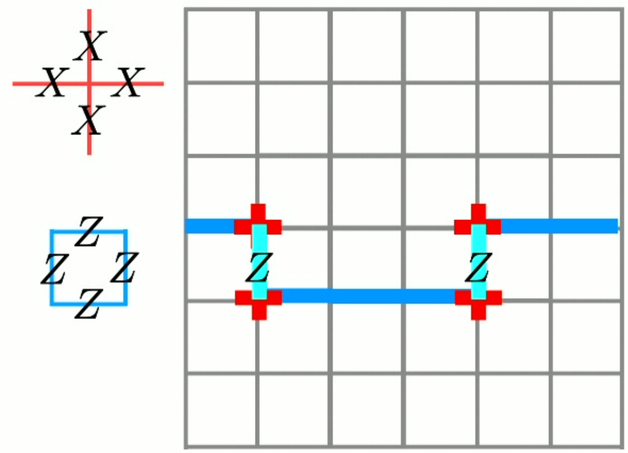
- Qubits at known locations are erased:
  — $\rho \mapsto \sum_P P\rho P^\dagger$.

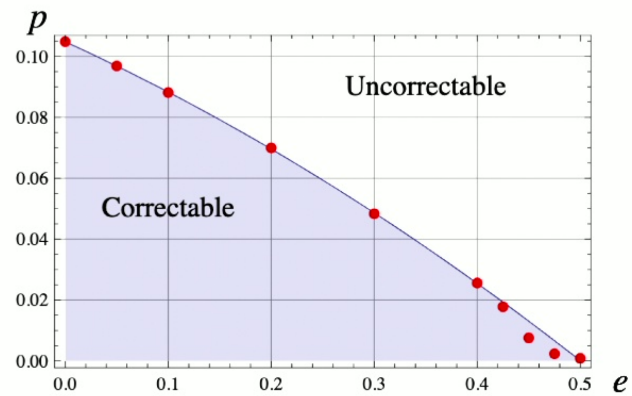
- Extremely high threshold $p_{\text{th}} = 0.5$:
  — bond-percolation threshold [Stace+09].

- We can also have Pauli noise.

- Computationally-efficient decoders:
  — min-weight perfect matching,
  — union-find [Delfosse,Nickerson21].

- Correctable region in the (e, p) plane.

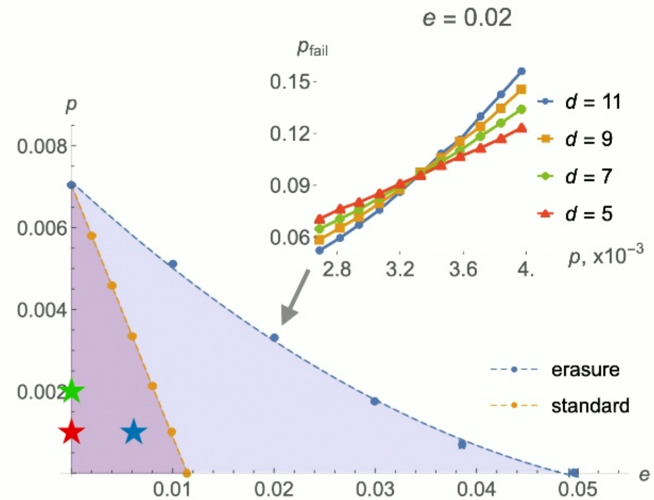




Stace et al., PRL'09

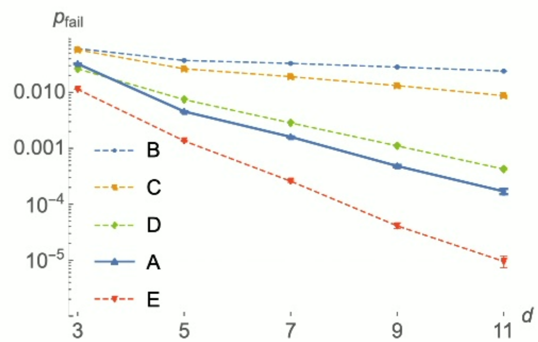
Numerical results

- Heralded erasures & Pauli noise:
  — standard circuit noise; rate p,
  — CNOT erases both qubits; rate e.



- Boost of the toric code performance:
  — correctable region increases by 3.5x!

- Subthreshold performance:
  — erasure (solid A),
  — standard (dashed B).



	p	p_M	e
B	0.1%	2%	0.6%
C	0.1%	0.1%	0.6%
D	0.2%	2%	0%
A	0.1%	2%	0.6%
E	0.1%	2%	0%

- We can lower the logical error rate & reduce the qubit overhead!

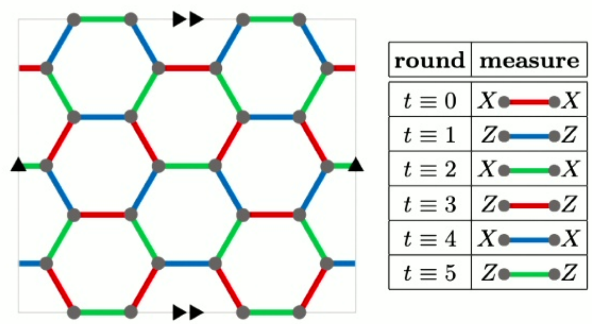
How to realize erasure qubits

A couple of comments

- Comparison of resources:

erasure scheme	standard scheme
2 transmons, 1 readout resonator	1 transmon, 1 readout resonator
$< \sqrt{n/4}$ erasures	$< \sqrt{n/8}$ unknown Paulis

- Erasure qubits benefit QEC codes:
  — Floquet codes [Gu,**AK**+23].



- Imperfect erasure detection:
  — false positive q^+ & negative q^-,
  — erasure spread.

- Rule of thumb target noise rates:
  $- \, e \lesssim 1\,\%\,, p \lesssim 0.1\,\%\,, q^\pm \lesssim 1\,\%\,.$

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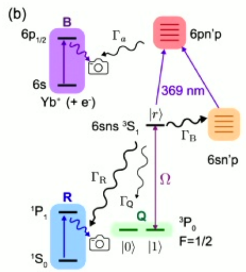
How to realize erasure qubits

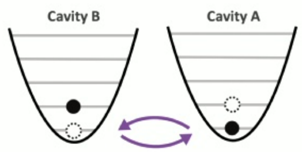
Neutral atoms

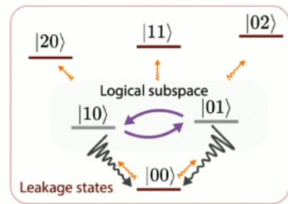


Y. Wu, et. al. *Nat. Comm.* **13**, 4657 (2022)
K. Sahay, et. al. *arXiv*:2302.03063 (2023)

Superconducting dual-rail encoding

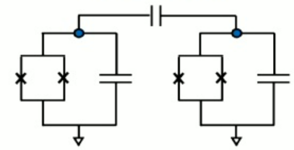
Microwave cavities





J. Teoh, ..., S. Puri, R. Schoelkopf. *arXiv*:2212.12077 (2022)

Transmons



A. Kubica*, A. Haim*, Y. Vaknin*, et. al. *arXiv*:2208.05461 (2022)

- Key idea: detectable leakage to the $|00\rangle$ state.

- Requirements on erasure qubits:
  — large erasure noise bias,
  — ability to detect erasure errors.

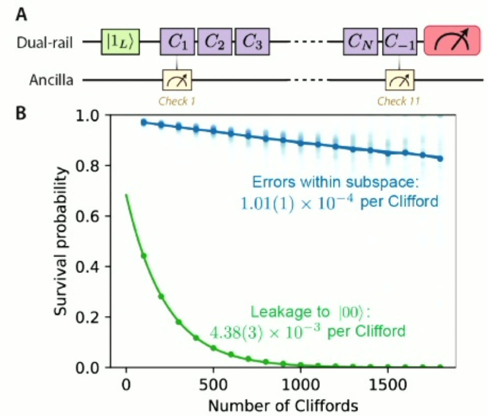
Experimental demonstration

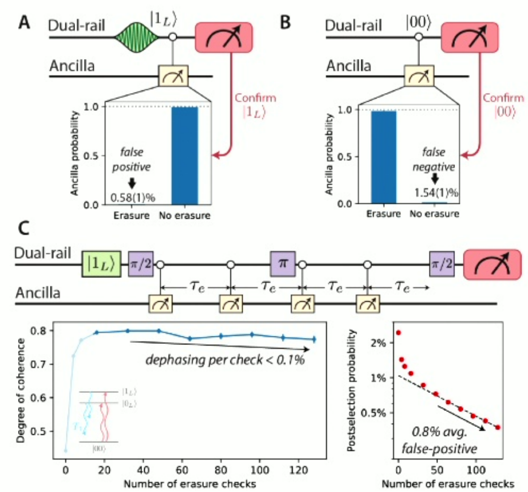
- Randomized benchmarking w/ erasure checks:
  — erasure rate 4.4×10^{-3},
  — residual error rate 10^{-4}.

- False positive and false negative detection:
  — rate around 10^{-2}.

- Repeated erasure checks within fixed time:
  — dephasing below 10^{-3}.

- Performance compatible with the QEC targets on the erasure qubits!





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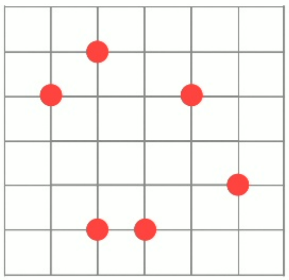
QEC itself is a noisy process…

- Parity-check outcomes may be incorrect:
  — matching impossible!



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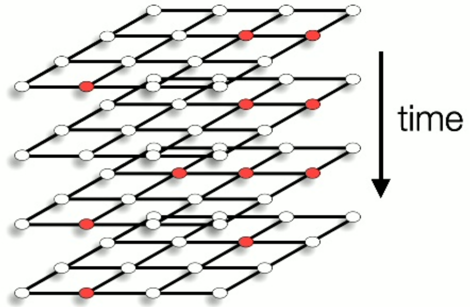


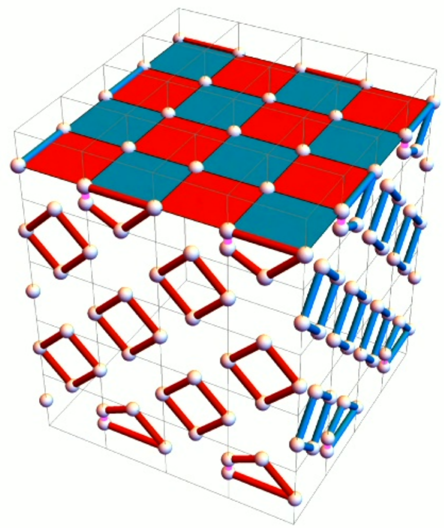
- Standard solution is to repeat measurements:
  — QEC-induced slowdown $t_{\text{logical}} \sim d \times t_{\text{physical}}$,
  — time-correlated errors.

- We can avoid repeating measurements [Fujiwara14;Campbell19;Delfosse+20;…]:
  — sacrificing geometric locality & const. weight of parity checks.

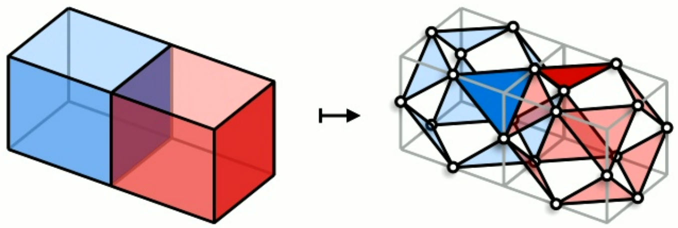
- The 3D subsystem toric code [**AK**,Vasmer22]:
  — new topological quantum code capable of single-shot QEC [Bombín15],
  — no QEC-induced slowdown!

3D subsystem toric code

- Simple realization on the cubic lattice:
  — qubits on edges.

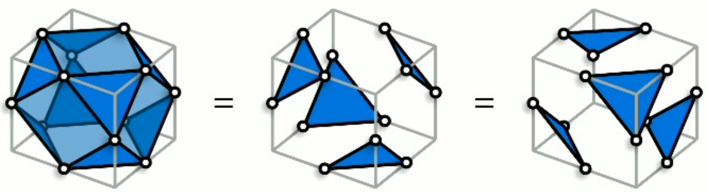


- Parity checks—Pauli Z (blue) & X (red).



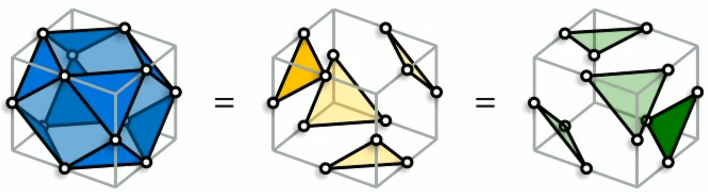
- Some parity checks do not commute!

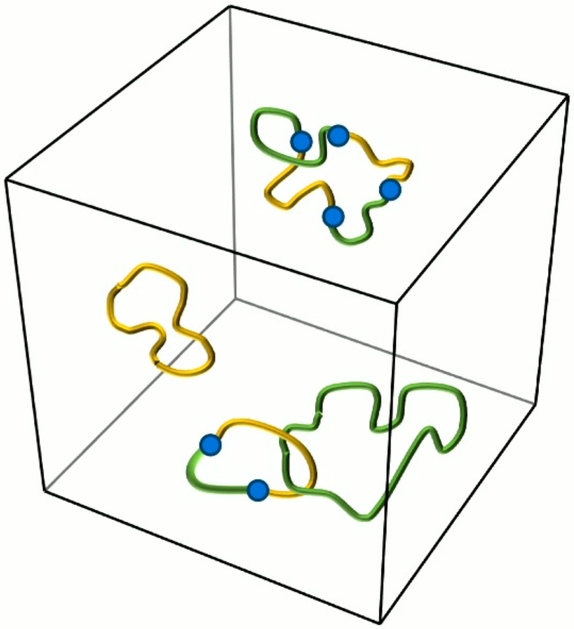
- Stabilizer operators:
  — commute w/ parity checks,
  — provide the syndrome.



Decoding problem

- Similar to the 2D toric code decoding:
  — the syndrome = endpoints of string-like errors.
- We measure stabilizer operators indirectly:

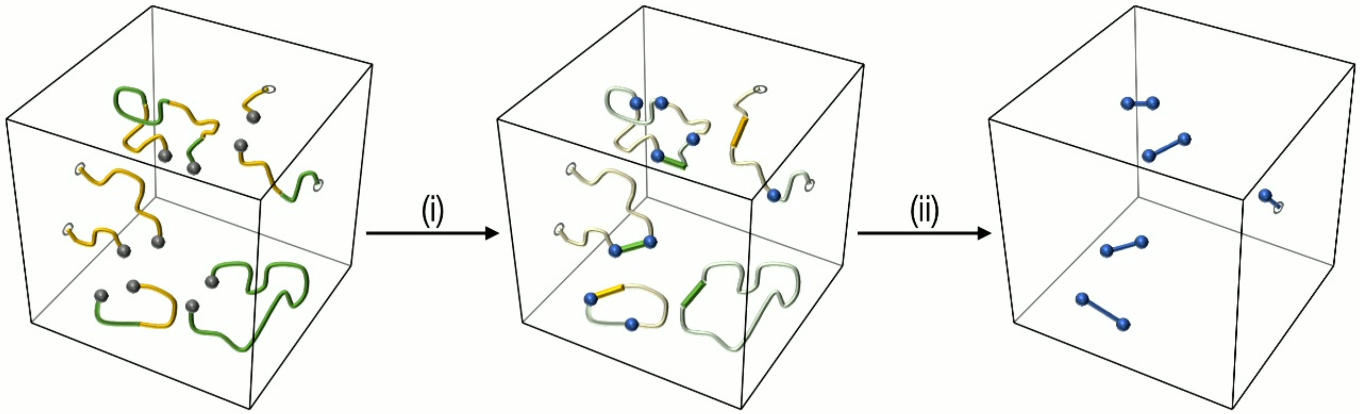




- Parity-check outcomes satisfy meta-checks.
- "Gauss's law":
  — the flux = loop-like objects,
  — the syndrome = locations where the flux changes color.

Single-shot decoder

- In the presence of measurement errors the flux does not form loop-like objects!



- Single-shot decoder:
  (i) repair loop-like structure,
  (iii) find an appropriate correction.

Min-Weight
Perfect Matching

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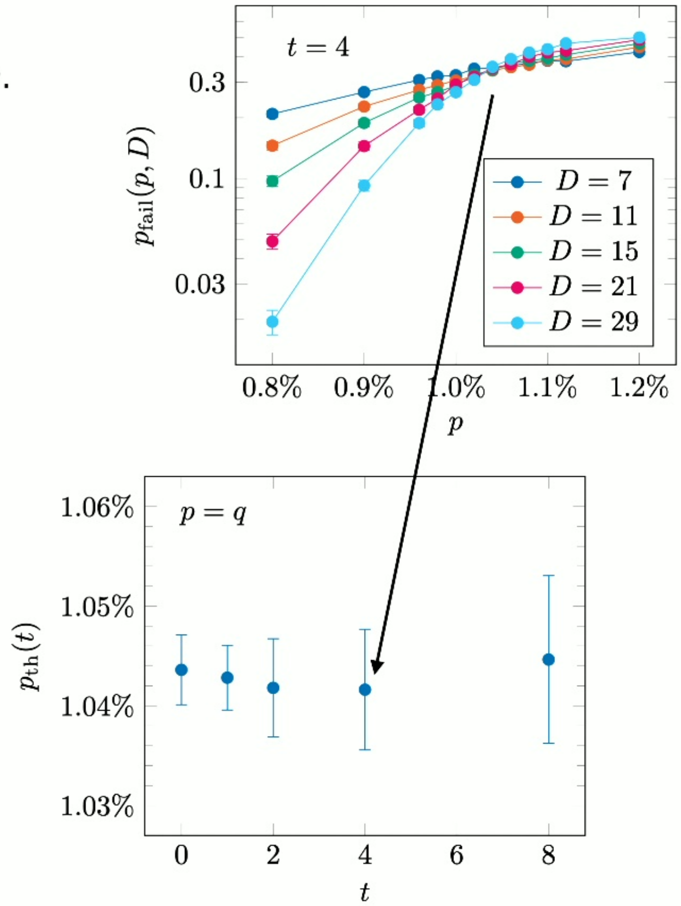
Numerical results

- IID Pauli X & measurement errors w/ rate p.
- We study how errors accumulate during repeated t rounds of QEC:
  — add new errors,
  — measure parity checks,
  — single-shot MWPM decoding.
- Storage threshold $p_{\text{STC}} = \lim_{t\to\infty} p_{\text{th}}(t)$.
- 3.5x the 3D gauge color code threshold [Brown+15]. The separation further increase for the circuit noise!



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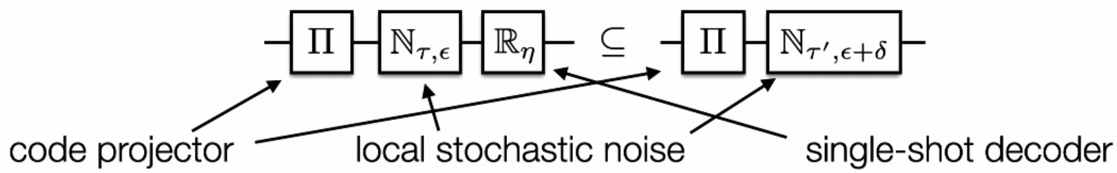
Proving single-shot QEC

- Single-shot decoder leaves residual noise, but keeps it under control.
- **Theorem**: for sufficiently small η & τ there exist τ' & δ, such that we have



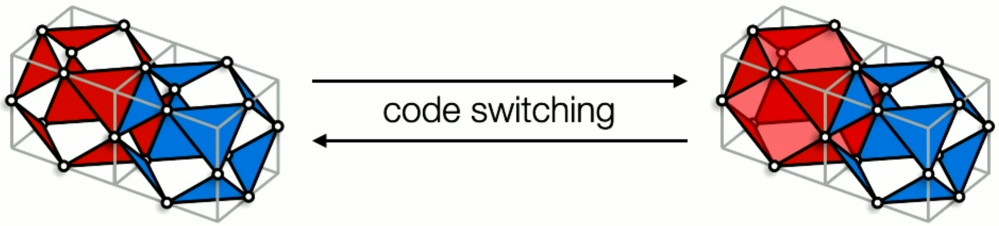
- Residual noise suppressed if measurement errors suppressed: $\lim_{\eta\to 0}\tau' = 0$.
- Increase in logical error rate vanishes as system size increases: $\lim_{L\to\infty}\delta = 0$.
- After t rounds of single-shot QEC we have $\left(\mathbb{R}_\eta \circ \mathbb{N}_{\tau,\epsilon}\right)^t \circ \Pi \subseteq \mathbb{N}_{\tau',t(\epsilon+\delta)} \circ \Pi$, i.e.,

— residual noise stays τ'-bounded,

— logical error rate grows linearly in t.

A couple of remarks

- What does the 3D subsystem toric code have to do w/ the 3D stabilizer toric code?



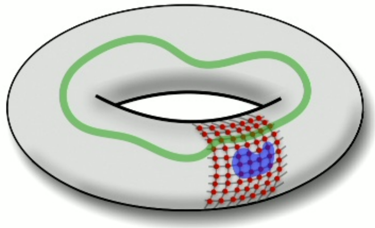
- Small weight is important—easier to realize & errors are not amplified by syndrome extraction circuits.

- The first genuine subsystem version of the toric code:
  — connection between the toric & color codes [**AK**+15].

- The subsystem toric code can be defined in $d \geq 3$ dimensions; also, for any finite Abelian group [Bridgeman,**AK**+23;Li+23].

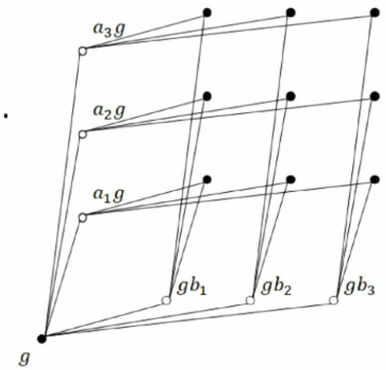
Beyond topological codes

- Limitations on QEC codes w/ geometrically-local checks in D dimensions:
  — [Bravyi+10;Haah21]: $n \sim L^D$, $k \sim L^{D-2}$, $d \sim L^{D-1}$.



- "Good" QEC codes exist [Calderbank,Shor96]:
  — code parameters $k \sim n$ & $d \sim n$.

- Quantum low-density parity-check (LDPC) codes:
  — const. weight of checks but no geometric locality!

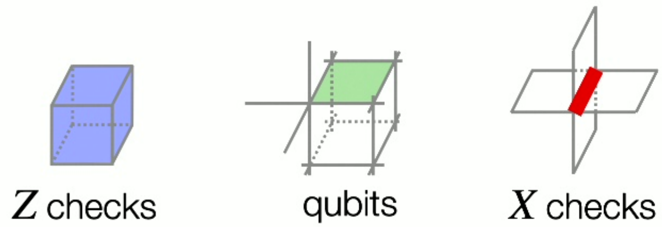


- "Good" quantum LDPC codes exist [Pantaleev,Kalachev22].

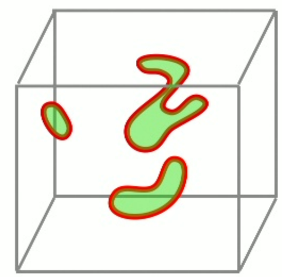
- Quantum Tanner codes [Leverrier,Zemor22]:
  — a left-right Cayley complex $\mathrm{Cay}_2(G, A, B)$,
  — a pair of local classical codes.

Idea behind the decoder

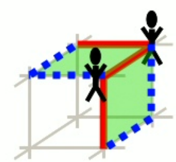
- Crude analogy:
  — the 3D stabilizer toric code.



- Decoding problem (for Z errors):
  — sheet-like errors & loop-like syndrome.



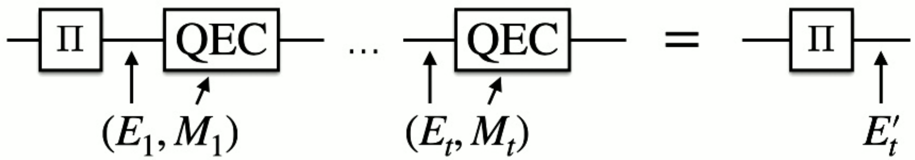
- Local decoders work [**AK**,Preskill19]:
  — require multiple iterations,
  — need to avoid local inconsistencies!



- Decoder for the quantum Tanner codes [Leverrier,Zemor22;Gu+22]:
  (i) local greedy decoder to reduce the syndrome,
  (ii) sequential/parallel decoder to reduce "the mismatch".

Single-shot QEC w/ quantum Tanner codes

- Setting: t repeated rounds of QEC, Pauli errors E_i & measurement errors M_i.



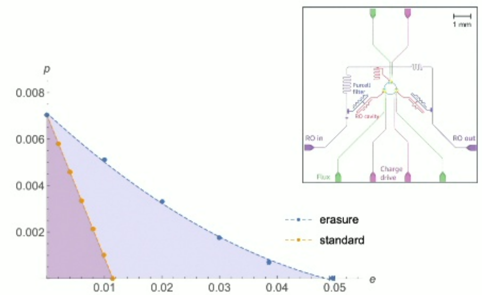
- **Theorem**: for any $\epsilon > 0$ there exists $\delta > 0$, s.t. $\max(|E_i|, |M_i|) \leq \delta n$ for each i, then the residual error satisfies $|E'_t| \leq \epsilon n$.

- Each QEC: const. number of iterations of a local decoder.

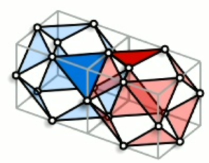
- No other assumptions about the noise:
  — it can be correlated,
  — adversarial & local stochastic noise on the same footing.

Summary

- Reducing the overhead of QEC.
- Erasure qubits:
  — conversion of dominant noise to erasures,
  — hardware-efficient QEC protocols.



- Single-shot QEC:
  — efficient method of handling measurement errors,
  — topological & quantum LDPC codes.



- Many questions still remain open.
- THANKS!