Title: The Corners of 1+1 Dimensional Quantum Gravity

Speakers: Ludovic Varrin

Series: Quantum Gravity

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Abstract: The concept of symmetries is crucial in our comprehension of modern theoretical physics. The Corner Proposal introduces a novel framework where symmetries are reinstated as foundational principles in our understanding of gravity. This aims to describe gravity using a language that is more adapted to quantization. In this presentation, I will start by providing an overview of the essential results leading to the main ideas the proposal. This will then allow me to state the proposal in the general case to then specialize to 1+1 dimensional gravity.

Finally, I will present elements of our recent research applying the proposal to the case of 1+1 dimensional gravity. I will demonstrate the framework's promising potential by calculating the entanglement entropy between two spatial regions--a significant challenge in quantum gravity. The result is the 1+1 dimensional equivalent of the well-established Bekenstein-Hawking area law governing the entropy of gravitational systems with the expected behavior of the quantum corrections.

Zoom link

The Corners of 1+1 Dimensional Quantum Gravity

Ludovic Varrin

National Centre for Nuclear Research, Warsaw

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Presentation Plan

- Introduction
- Symmetries and gauge theories
- The corner proposal
- (1+1)-d gravity from the corner proposal

Introduction: Symmetries

Symmetries play a crucial role in all of physics

- Great tool to find solutions of complicated problems (Cosmology, Newton gravity, classical mechanics,...)
- They describe fundamental interactions through gauge theories

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

- They are the defining property of fundamental physics
Poincaré $\rightarrow$ Quantum Field Theory and fundamental particles
Diffeomorphisms $\rightarrow$ General covariance and gravity

Introduction: Corner proposal

- Very interesting physics at the boundary of gauge theories (Quantum Hall effect, Holography, Black holes, ...)
- Gravity is a gauge theory of diffeomorphisms, at the boundary there exists a universal *corner symmetry group*
- The corner proposal states that this symmetry group is the fundamental ingredient of gravity

Symmetries and Gauge theories

Symmetries: Why?

From the Poincaré symmetry algebra $(\vec{J}, \vec{K}, \vec{P}, E)$

$$[J_m, P_n] = \epsilon_{mnk} P_k, \quad [K_i, P_k] = \eta_{ik} E, \quad [K_i, E] = -P_i,$$
$$[J_m, J_n] = \epsilon_{mnk} J_k, \quad [J_m, K_n] = \epsilon_{mnk} K_k, \quad [K_m, K_n] = -\epsilon_{mnk} J_k,$$

- Coadjoint orbit method $\rightarrow$ Classical relativistic particle
- Unitary irreducible representations $\rightarrow$ Quantum field theory

From only the symmetry algebra, one can get both the classical and quantum theory.

Symplectic geometry, Covariant Phase Space

The covariant phase space formalism provides a symplectic structure (on field-space $\mathcal{F}$) from the Lagrangian

$$\delta L[\varphi^a] = \mathrm{EOM}_a \delta\varphi^a + \mathrm{d}\theta[\varphi^a, \delta\varphi^a],$$

The field-space two-form

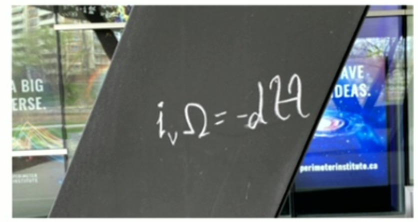
$$\Omega = \int_\Sigma \delta\theta,$$

is closed $\delta\Omega = 0$.

Noether Theorems

Let $V \in \mathrm{T}\mathcal{F}$ be a field-space vector, generating a symmetry. Then there is an associated charge

$$I_V \Omega = \delta H_V ,$$



such that

$$H_V = \int_\Sigma J_V .$$

If $V_\epsilon \in \mathrm{T}\mathcal{F}$ is associated to a gauge symmetry (gauge parameter ϵ)

$$H_{V_\epsilon} = \int_\Sigma \mathrm{d} Q_{V_\epsilon} = \int_{\partial\Sigma} Q_{V_\epsilon} .$$

Gauge theories and edge modes

In gauge theories without boundaries

$$I_{V_\epsilon}\Omega = 0 \quad \text{(no boundary)}$$

This identifies the gauge direction, and allows to perform symplectic reduction. In gauge theories *with* boundaries, we can introduce edge modes to the field space

$$\phi : \partial\Sigma \longrightarrow G$$

and an associated symplectic structure Ω^ϕ such that

$$I_{V_\epsilon}(\Omega + \Omega^\phi) = 0, \quad \text{(yes boundary)}$$

Example: Einstein-Hilbert gravity

Field-space is

$$\mathcal{F} = \{g_{\mu\nu}(x)\} .$$

The charge associated to a diffeomorphism $\xi \in \mathrm{T}M$ is

$$H_{\mathrm{EH}}[\xi] = \int_{\partial\Sigma} \mathrm{d}\sigma_{\mu\nu} \sqrt{-g} \nabla^{\mu} \xi^{\nu} .$$

One can introduce edge modes ϕ on the boundary $\partial\Sigma$

$$\mathcal{F}_{ext} = \{g_{\mu\nu}(x), \phi(\sigma)\} ,$$

such that

$$H_{\mathrm{EH,ext}}[\xi] = 0.$$

For more details

Review paper on edge modes in Yang-Mills theories and gravity written in an accessible mathematical language:

"*On the covariant formulation of gauge theories with boundaries*" (2312.01918)
M. Assanioussi, J. Kowalski-Glikman, I. Mäkinen, L. Varrin (2023)

Charge algebra

The symplectic form also generates the charge algebra

$$I_V I_W \Omega = \{H_V, H_W\},$$

for two symmetries V, W.

This algebra represents the symmetry algebra of the theory **up to central extensions**

$$\{H_V, H_W\} = H_{[V,W]} + \kappa_{V,W}.$$

Recap

- To each symmetry of the theory there exists an associated charge (Noether)
- For gauge symmetries, these charges vanish except in the presence of boundaries. In that case, some gauge symmetries become physical
- The set of gauge charges form a corner-algebra, which is a subalgebra of the original gauge algebra.

The Corner Proposal

"*Gravity is described by a set of charges and their algebra at corners*"

$$H_{\mathrm{EH}}^{\xi} = \int_{\partial\Sigma = S} \mathrm{d}\sigma_{\mu\nu}\sqrt{-g}\nabla^{\mu}\xi^{\nu}.$$

$$\left\{H_{\mathrm{EH}}^{\xi}, H_{\mathrm{EH}}^{\chi}\right\} = H_{\mathrm{EH}}^{[\xi,\chi]} + \kappa_{\xi,\chi}.$$

The extended corner symmetry

Considering finite distance corners, we have the *Extended Corner Symmetry* group

$$\boxed{\mathrm{ECS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S}$$

- $\mathrm{ECS} \subset \mathrm{Diff}(M)$, still infinite dimensional
- The rest of $\mathrm{Diff}(M)$ is left uncharged (pure gauge).

The Universal Corner Group

The ECS is a subgroup of the *Universal Corner Symmetry* group[1]

$$\mathrm{UCS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{GL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S$$

- Universal $\rightarrow$ does not depend on the dynamics
- Contains the BMSW asymptotic symmetry group

[1] Luca Ciambelli and Robert G. Leigh. "Isolated surfaces and symmetries of gravity". In: *Physical Review D* (Aug. 2021).

Corner proposal again

In the context of the proposal,

$$\mathrm{UCS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{GL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S$$

$$\mathrm{ECS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S$$

completely describe gravity.

Coadjoint orbits $\Longrightarrow$ Classical phase space
UIREP $\Longrightarrow$ Quantum Hilbert space

For more details

"*From Asymptotic Symmetries to the Corner Proposal*" (2212.13644), L. Ciambelli (2022)

"*Cornering quantum gravity*"(2307.08460)
L. Ciambelli, A. D'Alise, V. D'Esposito, D. Đorđevic, D. Fernández-Silvestre, L. Varrin (2023)

Corner proposal again

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$$\mathrm{UCS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{GL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S$$

$$\mathrm{ECS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S$$

completely describe gravity.

Coadjoint orbits $\Longrightarrow$ Classical phase space
UIREP $\Longrightarrow$ Quantum Hilbert space

The corner algebra

In 1+1 dimensions, the group becomes

$$\mathrm{ECS}_2 = \mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^2$$

The algebra has five generators $(L_0, L_\pm, P_\pm)$

$$[L_0, L_\pm] = \pm L_\pm, \qquad [L_-, L_+] = 2L_0$$
$$[L_0, P_\pm] = \pm\frac{1}{2} P_\pm\,, \quad [L_\pm, P_\mp] = \mp P_\pm$$

and one cubic Casimir

$$\mathcal{C}_3 = 2L_0P_-P_+ - L_+P_-^2 - L_-P_+^2$$

Central extensions

Remember

$$\left\{ H_{\mathrm{EH}}^{\xi}, H_{\mathrm{EH}}^{\chi} \right\} = H_{\mathrm{EH}}^{[\xi,\chi]} + \kappa_{\xi,\chi}.$$

The algebra admits one non-trivial central extension

$$[P_-, P_+] = \Lambda$$

Λ is a new Casimir of the algebra and we naturally choose the dimension

$$[\Lambda] = E^2$$

Central extensions: comments

$$[P_-, P_+] = \Lambda.$$

- In the Poincaré case there are no central extensions
- Similar to the Galilean case where

$$[P_i, K_j] = \delta_{ij} M$$

- The non-commutation of the translations is reminiscent of torsion.

Oscillator representation

We consider the oscillator representation

$$L_0 = \frac{1}{2}a^\dagger a + \frac{1}{4}, \quad L_- = \frac{1}{2}aa, \quad L_+ = \frac{1}{2}a^\dagger a^\dagger,$$
$$P_- = \sqrt{\Lambda}a, \qquad P_+ = \sqrt{\Lambda}a^\dagger.$$

Acting on the Fock space $|n\rangle$.

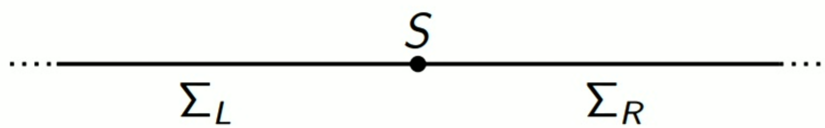
$$a|n\rangle = \sqrt{n}|n-1\rangle$$
$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\mathcal{H} = \{|n\rangle \mid n \in \mathbb{N}\}$$

The second Casimir $\mathcal{C}_3^{(\Lambda)}$ vanishes in this representation.

Local subsystems

In standard quantum theory



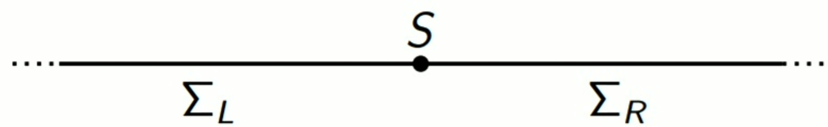
$$\mathcal{H}_G = \mathcal{H}_L \otimes \mathcal{H}_R.$$

But in gauge theories, gauge constraints relates data from Σ_L and Σ_R and

$$\mathcal{H}_G \subset \mathcal{H}_L \otimes \mathcal{H}_R.$$

The gauge constraints are expressed by the vanishing of the charges → Left and right charges must coincide

Gluing of two subregions (a)



$$L_0^L, L_\pm^L, P_\pm^L, \Lambda^L$$

$$\{|\alpha\rangle_L\}$$

such that

$$a_L|\alpha\rangle_L = \alpha|\alpha\rangle_L$$

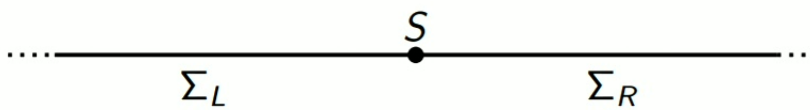
$$L_0^R, L_\pm^R, P_\pm^R, \Lambda^R$$

$$\{|\gamma\rangle_R\}$$

such that

$$a_R|\gamma\rangle_R = \gamma^*|\gamma\rangle_R$$

Gluing of two subregions (b)



We form the product space

$$\tilde{\mathcal{H}}_G = \{|\alpha\rangle_L \otimes |\gamma\rangle_R\}$$

and require that the left and right action of the maximal commuting subalgebra (Λ, L_-, P_-) on states coincides

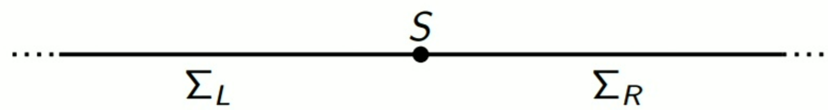
$$\Lambda^L|\tilde{\psi}\rangle_G = \Lambda^R|\tilde{\psi}\rangle_G$$
$$L_-^L|\tilde{\psi}\rangle_G = L_-^R|\tilde{\psi}\rangle_G$$
$$P_-^L|\tilde{\psi}\rangle_G = P_-^R|\tilde{\psi}\rangle_G$$

Gluing of two subregions (c)

The constrained glued Hilbert space is

$$\mathcal{H}_G = \{|\alpha\rangle_G = |\alpha\rangle_L \otimes |\alpha^*\rangle_R \mid \alpha \in \mathbb{C}\}$$

Gluing of two subregions (b)



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Gluing of two subregions (c)

The constrained glued Hilbert space is

$$\mathcal{H}_G = \{|\alpha\rangle_G = |\alpha\rangle_L \otimes |\alpha^*\rangle_R \mid \alpha \in \mathbb{C}\}$$

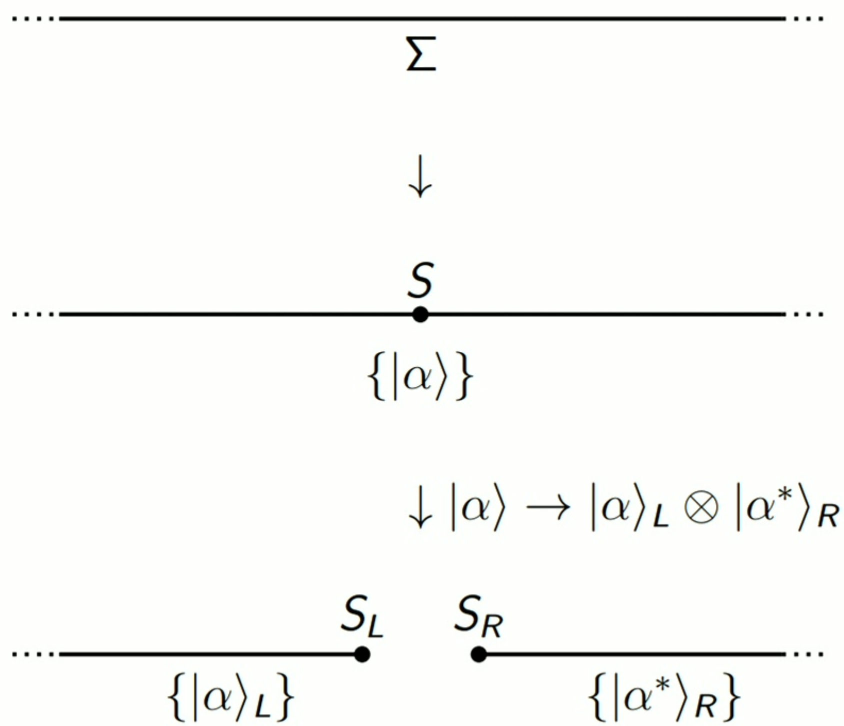
There is the correspondence

$$|\alpha\rangle_G \sim |\alpha\rangle_L$$
$$|\alpha\rangle_G \sim |\alpha\rangle_R$$

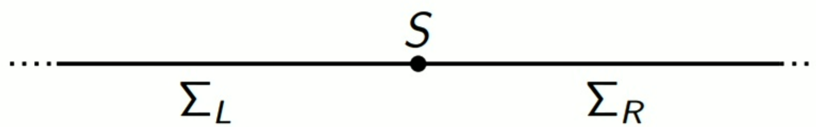
And we can construct the glued corner algebra from the left and right algebra

$$\mathcal{A}^G = \mathcal{A}^G\left(A^L, A^R\right)$$

Gluing complete!



Gluing of two subregions (a)



$$L_0^L, L_\pm^L, P_\pm^L, \Lambda^L$$

$$\{|\alpha\rangle_L\}$$

such that

$$a_L|\alpha\rangle_L = \alpha|\alpha\rangle_L$$

$$L_0^R, L_\pm^R, P_\pm^R, \Lambda^R$$

$$\{|\gamma\rangle_R\}$$

such that

$$a_R|\gamma\rangle_R = \gamma^*|\gamma\rangle_R$$

Entanglement entropy (a)

The conjugate doubling of the state

$$|\alpha\rangle \to |\alpha\rangle_L \otimes |\alpha^*\rangle_R$$

coincides very nicely with the framework of thermofield theory.
Consider the state

$$|\alpha, \beta\rangle_G = \exp[\theta(\beta)\left(a_L^\dagger a_R^\dagger - a_L a_R\right)]|\alpha\rangle_G$$

with

$$\cosh\theta(\beta) = (1 - \exp(-\beta\sqrt{\Lambda}))^{-\frac{1}{2}}$$

Entanglement entropy (b)

It can be shown[1] that the state $|\alpha, \beta\rangle_G$ is equivalent to the left state defined by the density matrix

$$\rho_L(\alpha, \beta) = D(\alpha) \exp(-\beta\sqrt{\Lambda} a_L^\dagger a_L) D^\dagger(\alpha)$$

The associated entanglement entropy can be computed via the replica trick

$$S_L = -\partial_q \ln(\mathrm{Tr}[\rho_L^q(\alpha, \beta)]) \,|_{q=1}$$

[1] Oz-Vogt, Mann, and Revzen, "Thermal Coherent States and Thermal Squeezed States".

Entanglement entropy (c)

At high temperature ($\beta << 1$), we get

$$S_L = 1 + \ln\left(\frac{1}{\beta\sqrt{\Lambda}}\right) + O\left((\beta\sqrt{\Lambda})^2\right)$$

This has the expected form:

- The first term is the Bekenstein-Hawking term in $1+1$d
- The second term arises from quantum corrections with the expected UV divergence

Conclusions and recap

- We found a family of Hilbert spaces of Quantum Gravity from the corner proposal
- We gave a concrete prescription to glue/separate two spatial subregions. The glued Hilbert space is

$$\mathcal{H}_G = \{|\alpha\rangle_L \otimes |\alpha\rangle_R \mid \alpha \in \mathbb{C}\}$$

- We gave a first consideration of the entanglement entropy between two spatial subregions