Title: Integrable Deformations on Twistor Space

Speakers: Joaquin Liniado

Series: Mathematical Physics

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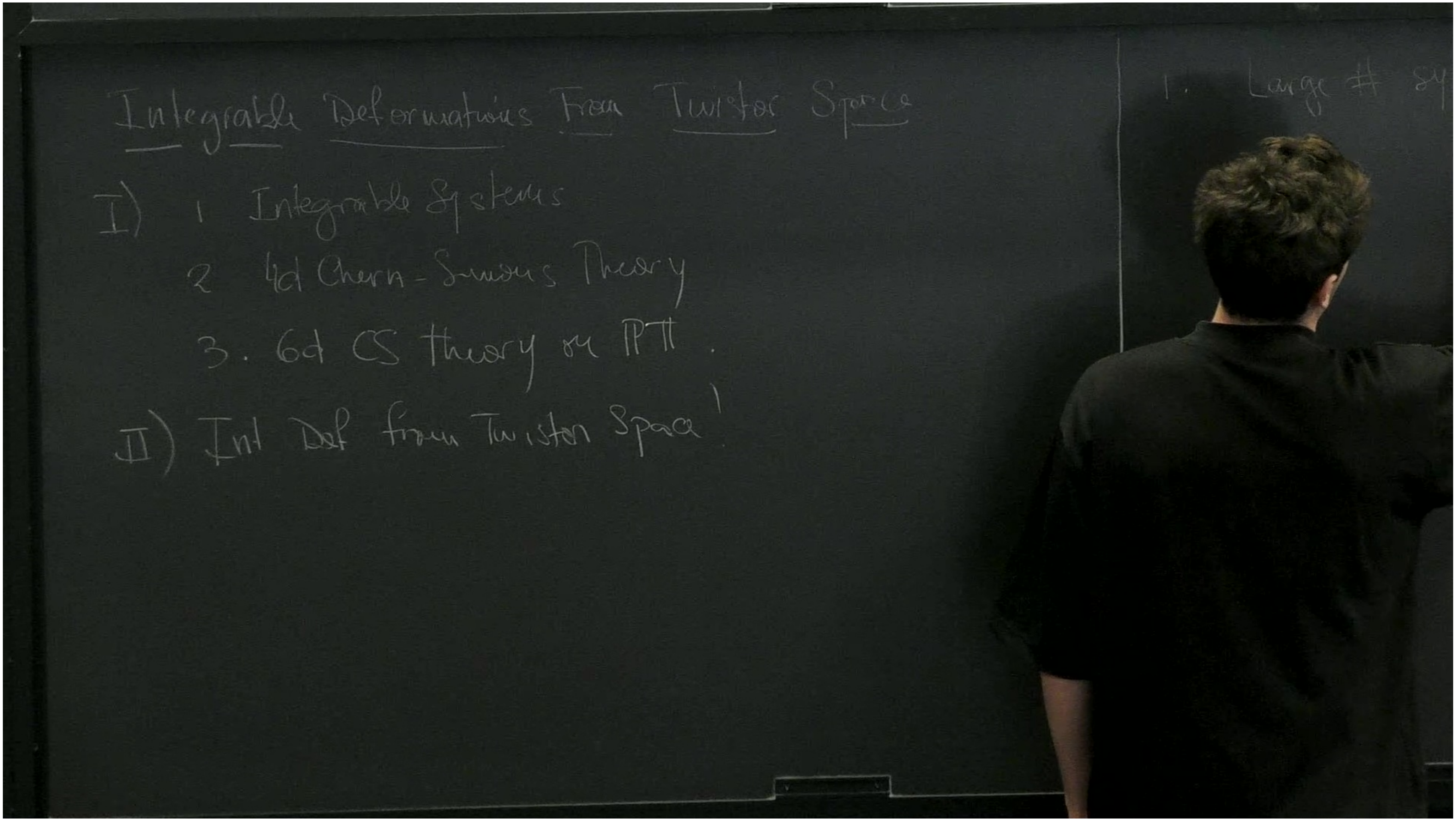
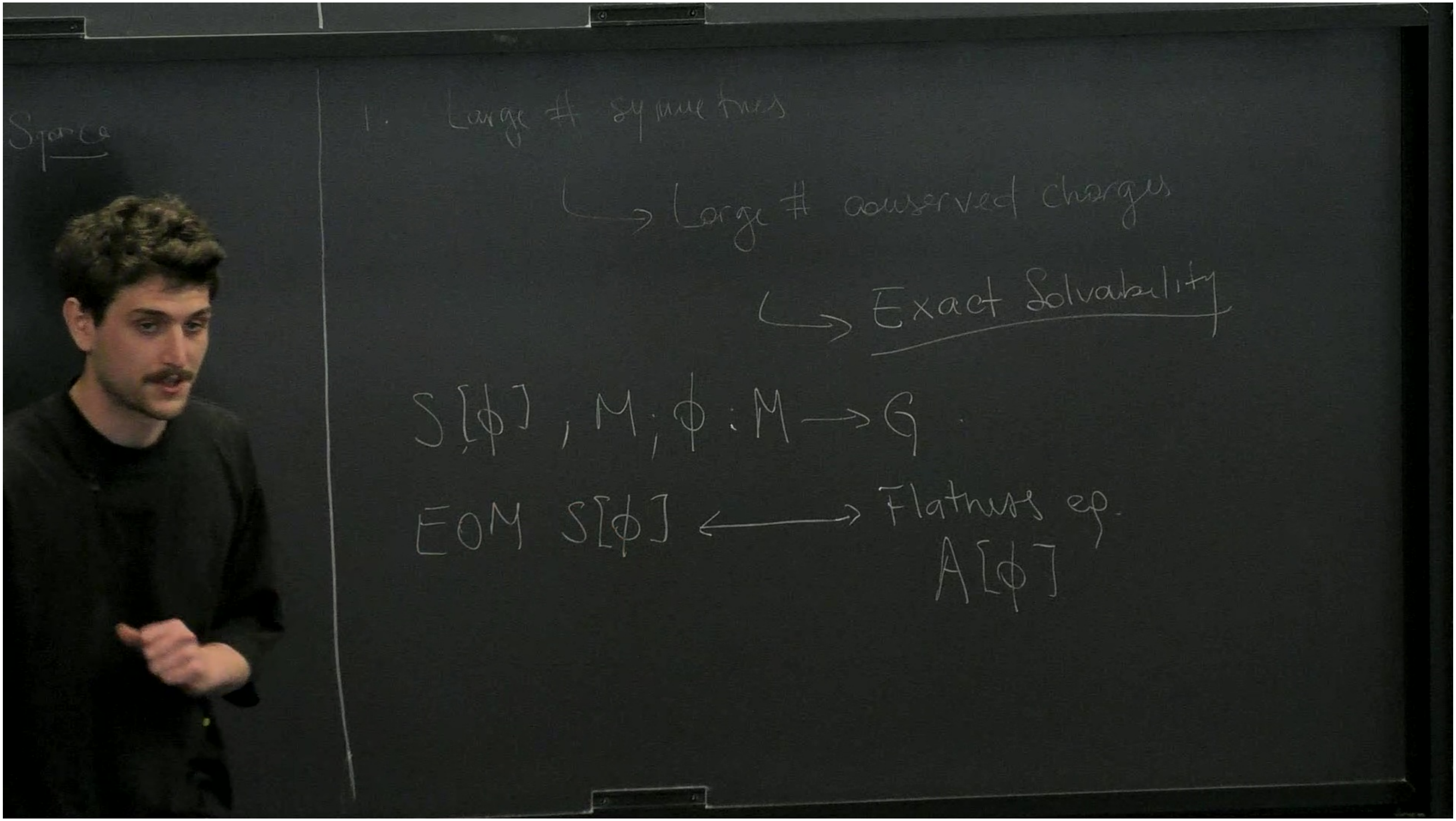
Abstract: Integrable field theories in two dimensions are known to originate as defect theories of 4d Chern-Simons theory and as symmetry reductions of the 4d anti-self-dual Yang-Mills equations. Based on ideas of Costello, it has been proposed in work of Bittleston and Skinner that these two approaches can be unified starting from holomorphic Chern-Simons theory in 6 dimensions. In this talk I will introduce the first complete description of this diamond of integrable theories for a family of deformed sigma models, going beyond the Dirichlet boundary conditions that have been considered thus far. The talk is based on the recent work https://arxiv.org/abs/2311.17551.

Zoom link


Integrable Deformations From Twistor Space
I) 1 Integrable Systems
2 4d Chern-Simons Theory
3. 6d CS theory on PT.
II) Int Def from Twistor Space
1. Large #


1. Large # symmetries
Large # conserved charges
Exact Solvability
S[φ], M; φ : M → G
EOM S[φ]
Flatness eq.
A[φ]

2d: Lax Connection, $S[\phi]$ on $\Sigma = (t, x)$

$$\mathcal{L} = \mathcal{L}_t(\phi, z)\,dt + \mathcal{L}_x(\phi, z)\,dx \qquad z \in \mathbb{CP}^1$$

$$\text{EOM } S[\phi] \longleftrightarrow F(\mathcal{L}) = 0 \quad \forall z \in \mathbb{CP}^1$$

$\Rightarrow S$ is (Lax) integrable

4d: $S[\phi]$ o…

$F = -\star F$

Penrose - Ward

Holomorphic

bundles o…

$\bar{\nabla}^2 = 0 \Rightarrow F($…

$= (t, x)$

$x \quad z \in \mathbb{CP}^1$

$\forall z \in \mathbb{CP}^1$

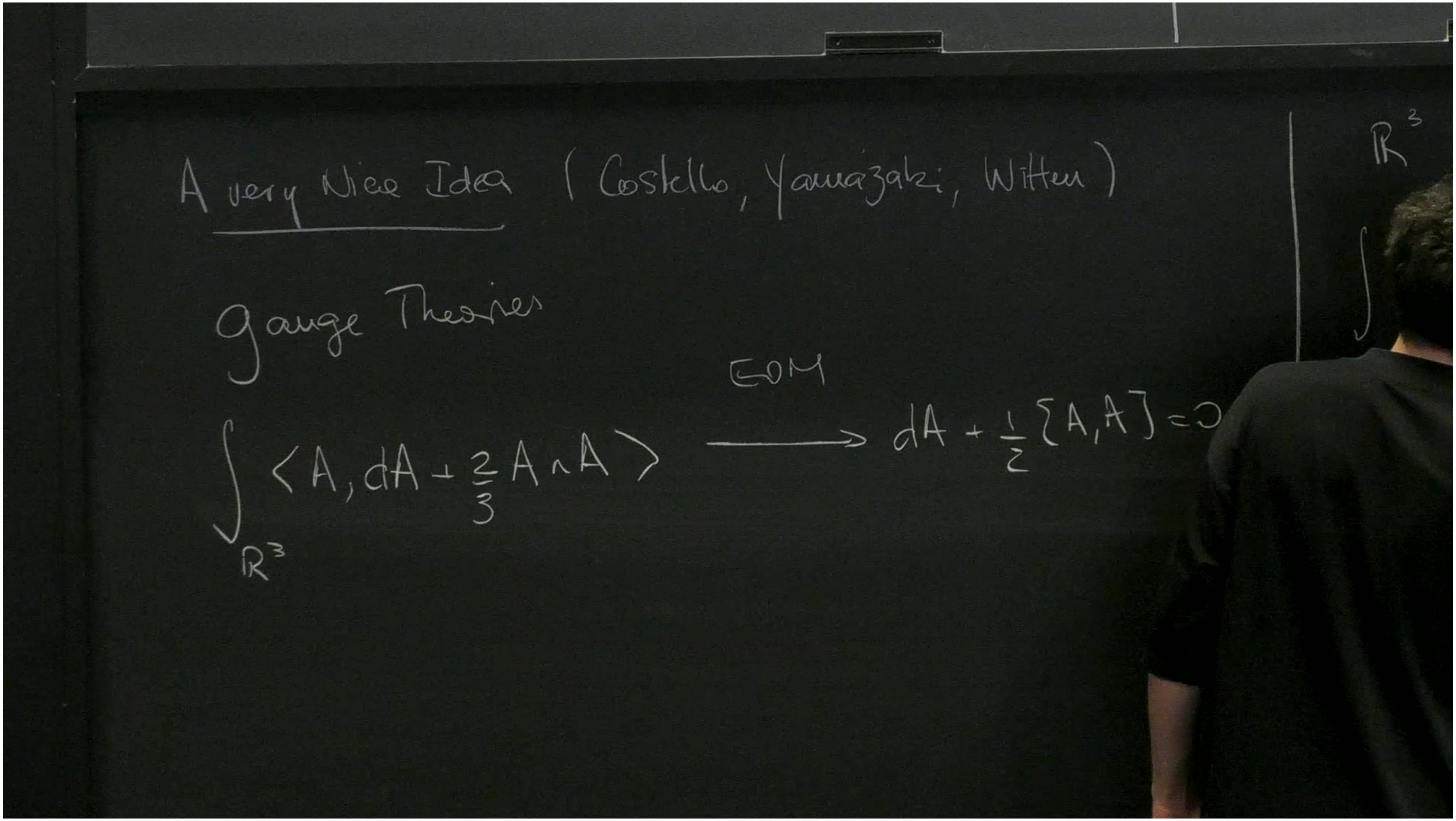
<u>4d</u> : $S[\phi]$ on $\mathbb{R}^4$; A ; ASDYM

$$F = -\star F$$

Penrose-Ward corr.

Holomorphic vect bundles on $\mathbb{PT}$ $\longleftrightarrow$ ASDYM

$$\bar{\nabla}^2 = 0 \rightarrow F(\bar{A}) = 0$$


A very Nice Idea (Costello, Yamazaki, Witten)
Gauge Theories
EOM
$\int_{\mathbb{R}^3} \langle A, dA + \frac{2}{3} A \wedge A \rangle \longrightarrow dA + \frac{1}{2}[A, A] = 0$
$\mathbb{R}^3$
$\int$

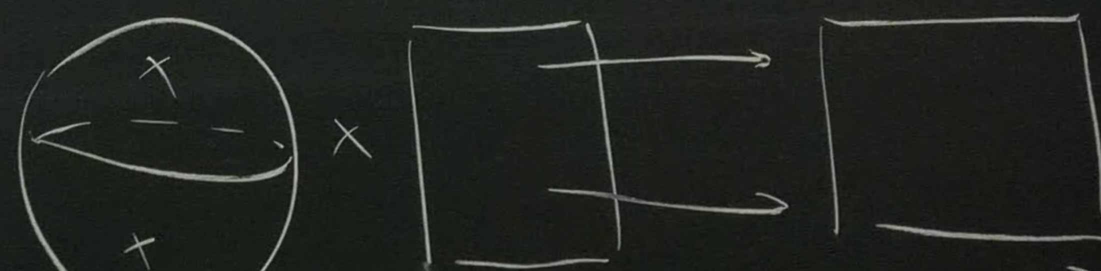
...azaki, Witten)

$$\Rightarrow dA + \frac{1}{2}\{A,A\} = 0$$

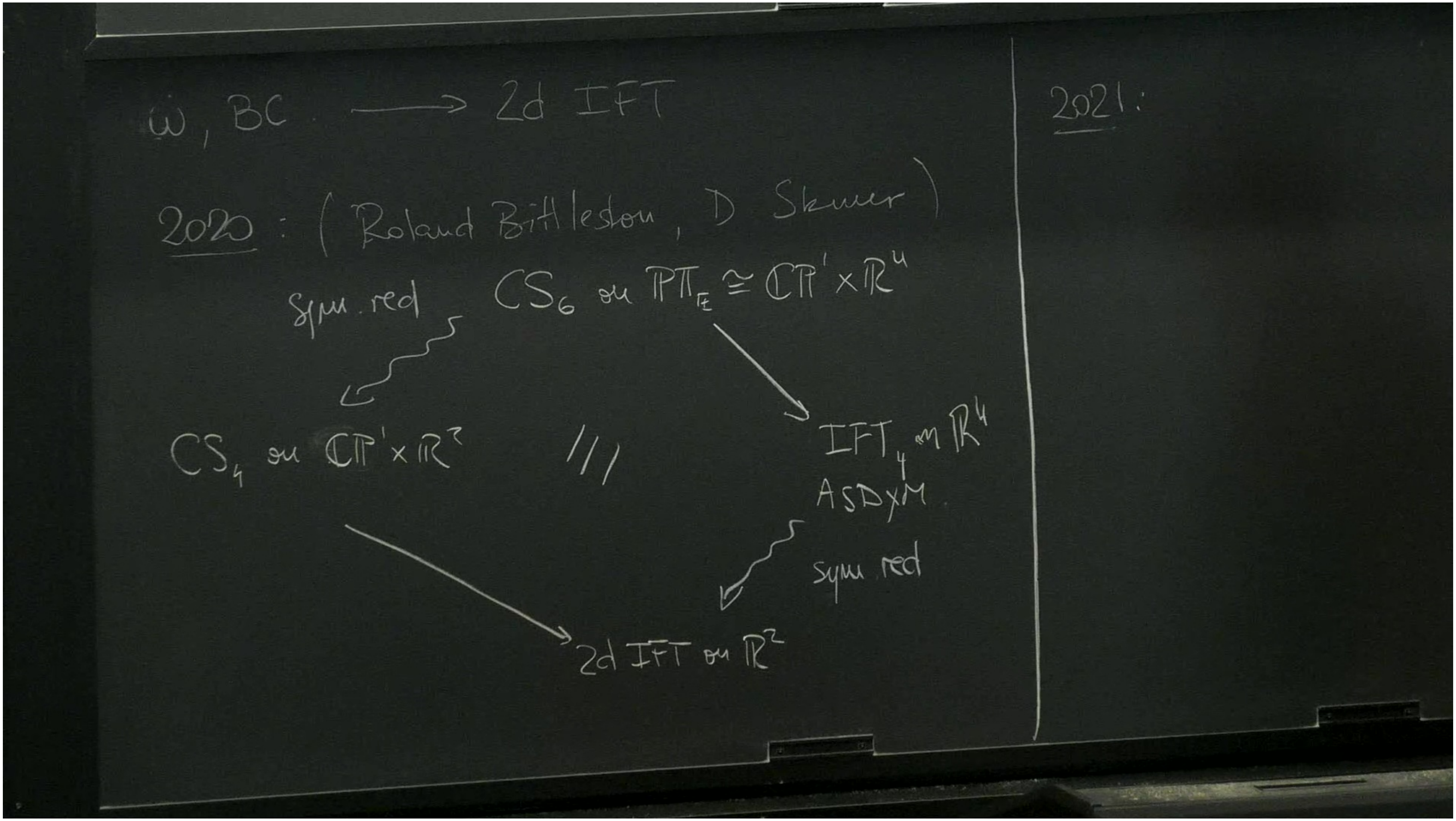
$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R}^2 \longrightarrow \mathbb{CP}^1 \times \mathbb{R}^2$$

$$\int_{\mathbb{CP}^1 \times \mathbb{R}^2} \omega \wedge \langle A, dA + \tfrac{2}{3} A \wedge A \rangle$$

Disorder Operator $\quad \omega = \dfrac{dz}{z}$



$\mathbb{R}^2$ EOM $\longrightarrow$ EOM $\mathbb{R}^2$


ω, BC ⟶ 2d IFT
2020 : (Roland Bittleston, D Skinner)
CS_6 on $\mathbb{PT}_E \cong \mathbb{CP}^1 \times \mathbb{R}^4$
symm. red
CS_4 on $\mathbb{CP}^1 \times \mathbb{R}^2$
///
IFT_4 on $\mathbb{R}^4$
ASDYM
symm. red
2d IFT on $\mathbb{R}^2$
2021 :

FT

on , D. Skinner)

$\mathbb{PT}_{\mathbb{E}} \cong \mathbb{CP}^1 \times \mathbb{R}^4$

IFT$_4$ on $\mathbb{R}^4$

ASDYM

sym red

IFT on $\mathbb{R}^2$

Integrable Def from Twistor Space

L. Cole, R. Cullinan, B. Hoare, D. Thompson

$$S_{4d} = \int_{\mathbb{PT}_{\mathbb{E}} \cong \mathbb{CP}^1 \times \mathbb{R}^4} \Omega \wedge \langle \bar{A}, \bar{\partial}\bar{A} + \frac{2}{3}\bar{A} \wedge \bar{A} \rangle$$

$+$ $\pm$

$+\times$

$$\Omega = \frac{e^{\dot{a}} \wedge e_{\dot{a}} \wedge e^{0}}{\langle \pi\alpha \rangle \langle \pi\tilde{\alpha} \rangle \langle \pi\beta \rangle^2} \in \Omega^{(3,0)}(\mathbb{PT})$$

π^a is $\mathbb{CP}^1$ coordinate ; $\mu^{\dot{a}} = \pi^a x_{a\dot{a}}$

$$\delta S = \int \Omega \wedge \langle \delta \bar{A}, \mathcal{F}(\bar{A}) \rangle$$

$$+ \int \bar{\partial}\Omega \wedge \langle \bar{A}, \delta A \rangle$$

$$\bar{\partial}\Omega = \# \delta \left(\langle \pi \alpha \rangle \right) + \delta \left(\langle \pi \tilde{\alpha} \rangle \right) + \bar{\partial}_0 \delta \left(\langle \pi \beta \rangle \right)$$

1. Large # sy

$S[\phi]$, M;

EOM $S[\phi$

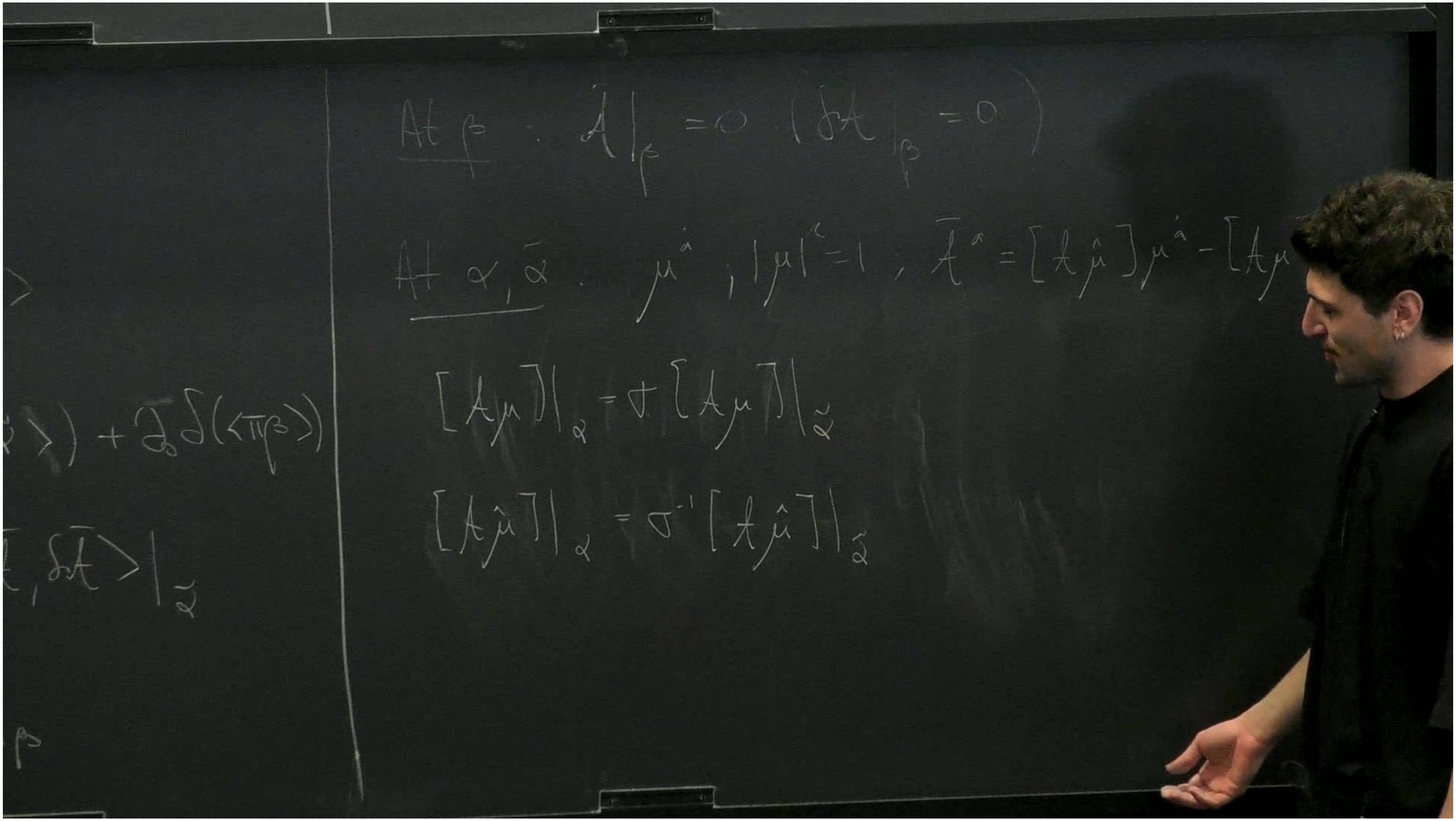
$$\delta S = \int \Omega \wedge \langle \delta \bar{A}, \mathcal{F}(\bar{A}) \rangle$$

$$+ \int_{\mathbb{CP}^1 \times \mathbb{R}^4} \bar{\partial}\Omega \wedge \langle \bar{A}, \delta A \rangle$$

$$\bar{\partial}\Omega = \# \delta(\langle \pi \alpha \rangle) + \delta(\langle \pi \tilde{\alpha} \rangle) + \partial_0 \delta(\langle \pi \beta \rangle)$$

$$\delta S_{bdry} = \int \# \langle \bar{A}, \delta \bar{A} \rangle \big|_{\alpha} + \# \langle \bar{A}, \delta \bar{A} \rangle \big|_{\tilde{\alpha}}$$

$$\# \, \partial_0 \langle \bar{A}, \delta \bar{A} \rangle \big|_{\beta}$$


At β : Ā|β = 0 (δA|β = 0)
At α, α̃ : μ^a, |μ|^2 = 1, Ā^a = [A μ̂] μ^a − [A μ
[A μ]|α = σ [A μ]|α̃
[A μ̂]|α = σ^-1 [A μ̂]|α̃

$$A_i\big|_{\alpha} = A_i\big|_{\tilde{\alpha}}$$

Symmetries

$$\bar{A} \to g^{-1}\bar{A}g + g^{-1}\bar{\partial}g$$

$$\bar{A}\big|_{\beta} = 0 \Rightarrow \bar{A}^{g}\big|_{\beta} = 0 \to g^{-1}\bar{\partial}g\big|_{\beta} = 0$$

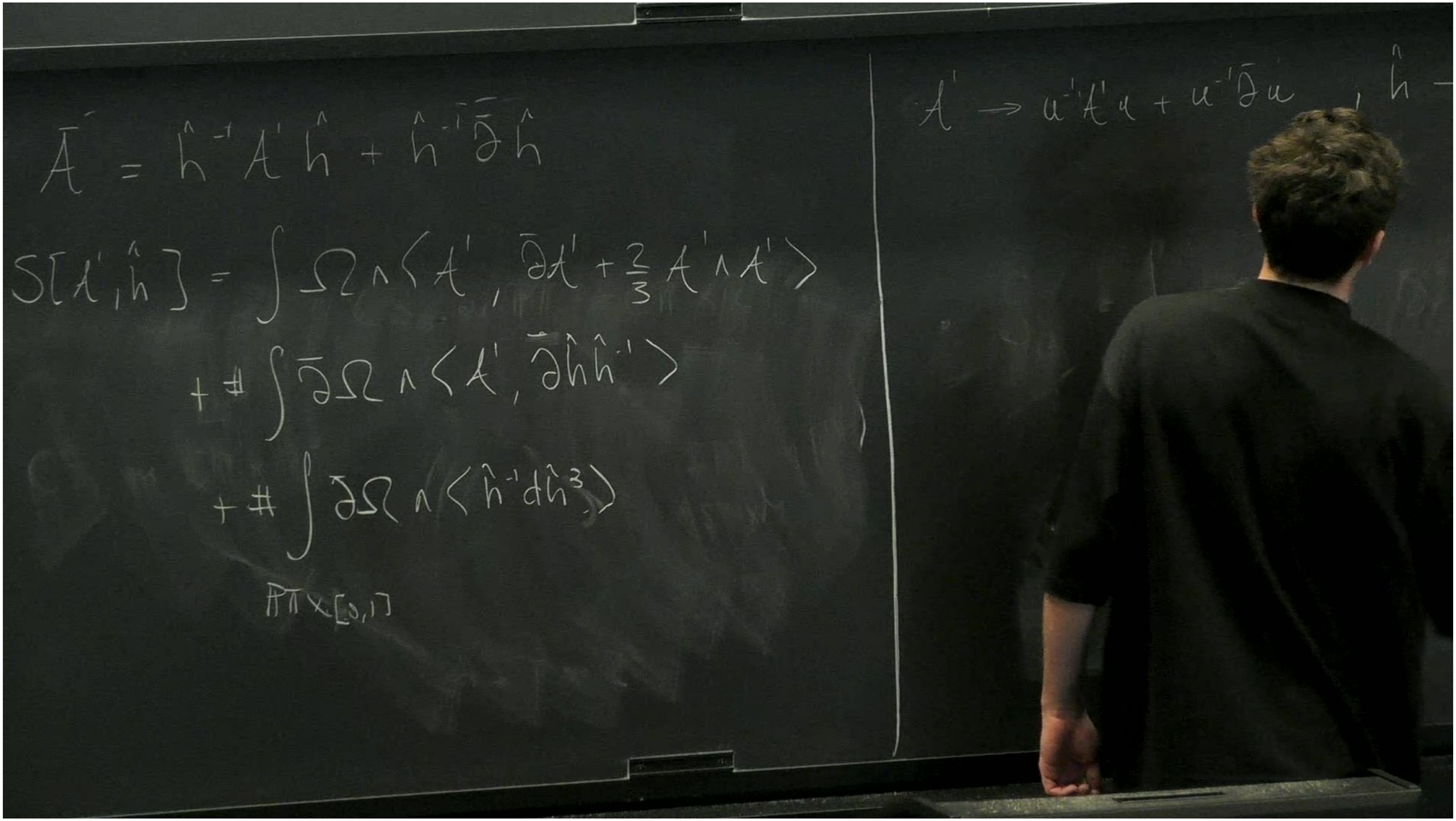
$$F = -*F$$

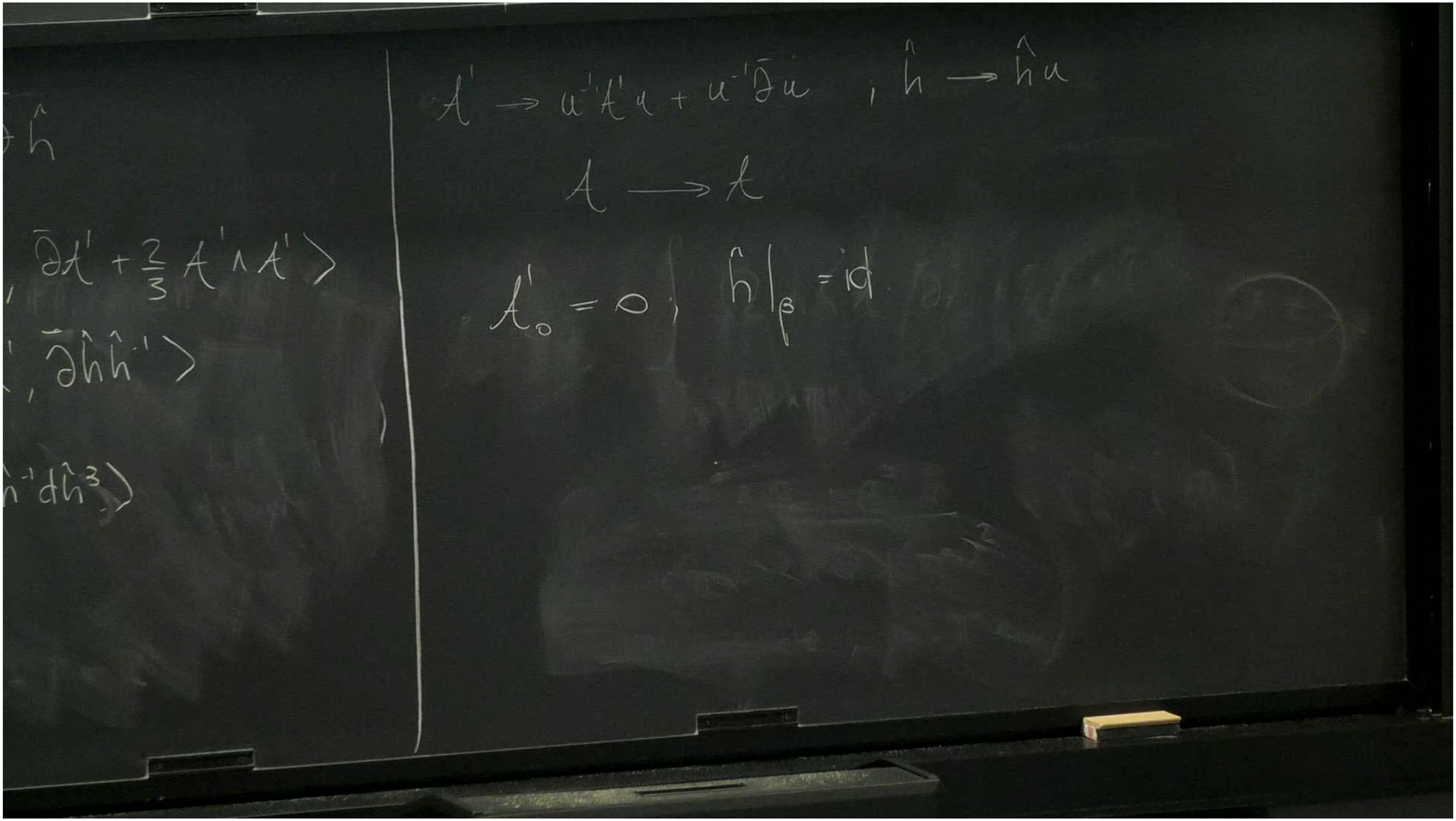
Penrose-Ward

Holomorphic

bundles or

$$\bar{\nabla}^2 = 0 \to F(A$$


$\bar{A}' = \hat{h}^{-1} A' \hat{h} + \hat{h}^{-1} \bar{\partial} \hat{h}$
$S[A', \hat{h}] = \int \Omega \wedge \langle A', \bar{\partial} A' + \frac{2}{3} A' \wedge A' \rangle$
$+ \# \int \bar{\partial}\Omega \wedge \langle A', \bar{\partial} \hat{h} \hat{h}^{-1} \rangle$
$+ \# \int \partial\Omega \wedge \langle \hat{h}^{-1} d\hat{h}^{3} \rangle$
$\mathbb{PT} \times [0,1]$
$A' \to u^{-1} A' u + u^{-1} \bar{\partial} u$, $\hat{h} \to$


$A' \to u^{-1}A'u + u^{-1}\bar{\partial}u$, $\hat{h} \to \hat{h}u$
$A \to A$
$A'_0 = 0$; $\hat{h}|_\beta = \mathrm{id}$
$\bar{\partial}A' + \frac{2}{3}A' \wedge A' \rangle$
$\bar{\partial}\hat{h}\hat{h}^{-1} \rangle$
$\hat{h}^{-1}d\hat{h}^3 \rangle$

$$\bar{\partial}_0 A'_\alpha = 0$$

$$S_{4d} = \int_{\mathbb{R}^4} \langle A', \bar{\partial}\hat{h}\hat{h}^{-1}\rangle|_\alpha + \int_{\mathbb{R}^4\times[0,1]} \langle \hat{h}^{-1} d\hat{h} \rangle|_\alpha$$

$$- \int_{\mathbb{R}^4} \langle A', \bar{\partial}\hat{h}\hat{h}^{-1}\rangle|_{\tilde{\alpha}} - S_{WZ}[\hat{h}]|_{\tilde{\alpha}}$$

$$\hat{h}|_\alpha = h, \quad \hat{h}|_{\tilde{\alpha}} = \tilde{h}$$

$$\mathbb{R}^3 = ($$

$$\int_{\mathbb{CP}^1\times\mathbb{R}^2} \omega \wedge \langle$$

$\langle \hat{h}^{-1} d\hat{h} \rangle |_\alpha$

$[0,1]$

$z[\hat{h}]_\alpha$

$$\bar{A}|_\beta = 0 \Rightarrow \cancel{\hat{h}^{-1}} A' \cancel{\hat{h}} + \cancel{\hat{h}^{-1} \bar{\partial} \hat{h}}|_\beta = 0$$

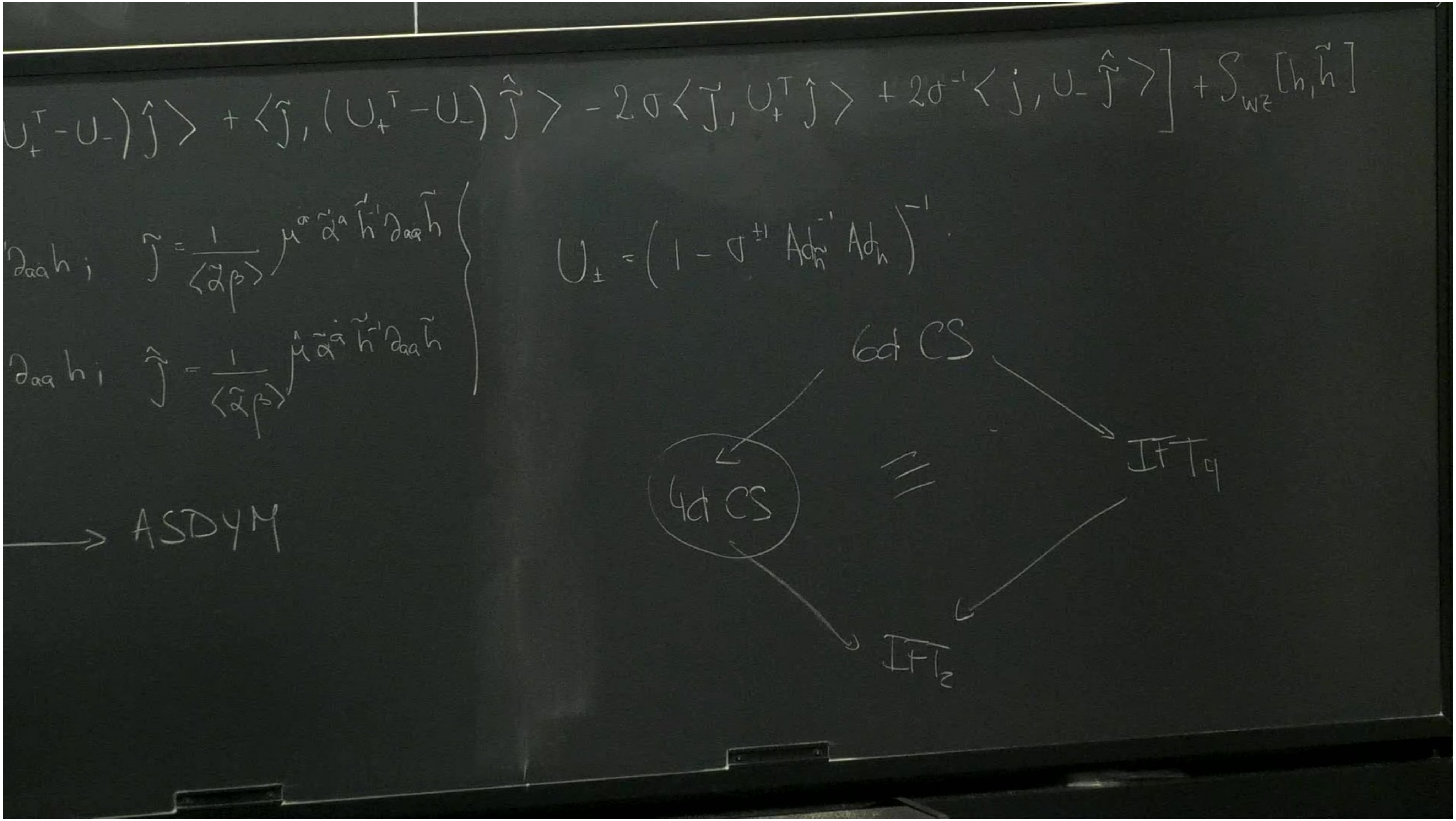
$$\Rightarrow A'|_\beta = 0 .$$

Using BC : $A' = A'[h, \tilde{h}]$

$$S = \frac{K}{\langle \alpha \tilde{\alpha} \rangle} \int_{\mathbb{R}^4} \mathrm{vol}_4 \Big[\langle j, (U_+^T - U_-)\hat{j} \rangle + \langle \tilde{j}, (U_+^T - U_-)\hat{\tilde{j}} \rangle - 2\sigma \langle \tilde{j}, U_+^T \hat{j} \rangle + 2\sigma^{-}$$

$$\left\{ \begin{array}{ll} j = \dfrac{1}{\langle \alpha \beta \rangle} \mu^a \alpha^a h^{-1} \partial_{aa} h \, ; & \tilde{j} = \dfrac{1}{\langle \tilde{\alpha} \beta \rangle} \mu^a \tilde{\alpha}^a \tilde{h}^{-1} \partial_{aa} \tilde{h} \\ \hat{j} = \dfrac{1}{\langle \alpha \beta \rangle} \hat{\mu}^a \alpha^a h^{-1} \partial_{aa} h \, ; & \hat{\tilde{j}} = \dfrac{1}{\langle \tilde{\alpha} \beta \rangle} \hat{\mu}^a \tilde{\alpha}^a \tilde{h}^{-1} \partial_{aa} \tilde{h} \end{array} \right\}$$

$$U_\pm = \left(1 - \sigma^{\pm 1} \mathrm{Ad}_{\tilde{h}}^{-1} \mathrm{Ad}_h \right)^{-1}$$


$U_+^T - U_-)\hat{j}\rangle + \langle \tilde{j}, (U_+^T - U_-)\hat{j}\rangle - 2\sigma\langle \tilde{j}, U_+^T \hat{j}\rangle + 2\sigma^{-1}\langle j, U_- \hat{j}\rangle] + S_{WZ}[h,\tilde{h}]$
$\partial_{a\dot{a}} h;\quad \tilde{j} = \frac{1}{\langle\tilde{\alpha}\beta\rangle}\mu^{\dot{a}}\tilde{\alpha}^a \tilde{h}^{-1}\partial_{a\dot{a}}\tilde{h}$
$\partial_{a\dot{a}} h;\quad \hat{j} = \frac{1}{\langle\tilde{\alpha}\beta\rangle}\hat{\mu}\tilde{\alpha}^a \tilde{h}^{-1}\partial_{a\dot{a}}\tilde{h}$
$U_\pm = \left(1 - \sigma^{\pm 1}\mathrm{Ad}_{\tilde{h}}^{-1}\mathrm{Ad}_h\right)^{-1}$
$\longrightarrow$ ASDYM
6d CS
4d CS
$\equiv$
IFT_4
IFT_2

1. Gauged WZW_4

$$S[A] - S[B] + \int \bar{\partial}\Omega \wedge \langle A, B \rangle$$

$\bar{A}|_{\beta} = \ldots$

Using