

Title: Integrable Deformations on Twistor Space

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Series: Mathematical Physics

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Abstract: Integrable field theories in two dimensions are known to originate as defect theories of 4d Chern-Simons theory and as symmetry reductions of the 4d anti-self-dual Yang-Mills equations. Based on ideas of Costello, it has been proposed in work of Bittleston and Skinner that these two approaches can be unified starting from holomorphic Chern-Simons theory in 6 dimensions. In this talk I will introduce the first complete description of this diamond of integrable theories for a family of deformed sigma models, going beyond the Dirichlet boundary conditions that have been considered thus far. The talk is based on the recent work <https://arxiv.org/abs/2311.17551>.

Zoom link

Integrable Deformations From Twistor Space

- I)
1. Integrable Systems
 2. 4d Chern-Simons Theory
 3. 6d CS theory on \mathbb{P}^2 .
- II) Int Def from Twistor Space!

1. Large # sy

Sparca

1. Large # symmetries

↳ Large # conserved charges

↳ Exact Solvability

$$S[\phi], M; \phi: M \rightarrow G$$

$$\text{EOM } S[\phi] \longleftrightarrow \begin{array}{l} \text{Flatness eq.} \\ A[\phi] \end{array}$$

2d, lax Connection, $S[\phi]$ on $\Sigma = (t, x)$

$$\mathcal{L} = \mathcal{L}_t(\phi, z) dt + \mathcal{L}_z(\phi, z) dz \quad z \in \mathbb{CP}^1$$

$$\text{EOM } S[\phi] \longleftrightarrow F(\mathcal{L}) = 0 \quad \forall z \in \mathbb{CP}^1$$

$\Rightarrow S$ is (lax) integrable

4d: $S[\phi]$ on

$$F = - * \bar{F}$$

Penrose - Ward

Holomorphic
bundles on

$$\bar{\nabla}^2 = 0 \rightarrow F(A)$$

$$x = (t, x)$$

$$z \in \mathbb{C}P^1$$

$$\forall z \in \mathbb{C}P^1$$

4d : $S[\phi]$ on \mathbb{R}^4 , A , ASDYM

$$F = - * F$$

Penrose - Ward corr.

Holomorphic vect bundles & PT \longleftrightarrow ASDYM

$$\bar{\nabla}^2 = 0 \rightarrow F(\bar{A}) = 0$$

A very Nice Idea (Costello, Yamazaki, Witten)

Gauge Theories

$$\int_{\mathbb{R}^3} \langle A, dA + \frac{2}{3} A \wedge A \rangle \xrightarrow{\text{EOM}} dA + \frac{1}{2} [A, A] = 0$$

\mathbb{R}^3

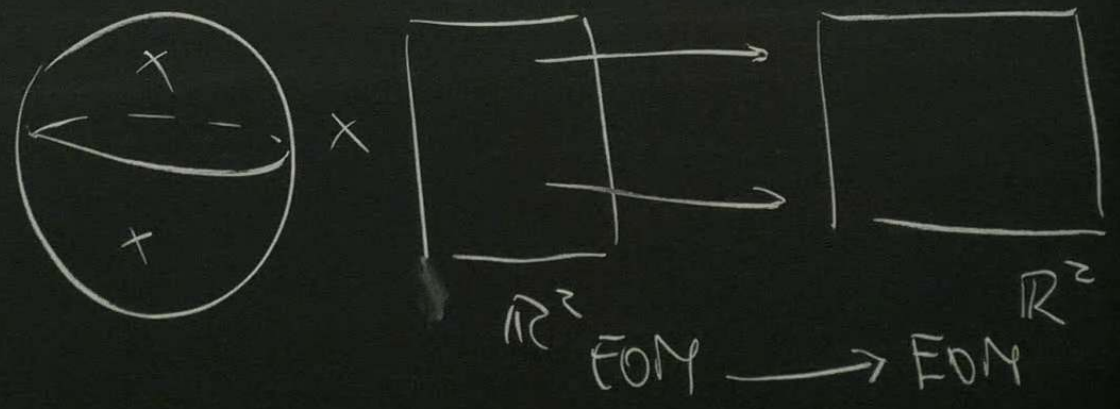
azaki, Witten)

$$dA + \frac{1}{2} [A, A] = 0$$

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R}^2 \longrightarrow \mathbb{C}P^1 \times \mathbb{R}^2$$

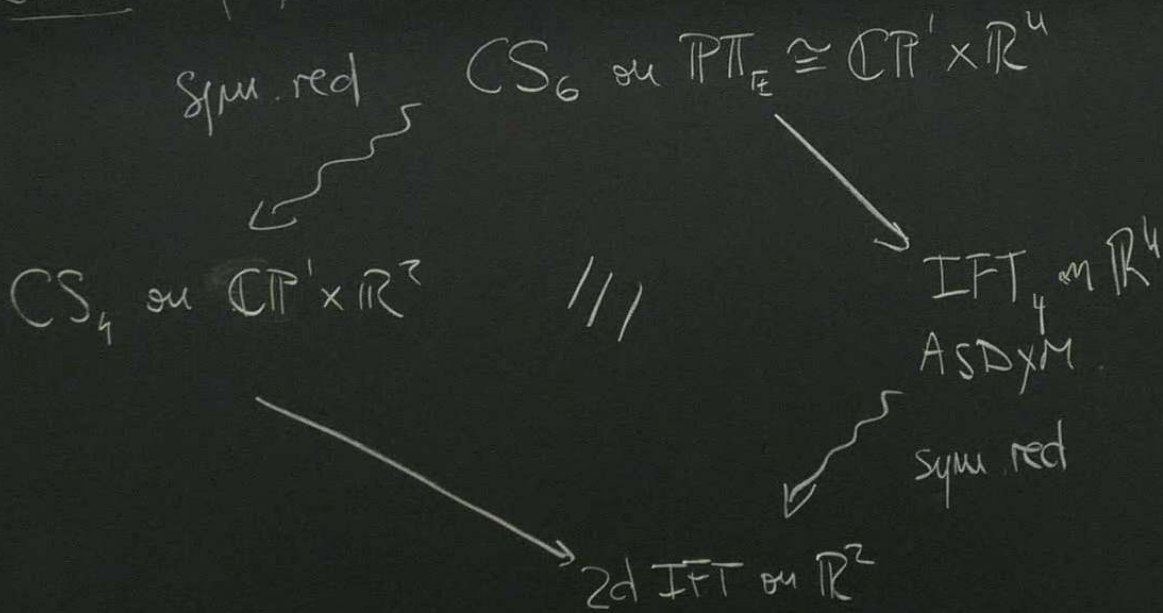
$$\int \omega \wedge \left\langle A, dA + \frac{2}{3} A \wedge A \right\rangle$$

$\mathbb{C}P^1 \times \mathbb{R}^2$ — Disorder Operator $\omega = \frac{d\bar{z}}{z}$



$\omega, BC \longrightarrow 2d$ IFT

2020: (Roland Bittlston, D Skemer)



2021:

T
 sym \perp D Skewer
 $PT_E \cong CP^1 \times R^4$
 IFT on R^4
 ASDYM
 sym red
 IFT on R^2

Integrable Def from Twistor Space

L. Cole, R. Cullinane, B. Hope, D. Thompson

$$S_{\text{def}} = \int_{PT_E \cong CP^1 \times R^4} \Omega \wedge \langle \bar{\lambda}, \bar{\partial} \bar{\lambda} + \frac{2}{3} \bar{\lambda} \wedge \bar{\lambda} \rangle$$



$$\Omega = \frac{e^a \wedge e_a \wedge e^0}{\langle \pi \alpha \rangle \langle \pi \tilde{\alpha} \rangle \langle \pi \beta \rangle^2} \in \Omega^{(3,0)}(RT)$$

π^a is CP^1 coordinate ; $\mu^a = \pi^a \kappa_{a\dot{a}}$

$$\delta S = \int \Omega \wedge \langle \delta \bar{A}, F(\bar{A}) \rangle + \int \bar{\Theta} \Omega \wedge \langle \bar{A}, \delta A \rangle$$

$$\bar{\Theta} \Omega = \# \delta (\langle \pi \alpha \rangle) + \delta (\langle \pi \vec{\alpha} \rangle) + \bar{\Theta}_0 \delta (\langle \pi \beta \rangle)$$

1. Large # sy



$S[\phi], M,$

EOM $S[\phi]$

$$\delta S = \int \Omega \wedge \langle \delta \bar{A}, F(\bar{A}) \rangle$$

$$+ \int_{\mathbb{CP}^1 \times \mathbb{R}^4} \bar{\Theta} \Omega \wedge \langle \bar{A}, \delta A \rangle$$

$$\bar{\partial} \Omega = \# \delta \langle \pi \alpha \rangle + \delta \langle \pi \vec{\alpha} \rangle + \bar{\partial}_0 \delta \langle \pi \beta \rangle$$

$$\delta S_{\text{bdry}} = \int \# \langle \bar{A}, \delta \bar{A} \rangle \Big|_{\alpha} + \# \langle \bar{k}, \delta \bar{k} \rangle \Big|_{\vec{\alpha}}$$

$$\# \bar{\partial}_0 \langle \bar{k}, \delta \bar{k} \rangle \Big|_{\beta}$$

$$\underline{At \beta} \quad \bar{A}|_{\beta} = 0 \quad (\delta A|_{\beta} = 0)$$

$$\underline{At \alpha, \bar{\alpha}} \quad \mu^{\hat{\alpha}}, |\mu^{\hat{\alpha}}| = 1, \bar{A}^{\hat{\alpha}} = [A_{\hat{\mu}}] \mu^{\hat{\alpha}} - [A_{\mu}]$$

$$[A_{\mu}]|_{\alpha} = \sigma [A_{\mu}]|_{\bar{\alpha}}$$

$$[A_{\hat{\mu}}]|_{\alpha} = \sigma^{-1} [A_{\hat{\mu}}]|_{\bar{\alpha}}$$

$$A_i|_{\alpha} = A_i|_{\tilde{\alpha}}$$

P'

$$+ = - * F$$

Symmetries

$$\bar{A} \rightarrow g' \bar{A} g + g^{-1} \bar{\partial} g$$

$$\bar{A}|_{\rho} = 0 \rightarrow \bar{A}^g|_{\rho} = 0 \rightarrow g^{-1} \bar{\partial} g|_{\rho} = 0$$

Penrose - Weyl

Holomorphic
bundles \mathcal{E}

$$\bar{\nabla}^2 = 0 \rightarrow F(A)$$

$$\bar{A} = \hat{h}^{-1} A' \hat{h} + \hat{h}^{-1} \bar{\partial} \hat{h}$$

$$S[A', \hat{h}] = \int \Omega \wedge \langle A', \bar{\partial} A' + \frac{2}{3} A' \wedge A' \rangle$$

$$+ \# \int \bar{\partial} \Omega \wedge \langle A', \bar{\partial} \hat{h} \hat{h}^{-1} \rangle$$

$$+ \# \int \bar{\partial} \Omega \wedge \langle \hat{h}^{-1} d\hat{h}^3 \rangle$$

$\mathbb{R}^4 \times [0, 1]$

$$A' \rightarrow u^{-1} A' u + u^{-1} \bar{\partial} u, \quad \hat{h}^{-1}$$

\hat{h}

$$A' \rightarrow u'^{\alpha} A'_{\alpha} + u'^{\beta} \bar{\partial}_{\beta} u \quad , \quad \hat{h} \rightarrow \hat{h}^{\alpha} u$$

$$A \rightarrow A$$

$$A'_0 = 0 \quad , \quad \hat{h}|_{\beta} = \text{id}$$

$$\langle \bar{\partial} A' + \frac{2}{3} A' \wedge A' \rangle$$

$$\langle \bar{\partial} \hat{h} \hat{h}' \rangle$$

$$\langle \hat{h}' d\hat{h}^3 \rangle$$

$$\bar{\partial}_0 \lambda'_\alpha = 0$$

$$S_{4\mathbb{H}} = \int_{\mathbb{R}^4} \langle \lambda', \bar{\partial} \hat{h} \hat{h}^{-1} \rangle |_\alpha + \int_{\mathbb{R}^4 \times [0,1]} \langle \hat{h}^{-1} d\hat{h} \rangle |_\alpha$$

$$- \int_{\mathbb{R}^4} \langle \lambda', \bar{\partial} \hat{h} \hat{h}^{-1} \rangle |_{\tilde{\alpha}} - S_{WZ} [\hat{h}] |_{\tilde{\alpha}}$$

$$\hat{h}|_\alpha = h, \quad \hat{h}|_{\tilde{\alpha}} = \tilde{h}$$

$$\mathbb{R}^3 =$$

$$\int \omega \wedge \langle \dots \rangle$$

$\mathbb{CP}^1 \times \mathbb{R}^2$



$$\langle \hat{h}^{-1} d\hat{h} \rangle |_{\alpha}$$

[0,1]

$$z \begin{bmatrix} \uparrow \\ \hat{h} \\ \downarrow \\ \alpha \end{bmatrix}$$

$$\bar{A} |_{\beta} = 0 \Rightarrow \cancel{\hat{h}^{-1}} A' \hat{h} + \hat{h}^{-1} \cancel{\partial \hat{h}} |_{\beta} = 0$$
$$\Rightarrow A' |_{\beta} = 0$$

Using BC, $A' = A' [h, \hat{h}]$

$$S = \frac{K}{\langle \alpha \tilde{\alpha} \rangle} \int_{\mathbb{R}^4} \text{vol}_4 \left[\langle j, (U_+^T - U_-) \hat{j} \rangle + \langle \hat{j}, (U_+^T - U_-) \tilde{j} \rangle - 2\sigma \langle \tilde{j}, U_+^T \hat{j} \rangle + 2\sigma \right]$$

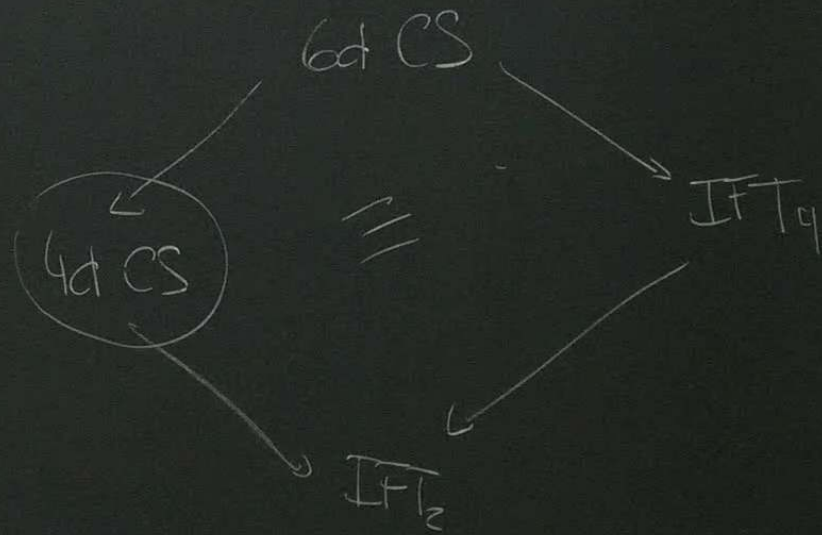
$$\left\{ \begin{array}{l} j = \frac{1}{\langle \alpha \beta \rangle} \mu^a \alpha^a h^{-1} \partial_{aa} h; \quad \tilde{j} = \frac{1}{\langle \tilde{\alpha} \tilde{\beta} \rangle} \mu^{\tilde{a}} \tilde{\alpha}^{\tilde{a}} \tilde{h}^{-1} \partial_{\tilde{a}\tilde{a}} \tilde{h} \\ \hat{j} = \frac{1}{\langle \alpha \beta \rangle} \hat{\mu}^a \alpha^a h^{-1} \partial_{aa} h; \quad \hat{\tilde{j}} = \frac{1}{\langle \tilde{\alpha} \tilde{\beta} \rangle} \hat{\mu}^{\tilde{a}} \tilde{\alpha}^{\tilde{a}} \tilde{h}^{-1} \partial_{\tilde{a}\tilde{a}} \tilde{h} \end{array} \right. \quad U_{\pm} = \left(1 - \sigma^{\pm 1} \text{Ad}_h^{-1} \text{Ad}_{\tilde{h}} \right)^{-1}$$

$$\langle \hat{j}, (U_+^T - U_-) \hat{j} \rangle + \langle \hat{j}, (U_+^T - U_-) \hat{j} \rangle - 2\sigma \langle \hat{j}, U_+^T \hat{j} \rangle + 2\sigma^{-1} \langle \hat{j}, U_- \hat{j} \rangle \Big] + \int_{WZ} [h, \tilde{h}]$$

$$\left. \begin{aligned} \partial_{aa} h; \quad \hat{j} &= \frac{1}{\langle \tilde{\alpha} \rangle} \mu^{\alpha} \tilde{\alpha}^{\alpha} \tilde{h}^{\alpha} \partial_{aa} \tilde{h} \\ \partial_{aa} h; \quad \hat{j} &= \frac{1}{\langle \tilde{\alpha} \rangle} \mu^{\alpha} \tilde{\alpha}^{\alpha} \tilde{h}^{\alpha} \partial_{aa} \tilde{h} \end{aligned} \right\}$$

$$U_{\pm} = \left(1 - \sigma^{\pm 1} Ad_h^{-1} Ad_h \right)^{-1}$$

→ ASDYM



1. Gauged WZW₄

$$S[A] - S[B] + \int \bar{\partial} \Omega_n(A, B)$$

$$\bar{A}|_{\beta} = 0$$

using