Title: Landscape of Measurement-Prepared Tensor Networks and Decohered Non-Abelian Topological Order

Speakers: Ruben Verresen

Collection: Physics of Quantum Information

Date: May 31, 2024 - 9:00 AM

URL: https://pirsa.org/24050042

Abstract: What is the structure of many-body quantum phases and transitions in the presence of non-unitary elements, such as decoherence or measurements? In this talk we explore two new directions. First, recent works have shown that even if one starts with an ideal preparation of topological order such as the toric code, decoherence can lead to interesting mixed states with subtle phase transitions [e.g., Fan et al, arXiv:2301.05689]. Motivated by a recent experimental realization of non-Abelian topological order [Iqbal et al, Nature 626 (2024)], we generalize this to decohered non-Abelian states, based on work with Pablo Sala and Jason Alicea [to appear]. Second, we study whether and how one can prepare pure states which are already detuned from ideal fixed-point cases---with tunable correlation lengths. This turns out to be possible for large classes of tensor network states which can be deterministically prepared using finite-depth measurement protocols. This is based on two recent works with Rahul Sahay [arXiv:2404.17087; arXiv:2404.16753].

Landscape of
Measurement-Prepared Tensor Networks
and
Decohered Non-Abelian Topological Order

Part I: arxiv:2404.16753 and arxiv:2404.17087
with **Rahul Sahay**

Part II: to appear soon
with **Pablo Sala** and **Jason Alicea**

Ruben Verresen
Harvard & MIT (→ UChicago)
(Funding: UQM Simons collaboration)

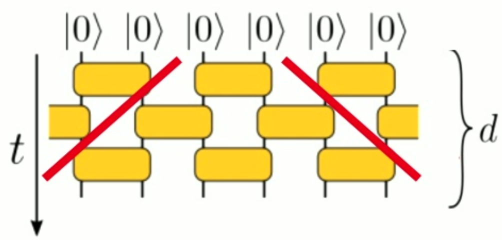
"Physics of Quantum Information" @ Perimeter Institute, May 31, 2024

Many-Body Quantum Entanglement from Non-Unitarity

Part I: wavefunctions from measurements in constant-depth

(with Rahul Sahay, arxiv:2404.16753, arxiv:2404.17087)

What is the power of measurement for preparing quantum states?



$$\langle \mathcal{O}_n \mathcal{O}_m \rangle_c = 0 \qquad |n-m| > 2d$$

Concurrent works:
Smith et al (arXiv:2404.16083), Stephens et al (arXiv:2404.16360), Zhang et al (arXiv:2405.09615)

Part II: decohering topological order

(with Pablo Sala + Jason Alicea, to appear)

What if we subject non-Abelian topological order to decoherence?

Article | Published: 14 February 2024

Non-Abelian topological order and anyons on a trapped-ion processor

Mohsin Iqbal, Nathanan Tantivasadakarn, Ruben Verresen, Sara L. Campbell, Joan M. Dreiling, Caroline Figgatt, John P. Gaebler, Jacob Johansen, Michael Mills, Steven A. Moses, Juan M. Pino, Anthony Ransford, Mary Rowe, Peter Siegfried, Russell P. Stutz, Michael Foss-Feig, Ashvin Vishwanath & Henrik Dreyer

Nature **626**, 505–511 (2024) | Cite this article

arxiv:2305.03766

Findings:

1. remarkable robustness
2. decoherence-induced critical phases and loop models

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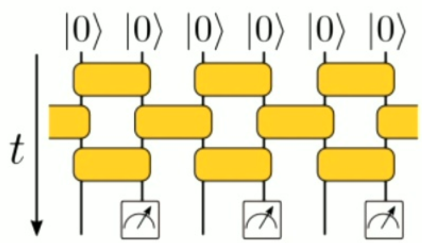

Briegel-Raussendorf '01; Raussendorf-Bravyi-Harrington '05; Piroli-Styliaris-Cirac '21; RV-Tantivasadakarn-Vishwanath '21; Tantivasadakarn-Thorngren-Vishwanath-RV '21; Lu-Lessa-Kim-Hsieh '22; Bravyi-Kim-Kliesch-Koenig '22; ...

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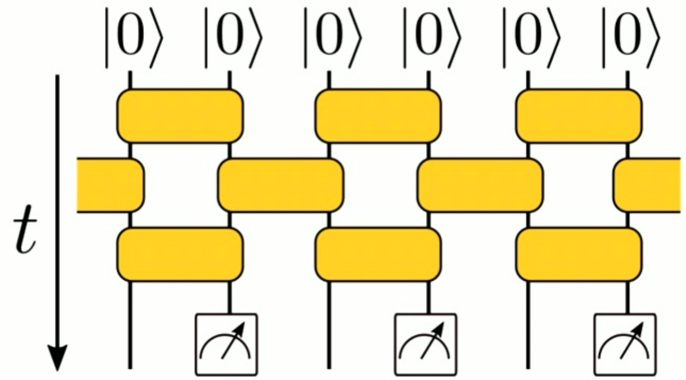
Constant-Depth Prep ⊆ Tensor Networks

Tensor Network States (with finite bond dimension)

=

{finite-depth circuits with forced measurement outcome}



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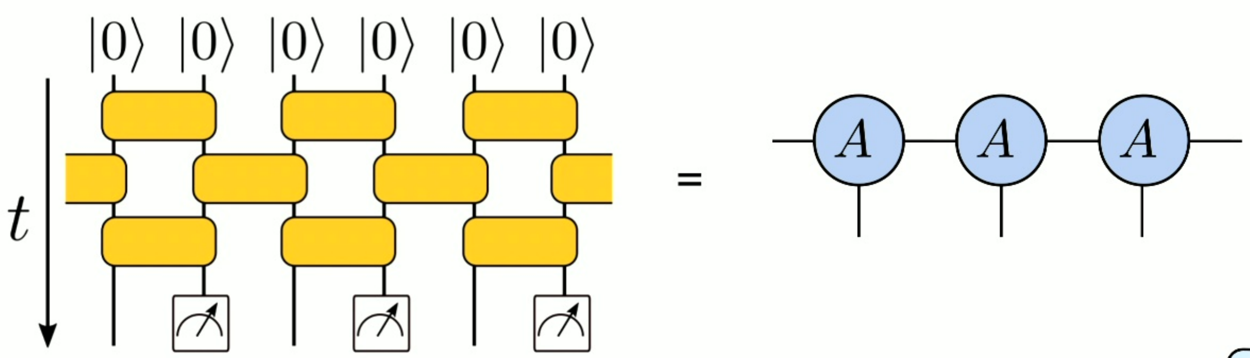
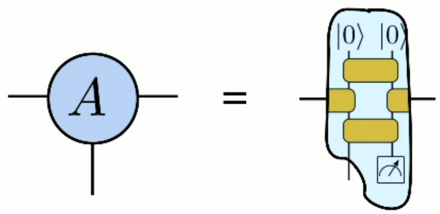


where 

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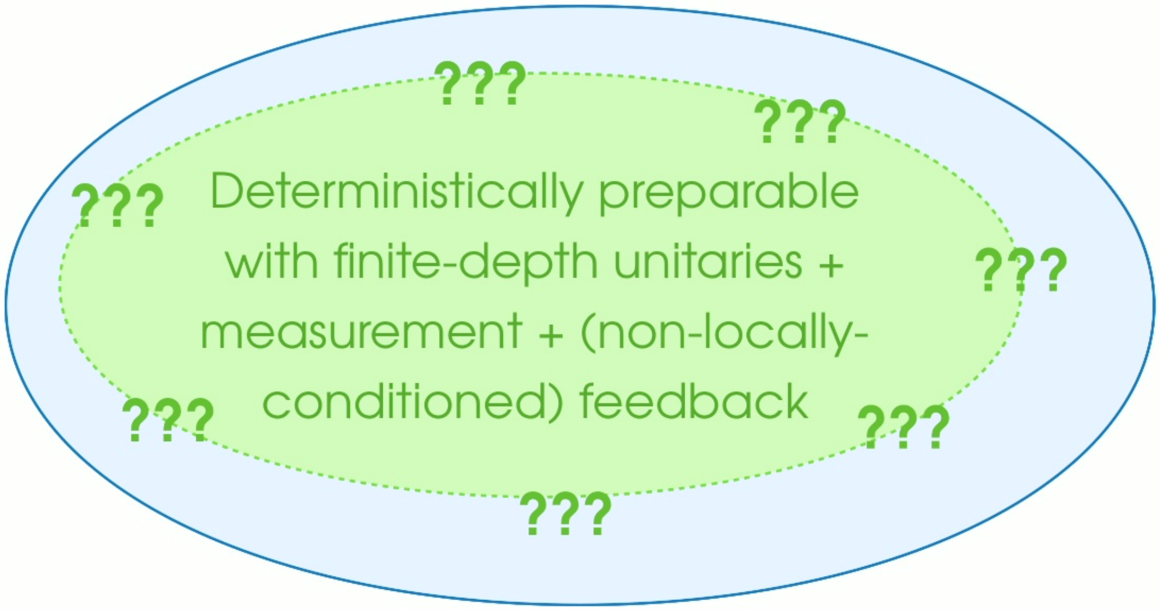

Tensor Network States
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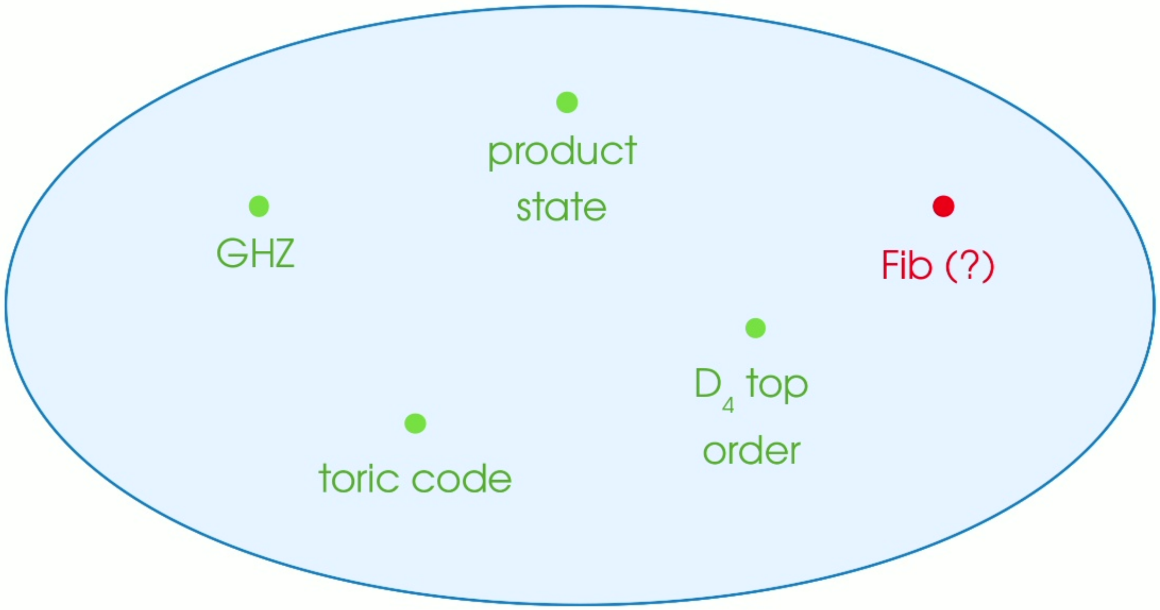
Constant-Depth Prep ⊆ Tensor Networks



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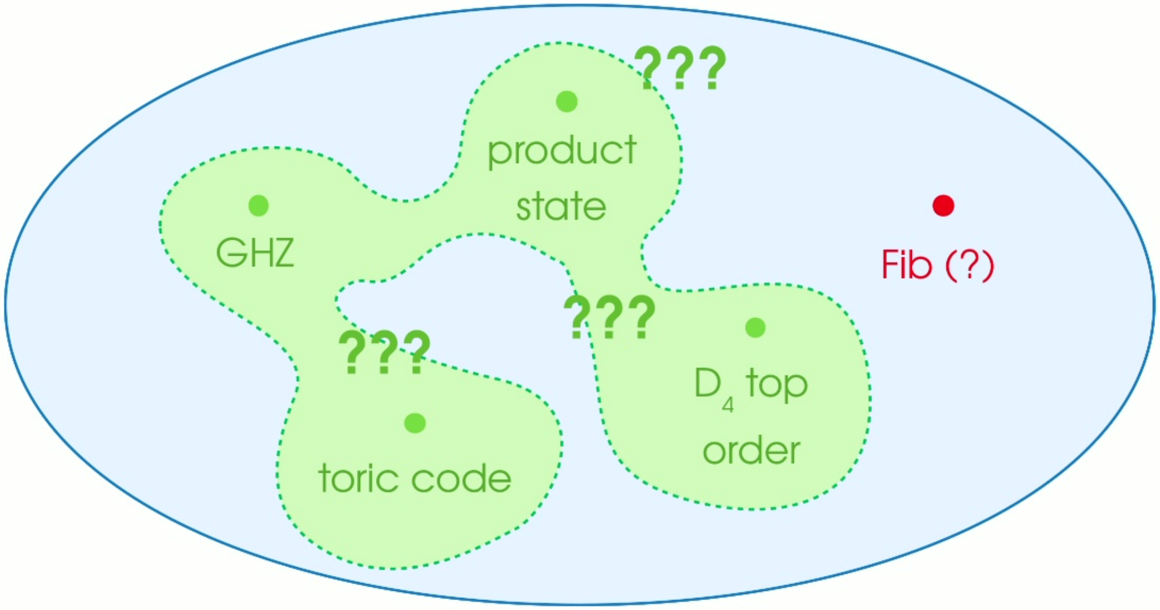

Tensor Network States
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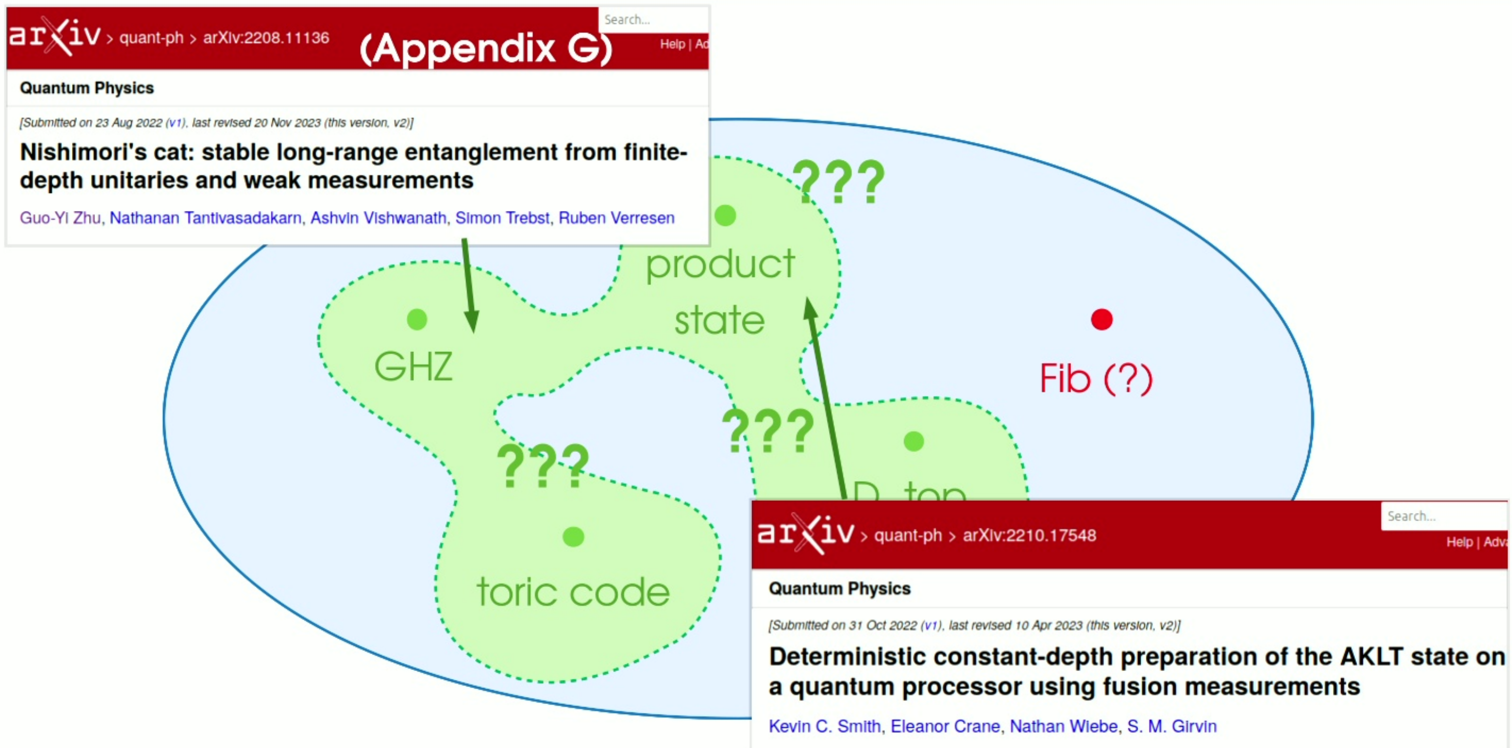


Tensor Network States

(= forced measurement outcome)

Outline for Part I

1. Motivating Example

Beyond-Fixed-Point SSB

2. Setup and General Formalism

Local Tensor Criteria, Resource Theorems, and Classification

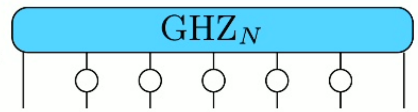
3. Phenomenology of Preparable States

Trade-Off between Entanglement and Correlations

Preparing deformed GHZ state

○ $= e^{\beta X}$

$$|\Psi(\beta)\rangle \propto \exp\left(\beta \sum_{x=2}^{N-1} X_x\right) |\mathrm{GHZ}_N\rangle \; =$$

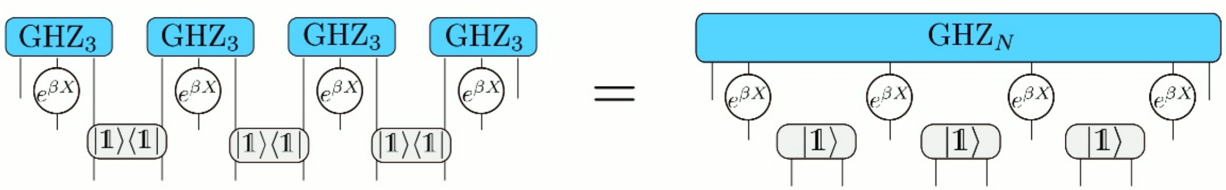


We start with decoupled clusters:

$$|\Psi_0(\beta)\rangle \propto \bigotimes_{x=2}^{N-1} e^{\beta X_{c_x}} |\mathrm{GHZ}_3\rangle_{l_x, c_x, r_x} \; = \cdots$$

When then 'glue' them together with Bell pair measurements:

$$|\mathbb{1}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



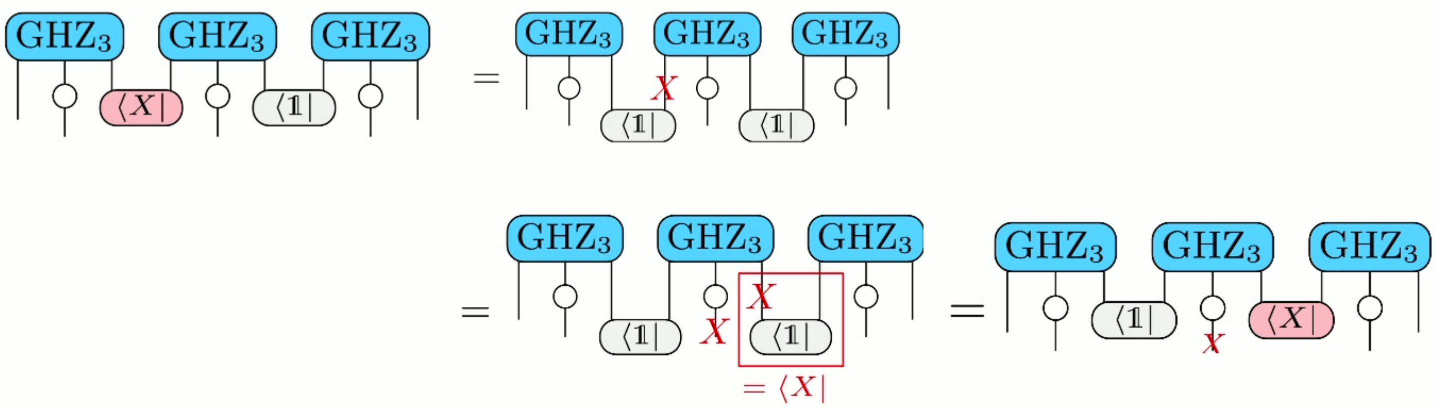
Preparing deformed GHZ state

How do we correct 'wrong' measurement outcomes?

$$|\mathbb{1}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad |Z\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|X\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \qquad |ZX\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$$

Note: $|X\rangle = X|\mathbb{1}\rangle$



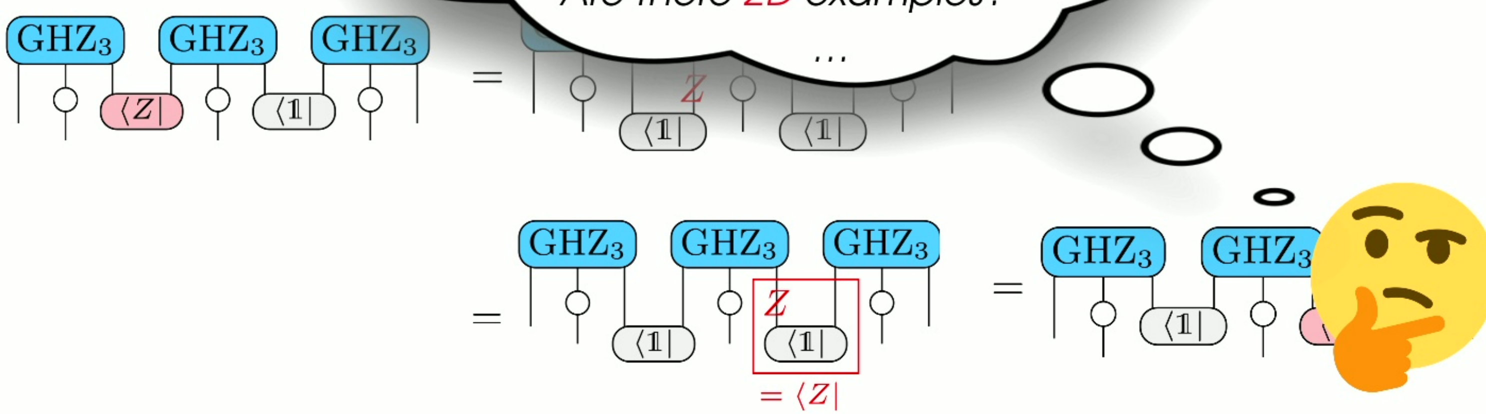
Preparing deformed GHZ state

How do we correct 'wrong' measurement outcomes?

$|\mathbb{1}\rangle = \frac{1}{\sqrt{2}}(|00\rangle$ … $\frac{1}{\ }(|00\rangle - |11\rangle)$

$|X\rangle$ …

Which states can be prepared in this way?
Was it important that we used the Bell basis?
Was it important that the state has flat entanglement spectrum?
Are there 2D examples?
…



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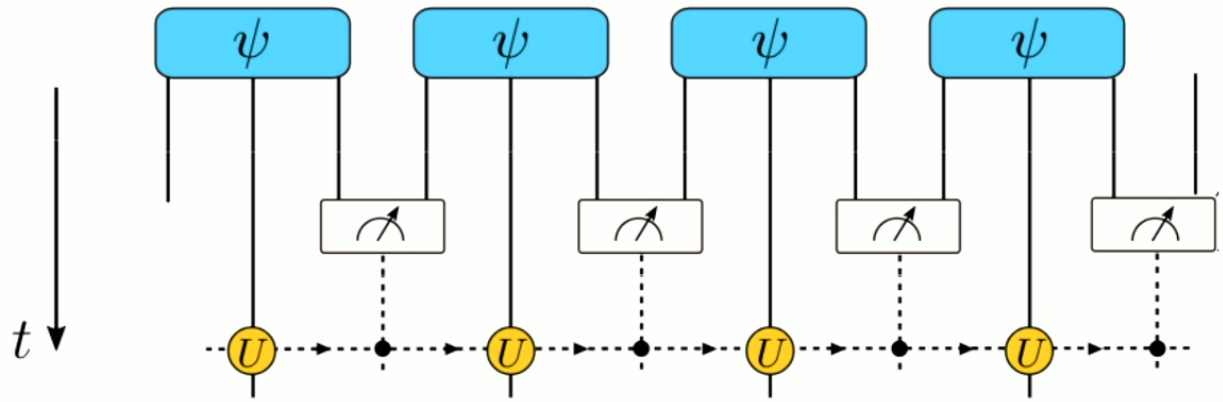
Preparation protocol

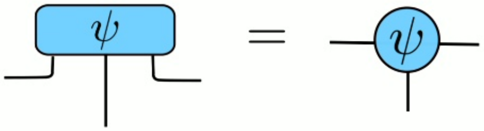
Definition (Gluable Quantum State) We say a quantum state is gluable if it can be deterministically prepared from:

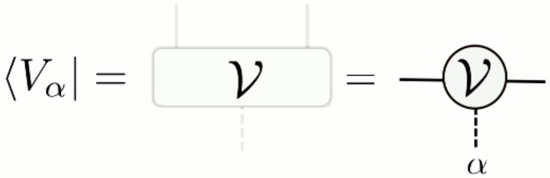
1. a product state of entangled clusters
2. finite-range measurements in a complete basis
3. tensor-product unitary feedback (left-conditioned)

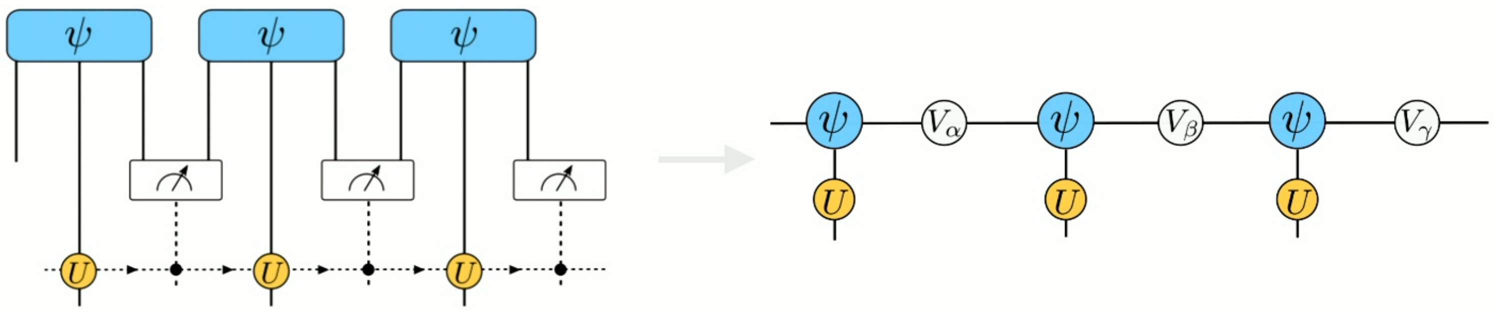


Any preparable state is a matrix product state

Reshape into matrix product state (MPS)







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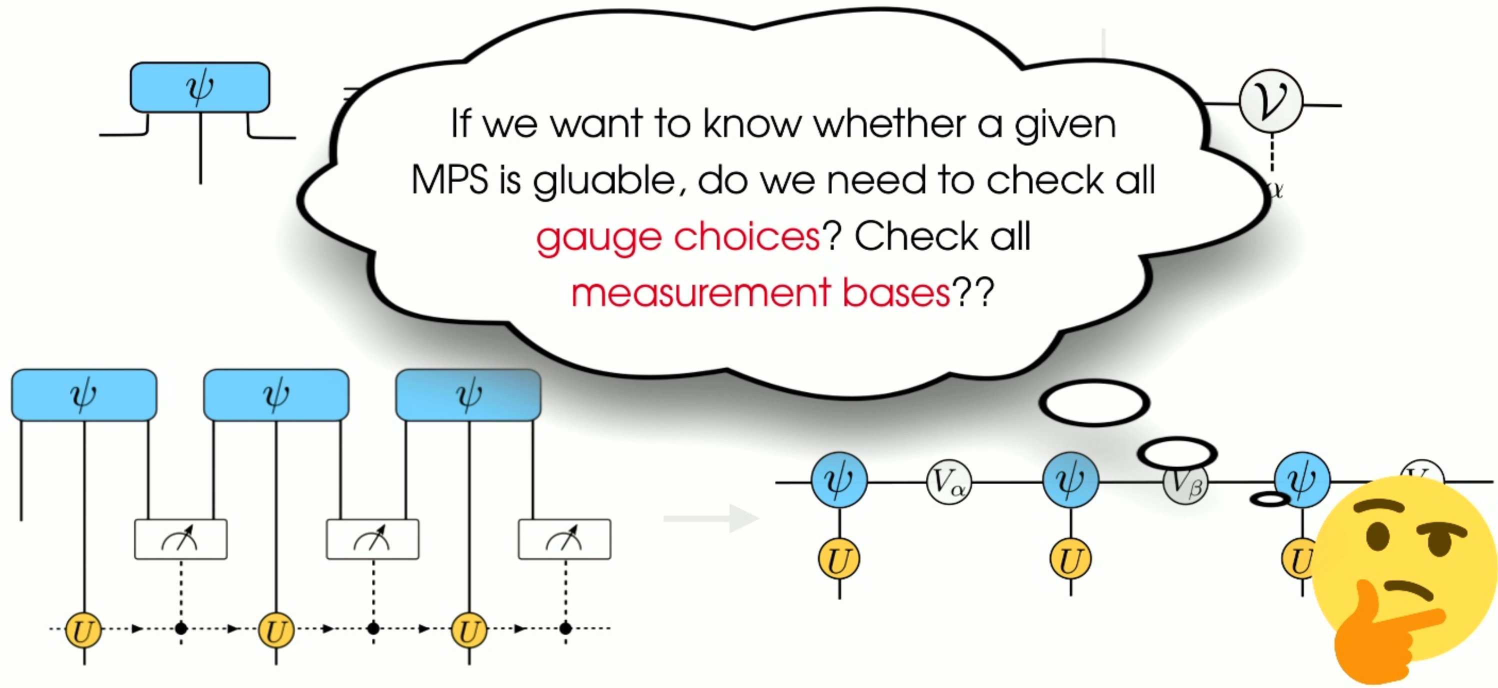


Resource theorem: where to look

Rahul Sahay

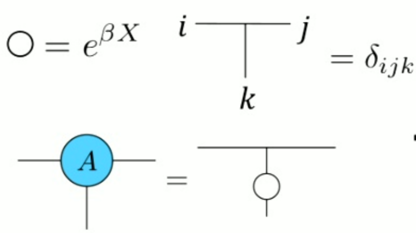
Theorem (Resource Theorem) Suppose that $|\Psi\rangle$ is a translation-invariant gluable quantum state. Then the following are true:

1. The wavefunction $|\Psi\rangle$ has an exact matrix product state (MPS) description
2. The clusters used for prep. have to be the MPS tensors in canonical form*
3. The measurement basis is maximally entangled (equiv. V operators are unitary).

Illustration for deformed GHZ state:

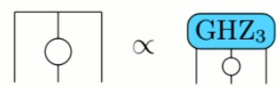
$$|\psi(\beta)\rangle = e^{\beta \sum_n X_n} |\mathrm{GHZ}\rangle$$

Tensors Related to Clusters



Can easily check this is in canonical form

Note initial clusters



Given by MPS tensor

Measurement Basis

We used the states

$|\mathbb{1}\rangle \quad |Z\rangle$

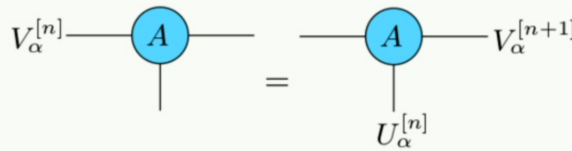
$|X\rangle \quad |ZX\rangle$

Maximally entangled 2-qubit Bell states

Local tensor criterion: how to tell

Rahul Sahay

Theorem (Local Tensor Criteria) A state $|\Psi\rangle$ is a translation-invariant gluable quantum state if and only if its matrix product state representation a complete error basis of operators $\{V_\alpha^{[0]}\}$ that "pushes through" the MPS:



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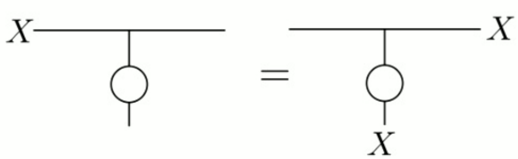

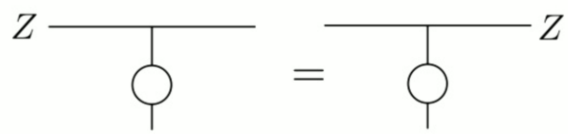

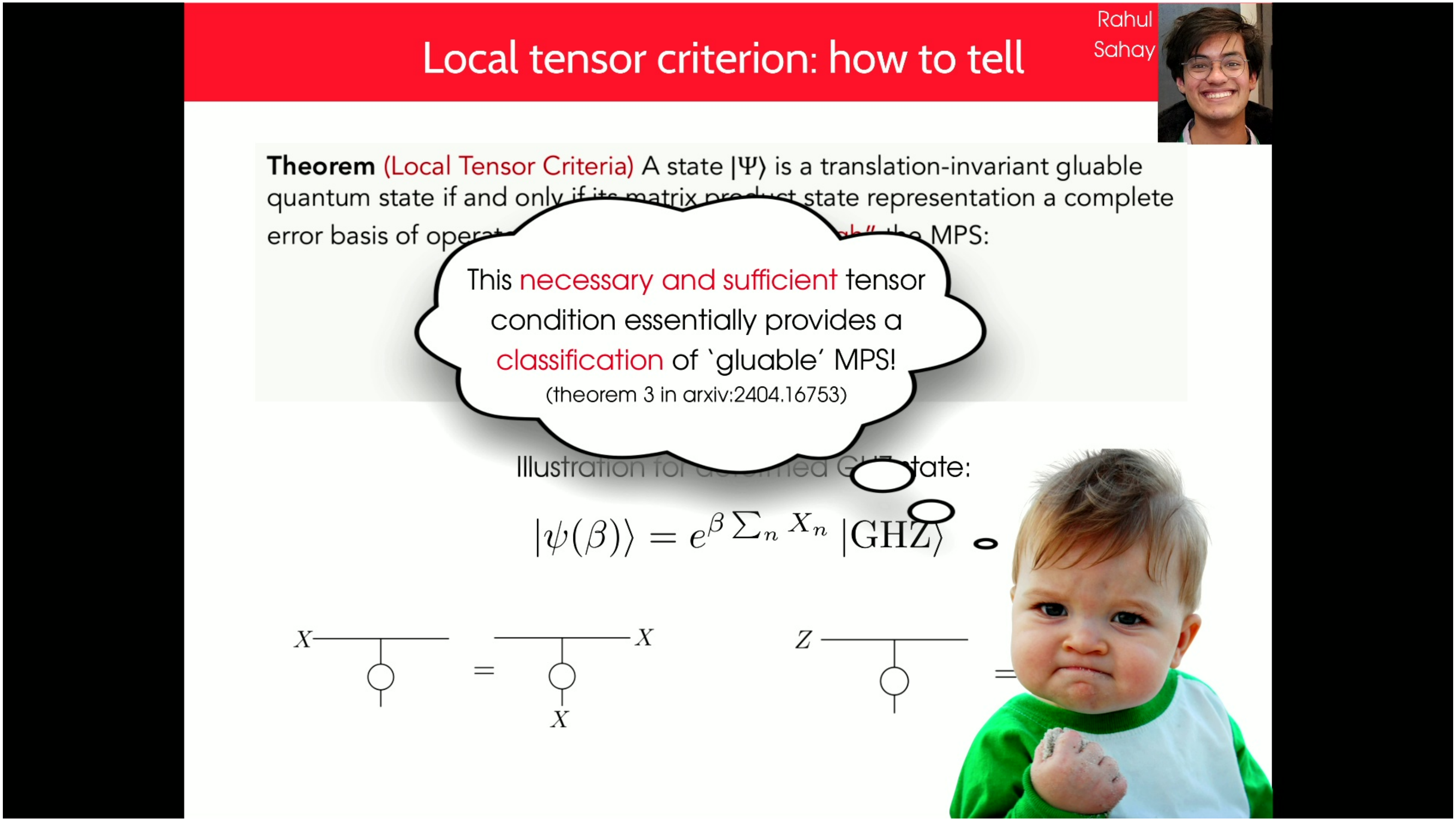
Local tensor criterion: how to tell
Rahul Sahay
Theorem (Local Tensor Criteria) A state $|\Psi\rangle$ is a translation-invariant gluable
quantum state if and only
state representation a complete
error basis of ope
MPS:
This necessary and sufficient tensor condition essentially provides a classification of `gluable' MPS!
(theorem 3 in arxiv:2404.16753)
Illustration fo
tate:
$|\psi(\beta)\rangle = e^{\beta \sum_n X_n} |\mathrm{GHZ}\rangle$
X
=
X
X
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Nonlocal correction → flat ES

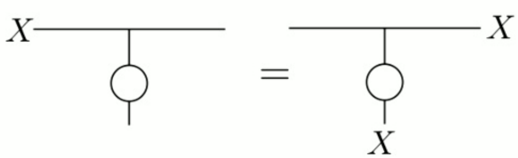
Rahul Sahay

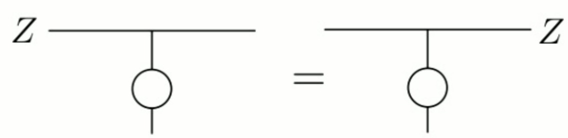
Theorem (paraphrased) If all errors are corrected by string operators (i.e., push through indefinitely in the MPS), then the entanglement spectrum of a semi-infinite bipartition must be flat.

Intuition: think of measurement-based quantum computation

Illustration for deformed GHZ state:

$$|\psi(\beta)\rangle = e^{\beta \sum_n X_n} |\mathrm{GHZ}\rangle \quad \rightarrow \quad \Lambda^2 = \left(\frac{1}{2}, \frac{1}{2}\right)$$





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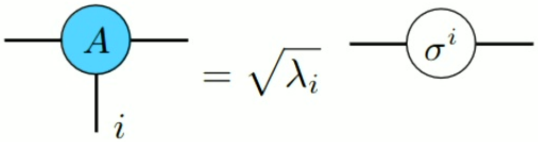
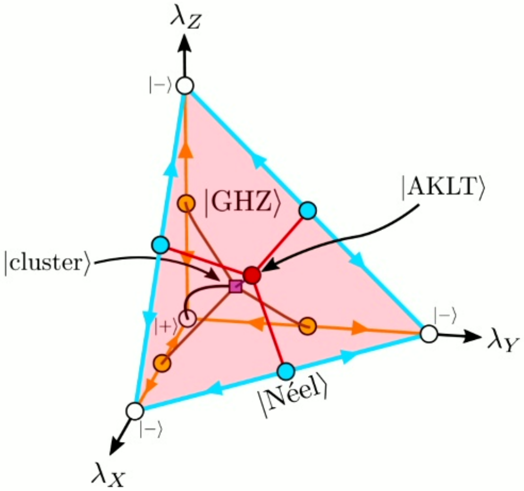
Despite this constraining property, still rich landscape of such states:

Example of complete classification for case χ=2:



$\sigma^i \in \{\mathbb{1}, X, Y, Z\}$



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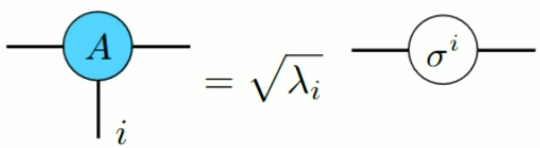

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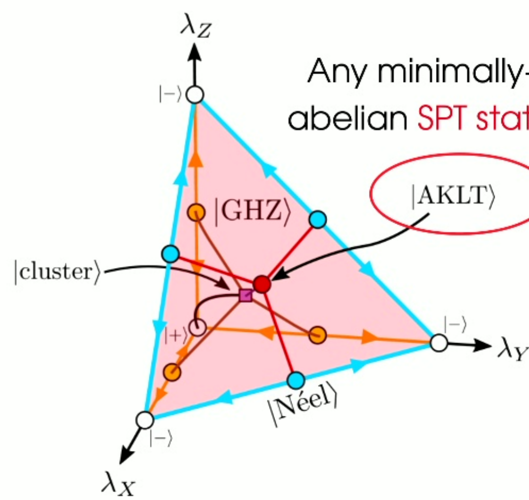
Any minimally-entangled abelian SPT state is gluable!



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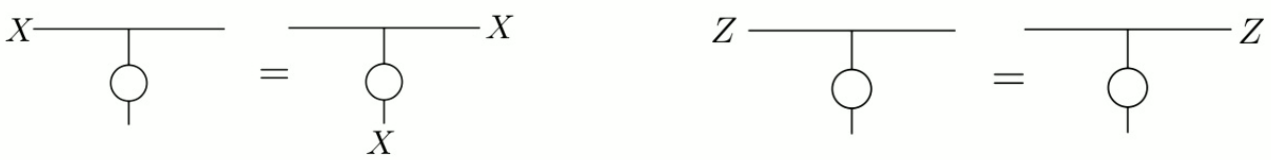


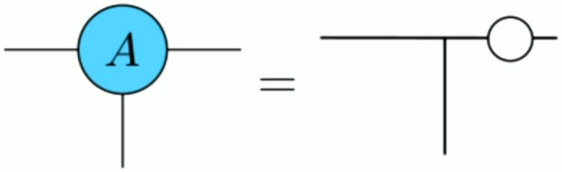
Local correction → constrains correlation

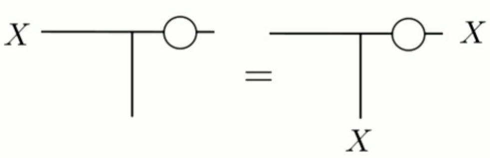
Rahul Sahay

$$|\Psi(\beta)\rangle \propto \exp\left(\beta \sum_{x=2}^{N-1} Z_x Z_{x+1}\right) |+\rangle^{\otimes N}$$

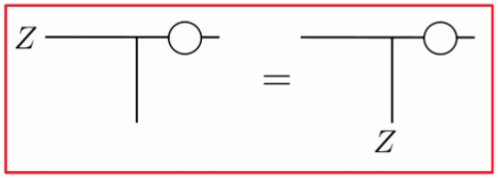
$$\Lambda^2 = \left(\frac{1+\delta}{2}, \frac{1-\delta}{2}\right)$$

$$\delta^{-1} = \cosh(2\beta)$$





$\bigcirc = e^{\alpha X}$ $\quad \alpha = \mathrm{arctanh}(e^{-2\beta})$



Theorem (Local Errors Constrain Correlations) If $|\Psi\rangle$ is gluable and there exists a measurement error that can be locally corrected, then there exists an operator with zero correlation length.

Summary for Part I

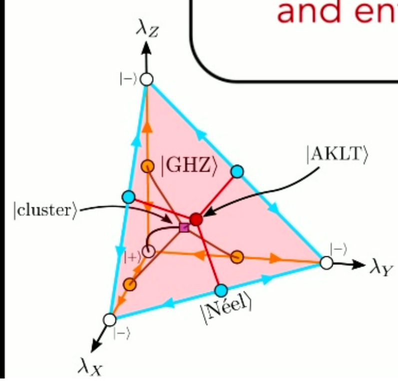
By focusing on measurement-only circuits:

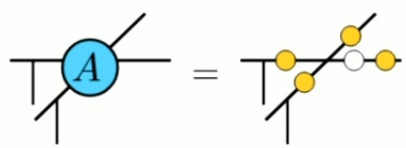
Key Results and Ideas

→ Classification of preparable quantum states in this setting

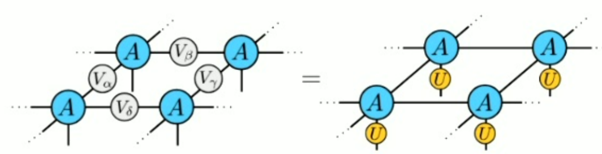
→ "Resource" theorem that almost fully constrains the preparation protocol for creating a quantum state

→ Phenomenological constraints on the properties of preparable states → trade-off b/w preparable correlations and entanglement





$$e^{\beta \sum X} |\mathrm{TC}\rangle$$



No-go theorem

Rahul Sahay

Putting the above results together:

Theorem (No-Go Theorem) If $|\Psi\rangle$ has a non-flat entanglement spectrum and no zero correlation length operators, it is not gluable.

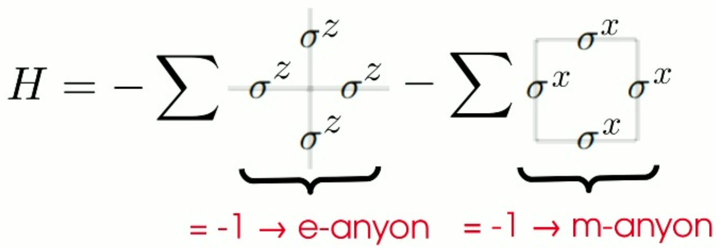
e.g., $$|\psi(\beta)\rangle = e^{\beta \sum_n X_n} e^{\beta \sum_n Z_n Z_{n+1}} |+\rangle^{\otimes N}$$

"We can create interesting entanglement and interesting correlation functions, but not at the same time!"

Part II (briefly)

Decohering topological order can lead to mixed states with interesting phase diagrams

(Fan-Bao-Altman-Vishwanath '23; Bao-Fan-Vishwanath-Altman '23; Li-Jian-Xu '23; Wang-Wu-Wang '23; Chen-Grover '23; Li-Mong '24; Lu '24; Sohal-Prem '24; Sang-Hsieh '24; Ellison-Ceng '24; Chen-Grove '24; Li-Lee-Yoshida '24; Lessa-Ma-Zhang-Cheg-Wang '24; … and more!!)



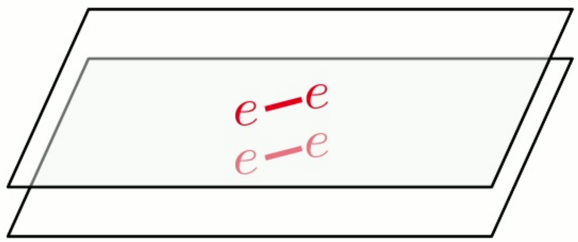
(Kitaev 1997)

Which statistical mechanical models (if any) might replace the role of the Ising model in the case of non-Abelian topological order?

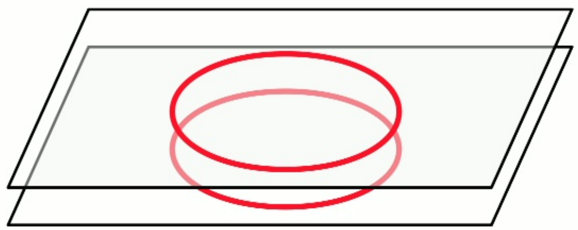
O(1) loop model describing toric code decoherence

Decohering toric code with e.g. X-noise leads to

$$|\rho\rangle \propto e^{\beta \sum_l X_l^A X_l^B} \, |\mathrm{TC}\rangle_A \otimes |\mathrm{TC}\rangle_B$$



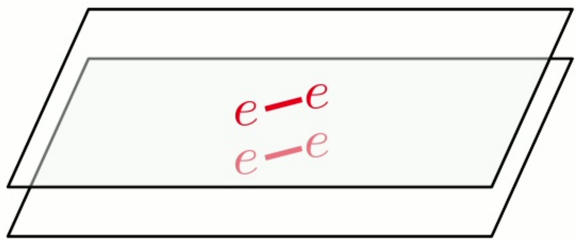
$$\langle \rho | \rho \rangle \propto \sum_{\text{loops } \gamma} \tanh(2\beta)^{|\gamma|}$$



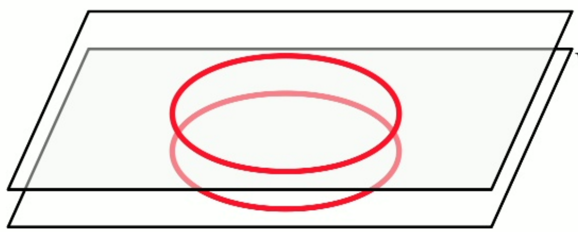
O(1) loop model describing toric code decoherence

Decohering toric code with e.g. X-noise leads to

$$|\rho\rangle \propto e^{\beta \sum_l X_l^A X_l^B} |\mathrm{TC}\rangle_A \otimes |\mathrm{TC}\rangle_B$$



$$\langle \rho | \rho \rangle \propto \sum_{\text{loops } \gamma} \tanh(2\beta)^{|\gamma|}$$



Special case of O(n) loop models:

$$\sum_{\text{loops } \gamma} t^{|\gamma|} n^{C_\gamma}$$

where C_γ is number of components

(Nienhuis '82)

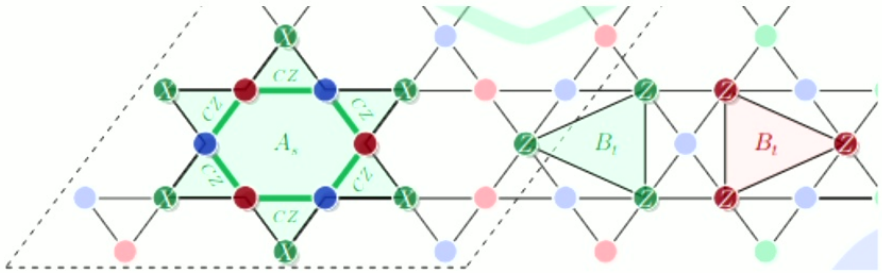
Simple Model for Non-Abelian Topological Order

D_4 topological order for qubits on kagome lattice

Yoshida, PRB (2016)

$$H = -\sum_s A_s - \sum_t B_t$$

States experimentally realized in Iqbal et al, arxiv:2305.03766



Ground state: $A_s = B_t = 1$

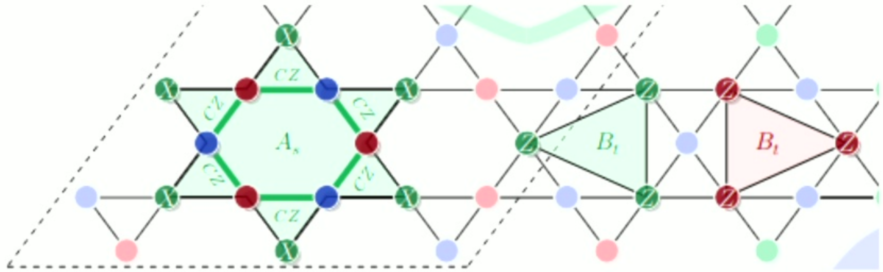
Anyons: $A_s = -1$ is `e-anyon' with d=1

$B_t = -1$ is `m-anyon' with d=2

$$m_R \times m_R = 1 + e_B + e_G + e_B e_G$$

Deforming D_4 topological order

Pablo
Sala



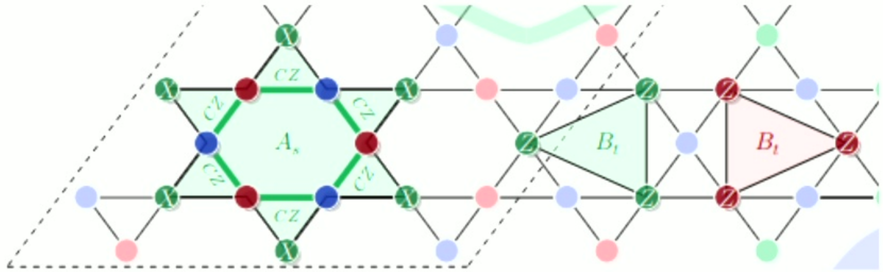
As warm-up for decoherence, let's deform the wavefunction

$$|\psi(\beta)\rangle = e^{\beta \sum_{r \in R} Z_r} |D_4\rangle$$

→ fluctuates abelian anyon with d=1
→ O(1) loop model

Deforming D_4 topological order

Pablo Sala



As warm-up for decoherence, let's deform the wavefunction

$$|\psi(\beta)\rangle = e^{\frac{\beta}{2}\sum_{r\in R} X_r} |D_4\rangle$$

→ fluctuates non-abelian anyon with d=2

$$\langle\psi(\beta)|\psi(\beta)\rangle \propto \sum_{\gamma} \tanh(\beta)^{|\gamma|} \langle D_4| \prod_{l\in\gamma} X_l |D_4\rangle$$

Deforming D_4 topological order

Pablo Sala

$$|\psi(\beta)\rangle = e^{\frac{\beta}{2}\sum_{r\in R} X_r} |D_4\rangle$$

→ fluctuates non-abelian anyon with d=2

$$\langle\psi(\beta)\,|\psi(\beta)\rangle \propto \sum_{\gamma} \left(\frac{\tanh(\beta)}{\sqrt{2}}\right)^{|\gamma|} 2^{C_\gamma}$$

Phase diagram:

Intuition: condensing m_R without e_B or e_G is difficult!

(see Chen-Grover arxiv:2403.06553 for analogue with abelian fermions)

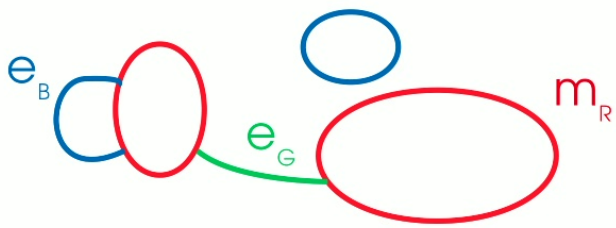
Deforming D_4 topological order

Pablo Sala

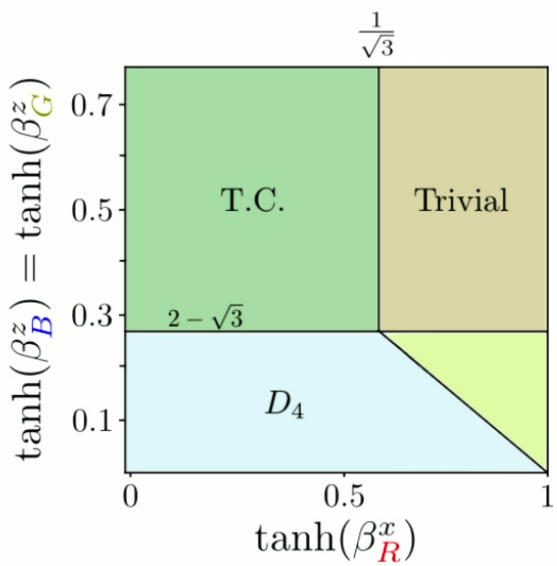
$$|\psi(\beta)\rangle = e^{\frac{\beta_r}{2}\sum_{r\in R} X_r + \frac{\beta_b}{2}\sum_{b\in B} Z_b + \frac{\beta_g}{2}\sum_{g\in G} Z_g} |D_4\rangle$$

→ fluctuates both abelian and non-abelian anyons

→ coupled O(1) and O(2) loop models with branching



Rewrite as local stat. mech. model and do Monte Carlo →



(similar result for $\langle\rho|\rho\rangle$)

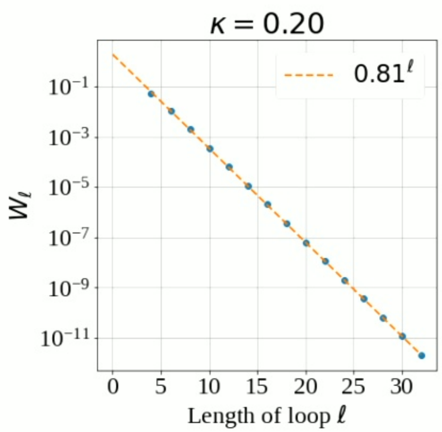
Deforming and decohering Ising anyons

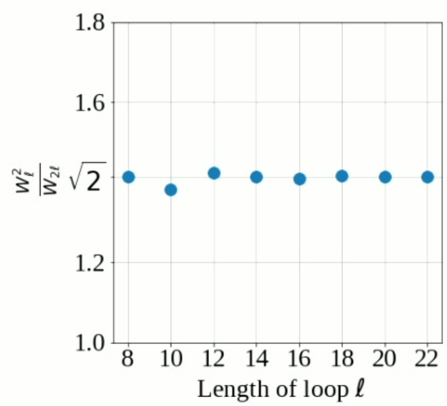
Pablo Sala

We also studied non-Abelian phase of Kitaev honeycomb model

→ we obtain $\sum_{\gamma} t^{|\gamma|} W(\gamma)$ where W is a free-fermion determinant

$$W(\gamma) \sim r^{|\gamma|} n^{C_\gamma} + \cdots$$





Jason Alicea

Summary for Part II

Pablo Sala

O(n) loop and net models natural for deformed and decohered non-Abelian topological order

Robust phases due to difficulty of condensing non-abelions

Can result in phases with algebraic correlations!

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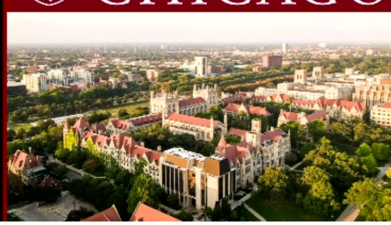


Starting my group at UChicago in the Fall of 2024

Group website: www.verresengroup.com

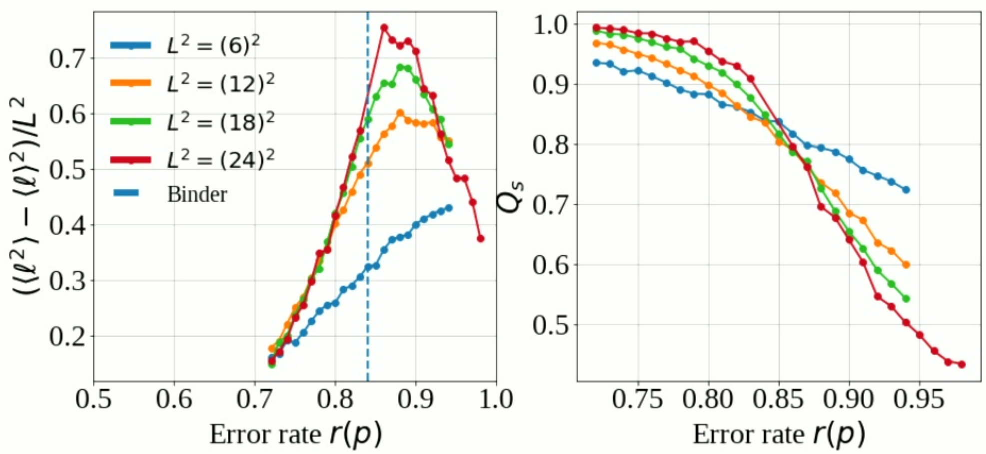
Deforming and decohering Ising anyons

Pablo Sala

We also studied non-Abelian phase of Kitaev honeycomb model

→ we obtain $\sum_{\gamma} t^{|\gamma|} W(\gamma)$ where W is a free-fermion determinant

For $|W(\gamma)|$ we see transition consistent with $O(\sqrt{2})$ loop model



Deforming D_4 topological order

Pablo Sala

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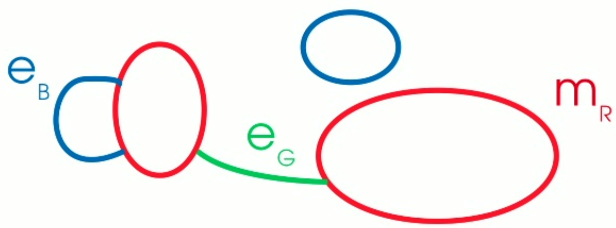

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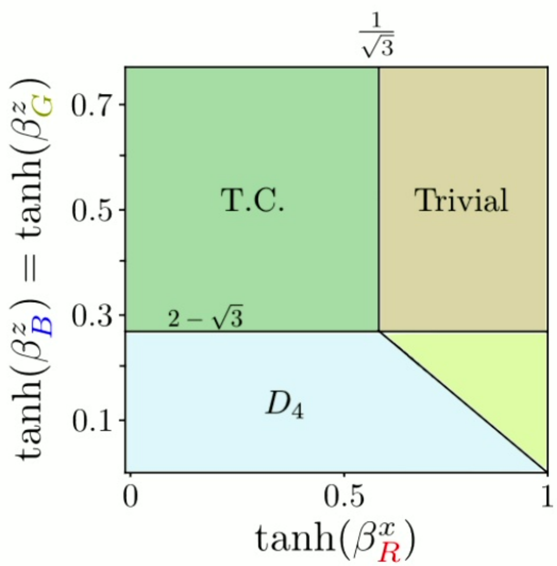

(similar result for $\langle\rho|\rho\rangle$)