Title: Unraveling quantum many-body scars: Insights from collective spin models

Speakers: Meenu Kumari

Collection: Physics of Quantum Information

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Abstract: Quantum many-body scars (QMBS) are atypical eigenstates of chaotic systems that are characterized by sub-volume or area law entanglement as opposed to the volume law present in the bulk of the eigenstates. The term, QMBS, was coined using heuristic correlations with quantum scars - eigenstates with high probability density around unstable classical periodic orbits in quantum systems with a semiclassical description. Through the study of entanglement in a multi-qubit system with a semiclassical description, quantum kicked top (QKT), we show that the properties of QMBS states strongly correlate with the eigenstates corresponding to the very few stable periodic orbits in a chaotic system as opposed to quantum scars in such systems. Specifically, we find that eigenstates associated with stable periodic orbits of small periodicity in chaotic regime exhibit markedly different entanglement scaling compared to chaotic quantum states, while quantum scar eigenstates demonstrate entanglement scaling resembling that of chaotic quantum states. Our findings reveal that quantum many-body scars and quantum scars are distinct. This work is in collaboration with Cheng-Ju Lin and Amirreza Negari.

Unraveling quantum many-body scars: Insights from collective spin models

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Joint work with

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Amirreza Negari
PhD student @ PI

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Motivation

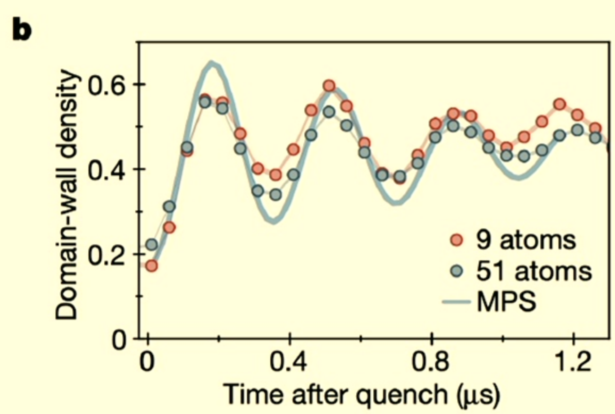
nature > articles > article

Published: 30 November 2017

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien, Sylvain Schwartz, Alexander Keesling, Harry Levine, Ahmed Omran, Hannes Pichler, Soonwon Choi, Alexander S. Zibrov, Manuel Endres, Markus Greiner, Vladan Vuletić & Mikhail D. Lukin

Nature **551**, 579–584 (2017) | Cite this article



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Motivation

- PXP model study - QMBS coined

nature > nature physics > articles > article

Article | Published: 14 May 2018

Weak ergodicity breaking from quantum many-body scars

C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn & Z. Papić

Nature Physics **14**, 745–749 (2018) | Cite this article

- Connections with Quantum Scars?

Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits

Eric J. Heller
Phys. Rev. Lett. **53**, 1515 – Published 15 October 1984

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Quantum Scars

Quantum eigenstates of a classically chaotic system with an enhanced probability density along **unstable periodic orbits** as compared to the statistically expected density.

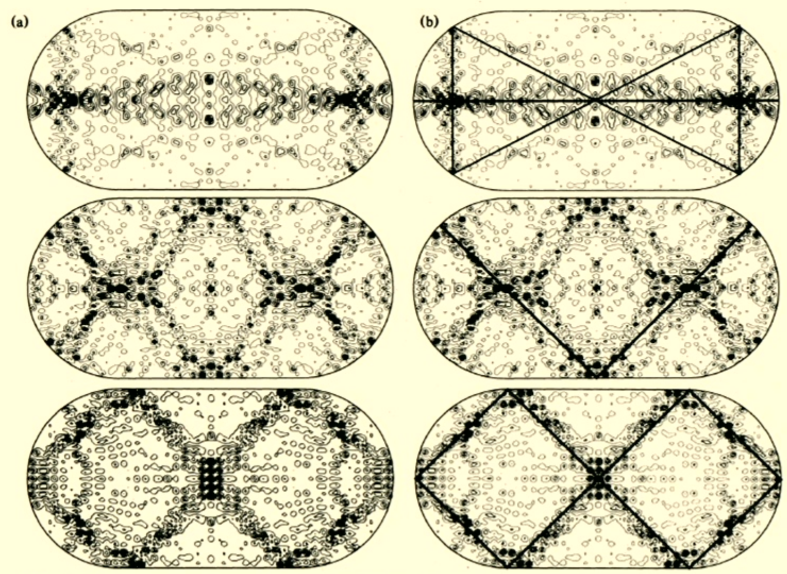
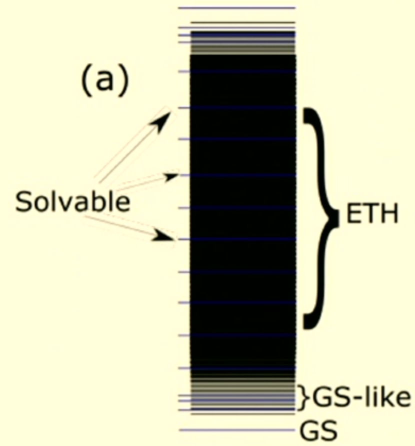


FIG. 2. Left column, three scarred states of the stadium; right column, the isolated, unstable periodic orbits corresponding to the scars.

Scar eigenfunctions in chaotic stadium billiards [Heller, PRL 1984].

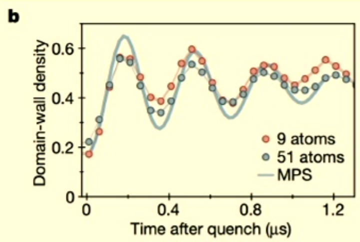
Quantum many-body scars (QMBS)

From "Quantum Many-Body Scars and Hilbert Space Fragmentation: A Review of Exact Results", S. Moudgalya *et al.*, Rep. Prog. Phys. (2022)

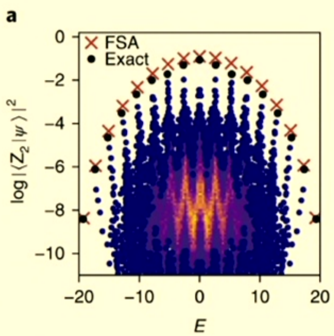


Summary of QMBS properties

- Revivals in dynamics

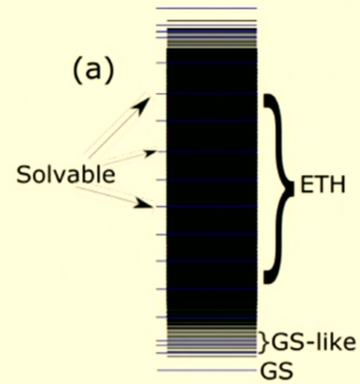


- Enhanced support in a few eigenstates

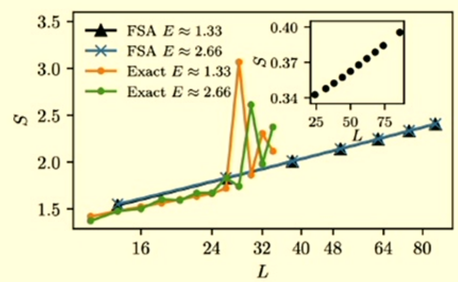


- Equidistant in energy

- Simple analytical form



- Entanglement scaling - different from random states



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Feature	QMBS	Quantum Scars
Revivals in dynamics	✓	✓
Enhanced support in a few eigenstates	✓	✓
Equidistant in energy	✓	?
Entanglement scaling - **different from random states?**	✓	?
Simple analytical form	✓	?
Stability		Unstable


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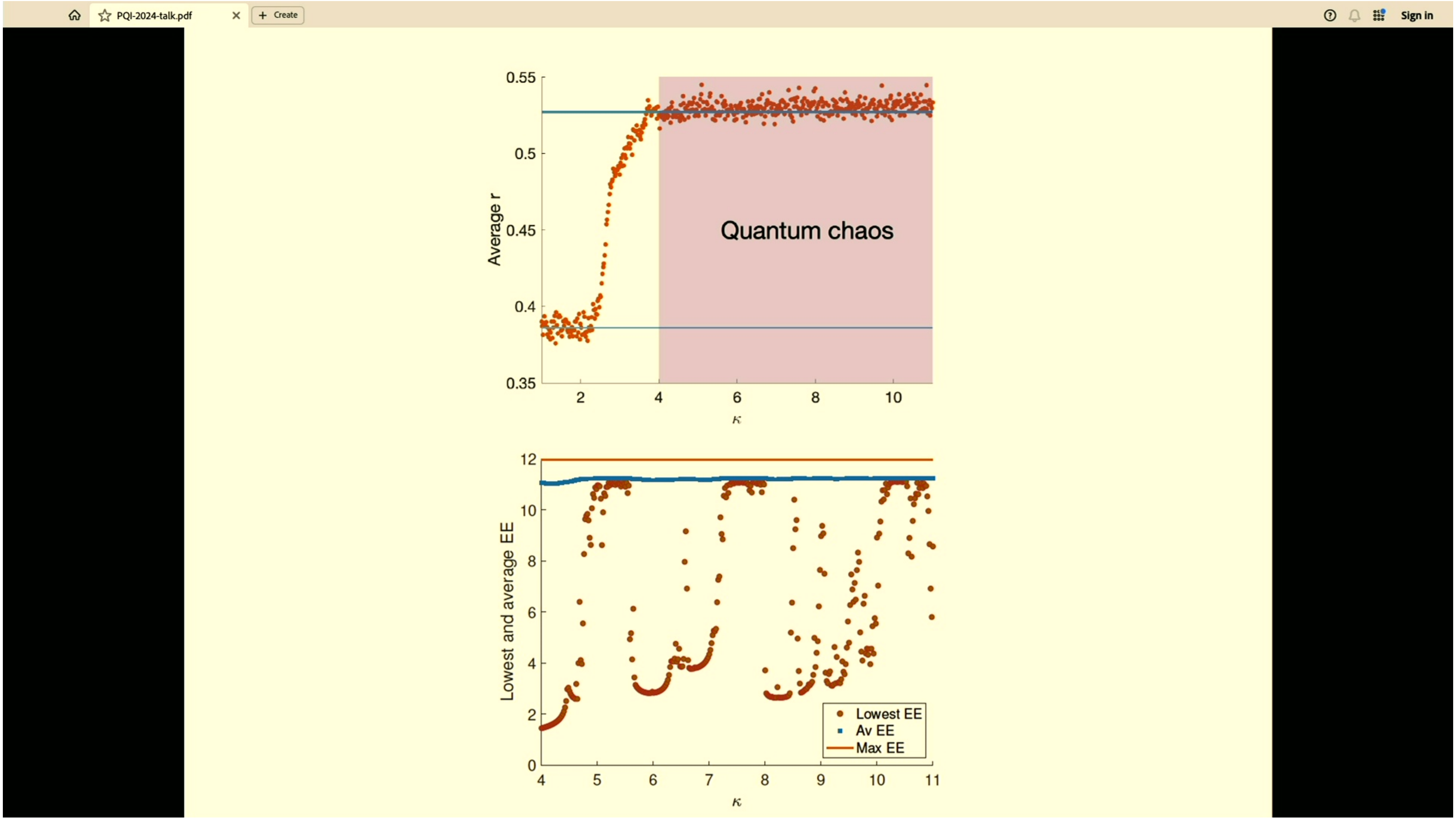
This talk
QMBS vs Quantum Scars
in Collective Spin Models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Quantum chaos
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Lowest and average EE
Lowest EE
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κ

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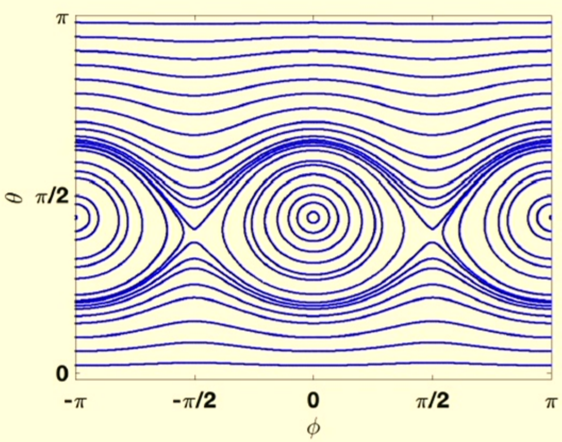
Collective spin models

Integrable model

Lipkin-Meshov-Glick (LMG) model

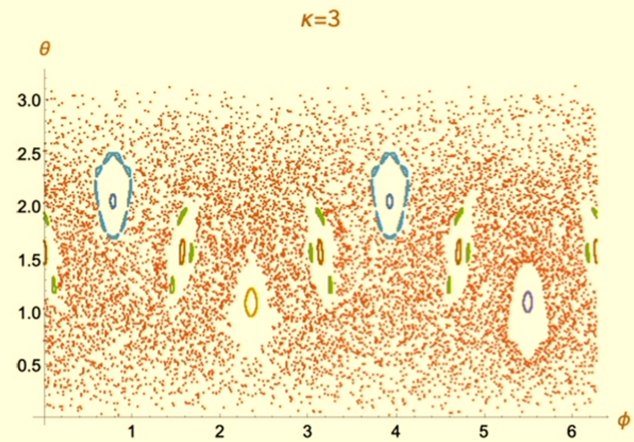
$$H = -\frac{1}{2j}(\gamma_x J_x^2 + \gamma_y J_y^2) - hJ_z$$



Non-integrable model

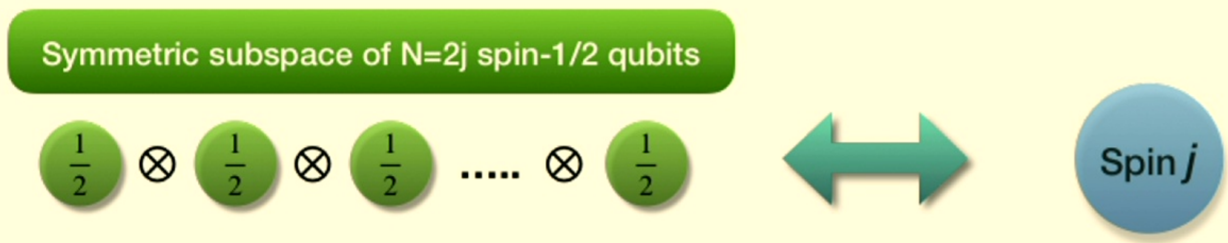
Quantum Kicked Top (QKT) model

$$H = \frac{p}{\tau}J_z + \frac{\kappa}{2j}J_x^2 \sum \delta(t - n\tau)$$



Why study collective spin models?

- They have well-defined classical limit.
- Unlike in quantum mechanics, periodic orbits and their stability, integrability, non-integrability and chaos is well-defined in classical mechanics.
  $\Rightarrow$ Ideal to study quantum scars.



- Multi-qubit picture $\Rightarrow$ Study of Quantum entanglement
  $\Rightarrow$ Possible to study quantum many-body scarring.

Classical Kicked Top

$$H = \frac{p}{\tau} J_z + \frac{\kappa}{2j} J_x^2 \sum \delta(t - n\tau)$$

$X = \frac{J_x}{j},\ Y = \frac{J_y}{j},\ Z = \frac{J_z}{j}$

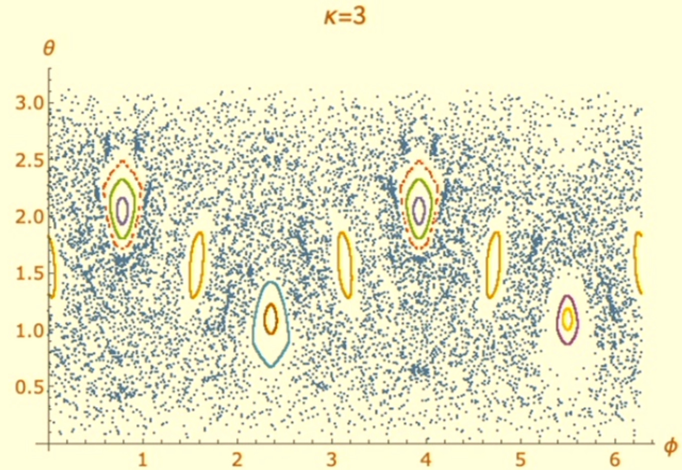
Classical equations of motion

$$X' = -Y$$
$$Y' = X\cos(\kappa Y) + Z\sin(\kappa Y)$$
$$Z' = Z\cos(\kappa Y) - X\sin(\kappa Y)$$

$X' \equiv X(n\tau + 1), X \equiv X(n\tau)$

$X^2 + Y^2 + Z^2 = 1.$

$\therefore$ Polar co-ordinates



Classical stroboscopic phase space.
$\kappa = 3.0$ and $p = \pi/2$

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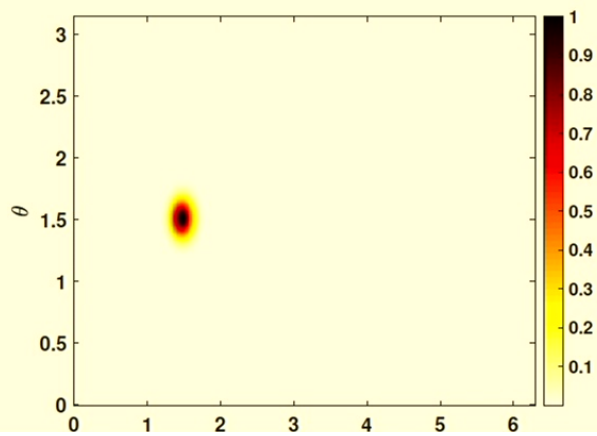
Quantum Kicked Top

$$H = \frac{p}{\tau} J_z + \frac{\kappa}{2j} J_x^2 \sum \delta(t - n\tau)$$

Floquet time evolution operator (for one time period, τ)

$$U = \exp\left(-\mathrm{i}\frac{\kappa}{2j} J_x^2\right) \exp\left(-ipJ_z\right)$$

Initial quantum states:
Spin Coherent States
(SCS) $|\theta, \phi\rangle = R(\theta, \phi)|j, j\rangle$



Kicked Top: Classical chaos measure

Lyapunov exponent, λ

$$\lambda(\mathbf{X_0}) = \lim_{t\to\infty} \lim_{d_0\to 0} \frac{1}{t} \ln\left(\frac{d_t(\mathbf{X_0}, \mathbf{X_0} + \mathbf{\Delta X_0})}{d_0(\mathbf{X_0}, \mathbf{X_0} + \mathbf{\Delta X_0})}\right)$$

$\mathbf{X_0} \in$ Phase space
$\lambda > 0 \Rightarrow$ Chaos

$$H = \frac{\pi}{2\tau} J_z + \frac{\kappa}{2j} J_x^2 \sum \delta(t - n\tau)$$

- $A \equiv \kappa$.
- $\lambda_+ \approx \ln(\kappa) - 1$, for large κ.

V. Constantoudis and N. Theodorakopoulos, PRE 1997.

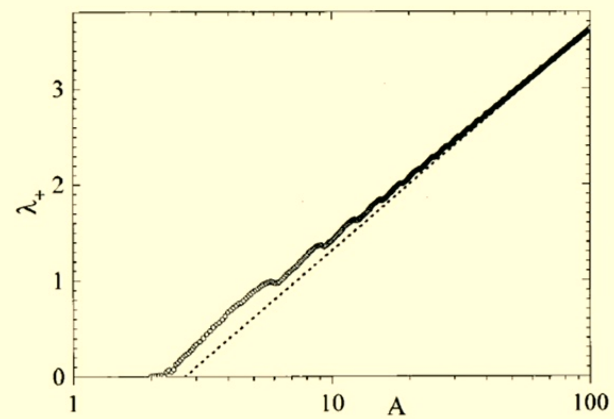


FIG. 1. (Maximum) Lyapunov exponent as a function of the anisotropy parameter for the kicked top. ($B=\pi/2$ has been used throughout the paper). The dotted line is the high-A approximation described in the text.

Kicked Top: Quantum chaos measure

$$\langle r \rangle = \text{mean}\left\{\frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})}\right\}$$

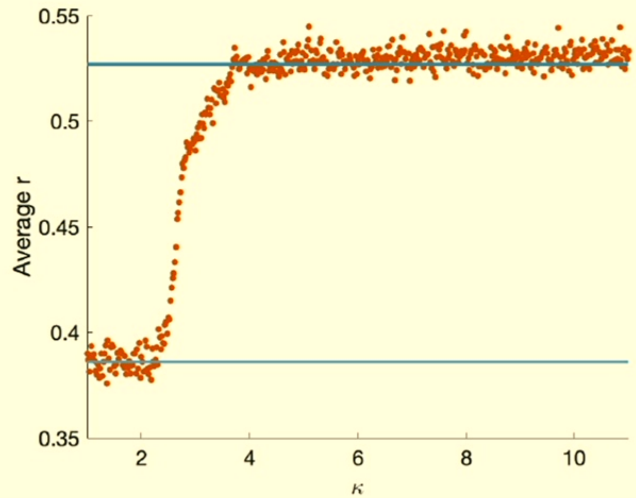
where $\delta_n = \phi_n - \phi_{n-1}$

- $\langle r \rangle \approx 0.38 \Rightarrow$ Integrability or near-integrability.
- $\langle r \rangle \approx 0.53 \Rightarrow$ Chaos or non-integrability.

$$U = \exp\left(-i\frac{\kappa}{2j}J_x^2\right)\exp\left(-ipJ_z\right)$$

$\{\phi_n\}$: Eigenphases of U

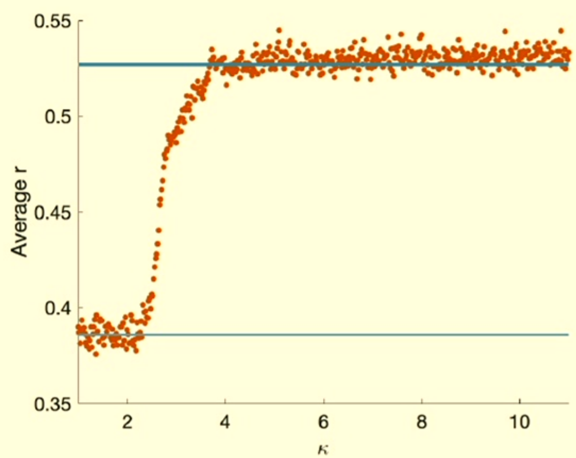
Figure for $p = \pi/2$



Kicked Top: Classical and Quantum chaos measures

For $\kappa \gtrsim 4$:
Ergodicity expected in both classical and quantum dynamics.

Quantum



Classical

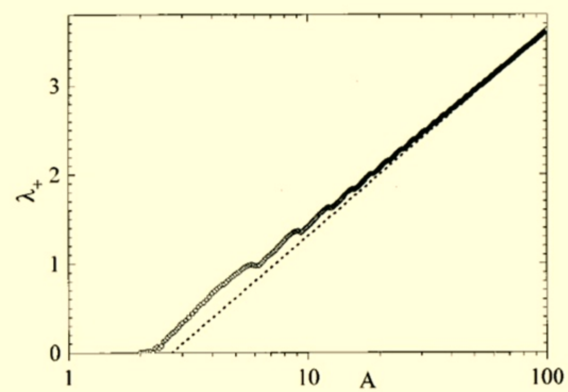


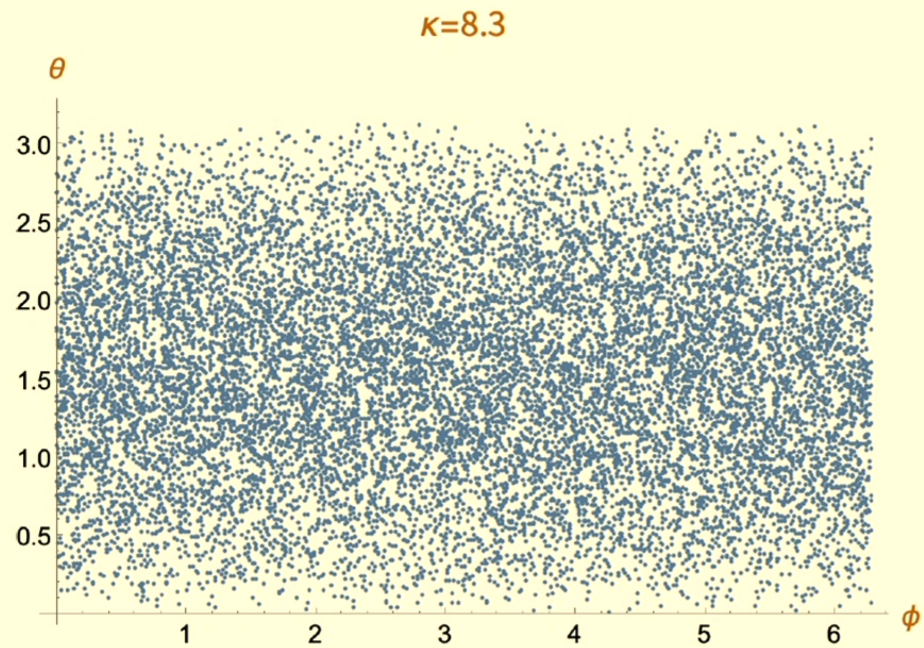
FIG. 1. (Maximum) Lyapunov exponent as a function of the anisotropy parameter for the kicked top. ($B=\pi/2$ has been used throughout the paper). The dotted line is the high-A approximation described in the text.

(PRE 1997)

Kicked Top: Classical and Quantum Dynamics

Classical dynamics

$$(\theta_0, \phi_0) = (2, 3)$$



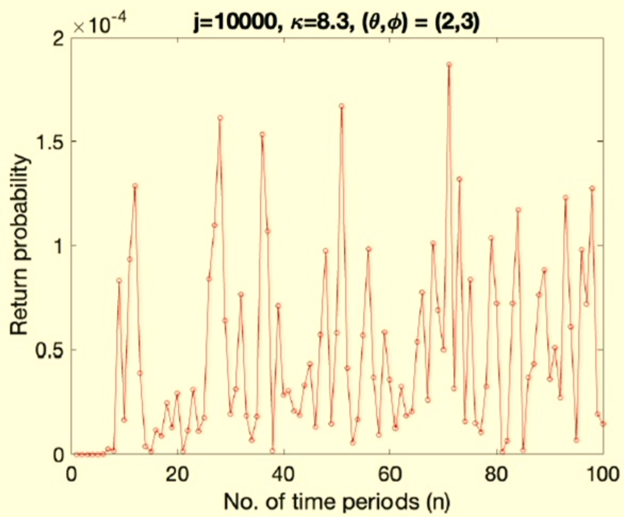
Kicked Top: Classical and Quantum Dynamics

Quantum dynamics

$$|\psi(0)\rangle = |\theta_0, \phi_0\rangle = \mathrm{SCS}|2,3\rangle$$

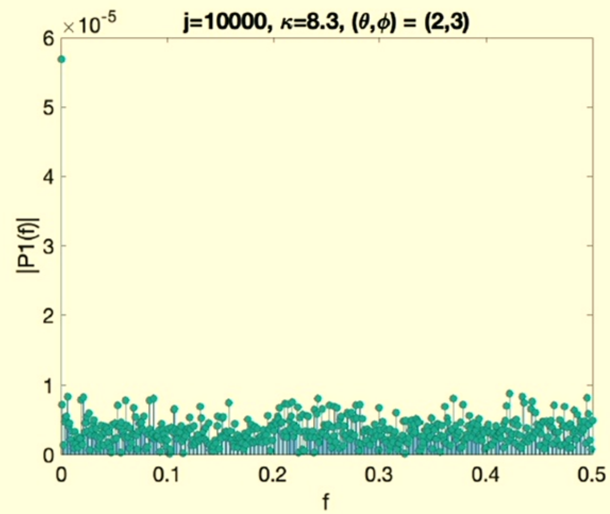
Survival probability

$$|\langle\psi(0)|\psi(n\tau)\rangle|^2$$

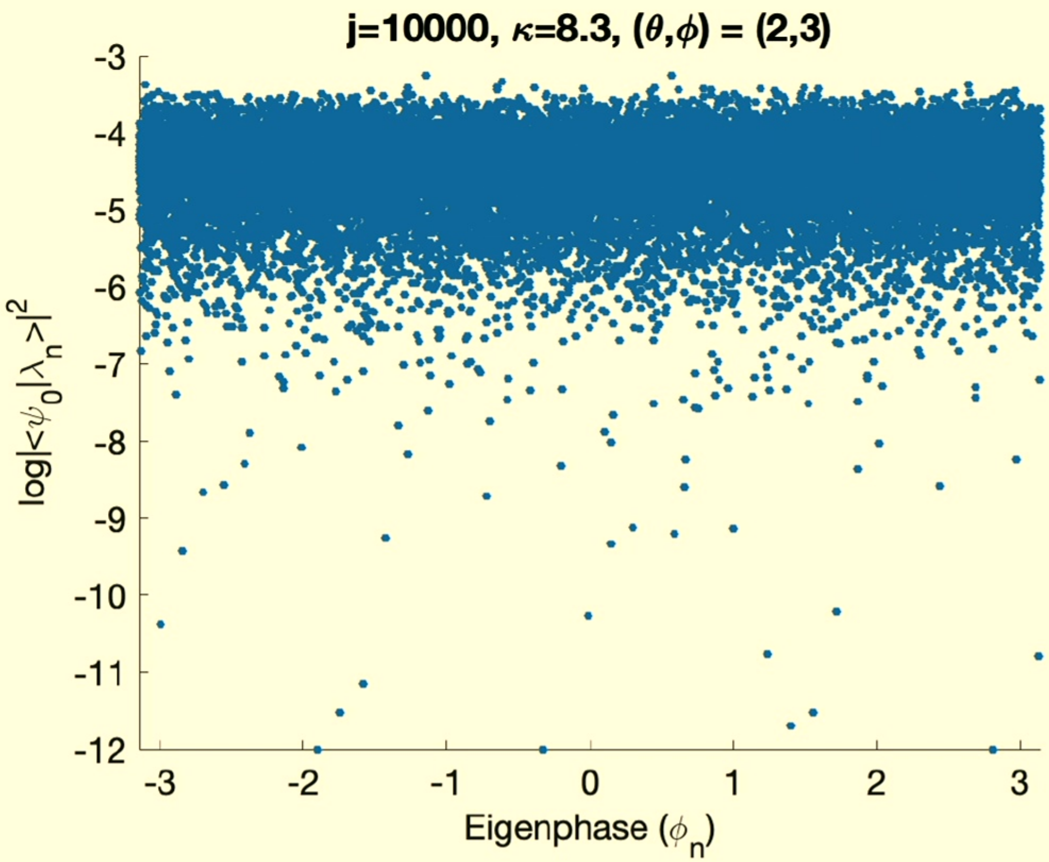


Fourier Transform

(of surv. prob.)



Support of $|\psi(0)\rangle = SCS|\theta_0, \phi_0\rangle$ in the eigenbasis of U $\{|\lambda_n\rangle\}$



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Ergodicity expected in chaotic quantum systems ...

However,

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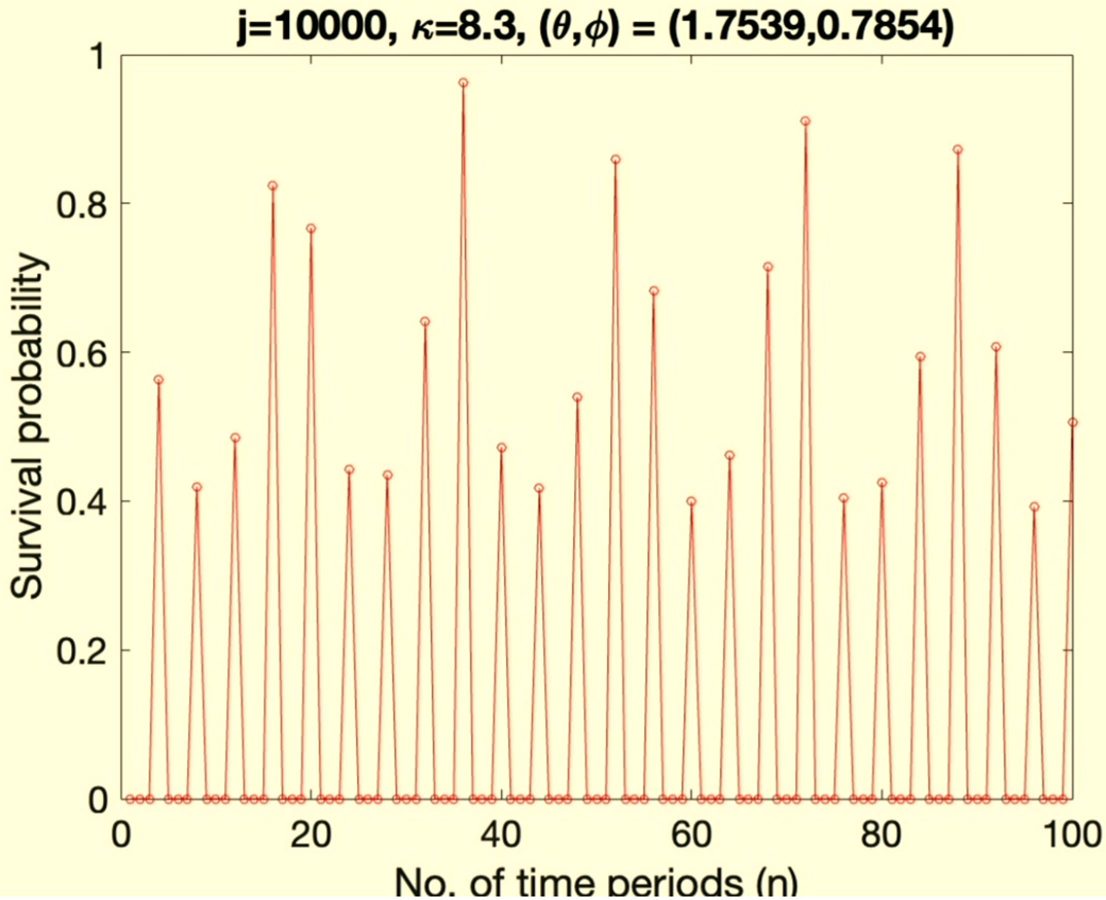
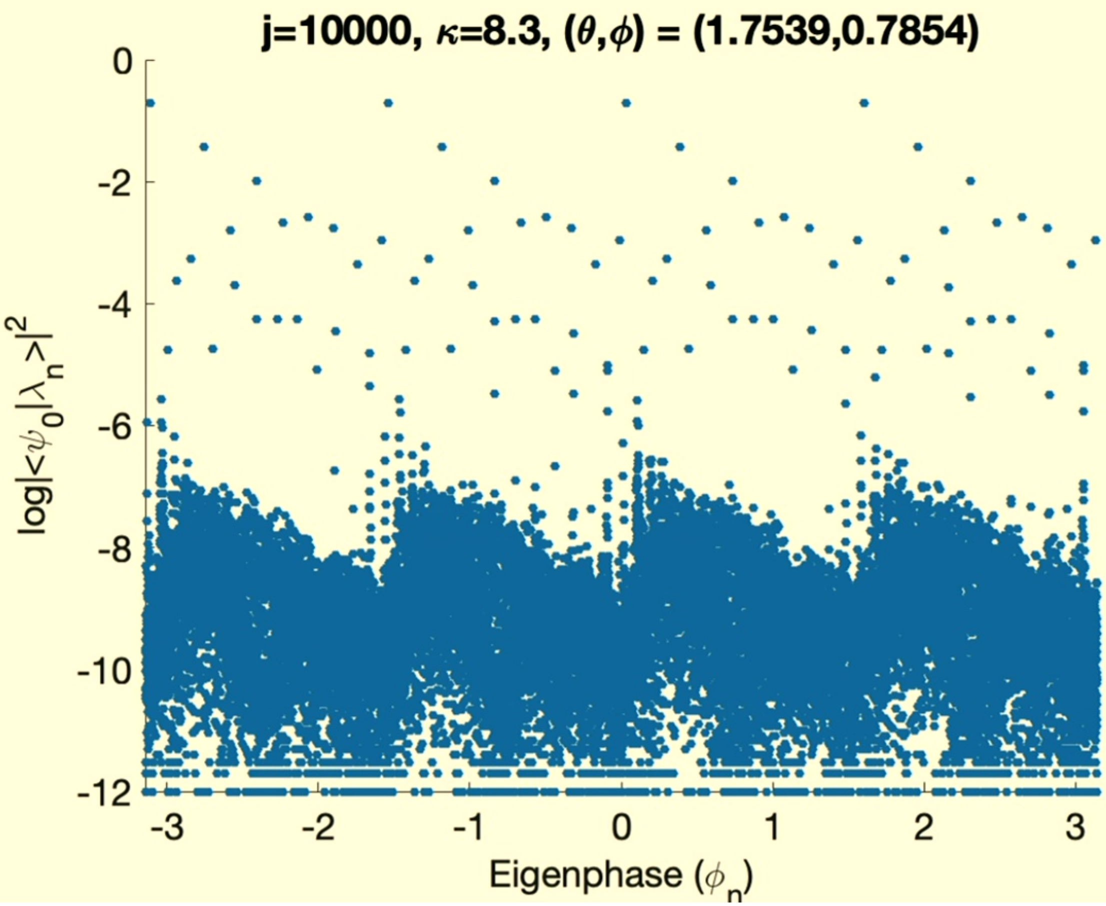
QKT: Weak breaking of ergodicity

$$|\psi(0)\rangle = SCS|1.7539, \frac{\pi}{4}\rangle$$



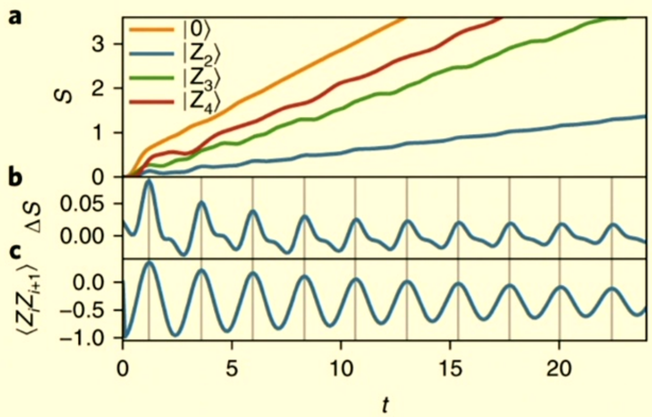
QKT: Weak breaking of ergodicity

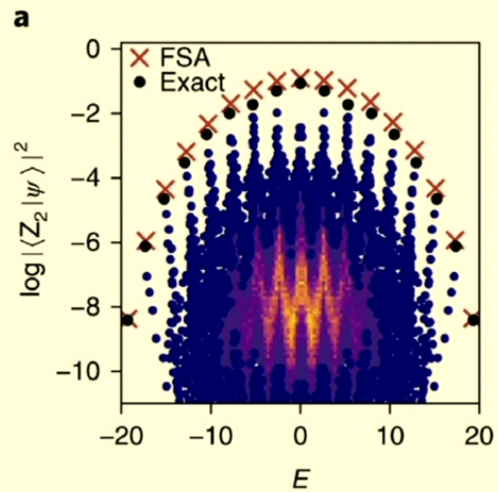
$|\psi(0)\rangle = SCS|1.7539, \frac{\pi}{4}\rangle$



QKT ergodicity breaking: Comparison with QMBS

PXP model





From "Weak ergodicity breaking from quantum many-body scars", C. J. Turner *et al.*, Nature Physics (2018)

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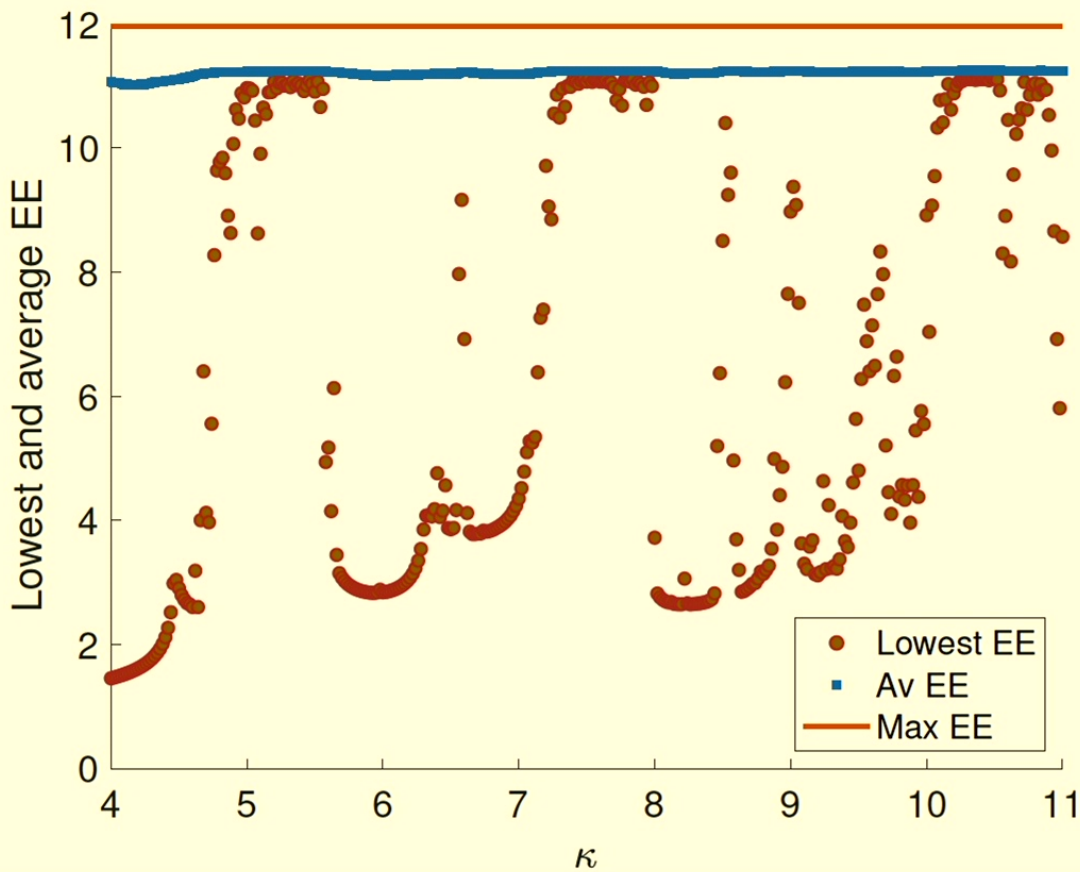
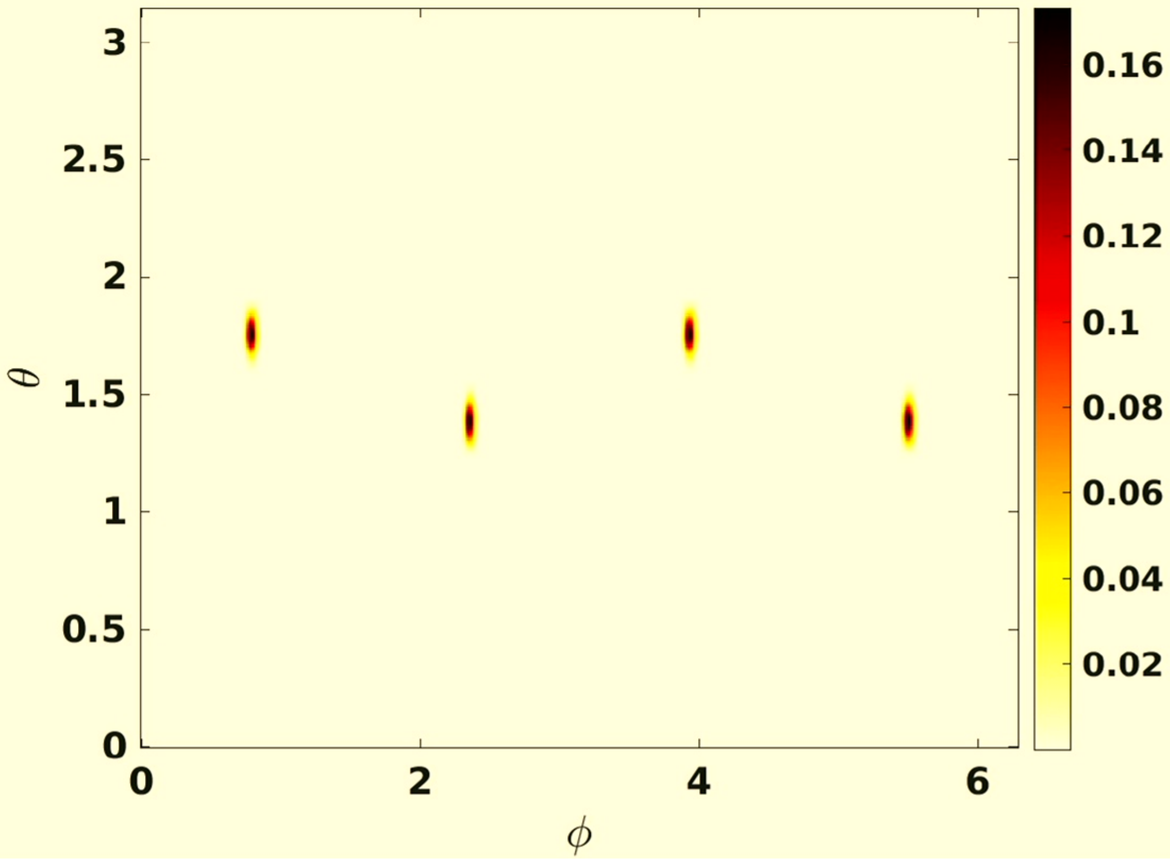
QKT: Entanglement in eigenstates

EE = Entanglement entropy



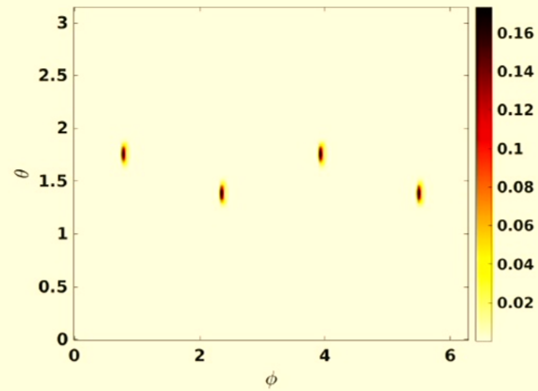
QKT: Lowest entangled eigenstate

Husimi distribution of the lowest entangled eigenstate



QKT: Variational wavefunction for eigenstate

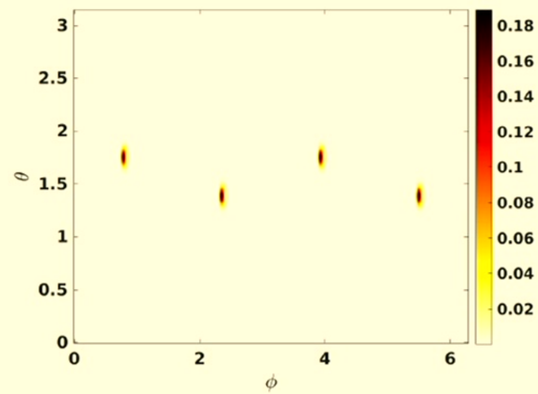
Top figure: Husimi of eigenstate with lowest EE in (R_z^+, R_y^+) sector.



Bottom figure: Husimi of variational wavefunction, $|\psi_{var}\rangle = \frac{1}{\mathcal{N}}\left(|\psi_1\rangle + R_z|\psi_1\rangle + R_y\left(|\psi_1\rangle + R_z|\psi_1\rangle\right)\right)$

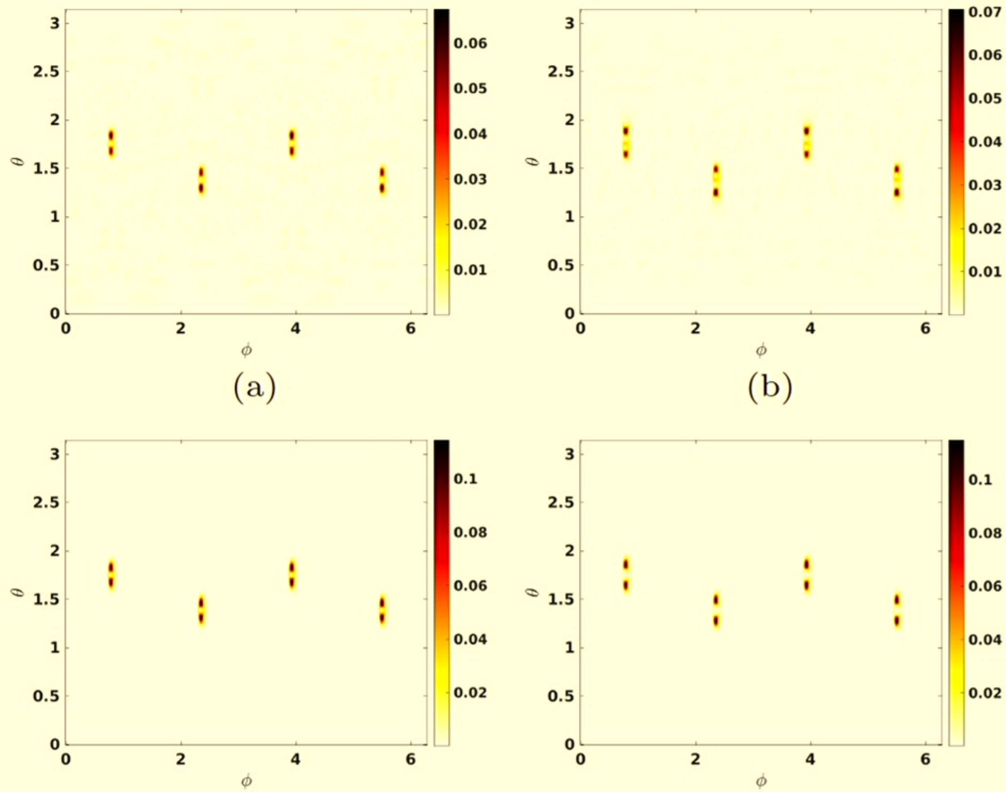
where $\mathcal{N}$ is the normalization factor, and

$$|\psi_1\rangle = R(\theta,\phi)S(\eta,\gamma)|j,j\rangle.$$



QKT: Variational wavefunction for eigenstate

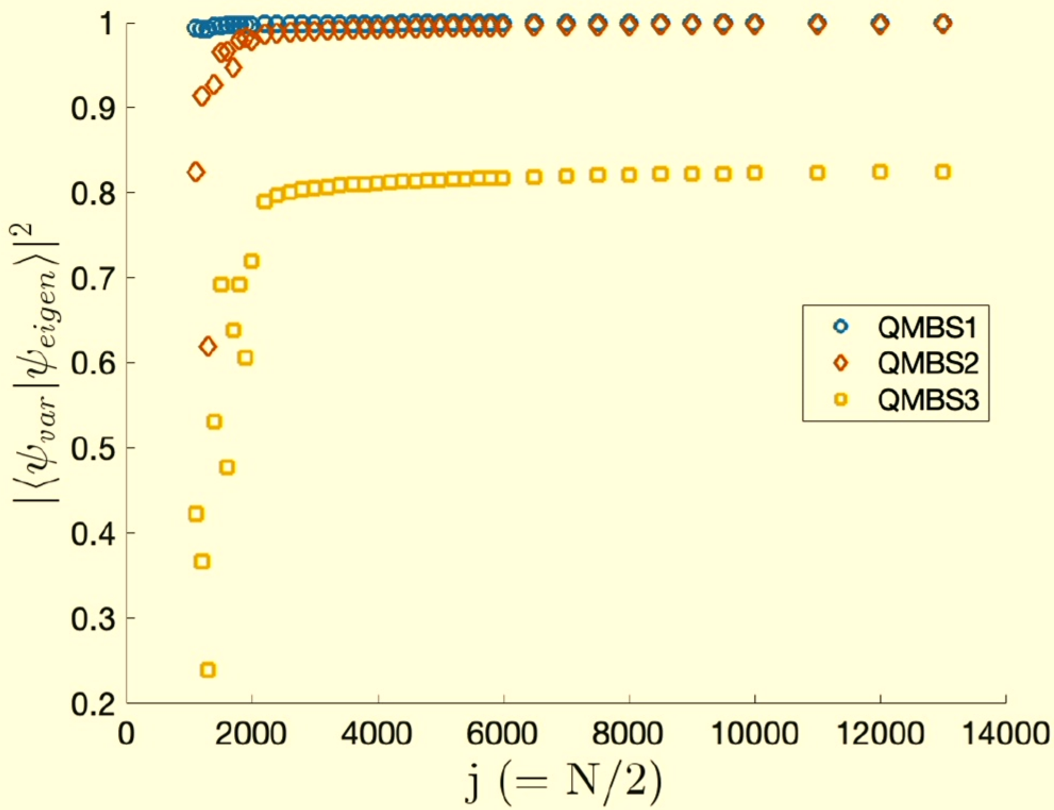
(a), (b): QMBS
(c), (d): Variational wavefunctions



(a)
(b)
(c)
(d)

QKT: Fidelity of variational wavefunction with eigenstate

As a function of spin quantum number, j. $\kappa = 8.3$.



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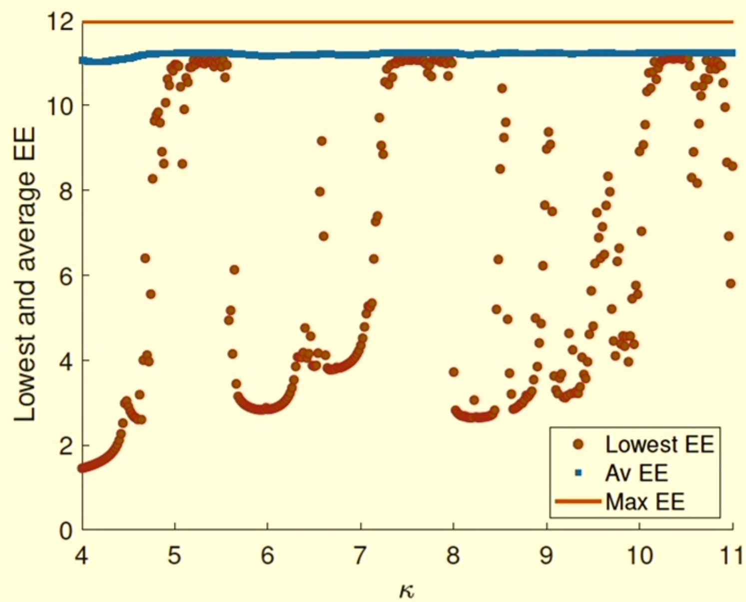
QKT: Weak ergodicity breaking and collective QMBS

- Weak ergodicity breaking in the chaotic QKT when there exists very low entangled eigenstates.
- Variational wavefunctions for collective QMBS.



In collective spin models

Feature	QMBS	Quantum Scars
Revivals in dynamics	✓	
Enhanced support in a few eigenstates	✓	
Equidistant in energy	$\approx$ ✓	
Entanglement scaling - **different from random states?**		
Simple analytical form	$\approx$ ✓	
Stability	✓	Unstable

Quantum Many-Body Scars: Entanglement scaling

Entanglement scaling in

- Typical eigenstates: volume law, similar to random states ($S(\rho_{L_A}) \sim L_A$).
- QMBS: sub-volume law, very different from random states ($S(\rho_L) \sim \log L$).

Figure from "Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations", C.J. Turner *et al.*, PRB (2018).

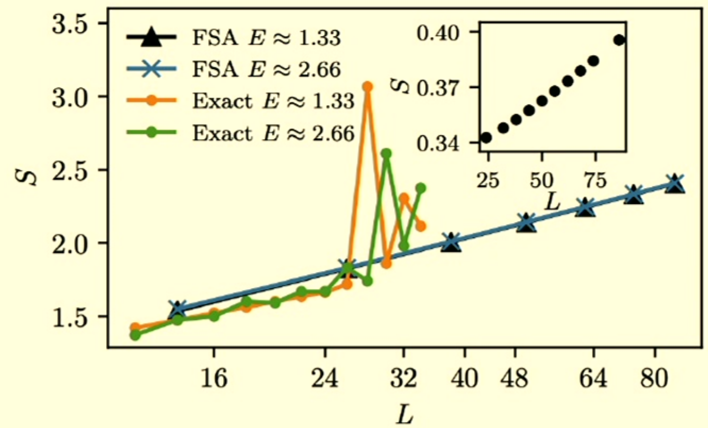


Figure 8. Logarithmic scaling of entropy for two adjacent FSA eigenstates in the middle of the spectrum. Black triangles correspond to the state at energy $E_1 \approx 1.33$ and blue crosses to $E_2 \approx 2.66$ (the two eigenstates have approximately the same entanglement entropy with difference $\Delta S \sim 0.1\%$). The fit gives $S \propto 0.48 \log(L)$. Green curve corresponds to the entropy of the exact special eigenstate at $E_1 \approx 1.33$. The non-monotonic behavior of entropy in this case is attributed to weak hybridization with volume-law entangled states at nearby energies. The inset displays the entropy of the FSA ground state. The weak growth of entropy with L is an artefact of the approximation, since the exact ground state is gapped and obeys area law for entropy.

Entanglement in pure random states

Let $|\psi^{AB}\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ with $\mathcal{D}_A \leq \mathcal{D}_B$

- Maximum entanglement in A:B $= \ln \mathcal{D}_A$.
- DN Page, PRL 1993: Random pure states are (nearly) maximally entangled.

  Entanglement$(|\psi^{AB}\rangle) = \mathrm{S}(\rho_A) \approx \ln \mathcal{D}_A$ - $\frac{1}{2}\frac{\mathcal{D}_A}{\mathcal{D}_B}$
- BGS conjecture $\Rightarrow$ The eigenstates of quantum chaotic Hamiltonians are essentially random states.

Entanglement scaling in

- QMBS in quantum-many body systems
- QMBS in collective spin models

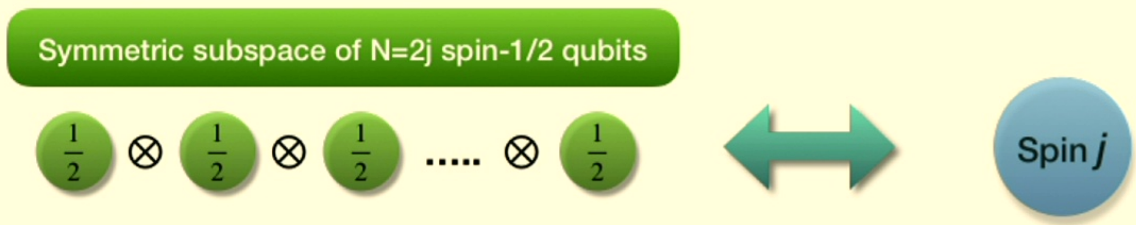
Entanglement in QKT

$$\hat{H} = \frac{p}{\tau}\hat{J}_z + \frac{\kappa}{2j}\hat{J}_x^2 \sum \delta(t - n\tau)$$

$$[\hat{J}_k, \hat{J}_l] = \mathrm{i}\epsilon_{klm}\hat{J}_m$$

$$[H, J^2] = 0 \quad \Rightarrow \quad j(j+1) = \text{constant}$$

Study of entanglement in QKT:



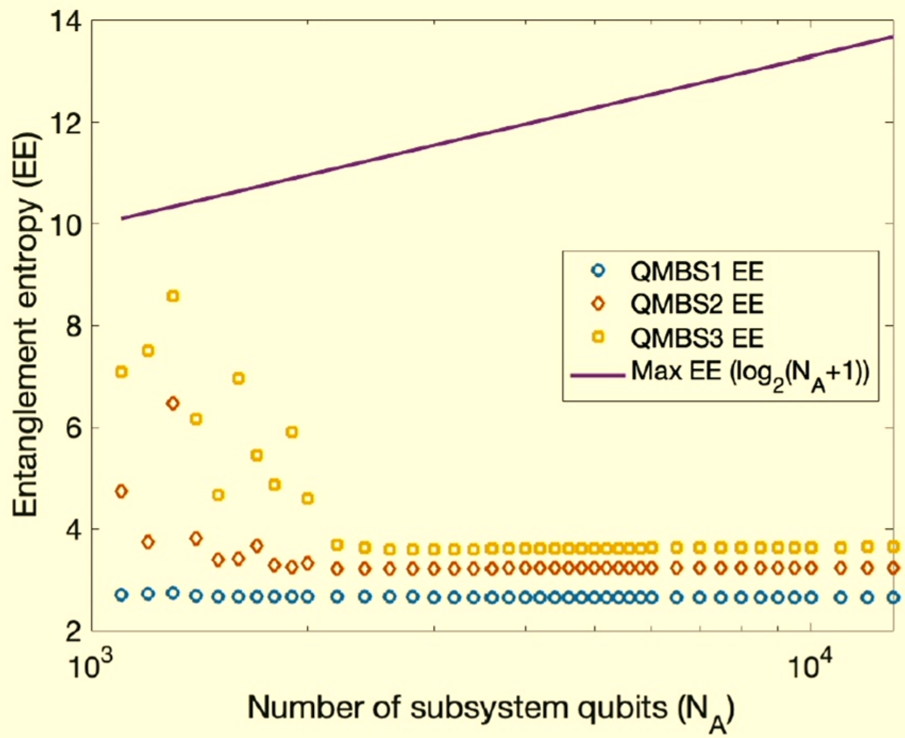
Entanglement scaling

Consider a system of N qubits, and bipartition N_A:$(N - N_A)$ qubits.

Entanglement scaling		
	Permutation symmetric subspace	Full Hilbert space
H.S. dimension	$N_A + 1$	2^{N_A}
Max EE	$\log(N_A + 1)$	N_A
EE in random states (for half bipartition)	$\log(N_A + 1) - 2/3$	$N_A - 1/2$

Entanglement scaling: QMBS in QKT

EE scaling of a few lowest entangled eigenstate for $\kappa = 8.3$



$N_A = N/2 = j.$

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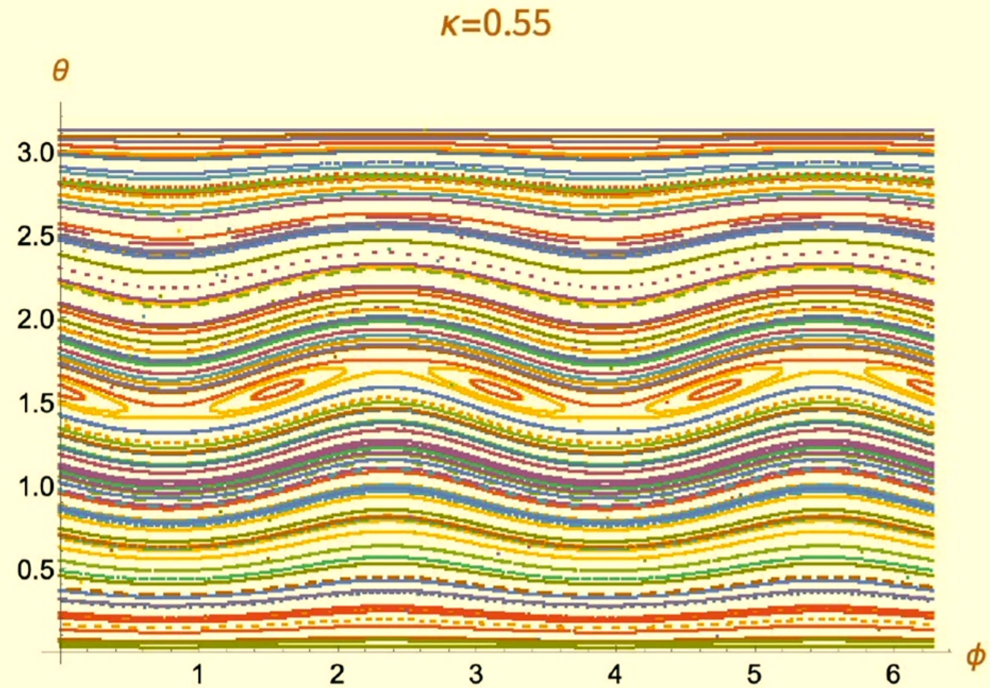
In collective spin models

Feature	QMBS	Quantum Scars
Revivals in dynamics	✓	
Enhanced support in a few eigenstates	✓	
Equidistant in energy	≈ ✓	
Entanglement scaling - **different from random states?**	✓	
Simple analytical form	≈ ✓	
Stability	✓	Unstable

Classical kicked top: Periodic orbit

Period-4 orbit:

$$(\theta, \phi) = (\tfrac{\pi}{2}, 0) \rightarrow (\tfrac{\pi}{2}, \tfrac{\pi}{2}) \rightarrow (\tfrac{\pi}{2}, \pi) \rightarrow (\tfrac{\pi}{2}, \tfrac{3\pi}{2}) \rightarrow (\tfrac{\pi}{2}, 0)$$



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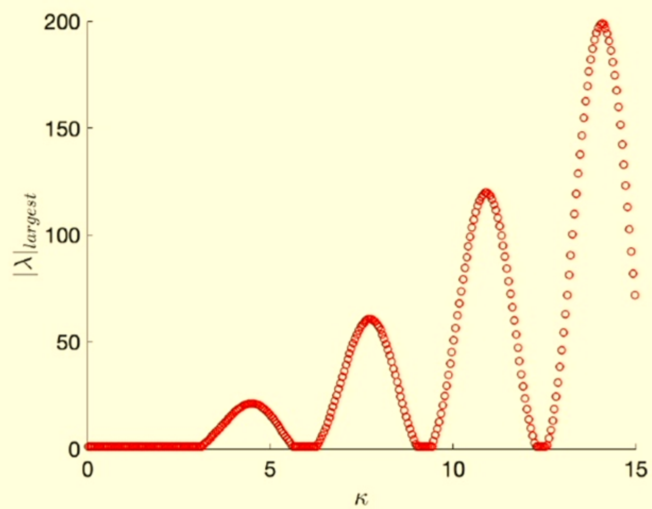
Classical kicked top: Periodic orbit

Period-4 orbit:

$$(\theta, \phi) = (\frac{\pi}{2}, 0) \to (\frac{\pi}{2}, \frac{\pi}{2}) \to (\frac{\pi}{2}, \pi) \to (\frac{\pi}{2}, \frac{3\pi}{2}) \to (\frac{\pi}{2}, 0)$$

Stability criterion: $(2\cos\kappa + \kappa \sin\kappa)^2 - 4 < 0$.

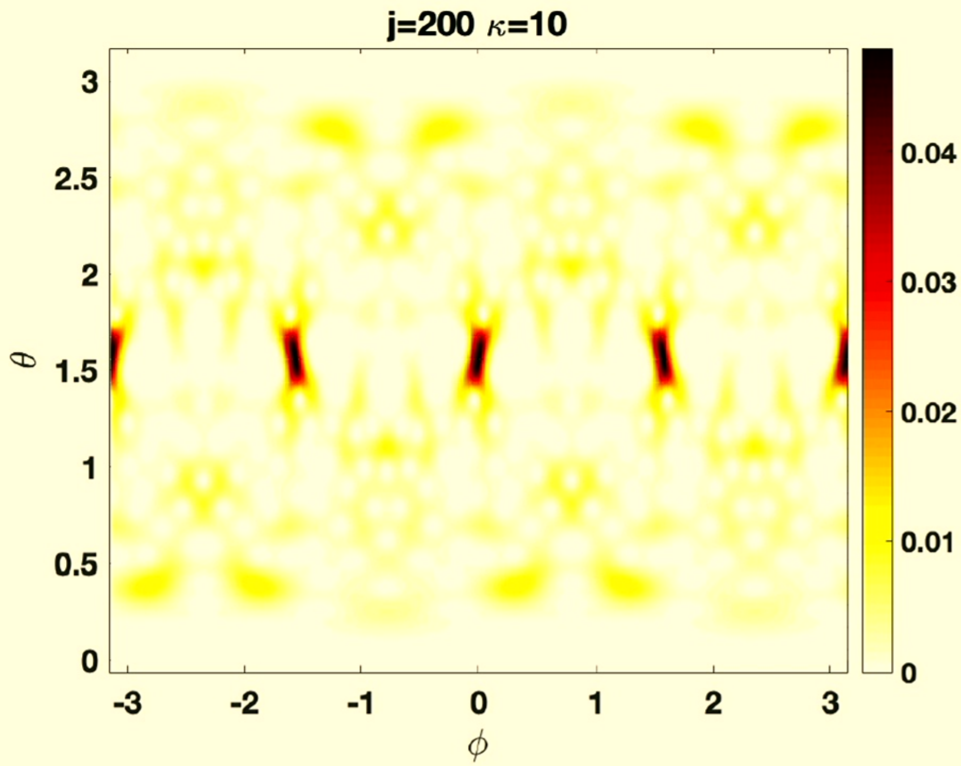
For $\kappa \in (n\pi - \epsilon_n)$ for $n \in \{2, 3, ...\}$, **period-4 orbit is stable even though kicked top is chaotic**.



$|\lambda|_{\text{largest}} > 1 \Rightarrow$ Period-4 orbit is unstable.

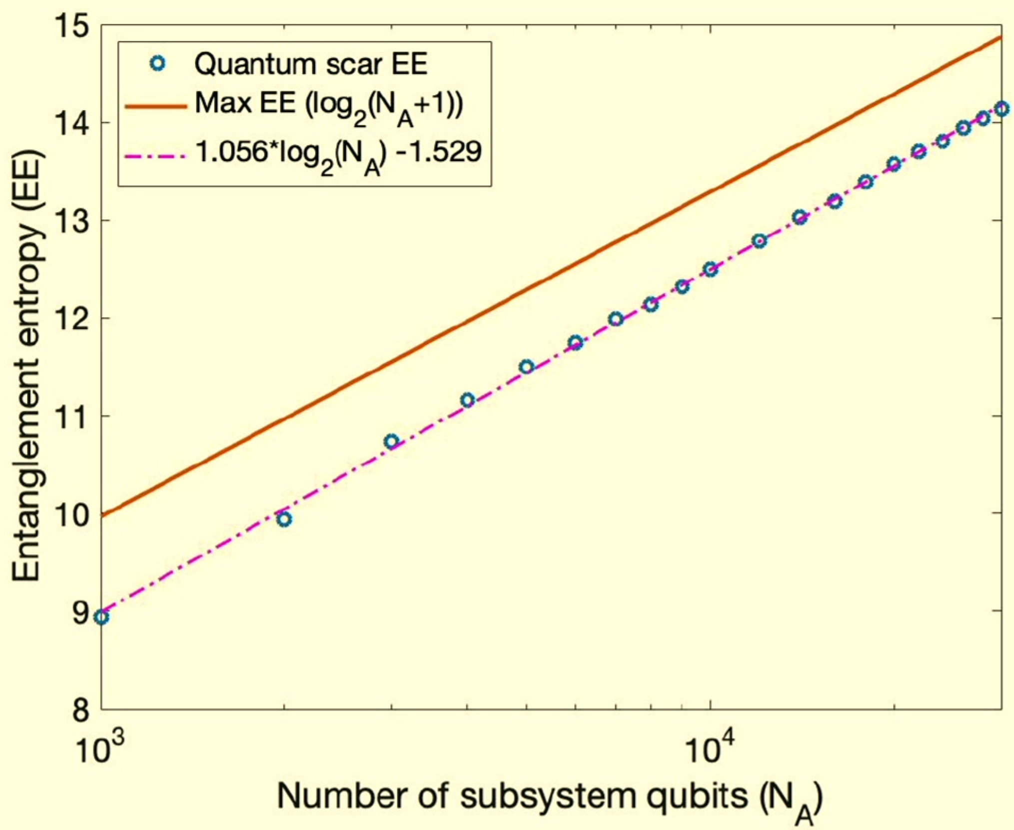
Quantum Scars in QKT

Husimi phase space distribution of period-4 scar for $\kappa = 10$.



Entanglement scaling in Period 4 scar

Half bipartition entanglement $N_A = N/2$. $\kappa = 5$



Entanglement scaling in Period 4 scar

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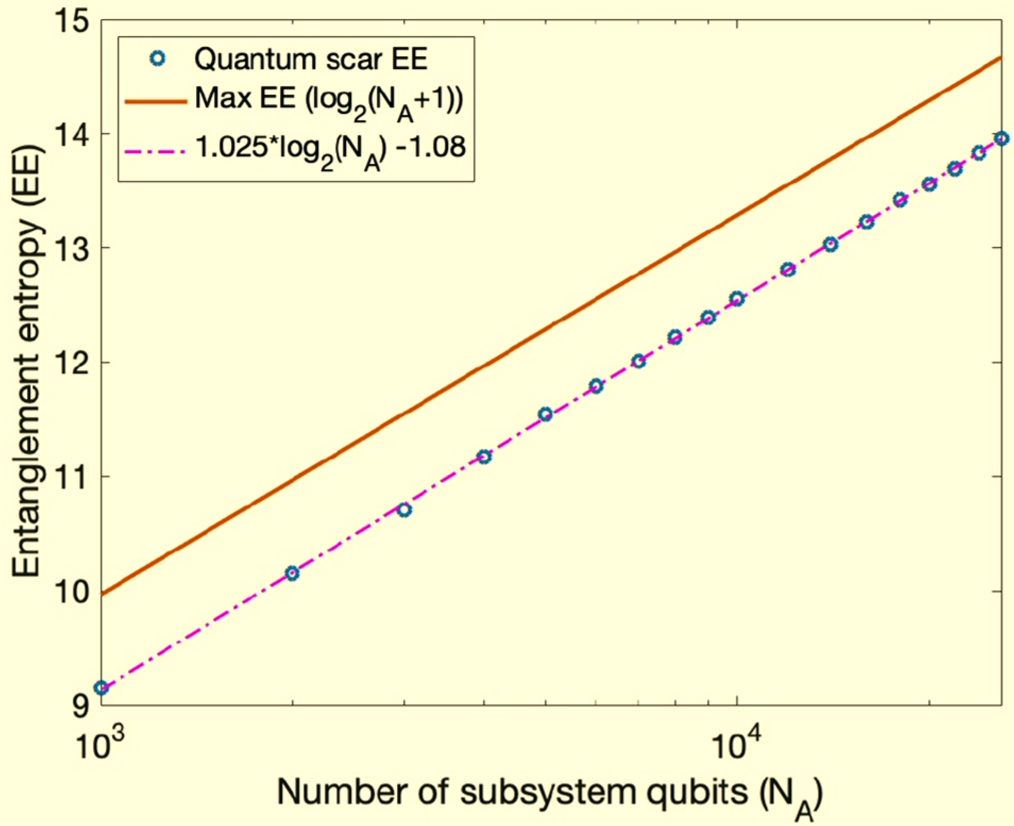

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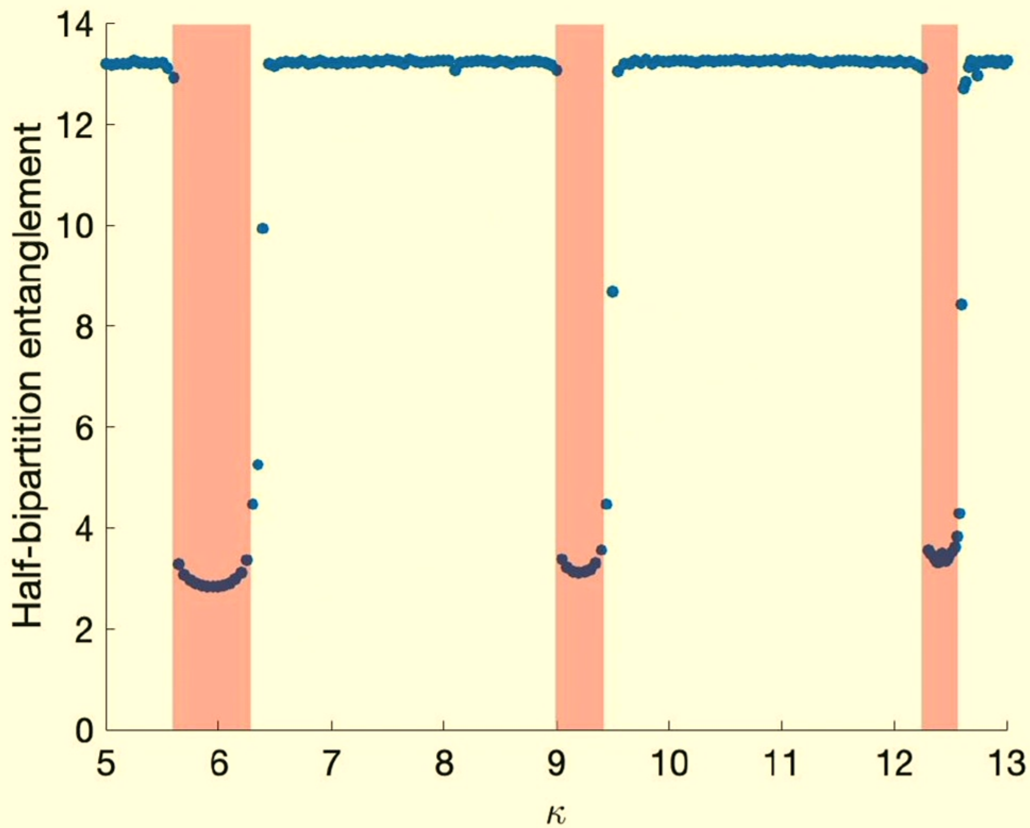
Entanglement in eigenstates corresponding to period-4 orbit.
$j = 16000$



Shaded region: Period-4 is stable classically.

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Summary

In collective spin models

Feature	QMBS	Quantum Scars
Revivals in dynamics	✓	
Enhanced support in a few eigenstates	✓	
Equidistant in energy	≈ ✓	
Entanglement scaling - **different from random states?**	✓	✗
Simple analytical form	≈ ✓	
Stability	✓	Unstable

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Summary

In collective spin models

Feature	**QMBS**	**Quantum Scars**
Revivals in dynamics	✓	
Enhanced support in a few eigenstates	✓	
Equidistant in energy	$\approx$ ✓	
Entanglement scaling - **different from random states?**	✓	✗
Simple analytical form	$\approx$ ✓	
Stability	✓	Unstable

Thank You