Title: Certifying almost all quantum states with few single-qubit measurements

Speakers: Hsin-Yuan Huang

Collection: Physics of Quantum Information

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Abstract: Certifying that an n-qubit state synthesized in the lab is close to the target state is a fundamental task in quantum information science. However, existing rigorous protocols either require deep quantum circuits or exponentially many single-qubit measurements. In this work, we prove that almost all n-qubit target states, including those with exponential circuit complexity, can be certified from only O(n^2) single-qubit measurements. This result is established by a new technique that relates certification to the mixing time of a random walk. Our protocol has applications for benchmarking quantum systems, for optimizing quantum circuits to generate a desired target state, and for learning and verifying neural networks, tensor networks, and various other representations of quantum states using only single-qubit measurements. We show that such verified representations can be used to efficiently predict highly non-local properties that would otherwise require an exponential number of measurements. We demonstrate these applications in numerical experiments with up to 120 qubits, and observe advantage over existing methods such as cross-entropy benchmarking (XEB).

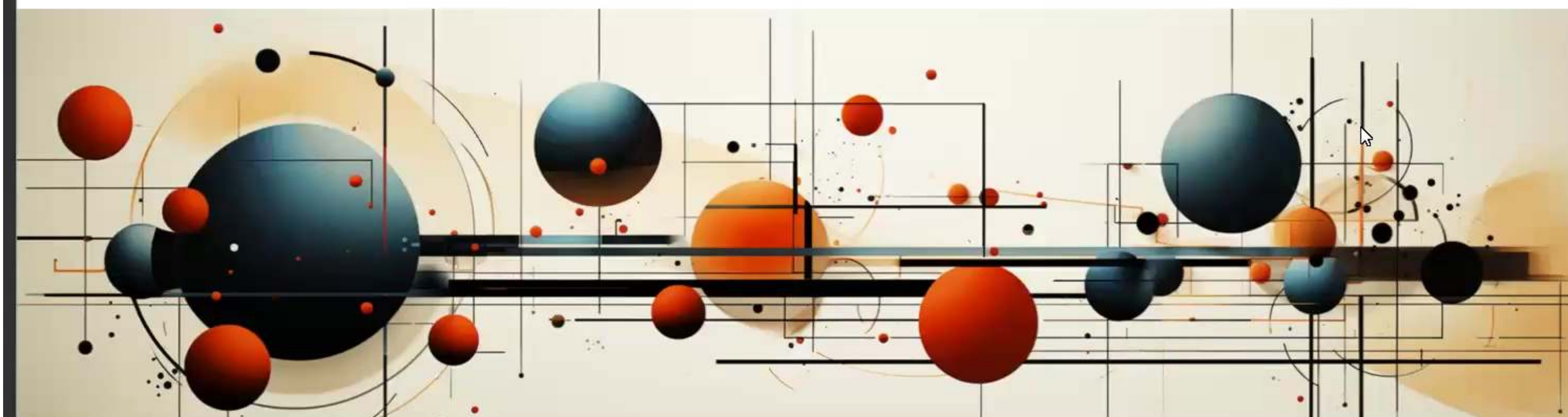
# Certifying almost all quantum states with few single-qubit measurements

Hsin-Yuan Huang (Robert)

with John Preskill and Mehdi Soleimanifar

# Motivation

- Quantum systems with **intricate entanglement** are pivotal in quantum information science.



# Motivation

- Quantum systems with intricate entanglement are pivotal in quantum information science.
- To understand if we have created the desired quantum system in the lab, we need to perform certification.

# What is Certification?

- We have a desired $n$-qubit state $|\psi\rangle$, which is our target state.
- We have an $n$-qubit state $\rho$ created in the experimental lab.
- **Task:** Test if $\rho$ is close to $|\psi\rangle\langle\psi|$ or not from data?

  ($\langle\psi|\rho|\psi\rangle$ is close to 1)
- A **fundamental task** in data science for quantum.

# Motivation

- Many techniques based on statistics & learning theory have been proposed for performing certification.
- However, it remains **experimentally challenging** to certify highly-entangled quantum many-body systems.

# What is Certification?

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# How to Certify?

- **Approach 1:** Random Clifford measurements (classical shadow)

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- **Advantage:**

  Only needs **depth-$n$** random Clifford circuits on $\rho$

- **Challenge:**

  Implementing depth-$n$ random Clifford circuits

  is still **experimentally challenging**.

# How to Certify?

- **Approach 2:** Random Pauli measurements (classical shadow)
- **Advantage:**

  Only needs single-qubit measurements on $\rho$

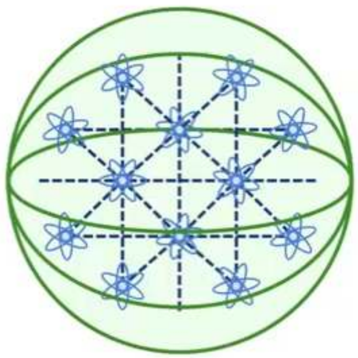
- **Challenge:**

  Requires **exp($n$)** measurements for most target $|\psi\rangle$

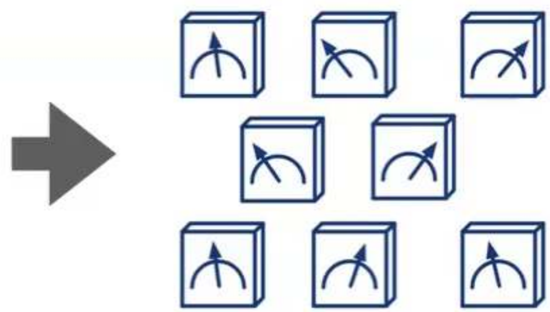
  especially when $|\psi\rangle$ is highly entangled.

# How to Certify?

- **Approach 3:** Cross-entropy benchmark (XEB)



Quantum state



Single-qubit
Measurement
(all Z bases)

# How to Certify?

- **Approach 3:** Cross-entropy benchmark (XEB)
- **Advantage:**

  Only needs **single-qubit** measurements (Z-basis) on $\rho$

- **Challenge:**

  Does not rigorously address the certification task.

  $\rho$ can be **far** from $|\psi\rangle\langle\psi|$ despite perfect XEB score.

# Existing Challenges

- All existing certification protocols either

  **a.** Require **deep** quantum circuits before measurements

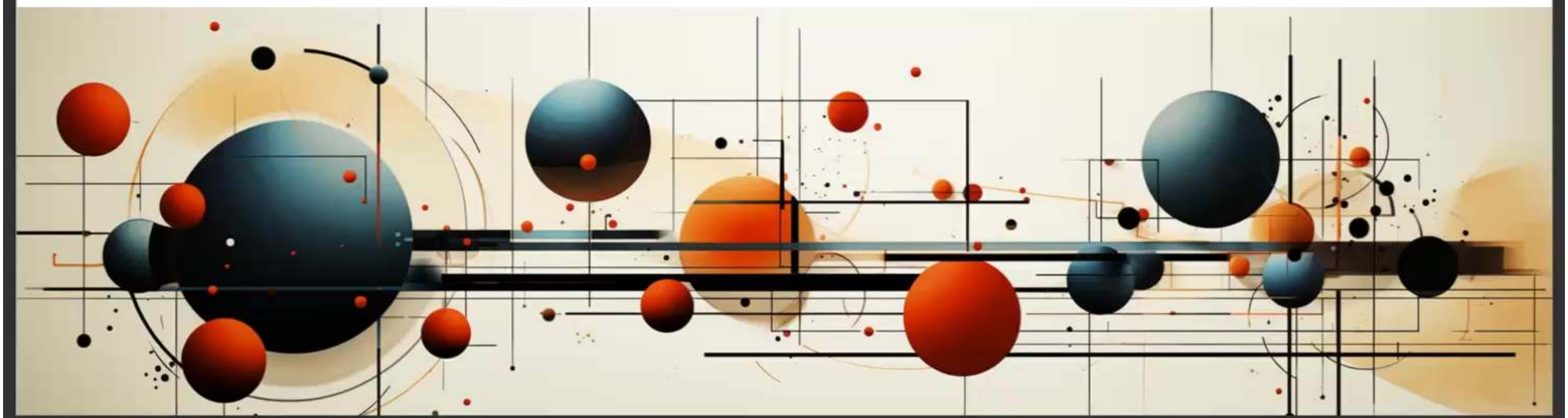
  **b.** Use **exponentially** many measurements

  **c.** Apply only for **low**-entanglement state $|\psi\rangle$

  **d.** **Lack** rigorous guarantees

# Question

Can we rigorously certify **almost all** quantum states from performing **few single-qubit** measurements?



# Outline

- Theorem
- Protocol
- Applications

# Certification

**Theorem 1**

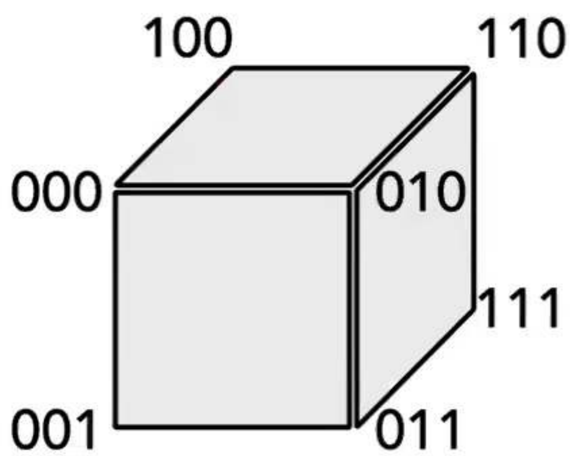
For almost all $n$-qubit state $|\psi\rangle$, we can certify that $\rho$ is close to $|\psi\rangle\langle\psi|$ using only $\mathcal{O}(n^2)$ single-qubit measurements.

- The certification procedure applies to any $\rho$.
- $\mathcal{O}(n^2)$ is enough even when $|\psi\rangle$ has $\exp(n)$ circuit complexity.

# Relaxation Time

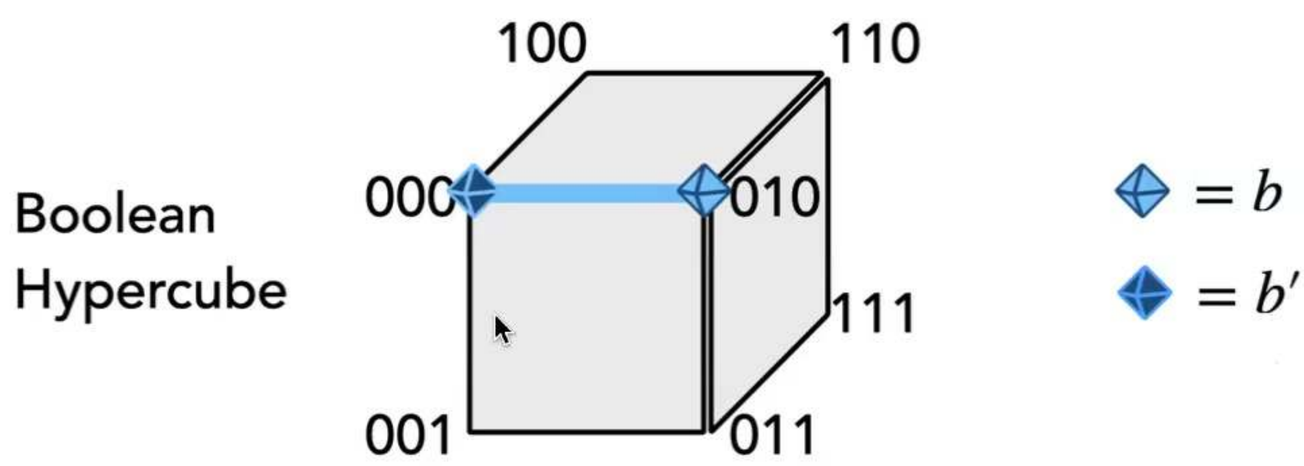
- Consider an $n$-qubit target state $|\psi\rangle$.
- Choose a basis $|b\rangle$, where $b \in \{0,1\}^n$ is a bitstring.
- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.

Boolean Hypercube



# Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on $n$-bit Boolean hypercube.

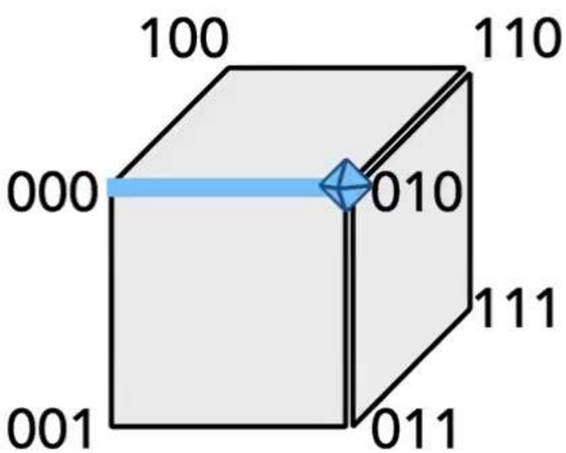


# Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the measurement distribution.
- Let $\tau$ be the time the random talk takes to relax to stationary $\pi$.

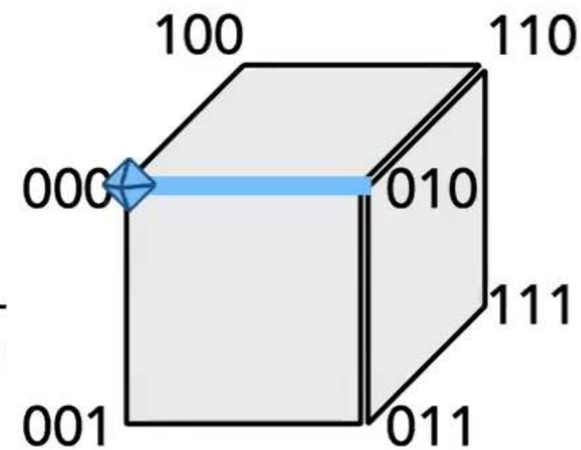
With prob.

$$\frac{\pi(b)}{\pi(b) + \pi(b')}$$




With prob.

$$\frac{\pi(b')}{\pi(b) + \pi(b')}$$



# Outline

- Theorem
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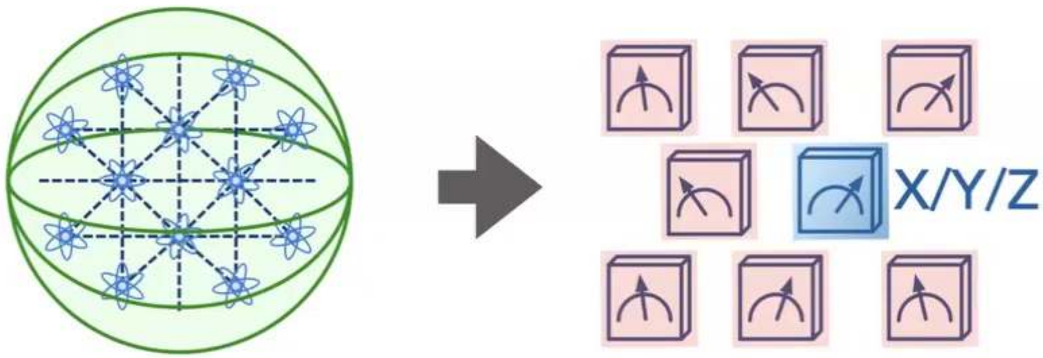
- Applications

# Measurement Protocol

- Pick a random qubit $x$. Measure $x$ in random X/Y/Z basis.




Quantum state

Single-qubit Measurement

# Relaxation Time

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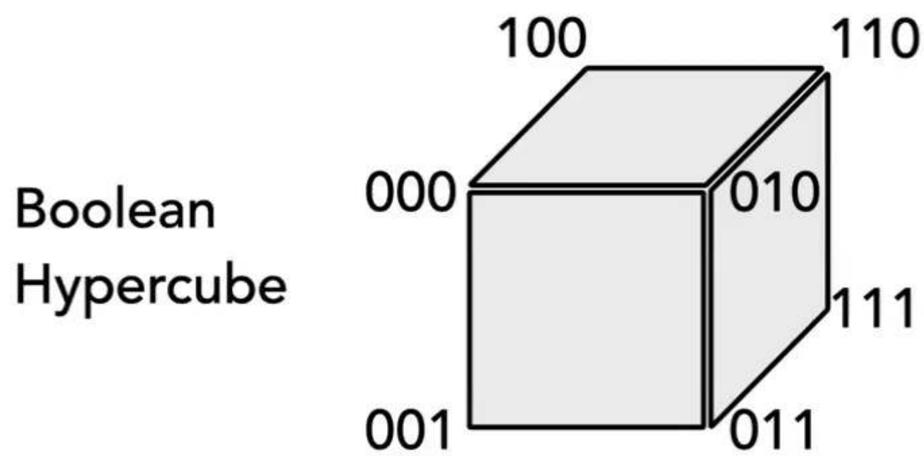
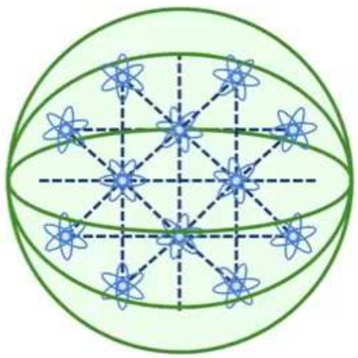
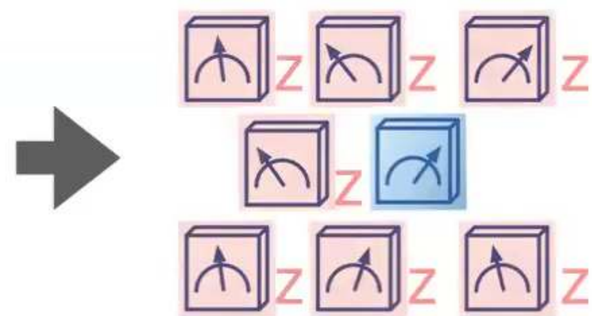


# Measurement Protocol

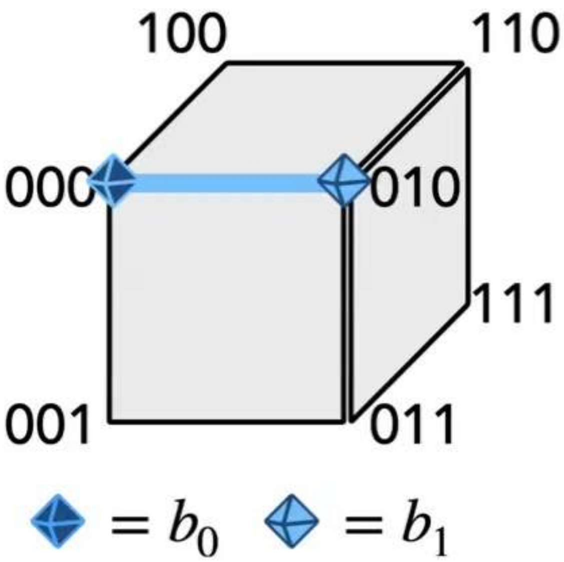
- Pick a random qubit $x$. Measure all except qubit $x$ in Z basis.

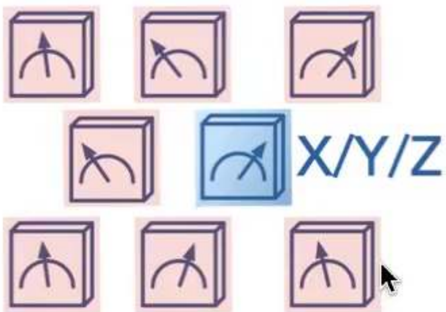


# Postprocessing

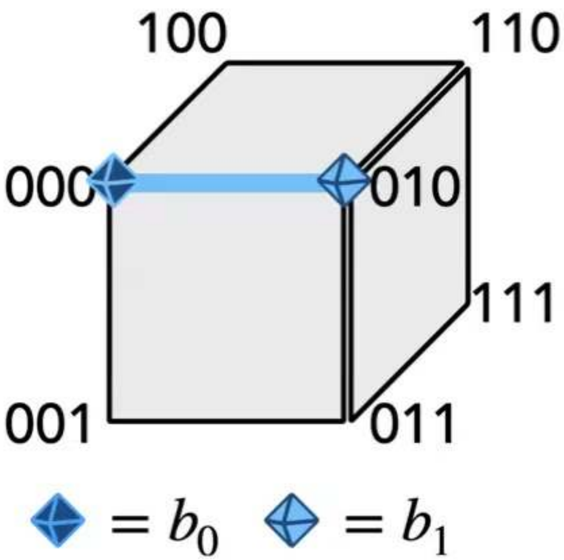
- The measurement outcomes on  specifies an edge $(b_0, b_1)$ on the Boolean hypercube.

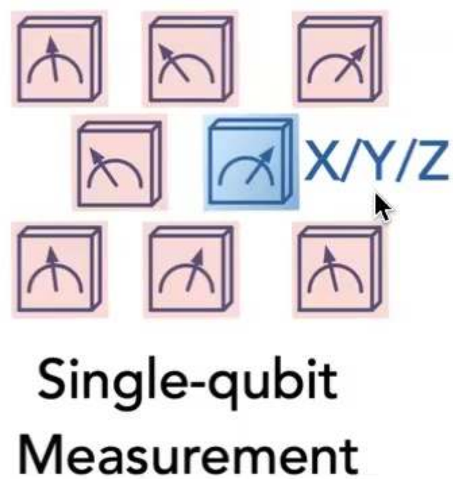


# Postprocessing

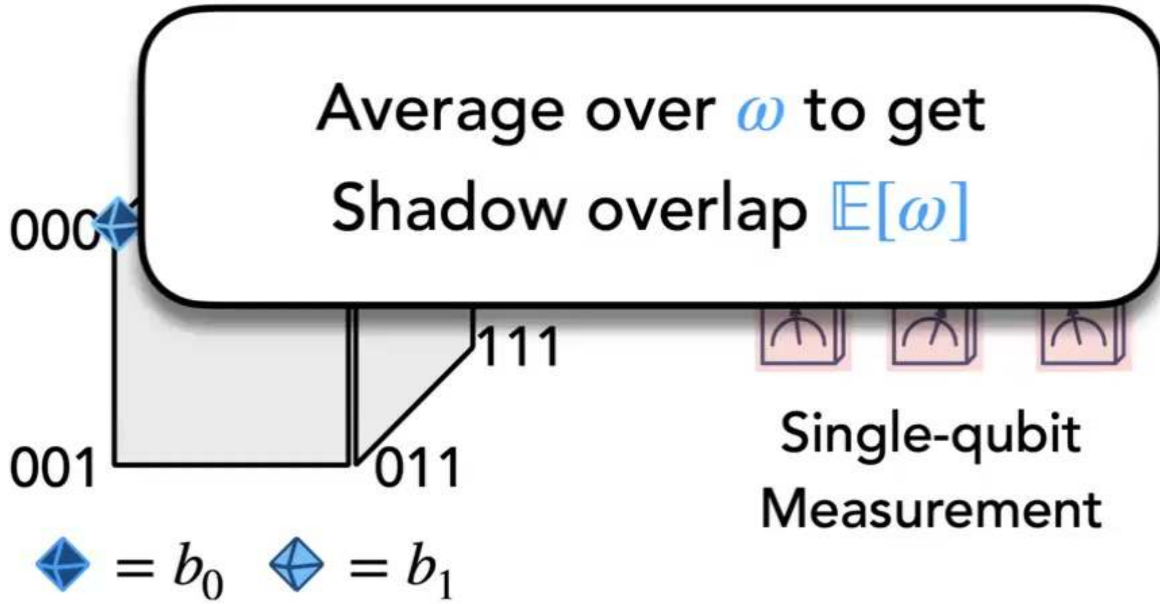
- The **ideal** post-measurement 1-qubit state $|\psi_{b_0,b_1}\rangle$ on **qubit** $x$ is proportional to $\langle b_0|\psi\rangle|0\rangle + \langle b_1|\psi\rangle|1\rangle$.



# Postprocessing

- Use randomized Pauli measurement (classical shadow) on **qubit** $x$ to predict the fidelity $\omega$ with the **ideal** 1-qubit state $|\psi_{b_0,b_1}\rangle$.



# Key Feature

Shadow overlap $\mathbb{E}[\omega]$ accurately tracks the fidelity $\langle\psi|\rho|\psi\rangle$.

$\mathbb{E}[\omega] \geq 1 - \epsilon$ **implies** $\langle\psi|\rho|\psi\rangle \geq 1 - \tau\epsilon$

$\langle\psi|\rho|\psi\rangle \geq 1 - \epsilon$ **implies** $\mathbb{E}[\omega] \geq 1 - \epsilon$

$\tau$ is the time the random talk takes to relax to stationary $\pi$

# Physical Intuition

Shadow overlap $\mathbb{E}[\omega] = \frac{1}{n}\sum_{i=1}^{n}\sum_{b_{\neq i}\in\{0,1\}^{n-1}} \mathrm{Tr}\left(\langle b_{\neq i}|\rho|b_{\neq i}\rangle \frac{\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}{\mathrm{Tr}\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}\right)$

- $|+\ldots+\rangle\langle+\ldots+|$ and $|-\ldots-\rangle\langle-\ldots-|$ has fidelity 0.
- $|+\ldots+\rangle\langle+\ldots+|$ and $|-\ldots-\rangle\langle-\ldots-|$ has $\mathbb{E}[\omega]$= 0.

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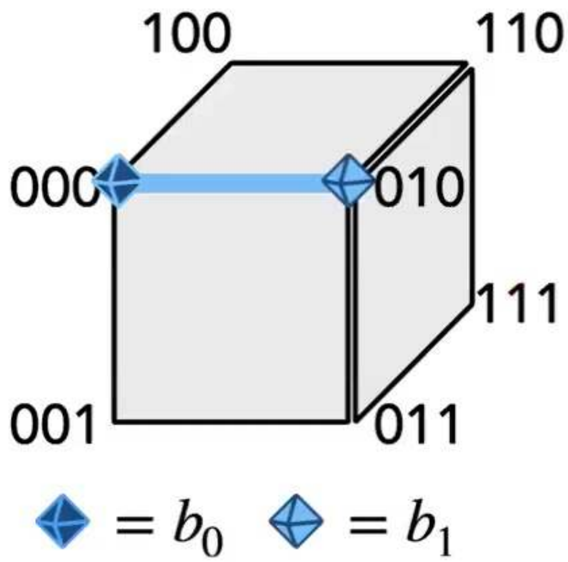
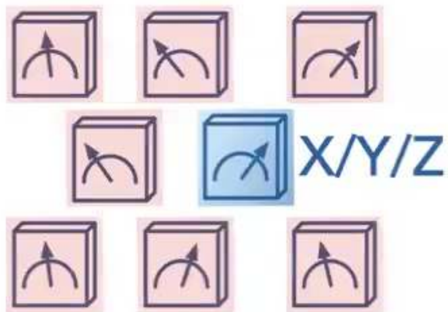
- $|+\ldots+\rangle\langle+\ldots+|$ and $|+\ldots+-\rangle\langle+\ldots+-|$ has $\mathbb{E}[\omega]= \frac{n-1}{n}$.
- Shadow overlap has a Hamming distance nature.

# Postprocessing

- The ideal post-measurement 1-qubit state $|\psi_{b_0,b_1}\rangle$ on qubit $x$ is proportional to $\langle b_0|\psi\rangle|0\rangle + \langle b_1|\psi\rangle|1\rangle$.




Single-qubit Measurement

# Measurement Protocol

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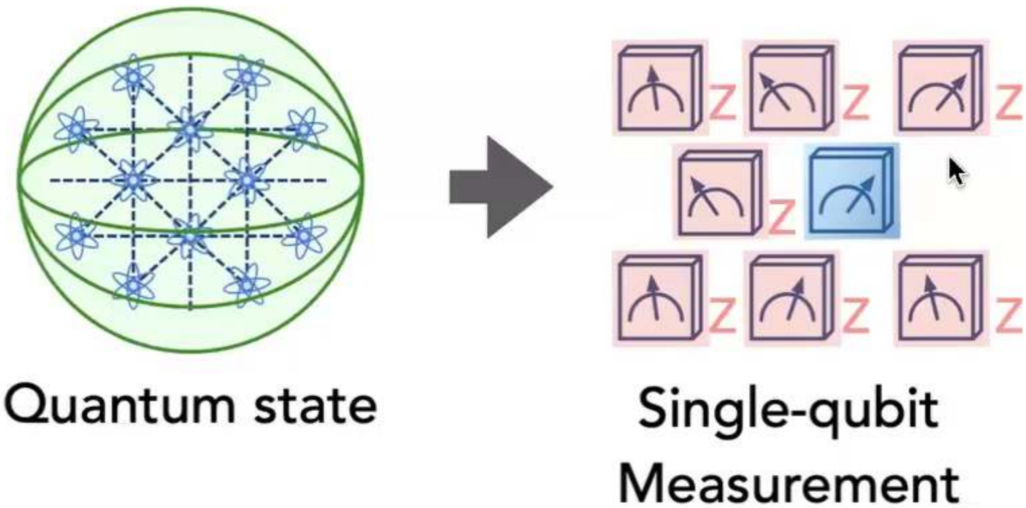

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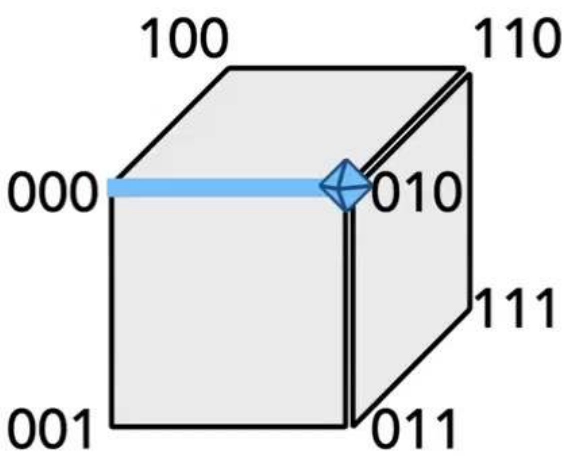


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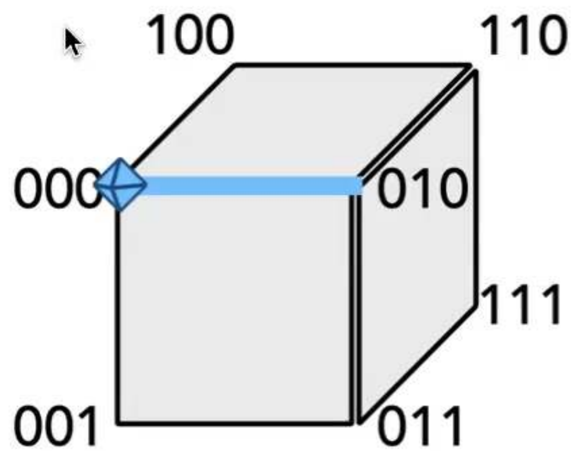


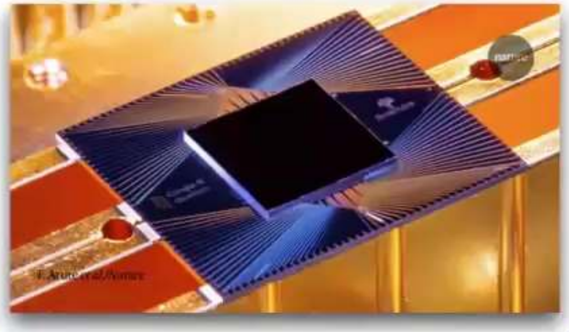
# Applications

What can we use this new certification protocol for?

**Example 1**

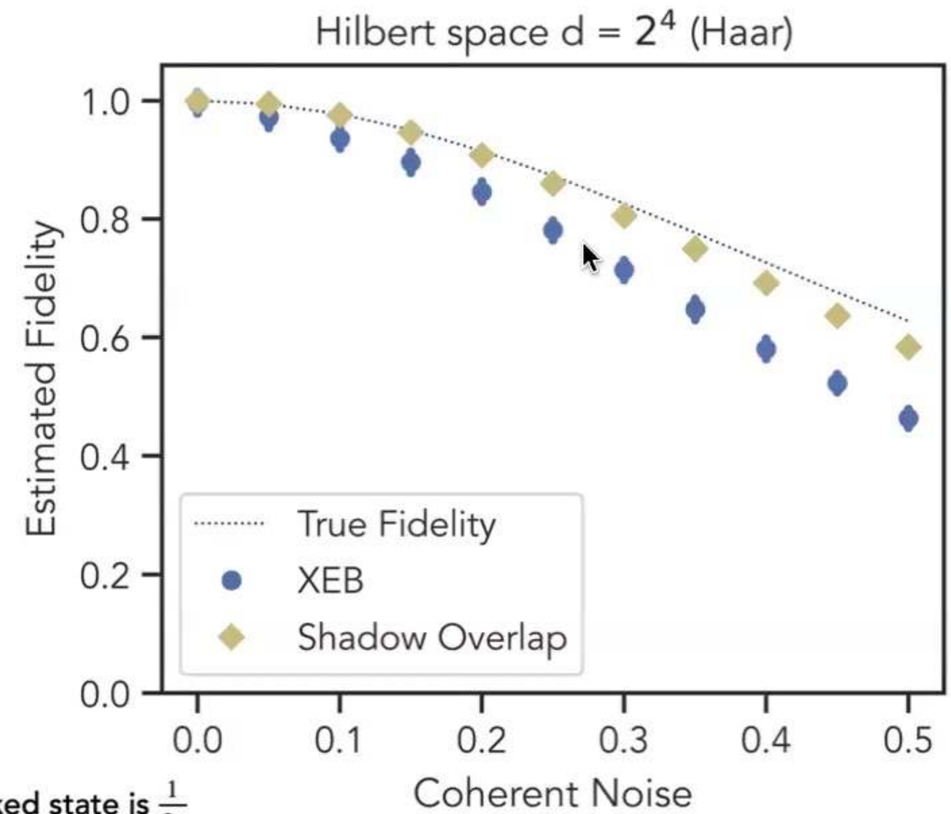
*Benchmarking*

Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity



# Benchmarking quantum devices

4-qubit Haar random state
*Coherent Noise*

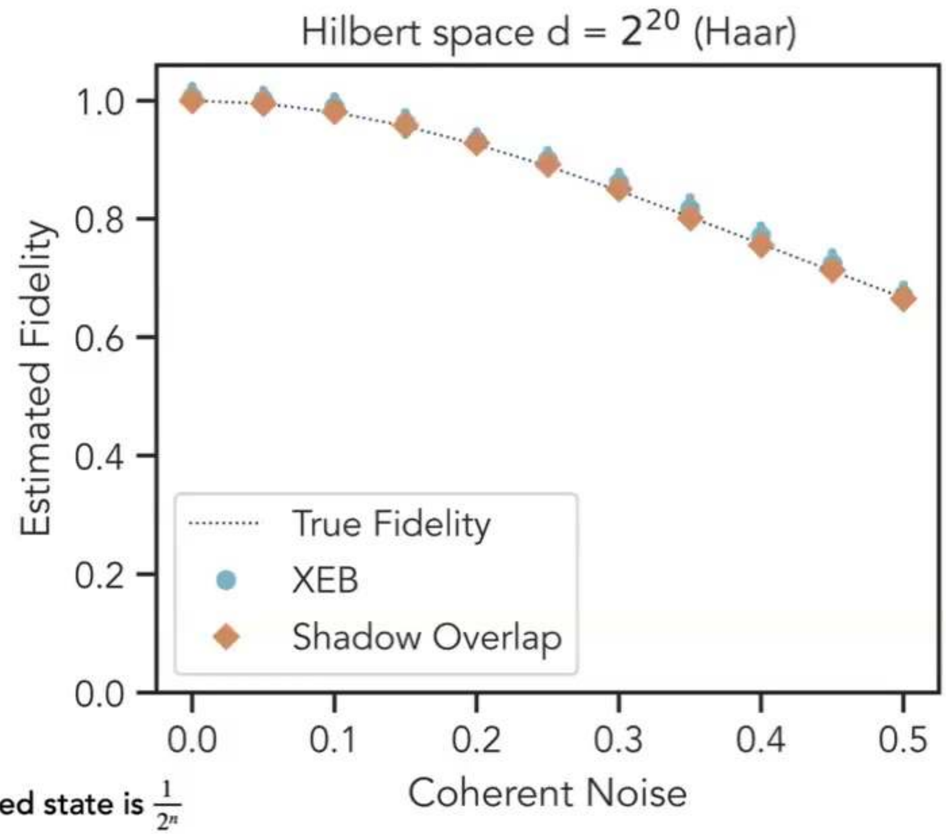



*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

20-qubit Haar random state

*Coherent Noise*

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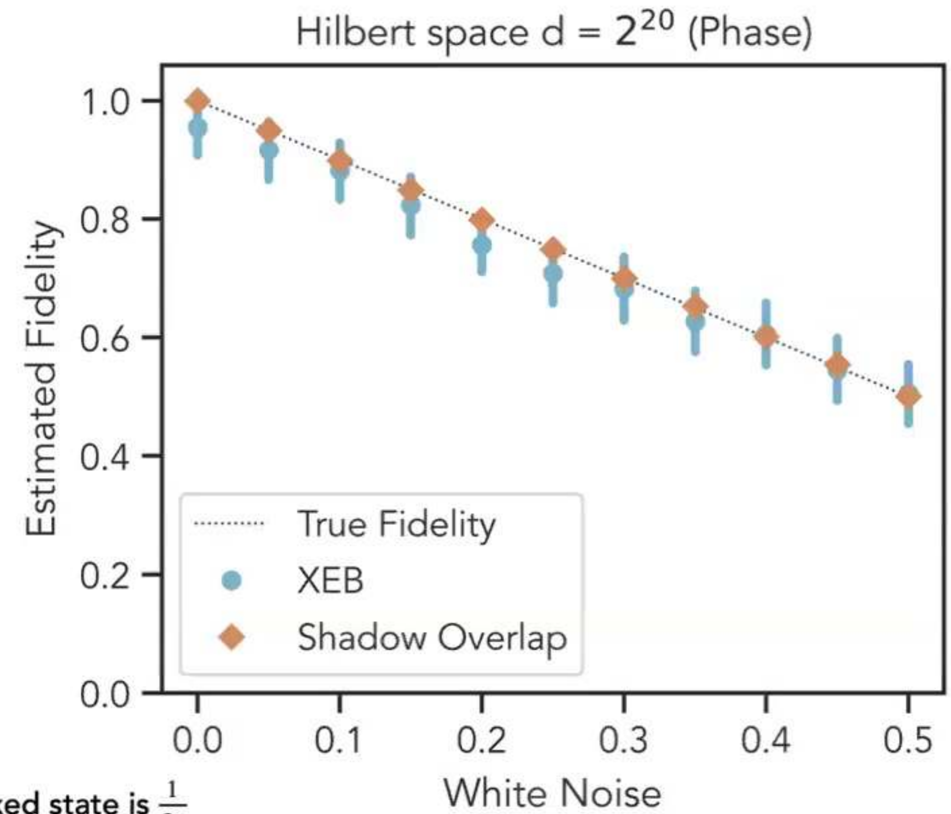
# Benchmarking quantum devices

20-qubit random structured state

*White Noise*

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{20} |\psi_i\rangle$$



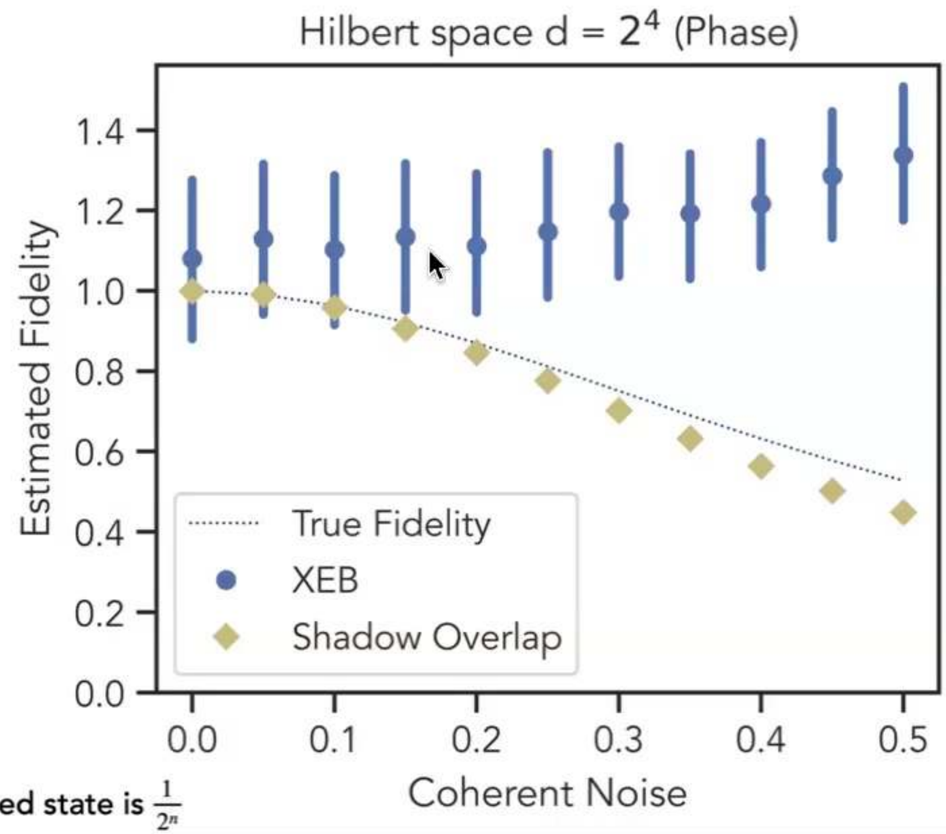

*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

**4-qubit random structured state**

***Coherent Noise***

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{4} |\psi_i\rangle$$




***Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$**

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20-qubit random structured state

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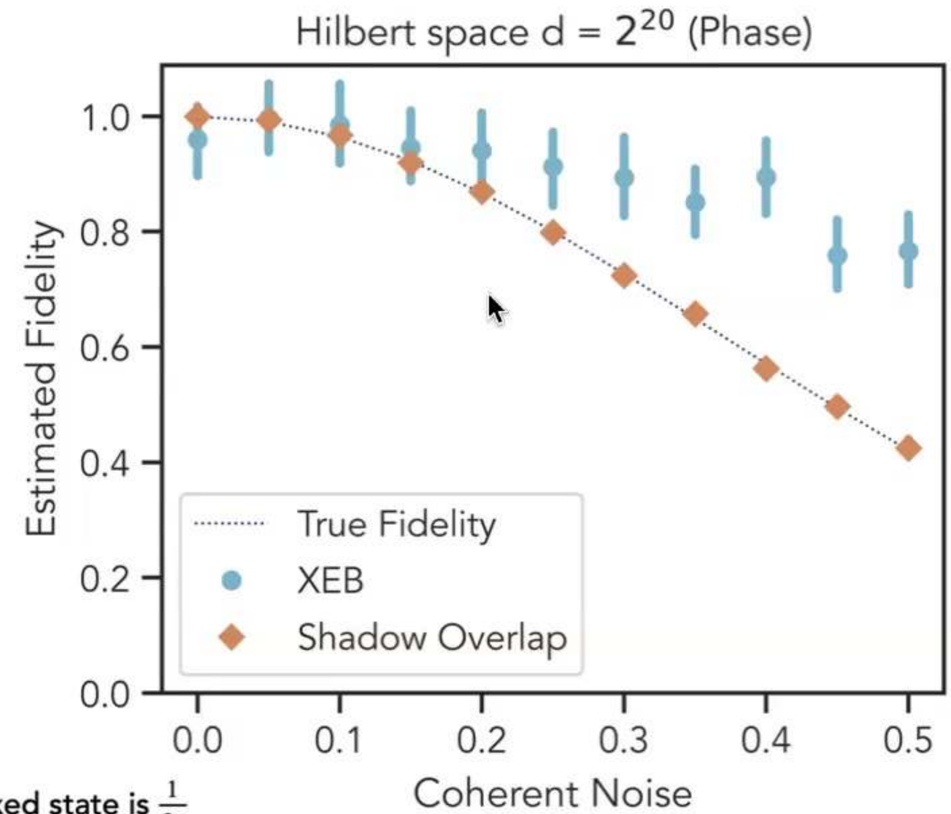
$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{20} |\psi_i\rangle$$

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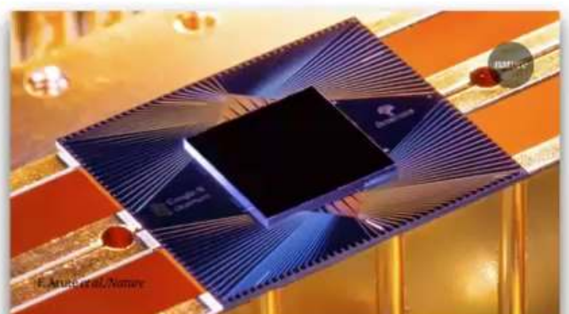
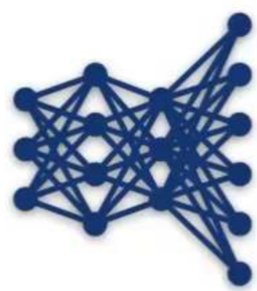


**Example 2**

***ML tomography***

Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



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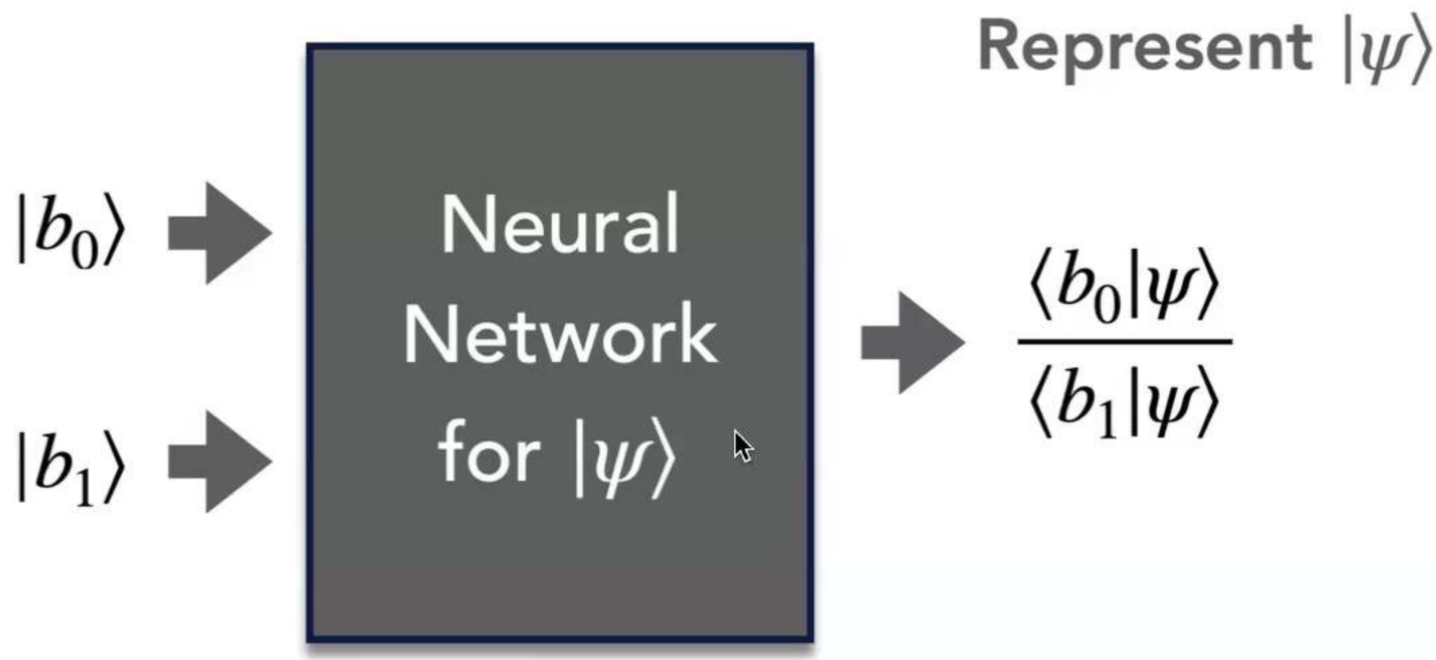
State-Cerification-long.pdf

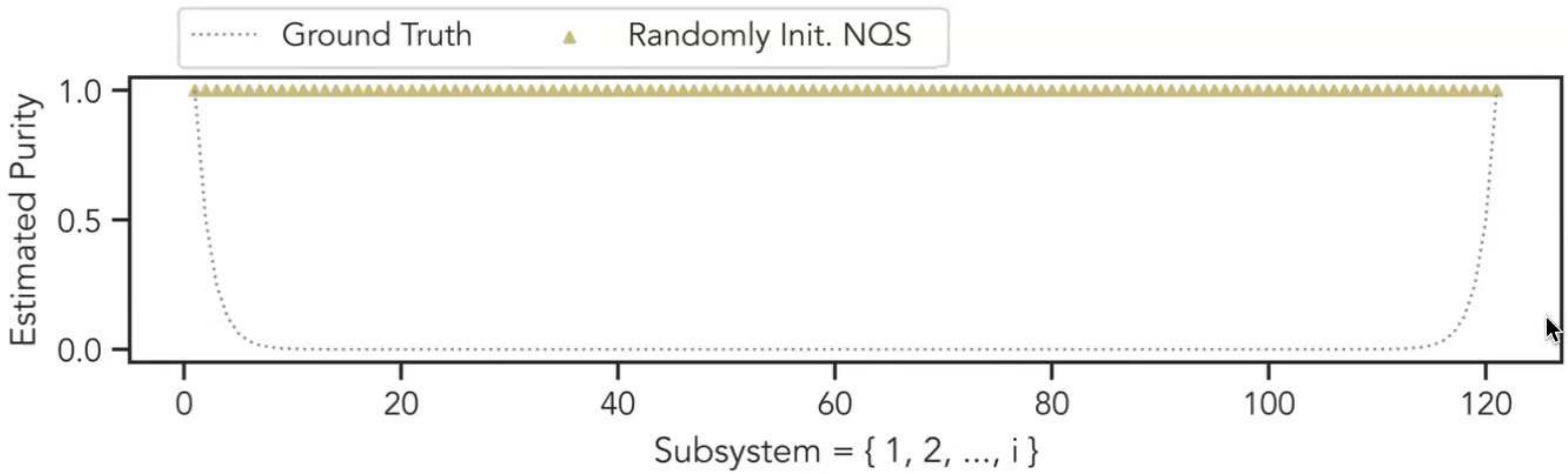
# Training/Certifying NN tomography




Relative Neural Quantum State

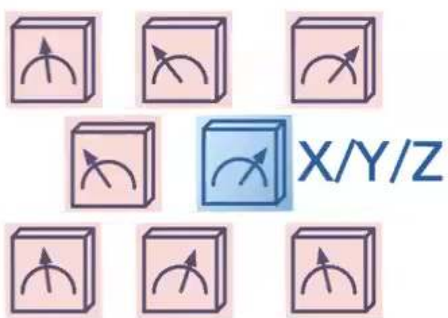
# Training/Certifying NN tomography

We consider learning a class of 120-qubit states with exponentially high circuit complexity.

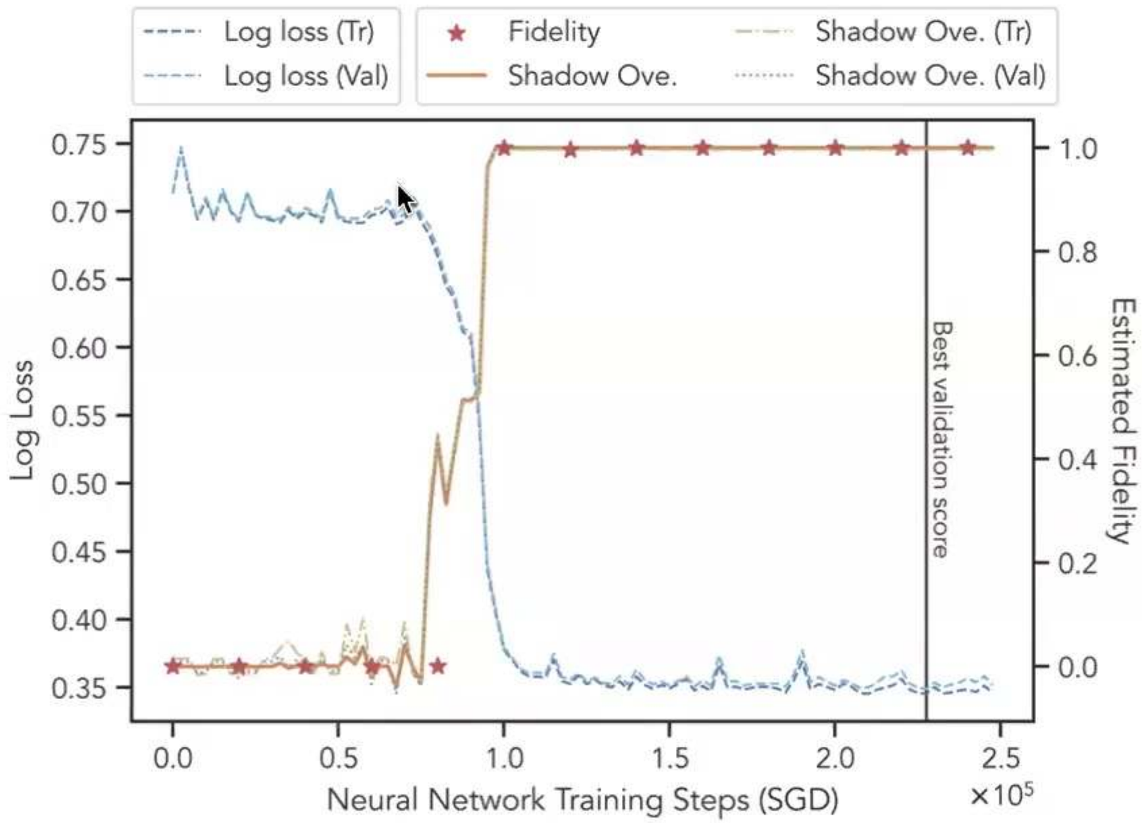


# Training/Certifying NN tomography

Trained using
shadow-overlap-based loss




Certified using
shadow overlap



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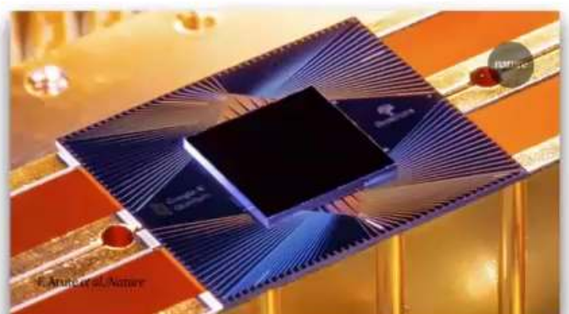

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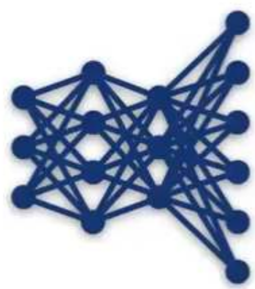
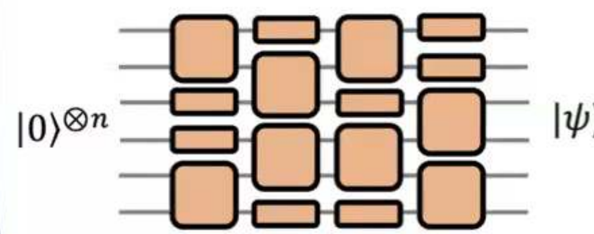


## Example 3

*Optimizing circuits*

To prepare a target state $|\psi\rangle$, we can optimize the circuit to max shadow overlap $\mathbb{E}[\omega]$



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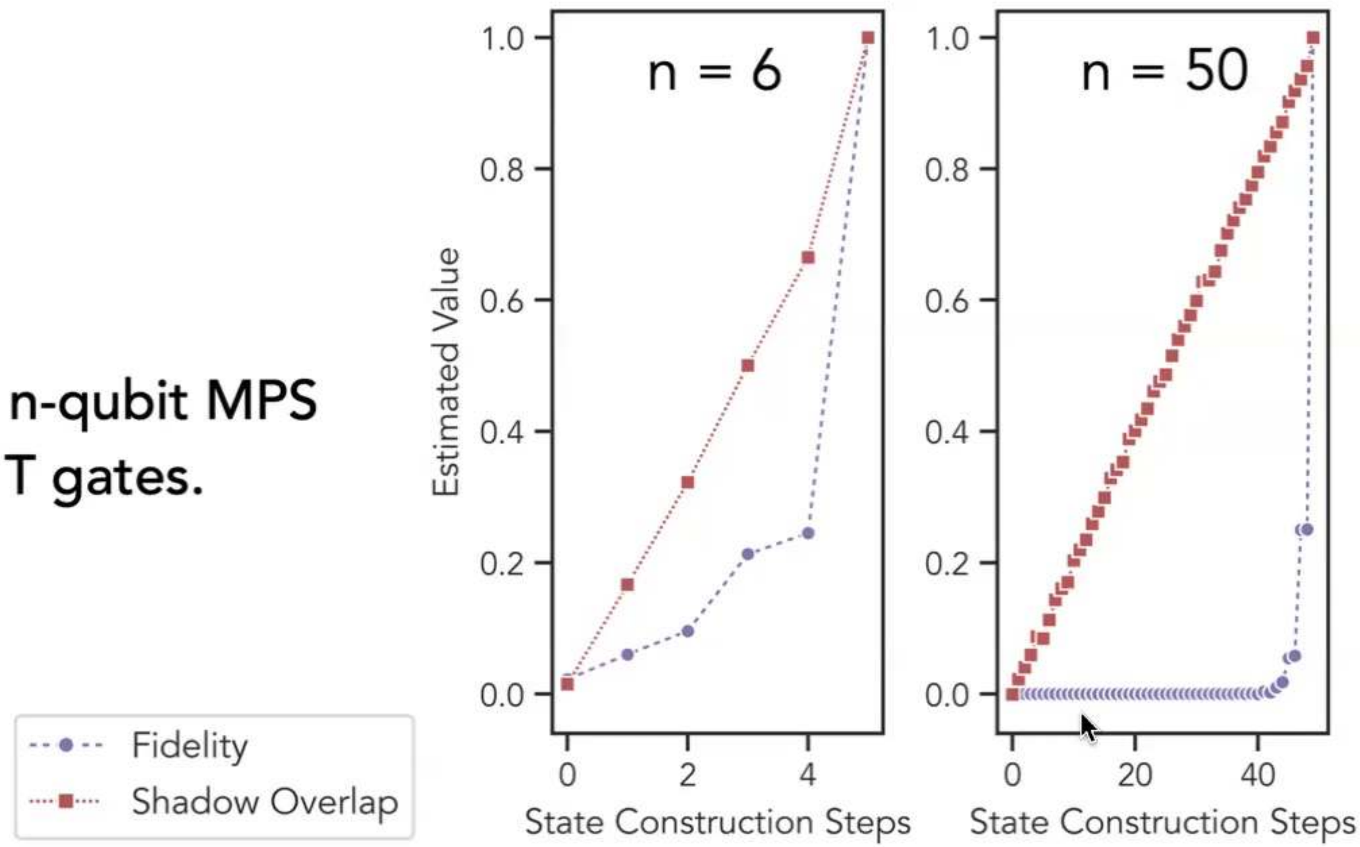
# Optimizing state-preparation circuit

Constructing an n-qubit MPS with H, CZ, T gates.



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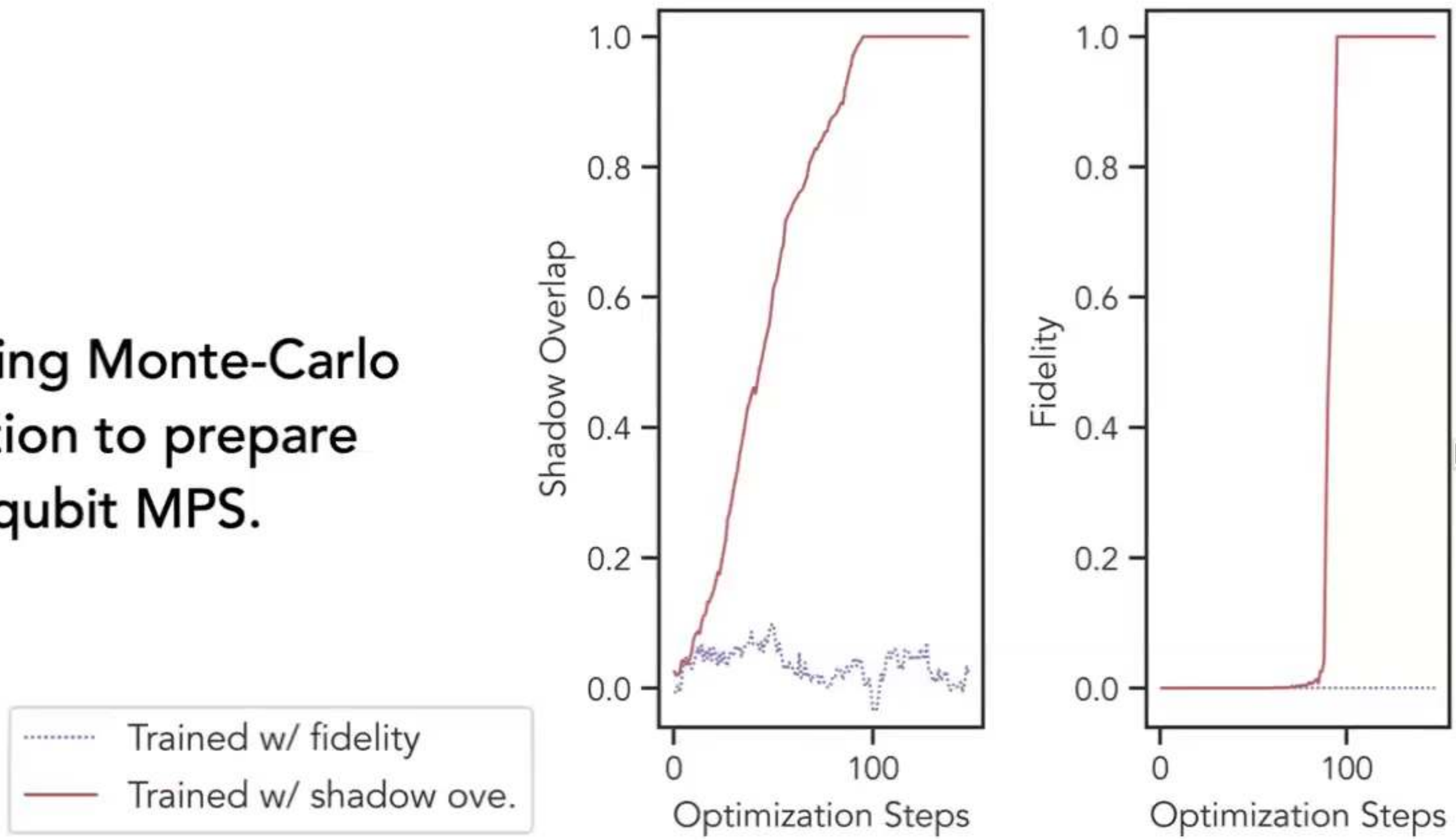
State-Cerification-long.pdf

# Optimizing state-preparation circuit

Training using Monte-Carlo optimization to prepare a 50-qubit MPS.



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# Conclusion

- We prove that almost all quantum states can be efficiently certified from few single-qubit measurements.
- Are there states not certifiable with few single-qubit measurements?