Title: Mapping ground states to string-nets

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Abstract: Two gapped ground states of lattice Hamiltonians are in the same quantum phase of matter, or topological phase, if they can be connected by a constant-depth circuit. It is conjectured that in two spatial dimensions, two gapped ground states with gappable boundary are in the same phase if and only if they have the same anyon contents, which are described by a unitary modular tensor category. We prove this conjecture for a class of states that obey a strict form of area law. Our main technical development is to transform these states into string-net wavefunctions using constant-depth circuits.

Classifying 2D topological phases: Mapping ground states to string-nets

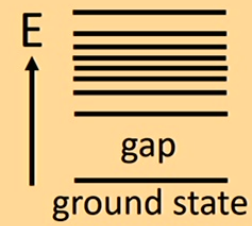
with Isaac Kim [UC Davis]
speaker: Daniel Ranard [MIT]

What can a ground state look like?

E.g. 1D spin chain.

$$H = \sum_i \sigma_i^z$$

$|0\rangle \; |0\rangle \; |0\rangle \; |0\rangle \; |0\rangle \; |0\rangle \; |0\rangle$

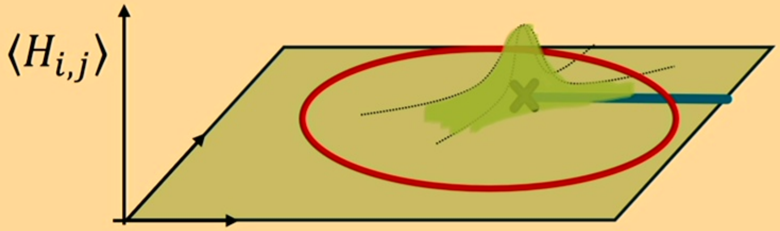


All gapped 1D ground states "look the same": like a product state.

Not true in higher dimensions!

2D toric code

$$H = \sum_{\text{vertex } v} A_v + \sum_{\text{plaquette } p} B_p$$



- Anyons
- *No* constant-depth circuit to product state

Trivial system

$$H = \sum_i \sigma_i^z$$

$$\psi_0 = |00 \ldots 0\rangle$$

- No anyons
- Product state

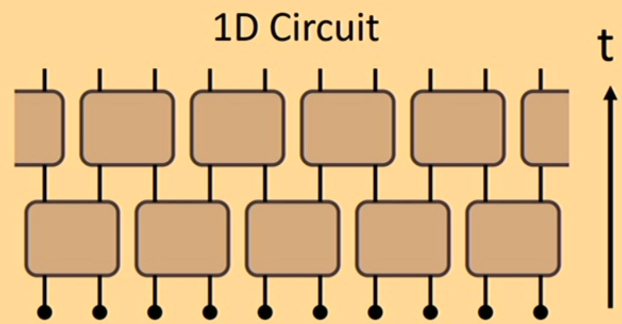
Quantum phases of matter

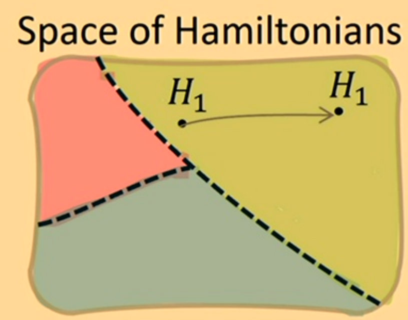
Two gapped systems $(|\psi_1\rangle, H_1)$ and $(|\psi_2\rangle, H_2)$ are in **the same phase** iff

Equivalent conditions:

1. **Circuits:** $|\psi_1\rangle = U\,|\psi_2\rangle$ for a constant-depth, local unitary circuit U.
"Same entanglement structure."

2. **Deformations:** ∃ path of gapped Hamiltonians from H_1 to H_2.
"No phase transition."





What are all the 2D gapped phases?

Don’t we already know?

Clearing up…

Two notions of topological phase:

1. Equivalence classes under constant-depth circuits (or adiabatic…)
2. "How the anyons behave" ← What people usually classify

Are these equivalent?

(a) Connected by circuit ⇒ same anyons ← Intuitive

(b) Same anyons ⇒ connected by circuit? ← **Why?**

Clearing up…

Two notions of topological phase:

1. Equivalence classes under constant-depth circuits (or adiabatic…)
2. "How the anyons behave" ← What people usually classify

Are these equivalent?

(a) Connected by circuit ⇒ same anyons ← Intuitive

(b) Same anyons ⇒ connected by circuit? ← Wrong

Actually, models with the same anyons **not** always in the same phase.

We consider: 2D gapped ground states with **gappable boundary**

Conjecture: For these systems, same anyons $\Leftrightarrow$ same phase
& all phases captured by a string-net model (Levin-Wen)

We prove: Something like that!

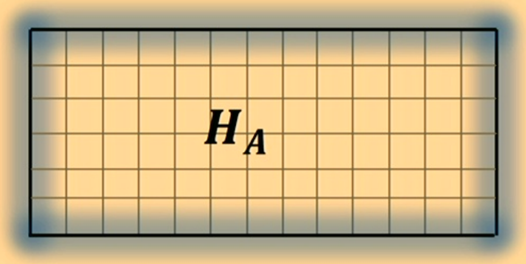
All 2D gapped ground states with gappable boundary
(that satisfy some assumptions)
can be mapped to a string-net by constant-depth circuit.

Gapped boundary

$H = \sum_i h_i$ Unique gapped ground state on the plane or sphere.

$H_A = \sum_{i \in A} h_i$ Truncate to disk A. (Maybe degenerate or gapless.)

$H'_A = H_A + H_{\partial A}$ Add boundary terms $H_{\partial A}$.
If H'_A gapped has unique gapped ground state,
then $H_{\partial A}$ called a **gapped boundary condition.**



New boundary terms $H_{\partial A}$

If ∃ gapped boundary condition,
H has **“gappable boundary.”**
Otherwise, If not,
H has **“ungappable boundary.”**

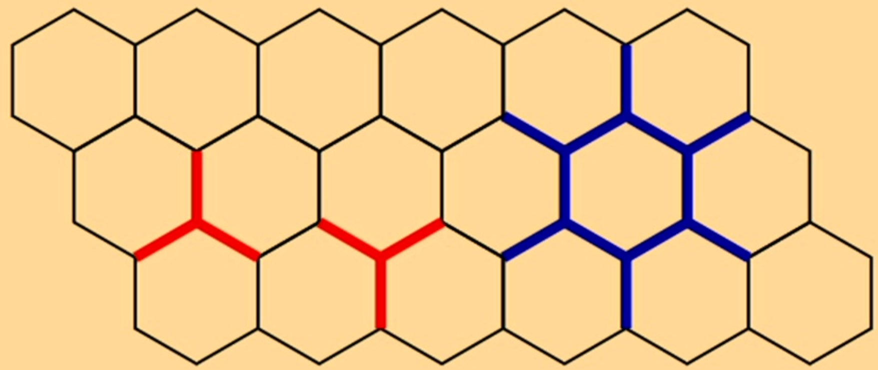
String-net Hamiltonians

$$H = \sum_{\text{vertex } v} Q_v + \sum_{\text{plaquette } p} B_p$$

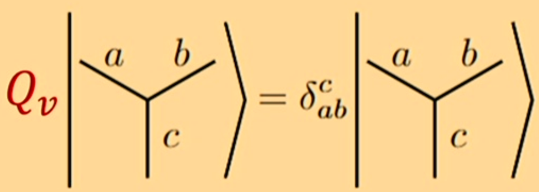
Model specified by choice of
$\mathcal{C} =$ unitary fusion category (UFC)

Q_v, B_p commuting projectors.

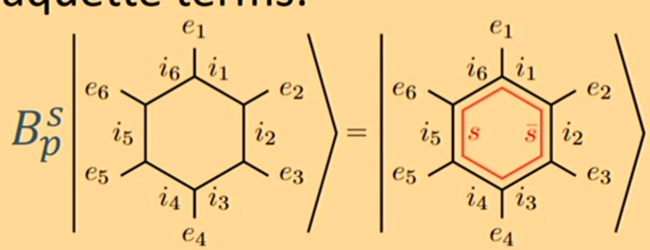
DOF on edges of honeycomb lattice
$\mathcal{H}_{\text{edge}} = \text{span}\{\, |a\rangle : a \in \mathcal{C}\}$



Vertex terms:



Plaquette terms:



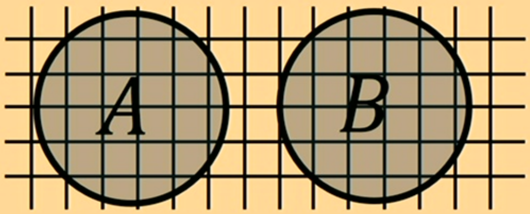
$B_p := \sum_{s \in \mathcal{C}} B_p^s$

Our assumptions: general philosophy

- Hard to work with arbitrary gapped ground states.
  Even proving the area law is an open problem!

- Want a framework where we *can* make rigorous arguments, but *don't* need to deal with "epsilons and deltas" (yet…)

- **Assume** the phases of interest have a representative state for which correlations of disjoint regions are **exactly** zero
  (+ a few more assumptions like that).



$$I(A\!:\!B) = 0$$

- Classify the above states under constant-depth circuits.

Entanglement bootstrap axioms

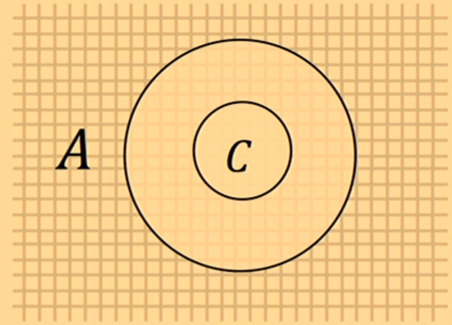
(Shi, Kato, Kim, `19)

We assume:

(1) No long-range mutual information, &

(2) No long-range *conditional* mutual information.

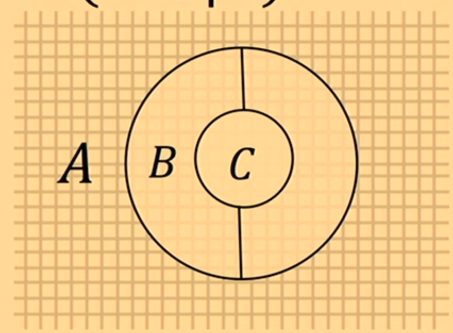
$I(A:C) = 0$



These states:

- Have zero correlation length
- Have gapped parent Hamiltonians
- Approximately encompass all gapped ground states after sufficient coarse-graining? (Well…)

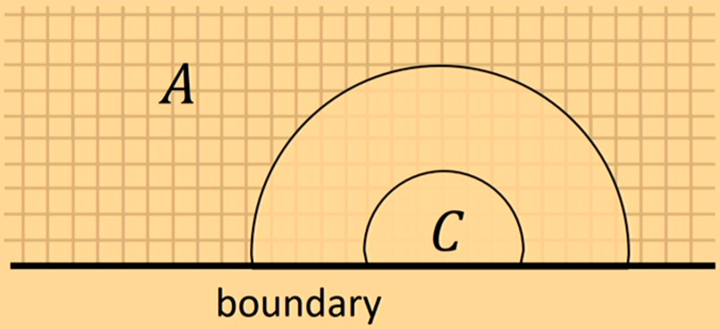
$I(A:C|B) = 0$

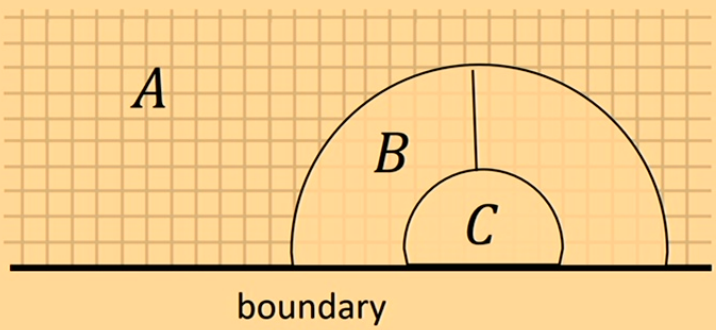


Gapped boundary

Assume the state satisfies analogous conditions near the physical boundary.

(Shi & Kim, `21)





$I(A:C) = 0$

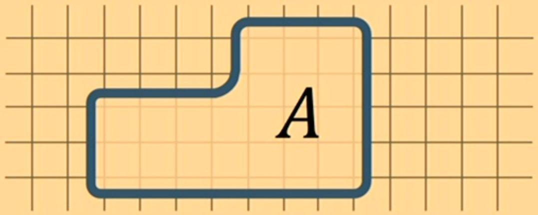
$I(A:C|B) = 0$

Alternative assumptions?

- **Sufficient** to assume **strict area law**,

$$S_A = \alpha|\partial A| - \gamma,$$

for some constants α, γ, independent of region A.



- Still working on robust *approximate* axioms.

Gappable boundaries

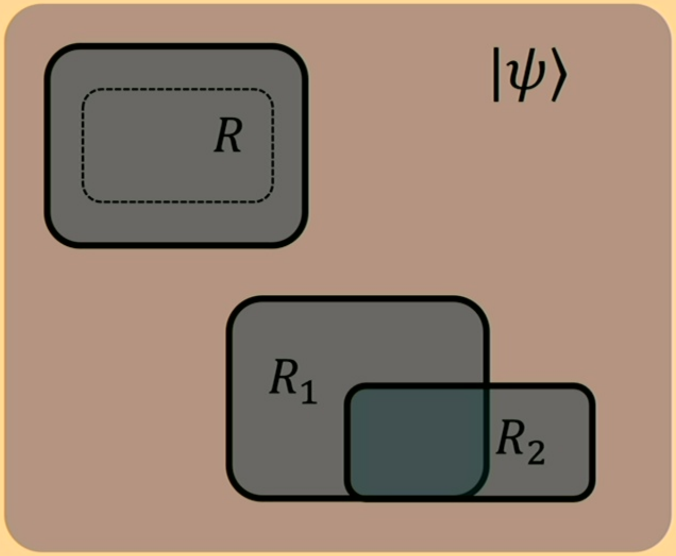
Want to assume gappable boundary.

Don't want to assume $|\psi\rangle$ has a physical boundary present.

We assume any disk subregion can be *given* a gapped boundary.

Assume:

$\forall$ subregions R, $\exists$ $|\phi_R\rangle$ on R that satisfies the boundary axioms and is **consistent:**

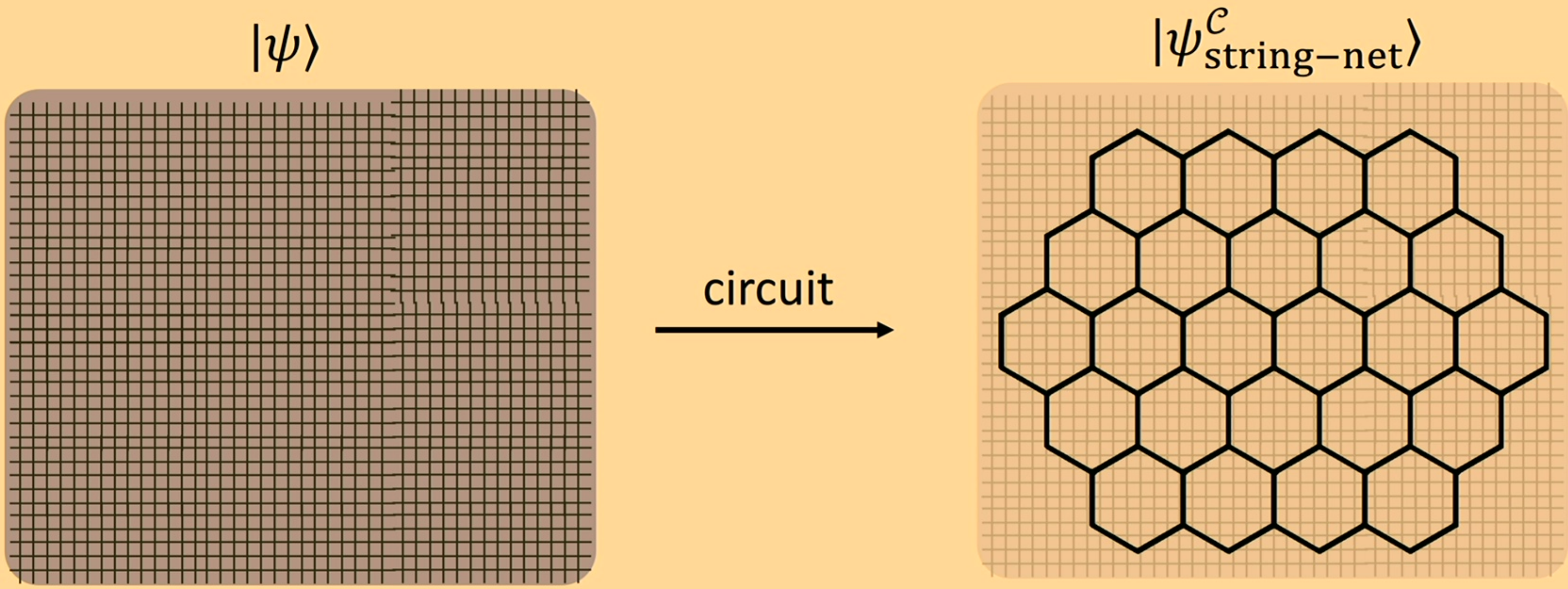


$|\psi\rangle, |\psi_R\rangle$ match on interior.

$|\psi_{R_1}\rangle, |\psi_{R_2}\rangle$ match on overlap.

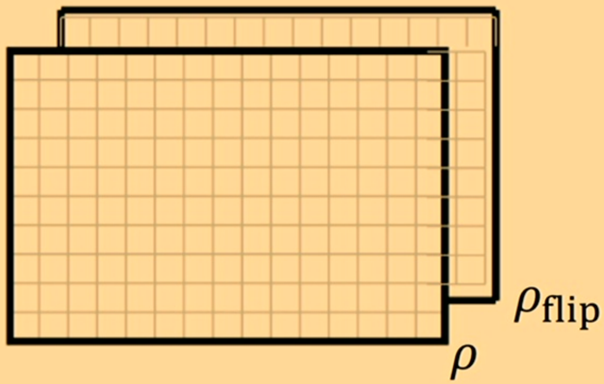
Main result

For any state $|\psi\rangle$ satisfying the axioms, there exists a unitary fusion category $\mathcal{C}$ and a constant-depth circuit such that…



Stacking two layers to get a string-net

Given a translation-invariant 2D state ρ with strict area law, the state $\rho \otimes \rho_{\text{flip}}$ is in the same phase as some string-net.



Don't need to assume gappable boundary here!

How do you find the circuit?

First: what string-net should we be looking for?

Given a ground state,
how do learn about “anyons”?

Reduced ground space

aka "information convex set"

Use parent Hamiltonian H of state σ.

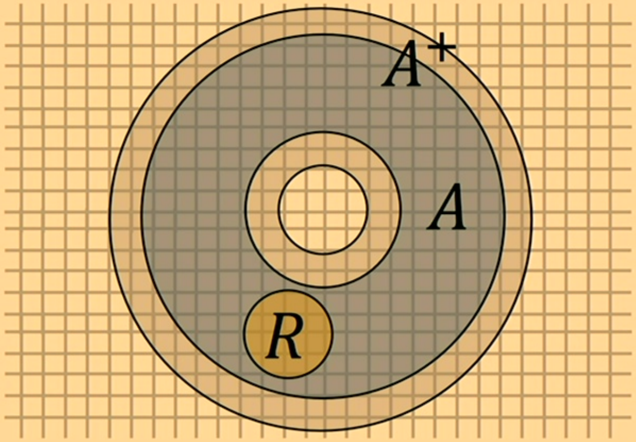
Take all states ρ_{A^+} in ground space of H_{A^+}.
Partial trace to obtain a **set of states** ρ_A on A.
Call this the **reduced ground space**, RGS(A).

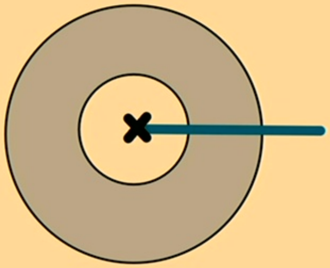
All states in $\mathrm{RGS}(A)$ match global ground state on disks $R \subset A$.

For toric code state σ, for A annulus,
$\mathrm{RGS}(A) = \{\text{mixtures of } \sigma_A, \sigma_A^e, \sigma_A^m, \sigma_A^{em}\}$.

Recovered list of anyons from the ground state!

Annulus $A \subset A^+$.



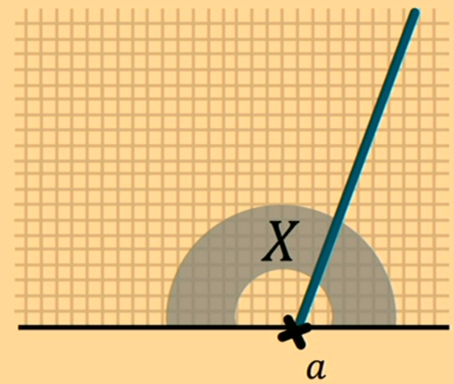


But the string-net we're looking for is more directly related to the **boundary anyons** than the **bulk anyons**.

E.g. toric code edge DOF have **two** labels, but there are **four** bulk anyons.

Bulk anyons = **braided tensor category**
Boundary anyons = **fusion category**
Data defining string-net = **fusion category**

Boundary anyons

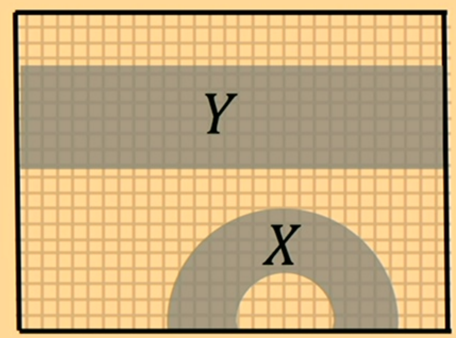


Can define reduced ground space for any region.

For "half-annulus" X attached to the boundary,

$$\mathrm{RGS}(X) = \mathrm{mixtures}\{\rho_A^a\}_a$$

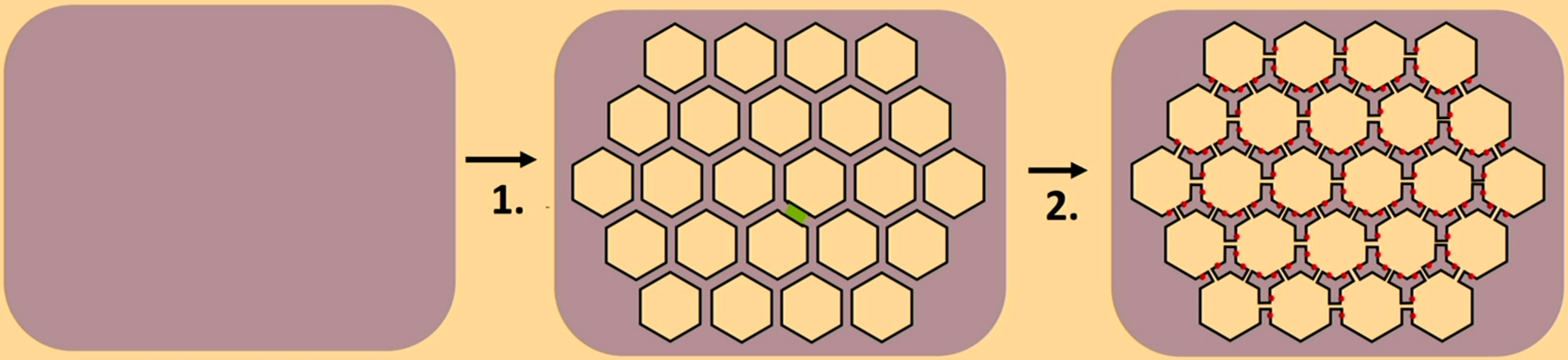
with one sector for each anyon type a that can live at the boundary.

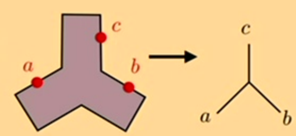


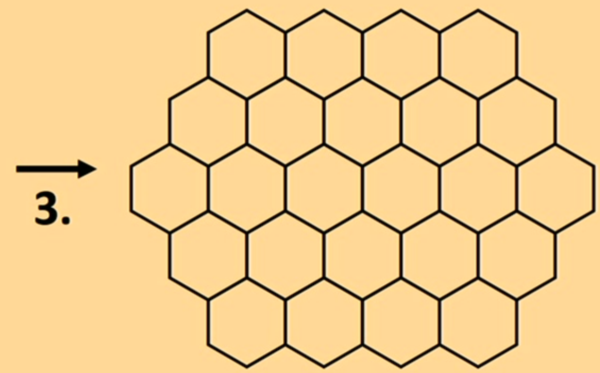
Structure of $\mathrm{RGS}(X)$ only depends on *topology* of X.
E.g. $\Sigma(X) \cong \Sigma(Y)$.

Building the circuit

Three-layer circuit, using coarse-grained regions. Builds string-net using ***vertex*** **DOF** convention.



1. Punch holes 
2. Disentangle vertex regions, conditional on the **edge** sectors 
3. Local change of basis within each vertex region: map to string-net's local basis. 



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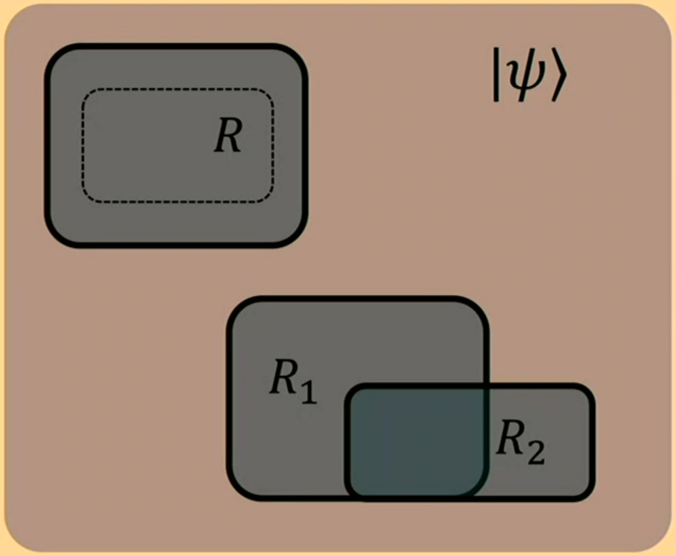

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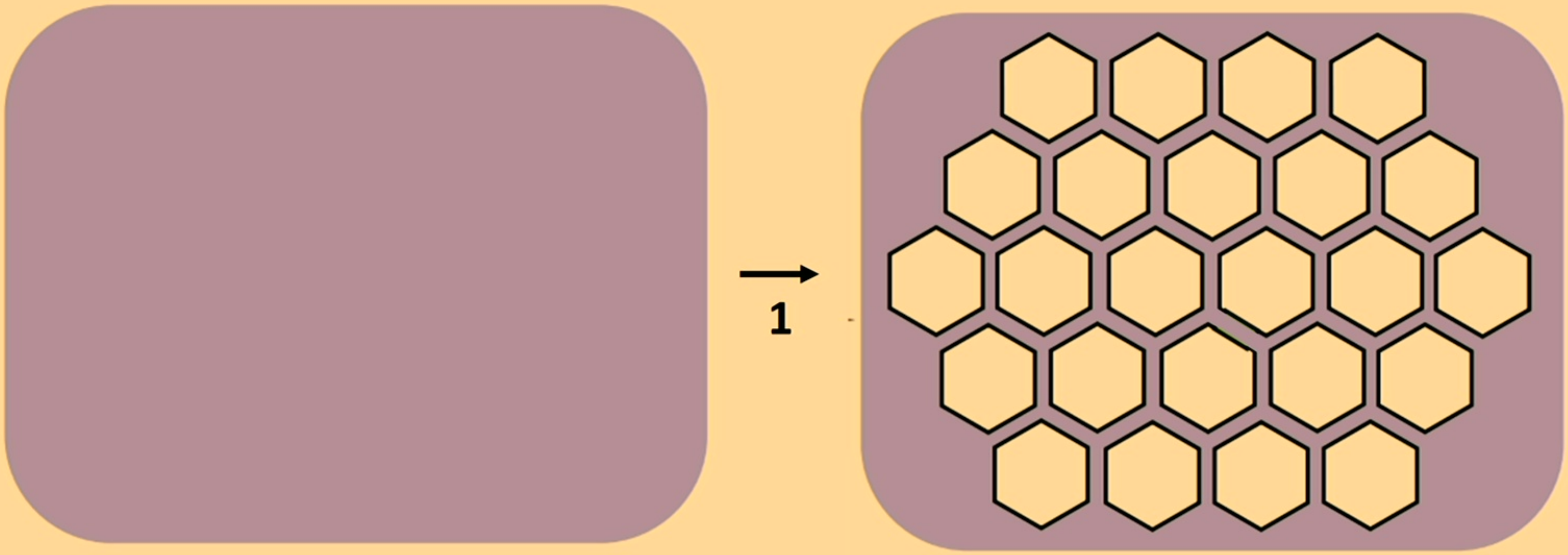
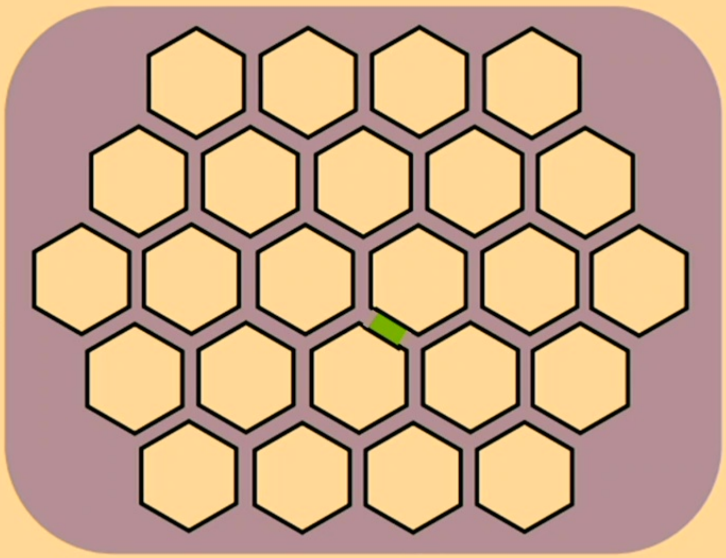
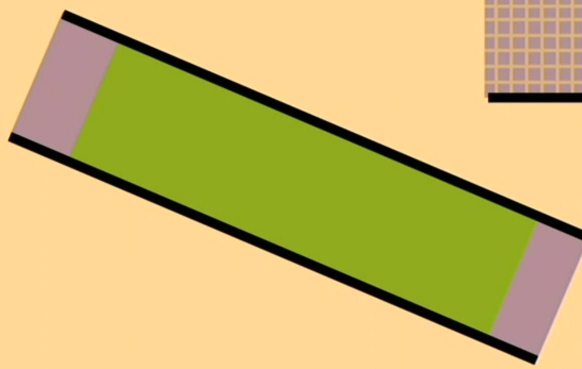
First circuit layer: Punching holes

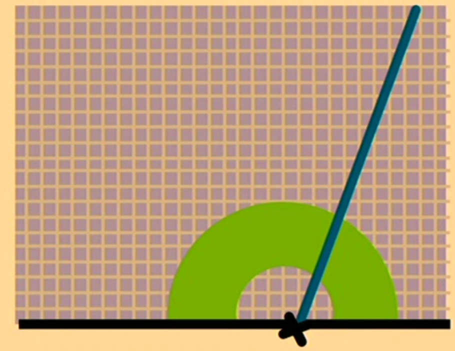


- Inside the hole, state is a pure product state. (Could discard.)
- Hole has gapped boundary.
- Hole created by unitary supported on neighborhood of hole.

Analyzing edge regions

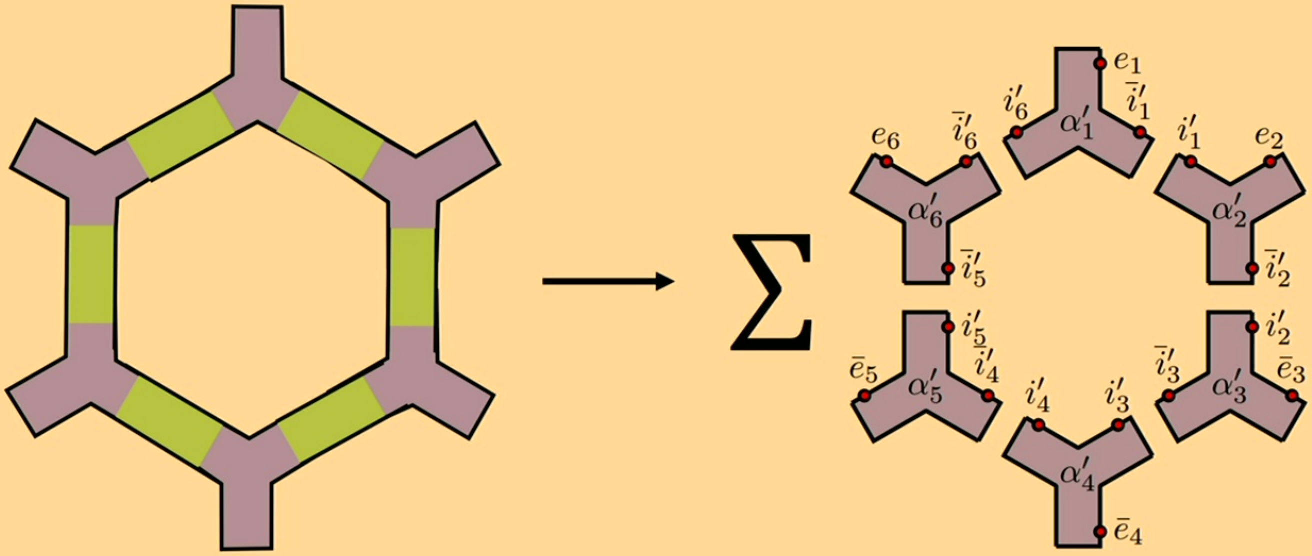






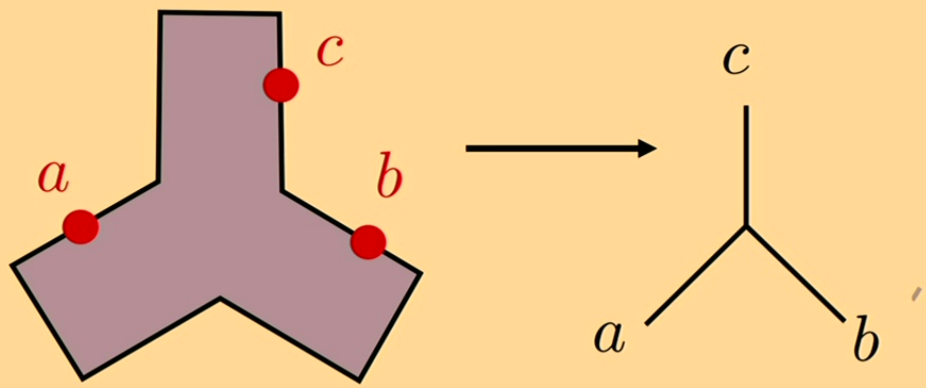
- Inside the hole, state is a pure product state. (Could discard.)
- Hole has gapped boundary.
- Hole created by unitary supported on neighborhood of hole.
- Reduced ground space of  has sectors labeled by boundary anyon types.

Second layer: disentangling vertex regions



- Edge regions are like 1D long-range correlated states, GHZ-ish.
- Individual sectors are short-range correlated, can be factorized.
- Factorize the edge regions conditional on sector.
  Creates **superposition of product states (product over vertices).**

Third layer: change of basis on vertex regions.

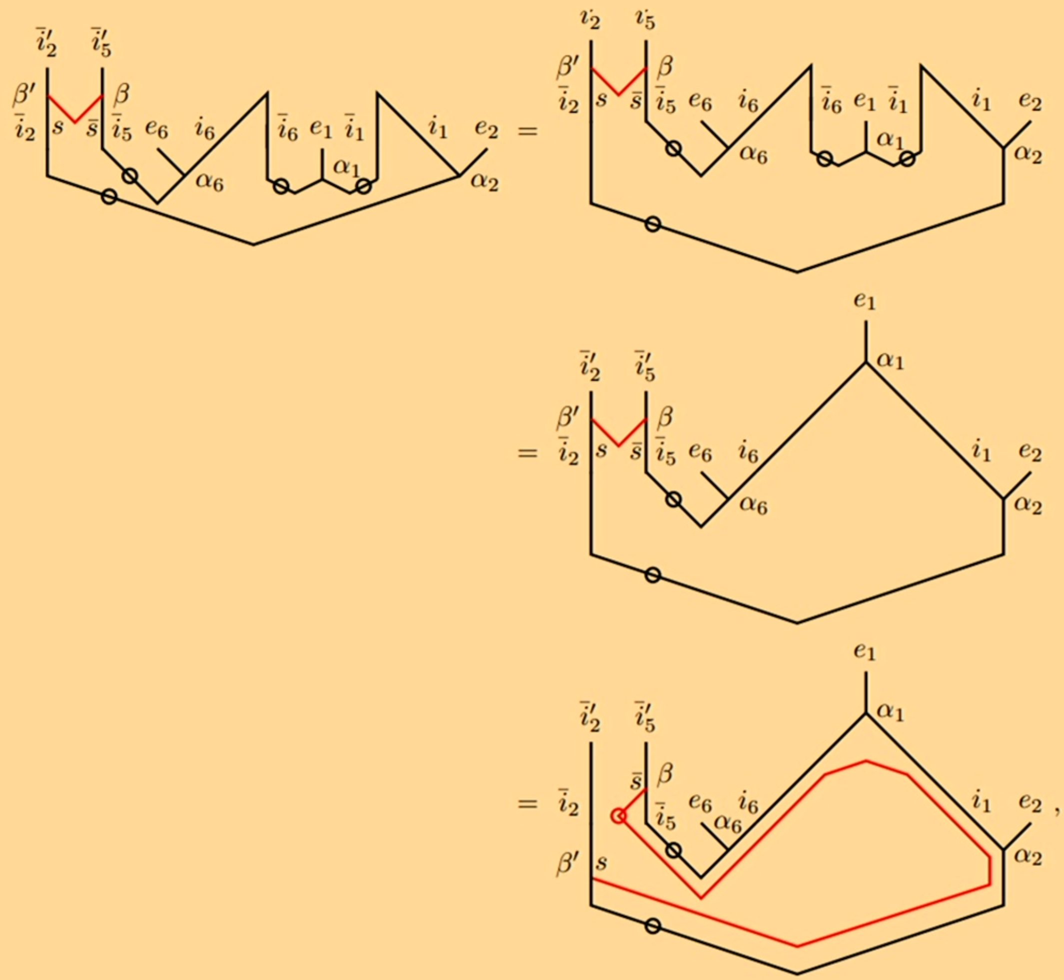


- Take convention that string-net DOF should live on vertices, not edges.
- In some sense, third layer is unnecessary: just a local change of basis.
- Identify a subspace of vertex region as string-net vertex space.
  (State is supported on this subspace; complement can be discarded.)

Now we have a state in the string-net Hilbert space.

Looks like a string-net, but need to check it’s actually the ground state of the Levin-Wen Hamiltonian.

The vertex terms of Hamiltonian are already satisfied.
Must check plaquette terms.


$\bar{i}'_2$ $\bar{i}'_5$
β' β
$\bar{i}_2$ s $\bar{s}$ $\bar{i}_5$ e_6 i_6 $\bar{i}_6$ e_1 $\bar{i}_1$ i_1 e_2 =
α_6 α_1 α_2
i_2 i_5
β' β
$\bar{i}_2$ s $\bar{s}$ $\bar{i}_5$ e_6 i_6 $\bar{i}_6$ e_1 $\bar{i}_1$ i_1 e_2
α_6 α_1 α_2
e_1
α_1
$\bar{i}'_2$ $\bar{i}'_5$
β' β
= $\bar{i}_2$ s $\bar{s}$ $\bar{i}_5$ e_6 i_6 i_1 e_2
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$\bar{s}$ β
= $\bar{i}_2$ $\bar{i}_5$ e_6 i_6 α_6 i_1 e_2 ,
β' s α_2

Thank you!