Title: Universal bound on topological gap

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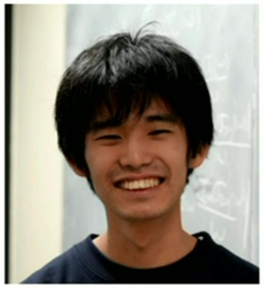
Abstract: I will show the existence of a universal upper bound on the energy gap of topological states of matter, such as (integer and fractional) Chern insulators, quantum spin liquids and topological superconductors. This gap bound turns out to be fairly tight for the Chern insulator states that were predicted and observed in twisted bilayer transition metal dichalcogenides. Next, I will show a universal relation between the energy gap and dielectric constant of solids. These results are derived from fundamental principles of physics and therefore apply to all electronic materials. I will end by outlining new research directions involving topology, quantum geometry and energy.

Topological Bounds on Energy Gap and Correlation Function

Liang Fu

with Yugo Onishi



Onishi & LF:

- topological bound on energy gap
  PRX 14, 011052 (2024)
- topological bound on structure factor
  arXiv:2401.13847 & to appear
- thermodynamic bound on energy gap
  arXiv:2401.04180

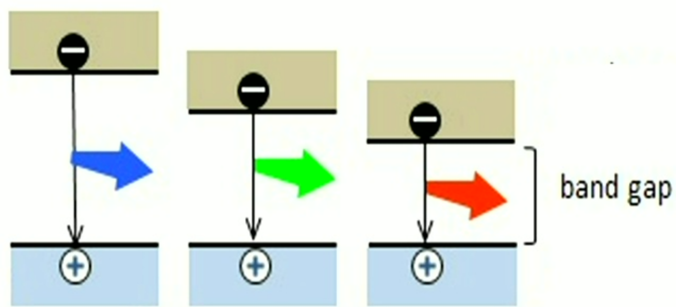
…

We consider general many-body systems with conserved U(1) charge, such as total electron number N or spin S_z.

We study bulk properties in the thermodynamic limit, such as energy gap and correlation function in the bulk.

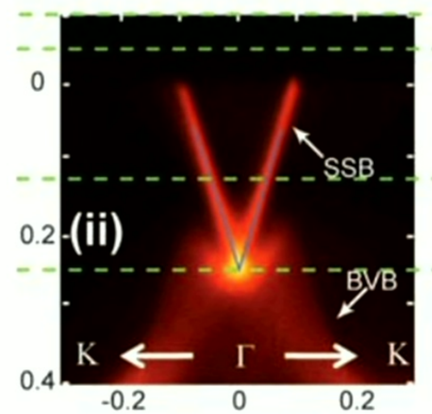
Systems with short-range and $1/r$ Coulomb interactions will both be considered.

Energy Gap of Insulating States



energy gap determines the color of absorbed and emitted light

Bi_2Te_3 : bulk gap 0.15eV

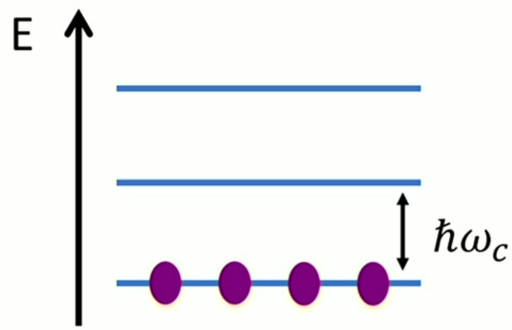


topological insulator

Energy Gap of Chern Insulators

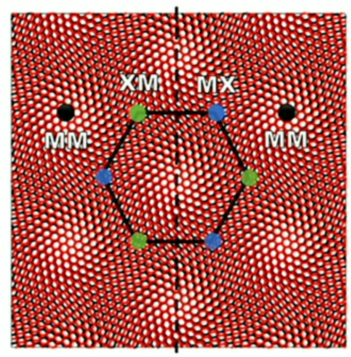
2D gapped states with quantized $\sigma_{xy} = C\frac{e^2}{h}$

Quantum Hall states



cyclotron gap

Twisted $MoTe_2$



QAH at ν=1 (B=0):

- activation gap ~ 30K
- thermodynamic gap ~ 6meV

(Seattle; Cornell, 2023)

Upper Bound on Energy Gap

$$\Delta \leq \frac{2\pi\hbar^2 n}{m|C|}$$

$C \equiv \frac{h\sigma_{xy}}{e^2}$: many-body Chern number (integer or fractional)

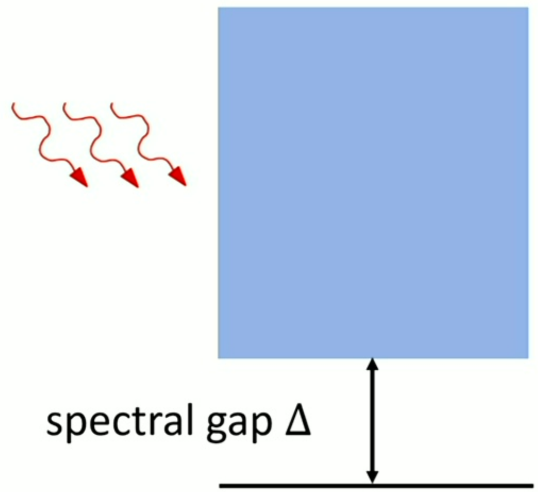
derived from basic principles of physics for Hamiltonians of the form:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \mathbf{p}_i \cdot \mathbf{A}(\mathbf{r}_i) + U(\mathbf{r}_i) + \sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j)$$

generalization to any systems with conserved U(1) charge

Onishi & LF, PRX 14, 011052 (2024)

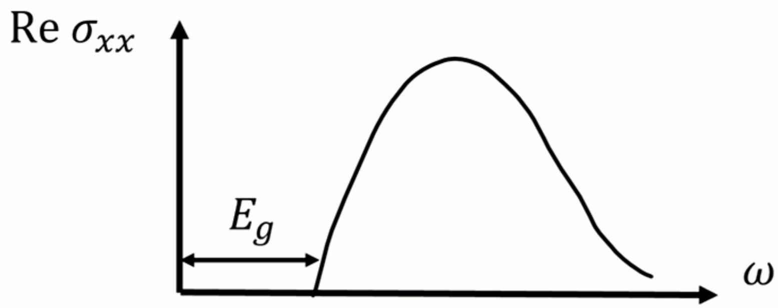
Spectral Gap and Optical Gap



optical gap E_g: energy gap between ground state and first excited state that is optically accessible.

By definition, $\Delta \leq E_g$.

Optical Conductivity and Optical Gap



- optical spectral weight: $W^0 \equiv \int_0^\infty d\omega \, \mathrm{Re}\, \sigma_{xx} = \frac{\pi n}{2m}$
- negative first moment: $W^1 \equiv \int_0^\infty d\omega \, \frac{\mathrm{Re}\, \sigma_{xx}}{\omega}$
- inequality: $W^1 \leq \frac{W^0}{E_g}$

Negative moments are more dominated by low-energy excitations.

Optical Absorption

$\mathrm{Re}\,\sigma_{xx}$ and $\mathrm{Im}\,\sigma_{xy}$ are dissipative parts of optical response
(T-even) (T-odd)

Under left/right circularly polarized light $\boldsymbol{E} = E(\cos\omega t, \pm\sin\omega t)$,

the absorbed power must be non-negative:

$$P_{\pm} = \boldsymbol{j}\cdot\boldsymbol{E} = \left(\mathrm{Re}\,\sigma_{xx}(\omega) \pm \mathrm{Im}\,\sigma_{xy}(\omega)\right)E^2 \geq 0$$

magnetic circular dichroism

$\Rightarrow\ \mathrm{Re}\,\sigma_{xx} \geq |\mathrm{Im}\,\sigma_{xy}|$ at all ω

Onishi & LF, PRX (2024)

Topological Bound on Optical Gap

$$W^1 \equiv \int_0^\infty d\omega \, \frac{\mathrm{Re}\, \sigma_{xx}}{\omega} \geq \int_0^\infty d\omega \, \frac{|\mathrm{Im}\, \sigma_{xy}|}{\omega}$$

$$\geq \left| \int_0^\infty d\omega \, \frac{\mathrm{Im}\, \sigma_{xy}}{\omega} \right| = \frac{\pi}{2} \sigma_{xy}(0) \equiv \frac{1}{4} C$$

(Kramers-Kronig relation)

$$W^1 \equiv \int_0^\infty d\omega \, \frac{\mathrm{Re}\, \sigma_{xx}}{\omega} \leq \frac{\int_0^\infty d\omega \, \mathrm{Re}\, \sigma_{xx}}{E_g}$$

optical absorption onsets at $\omega = E_g$

$$= \frac{\pi n/(2m)}{E_g}$$

(f-sum rule)

$$\boxed{E_g \leq \frac{2\pi \hbar^2 n}{m|C|}}$$

Topological Bound on Optical Gap

$$E_g \leq \frac{2\pi\hbar^2 n}{m|C|}$$

derived from

- non-negative dissipation
- causality (Kramers-Kronig)
- f-sum rule: valid for H with kinetic energy $p^2/2m$

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \mathbf{p}_i \cdot \mathbf{A}(\mathbf{r}_i) + U(\mathbf{r}_i) + \sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j)$$

$$\int_0^\infty d\omega \, \mathrm{Re}\, \sigma_{xx} = \frac{\pi}{2V}\left\langle \frac{\partial^2 H}{\partial p^2} \right\rangle = \frac{\pi n}{2m}$$

applicable to interacting and fractional Chern insulators

Quantum Hall States

effective Hamiltonian $H = \sum_i \frac{(p_i - eA_i)^2}{2m} + \sum_{ij} U(r_i - r_j)$

Optical gap bound: $E_g \leq \frac{2\pi\hbar^2 n}{m|C|}$

n: charge carrier
m: band mass

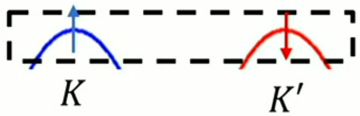
At Landau level filling factor ν,

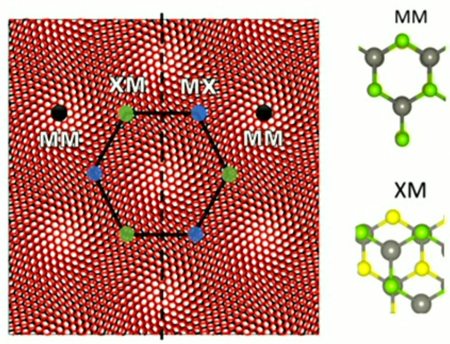
$$n = \frac{\nu B}{\Phi_0}, \; C = \nu \;\; \Rightarrow \;\; \frac{2\pi\hbar^2 n}{m|C|} = \hbar\omega_c$$

- optical absorption occurs only at $\hbar\omega_c$ for circularly polarized light of one handedness (Kohn's theorem)
- optical gap $E_g = \hbar\omega_c$ saturates the bound.

Gap bound on Chern insulators is tight!

Chern Insulators





Twisted semiconductor bilayers:

$$H_{\uparrow} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + V_1(r) & t(r) \\ t^{\dagger}(r) & \frac{\hbar^2 k^2}{2m} + V_2(r) \end{pmatrix}$$

$$= \frac{p^2}{2m} + V(\boldsymbol{r}) + \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{\sigma}$$

$$H_{\downarrow} = H_{\uparrow}^{*}$$

Wu et al (2019)

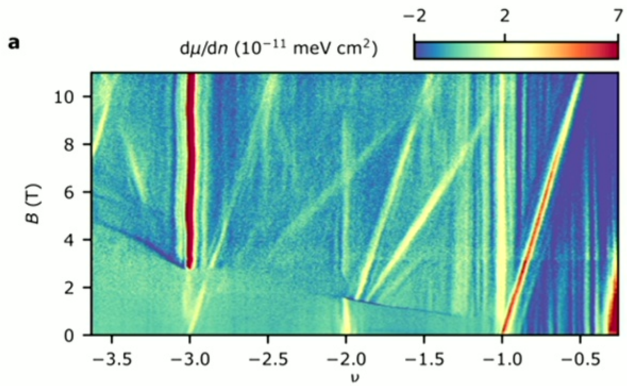
- time-reversed pair of spin ↑and ↓ Chern bands (Kane-Mele)
- theory of ferromagnetism & (fractional) Chern insulator at B=0

Devakul, Crepel, Zhang & LF (2021), Crepel & LF (2022)
Li, Kumar, Sun & Lin (2021)

Integer & Fractional Chern Insulators

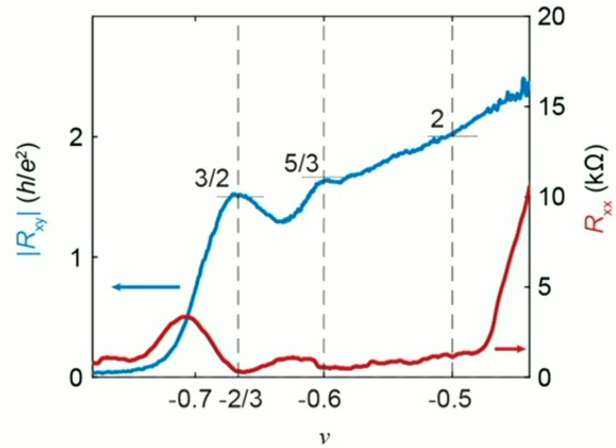
twisted bilayer WSe_2

$\theta = 1.23°$



Feldman et al, Science (2024)

twisted bilayer $MoTe_2$



Park … Xu, Nature (2023)

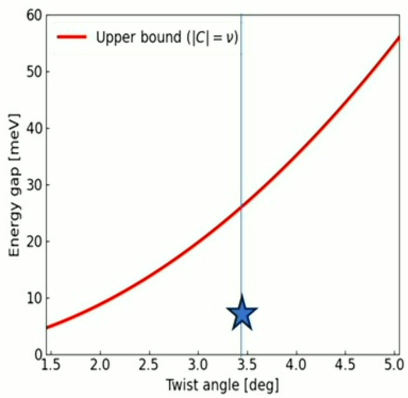
Xu … Li, PRX (2023)

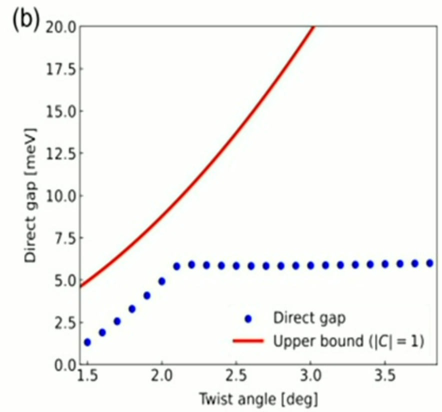
Pentalayer graphene/hBN: Long Ju et al, Nature (2024)

Chern Insulators

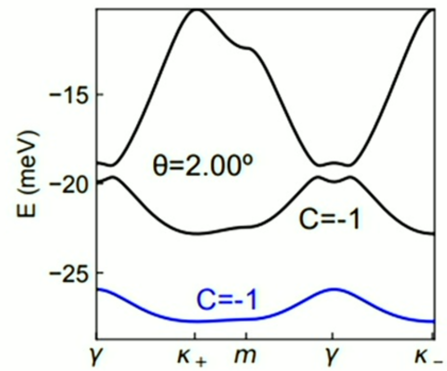
$$\text{Gap bound } \frac{2\pi\hbar^2 n}{m|C|} = \frac{2\pi\hbar^2}{mA_\theta} \text{ for } C = \nu \text{ (moire filling)}$$

MoTe$_2$: $m = 0.62m_0$





magic-angle
moire band



- Direct gap $\approx$ 60% of gap bound at magic angle $\theta \approx 2.1°$.
- Towards high-temperature QAH: large angle & small effective mass.

From Optical Conductivity to Structure Factor

$$\frac{1}{4}C \leq \int_0^\infty d\omega \, \frac{\text{Re}\,\sigma_{xx}}{\omega} \leq \frac{\int_0^\infty d\omega \text{Re}\,\sigma_{xx}}{E_g} = \frac{\pi n}{2mE_g}$$

Continuity equation $\frac{d\rho}{dt} + \nabla \cdot j = 0$
relates conductivity & density response: $\Pi(\boldsymbol{q}, \omega) = i\frac{q_\alpha q_\beta \sigma_{\alpha\beta}(\boldsymbol{q}, \omega)}{\omega}$

Fluctuation-dissipation theorem: $\text{Im}\,\Pi(\boldsymbol{q}, \omega) = S(\boldsymbol{q}, \omega)/(2\hbar)$

A new sum rule relating optical conductivity to static structure factor:

$$S(q) = \frac{\hbar}{\pi} q_\alpha q_\beta \int_0^\infty d\omega \, \frac{\text{Re}\,\sigma_{\alpha\beta}}{\omega}, \; q \to 0$$

with $S(q) = \langle \rho_q \rho_{-q} \rangle$

For gapped systems with short-range interaction,

$$S(q \to 0) = \frac{1}{4\pi} q^2 K + \cdots$$

"quantum weight"

Onishi & LF, arXiv:2401.13847

Topological Bound on Optical Gap

$$E_g \leq \frac{2\pi\hbar^2 n}{m|C|}$$

derived from

- non-negative dissipation
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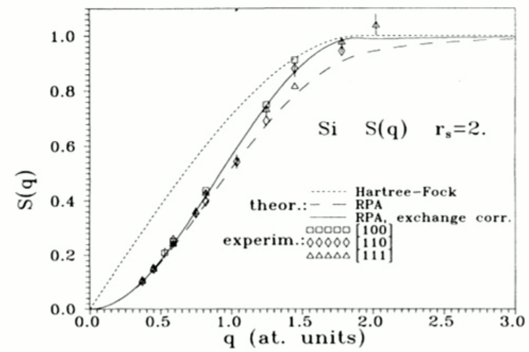
applicable to interacting and fractional Chern insulators

Quantum Weight

describes long-wavelength density fluctuations



In classical limit, electrons are point particles located at lattice sites, leading to $S(q) = 0$ for any $q \neq G$

Quantum weight arises from quantum fluctuation in electron position and vanishes as $\hbar \to 0$.

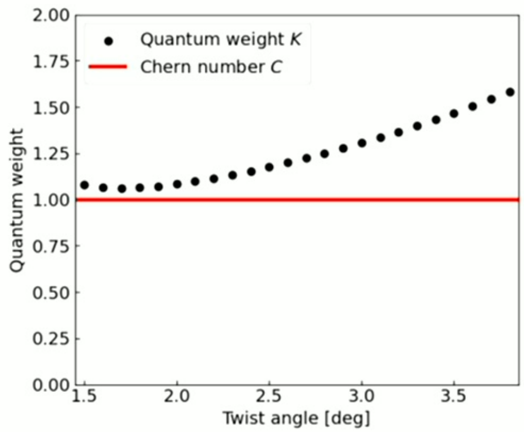
$S(q)$ can be measured by inelastic x-ray scattering.

Topological Bound on Structure Factor

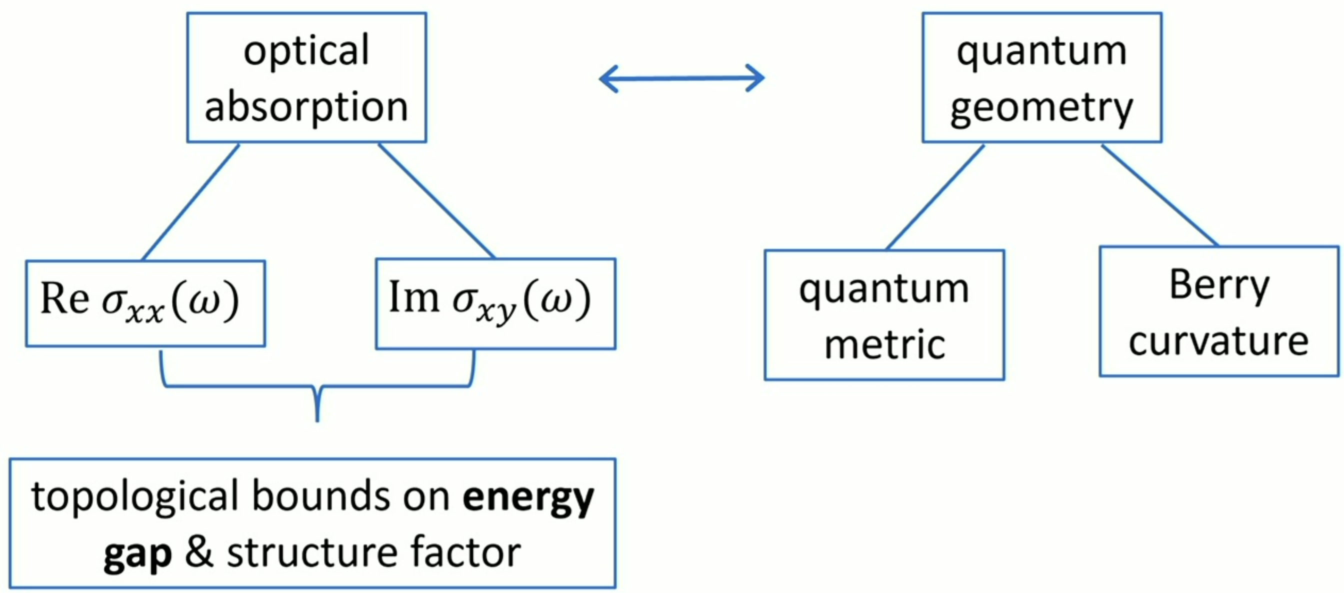
$$\int_0^\infty d\omega \, \frac{\mathrm{Re}\, \sigma_{xx}}{\omega} \geq \frac{C}{4} \;\; \Rightarrow \;\; K \geq C$$

Landau level systems saturate this bound: $S_{\boldsymbol{q}}^{\mathrm{LL}} = \nu q^2/(4\pi) + \ldots.$

Chern insulators in twisted semiconductors:



Relation to Quantum Geometry



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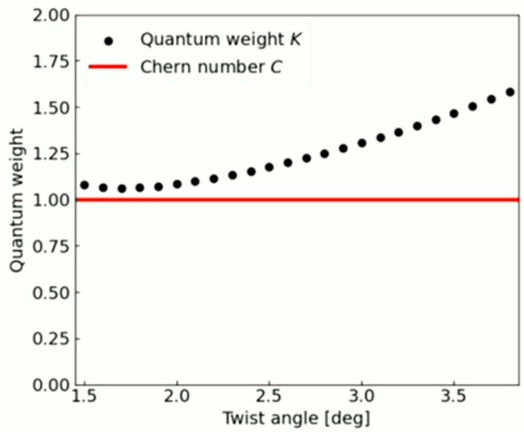
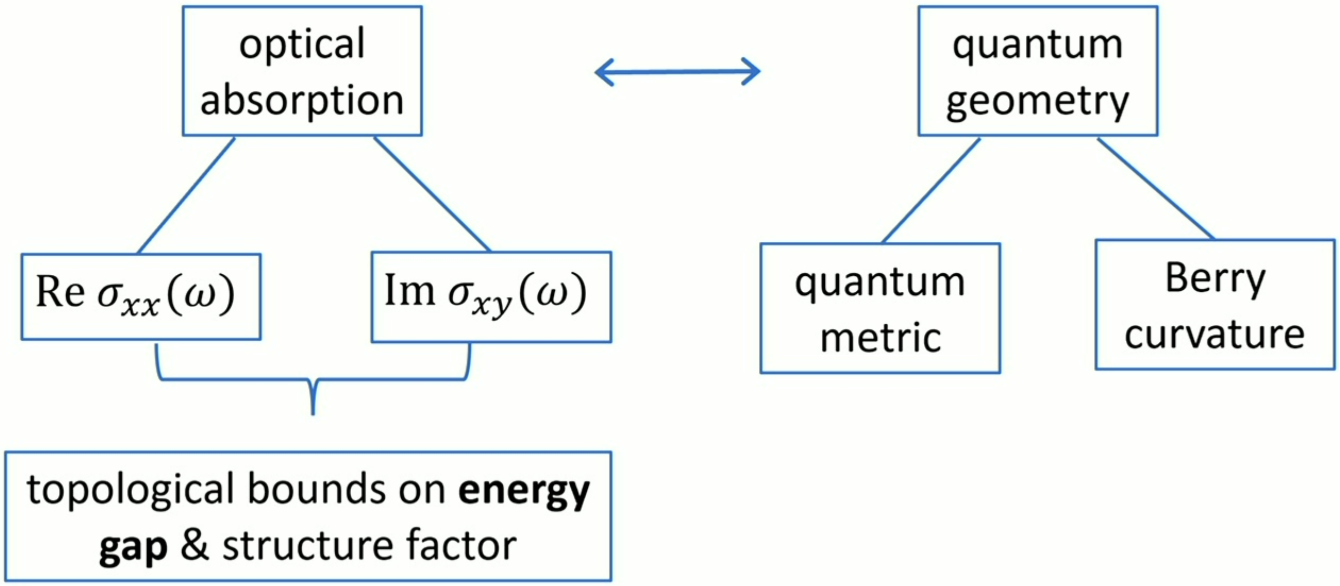
Chern insulators in twisted semiconductors:



Relation to Quantum Geometry

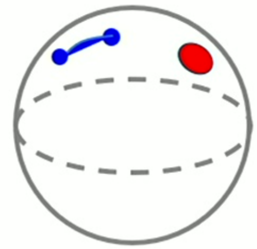


Quantum Geometry and Optical Absorption

$$\boldsymbol{Q} \equiv \langle \partial_i u_{\boldsymbol{k}} | (1 - P_{\boldsymbol{k}}) | \partial_j u_{\boldsymbol{k}} \rangle = \boldsymbol{g} + i\boldsymbol{\Omega}$$

quantum metric   Berry curvature

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{\boldsymbol{L}} + \sigma^H$$



$$\frac{4\hbar}{e^2}\int_0^\infty d\omega \, \frac{\mathrm{Re}\,\sigma_{xx}}{\omega} = 2\pi \int [d\boldsymbol{k}] \, \mathrm{Tr}\, g$$

Souza, Wilkins & Martin, PRB (2000)

$$\frac{4\hbar}{e^2}\int_0^\infty d\omega \, \frac{\mathrm{Im}\,\sigma_{xy}}{\omega} = 2\pi \int [d\boldsymbol{k}] \, \boldsymbol{\Omega} = C$$

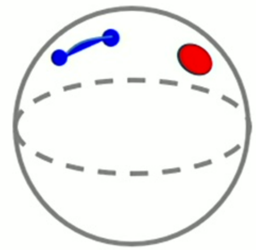
Chern number

Kramers-Kronig+ TKNN

For band insulators, integral of quantum metric over BZ is equal to q^2 coefficient of structure factor.

Quantum Geometry and Optical Absorption

Mathematical inequality:



$$\int [d\boldsymbol{k}]\, \mathrm{Tr}\, g \geq C$$

Roy (2014), Peotta & Thorma (2015) …

In Chern bands, the spread of electron Wannier function must be at least on the order of lattice constant.

Many-Body Quantum Metric

For interacting systems, quantum geometry can be defined for the many-body ground state manifold over twisted boundary condition.

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 30 April 1982)

The Hall conductance of a two-dimensional electron gas has been studied in a uniform magnetic field and a periodic substrate potential U. The Kubo formula is written in a form that makes apparent the quantization when the Fermi energy lies in a gap. Explicit expressions have been obtained for the Hall conductance for both large and small $U/\hbar\omega_c$.

Quantized Hall conductance as a topological invariant

Qian Niu, D. J. Thouless,* and Yong-Shi Wu†
Department of Physics FM-15, University of Washington, Seattle, Washington 98195
(Received 21 September 1984)

Whenever the Fermi level lies in a gap (or mobility gap) the bulk Hall conductance can be expressed in a topologically invariant form showing the quantization explicitly. The new formulation generalizes the earlier result by Thouless, Kohmoto, Nightingale, and den Nijs to the situation where many-body interaction and substrate disorder are also present. When applying to the fractional quantized Hall effect, we draw the conclusion that there must be a symmetry breaking in the many-body ground state. The possibility of writing the fractionally quantized Hall conductance as a topological invariant is also discussed.

$$G_{\alpha\beta} \equiv \mathrm{Re} \left\langle \partial_{\theta_\alpha} \Psi_{\boldsymbol{\theta}} \middle| (1 - P_{\boldsymbol{\theta}}) \middle| \partial_{\theta_\beta} \Psi_{\boldsymbol{\theta}} \right\rangle_{\boldsymbol{\theta}=0}$$

Quantum Metric and Quantum Weight

SWM sum rule: $$\int_0^{\infty} \mathrm{d}\omega \, \frac{\mathrm{Re}\, \sigma_{\alpha\alpha}(\omega)}{\omega} = \frac{\pi e^2}{\hbar} G_{\alpha\alpha}.$$

Our sum rule: $$\int_0^{\infty} \mathrm{d}\omega \, \frac{1}{\omega} \, \mathrm{Re} \left[\frac{\sigma_{\alpha\alpha}(\omega)}{\epsilon_{\alpha\alpha}(\omega)} \right] = \frac{e^2}{2\hbar} K_{\alpha\alpha}.$$

For gapped systems with short-range interactions or in reduced dimensions:

$$K = 2\pi G, \text{ when } \epsilon(\omega) = 1.$$

This relation allows the many-body quantum metric to be obtained from a single ground state wavefunction!

Generalized Optical Weights

$$W^n \equiv \int_0^\infty d\omega \, \frac{\mathrm{Re}\,\sigma_{xx}}{\omega^n} \leq \frac{1}{E_g} W^{n-1}$$

Inequalities: $E_g \leq \frac{W^1}{W^2} \leq \sqrt{\frac{W^0}{W^2}} \leq \frac{W^0}{W^1}$

- saturated if and only if absorption occurs at a single frequency

W^0: spectral weight

W^1: quantum weight
(lower bounded by Chern number)

W^2 ?

Energy Gap and Polarizability

W^2 is directly related to thermodynamic polarizability

$$\sigma(\omega) = \frac{j}{E} = \frac{\dot{P}}{E} = -i\omega\chi(\omega)$$

χ: polarizability

From Kramers-Kronig relation,

$$W^2 \equiv \int_0^\infty d\omega \, \frac{\mathrm{Re}\,\sigma_{xx}}{\omega^2} = \int_0^\infty d\omega \, \frac{\mathrm{Im}\,\chi_{xx}}{\omega} = \frac{\pi}{2}\chi(0)$$

thermodynamic response

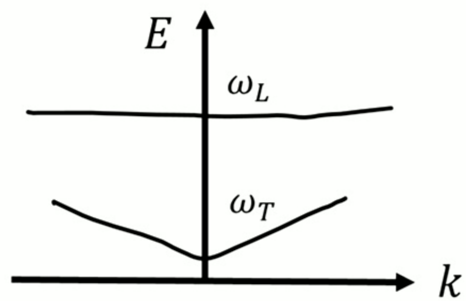
$$E_g \le \sqrt{\frac{W^0}{W^2}} = \hbar\sqrt{\frac{ne^2}{m\chi(0)}} = \frac{\hbar\omega_p}{\sqrt{\chi(0)}}$$

ω_p: plasma frequency at electron density n

Onishi & LF, arXiv:2401.04180

Long-Range Interaction

3D charged systems with long-range Coulomb interaction have two types of long-wavelength excitations that couple to longitudinal and transverse electric field respectively, with different excitation energies.



For 3D Wigner crystal in a periodic potential:

$$\omega_L = \sqrt{\omega_p^2 + \omega_0^2}, \qquad \omega_T = \omega_0.$$

Longitudinal and Transverse Gap Bounds

$$\frac{1}{\epsilon_0}\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\mathrm{Re}\,\sigma^T(\boldsymbol{q}\to 0,\omega)}{\omega^2} = \int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\mathrm{Im}\,\epsilon(\omega)}{\omega}$$
$$= \pi(\epsilon - 1)$$

$$\frac{1}{\epsilon_0}\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\mathrm{Re}\,\sigma^L(\boldsymbol{q}\to 0,\omega)}{\omega^2} = -\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\mathrm{Im}\,\epsilon^{-1}(\omega)}{\omega}$$
$$= \pi(1 - \epsilon^{-1})$$

$$E_g^T(q\to 0) \le \frac{\hbar\omega_p}{\sqrt{\epsilon - 1}}$$

$$E_g^L(q\to 0) \le \frac{\hbar\omega_p}{\sqrt{1 - \epsilon^{-1}}}$$

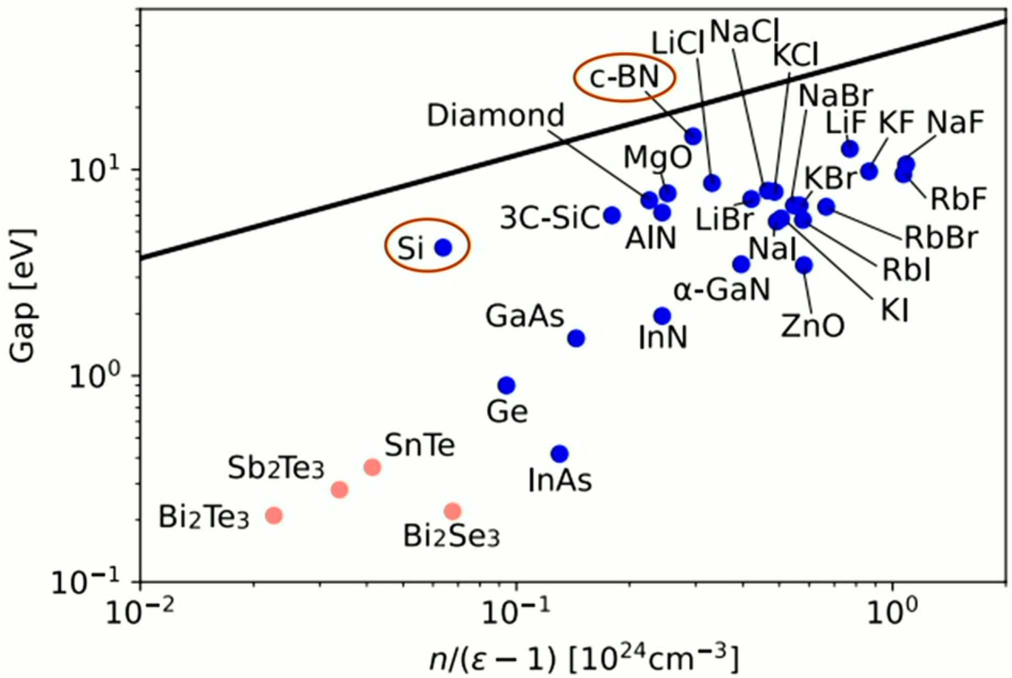
optical gap = transverse gap

Highly polarizable electronic materials have small transverse gap.

Onishi & LF, arXiv:2401.04180

Material	$\epsilon(\infty)$	n [m^{-3}]	E_g [eV]	$E_g^{\max}$ [eV]	$E_g/E_g^{\max}$	Comment
c-BN	4.46 [12]	1.02×10^{30} [9]	14.5 [9]	20.11	0.721	Direct gap
Si	11.97 [9]	7.00×10^{29} [9]	4.185 [9]	9.38	0.446	Direct gap
MgO	2.94 [9]	4.90×10^{29} [9]	7.672 [9]	18.65	0.411	Excitonic gap
Diamond	5.70 [9]	1.06×10^{30} [9]	7.1 [13]	17.62	0.403	Direct gap
LiCl	2.78 [14]	5.89×10^{29} [15]	8.6 [16]	21.36	0.403	E_g is obtained from the lowest absorption peak
LiF	1.96 [14]	7.39×10^{29} [15]	12.6 [17]	32.58	0.387	Excitonic gap
3C-SiC	6.38 [9]	9.65×10^{29} [9]	6.0 [9]	15.73	0.381	Direct gap
AlN	4.93 [9]	9.58×10^{29} [9]	6.19 [9]	18.34	0.338	Direct gap. $\epsilon(\infty) = \epsilon_{\parallel}(\infty)$
NaCl	2.34 [14]	6.24×10^{29} [18]	7.9 [16]	25.34	0.312	E_g is obtained from the lowest absorption peak
KCl	2.19 [14]	5.78×10^{29} [18]	7.8 [16]	25.87	0.301	E_g is obtained from the lowest absorption peak
LiBr	3.17 [14]	9.14×10^{29} [15]	7.2 [16]	24.09	0.299	E_g is obtained from the lowest absorption peak
KF	1.85 [14]	7.36×10^{29} [15]	9.8 [16]	34.54	0.284	E_g is obtained from the lowest absorption peak
NaF	1.74 [14]	8.04×10^{29} [18]	10.6 [16]	38.70	0.274	E_g is obtained from the lowest absorption peak
RbF	1.96 [14]	1.03×10^{30} [15]	9.5 [16]	38.38	0.248	E_g is obtained from the lowest absorption peak
NaBr	2.59 [14]	8.69×10^{29} [15]	6.7 [16]	27.45	0.244	E_g is obtained from the lowest absorption peak
KBr	2.34 [14]	7.58×10^{29} [15]	6.7 [16]	27.93	0.240	E_g is obtained from the lowest absorption peak
KI	2.62 [14]	8.18×10^{29} [15]	5.8 [16]	26.39	0.220	Optical absorption edge
RbBr	2.34 [14]	8.92×10^{29} [15]	6.6 [16]	30.30	0.218	E_g is obtained from the lowest absorption peak
NaI	2.93 [14]	9.50×10^{29} [15]	5.6 [16]	26.05	0.215	E_g is obtained from the lowest absorption peak
RbI	2.59 [14]	9.18×10^{29} [15]	5.7 [16]	28.21	0.202	E_g is obtained from the lowest absorption peak
α-GaN	5.20 [9]	1.66×10^{30} [9]	3.475 [9]	23.36	0.149	E_g is for A-exciton. $\epsilon(\infty) = \epsilon_{\perp}(\infty)$.
ZnO	3.75 [9]	1.60×10^{30} [9]	3.441 [9]	28.30	0.122	$\epsilon(\infty) = \epsilon_{\parallel}(\infty)$.
GaAs	10.86 [9]	1.42×10^{30} [9]	1.52 [9]	14.08	0.108	Direct gap
InN	8.40 [9]	1.80×10^{30} [9]	1.95 [9]	18.34	0.106	Direct gap
Ge	16.00 [9]	1.41×10^{30} [9]	0.898 [9]	11.40	0.079	Direct gap
SnTe	40 [19, 20]	1.61×10^{30} [9]	0.36 [9]	7.55	0.048	$\epsilon(\infty)$ varies among Ref. [19, 20] around 40-50.
Sb_2Te_3	51.00 [9]	1.69×10^{30} [9]	0.28 [9]	6.83	0.041	$\epsilon(\infty) = \epsilon_{\perp}(\infty)$.
InAs	12.37 [9]	1.48×10^{30} [9]	0.418 [9]	13.37	0.031	Direct gap
Bi_2Te_3	85.00 [9]	1.90×10^{30} [9]	0.21 [21]	5.58	0.023	$\epsilon(\infty) = \epsilon_{\perp}(\infty)$.
Bi_2Se_3	29.00 [9]	1.89×10^{30} [9]	0.22 [22]	9.65	0.017	$\epsilon(\infty) = \epsilon_{\perp}(\infty)$.

Optical Gap and Dielectric Constant



Onishi & LF, arXiv:2401.04180

Quantum Geometry & Energy Gap

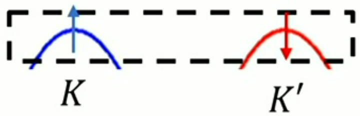
- gap and dielectric constant
  electron-phonon effect, ferroelectric soft mode
- gap and topology
  T-invariant topological insulators
- better bound on optical spectral weight

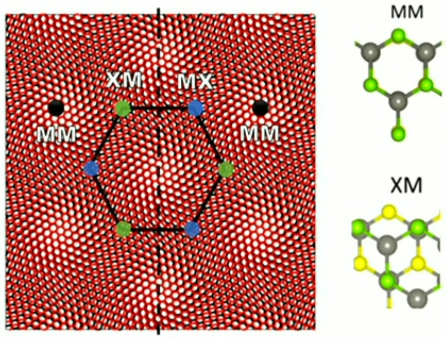
Topological bound on quantum weight: analog of minimal surface

Energy and length are related by quantum mechanics
=> Bound on energy gap by quantum geometry and topology

Bound on energy gap by thermodynamic response

Chern Insulators





Twisted semiconductor bilayers:

$$H_{\uparrow} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + V_1(r) & t(r) \\ t^{\dagger}(r) & \frac{\hbar^2 k^2}{2m} + V_2(r) \end{pmatrix}$$

$$= \frac{p^2}{2m} + V(\boldsymbol{r}) + \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{\sigma}$$

$$H_{\downarrow} = H_{\uparrow}^{*}$$

Wu et al (2019)

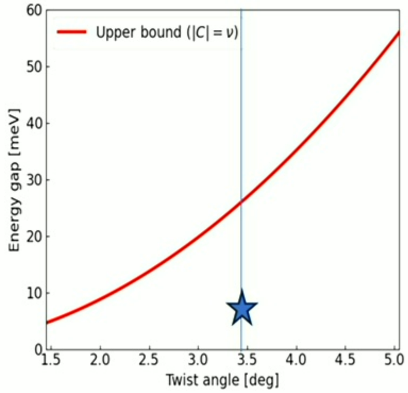
- time-reversed pair of spin ↑and ↓ Chern bands (Kane-Mele)
- theory of ferromagnetism & (fractional) Chern insulator at B=0

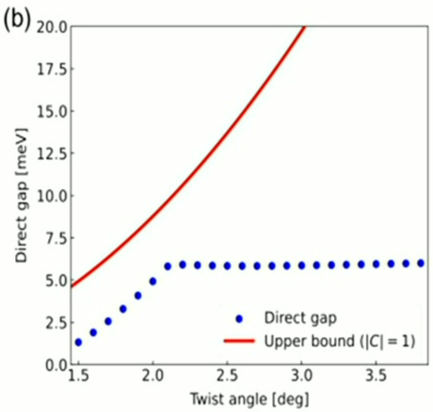
Devakul, Crepel, Zhang & LF (2021), Crepel & LF (2022)
Li, Kumar, Sun & Lin (2021)

Chern Insulators

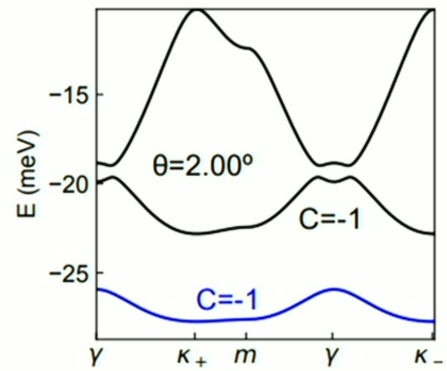
Gap bound $\frac{2\pi\hbar^2 n}{m|C|} = \frac{2\pi\hbar^2}{mA_\theta}$ for $C = \nu$ (moire filling)

$MoTe_2$: $m = 0.62m_0$



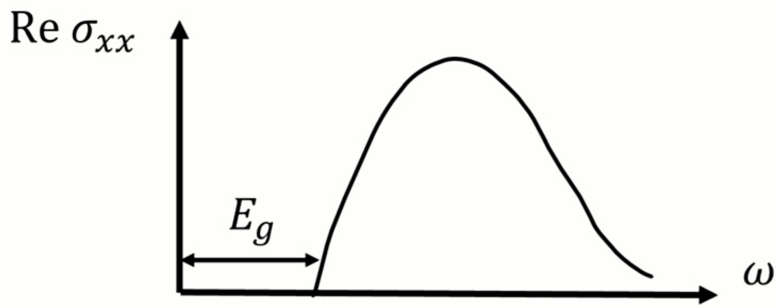


magic-angle
moire band



- Direct gap ≈ 60% of gap bound at magic angle $\theta \approx 2.1^\circ$.
- Towards high-temperature QAH: large angle & small effective mass.

Optical Conductivity and Optical Gap



- optical spectral weight: $W^0 \equiv \int_0^\infty d\omega \, \mathrm{Re}\, \sigma_{xx} = \frac{\pi n}{2m}$
- negative first moment: $W^1 \equiv \int_0^\infty d\omega \, \frac{\mathrm{Re}\, \sigma_{xx}}{\omega}$
- inequality: $W^1 \leq \frac{W^0}{E_g}$

Negative moments are more dominated by low-energy excitations.

Mind the Gap

How large can topological gap be ?