

Title: The rise and fall of mixed-state entanglement: measurement, feedback, and decoherence

Speakers: Peter Lu

Collection: Physics of Quantum Information

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URL: <https://pirsa.org/24050030>

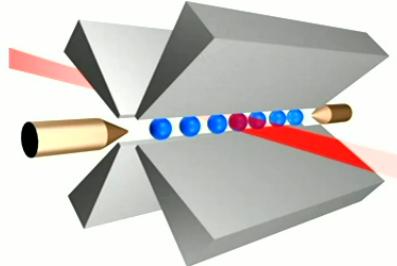
Abstract: Long-range entangled mixed states are exotic many-body systems that exhibit intrinsically quantum phenomena despite extensive classical fluctuations. In the first part of the talk, I will show how they can be efficiently prepared with measurements and unitary feedback conditioned on the measurement outcome. For example, symmetry-protected topological phases can be universally converted into mixed states with long-range entanglement, and certain gapped topological states such as Chern insulators can be converted into mixed states with critical correlations in the bulk. In the second part of the talk, I will discuss how decoherence can drive interesting mixed-state entanglement transitions. By focusing on the toric codes in various space dimensions subject to certain types of decoherence, I will present the exact results of entanglement negativity, from which the universality class of entanglement transitions can be completely characterized.

The rise and fall of **mixed-state long-range entanglement** measurement, feedback, and decoherence

Tsung-Cheng Peter Lu

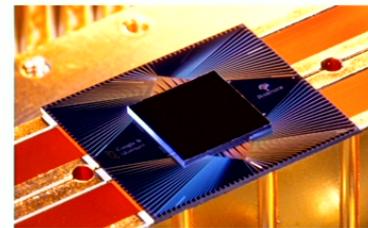


New possibilities via quantum devices



Trapped-ion quantum computer

E.g. Duke/IonQ
Quantinuum



Superconducting qubits

E.g. Google
IBM

What should we do with them?

New possibilities via quantum devices

Condensed matter
physicists



Realize interesting states of matter!

Traditional approach



Herbertsmithite
(A spin liquid candidate)

New (bottom-up) approach

$$|\psi\rangle = U|0\rangle^{\otimes N}$$

Realizing the target state
with quantum operations

New possibilities via quantum devices

Condensed matter
physicists



Realize interesting states of matter!

Quantum information
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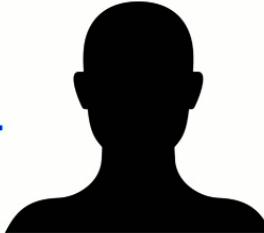


Perform quantum computing!

Quantum computing can beat classical computing

New possibilities via quantum devices

Condensed matter
physicists



Realize interesting states of matter!

Quantum information
physicists



Perform quantum computing!

Quantum computing can beat classical computing

**(Mixed-state)
Long-range entangled quantum matter**

Outline

Part I: State preparation

i.e. how to efficiently create long-range entanglement?

arXiv: 2303.15507 (PRX Quantum), joint work with



Zhehao Zhang
(UCSB)



Sagar Vijay
(UCSB)



Tim Hsieh
(Perimeter)

Part II: Stability of quantum matter subject to decoherence

i.e. how does decoherence kill long-range entanglement?

arXiv:2404.06514, TCL

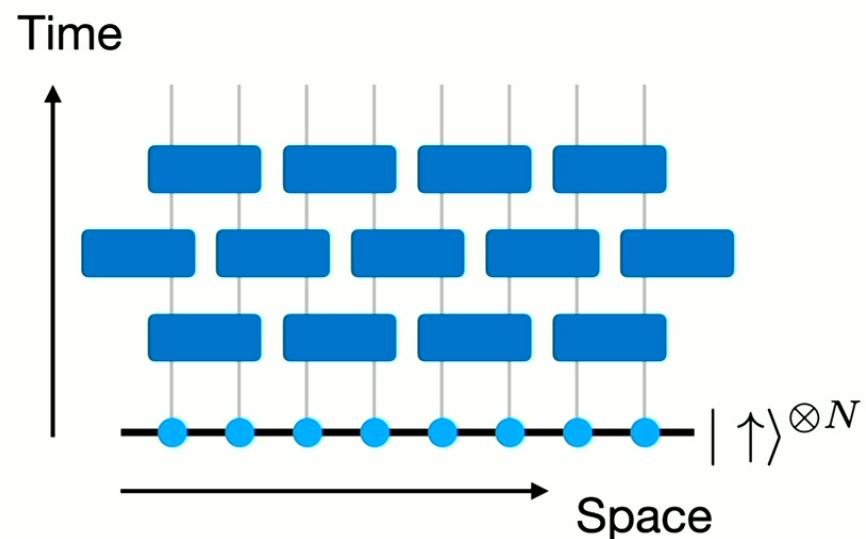
State preparation - bottleneck of quantum devices

Local unitary circuits

Realizing LRE requires time growing with the system size

[Lieb, Robinson (1972)] [Bravyi, Hastings, Verstraete (2006)]

Not scalable since qubits decohere quickly

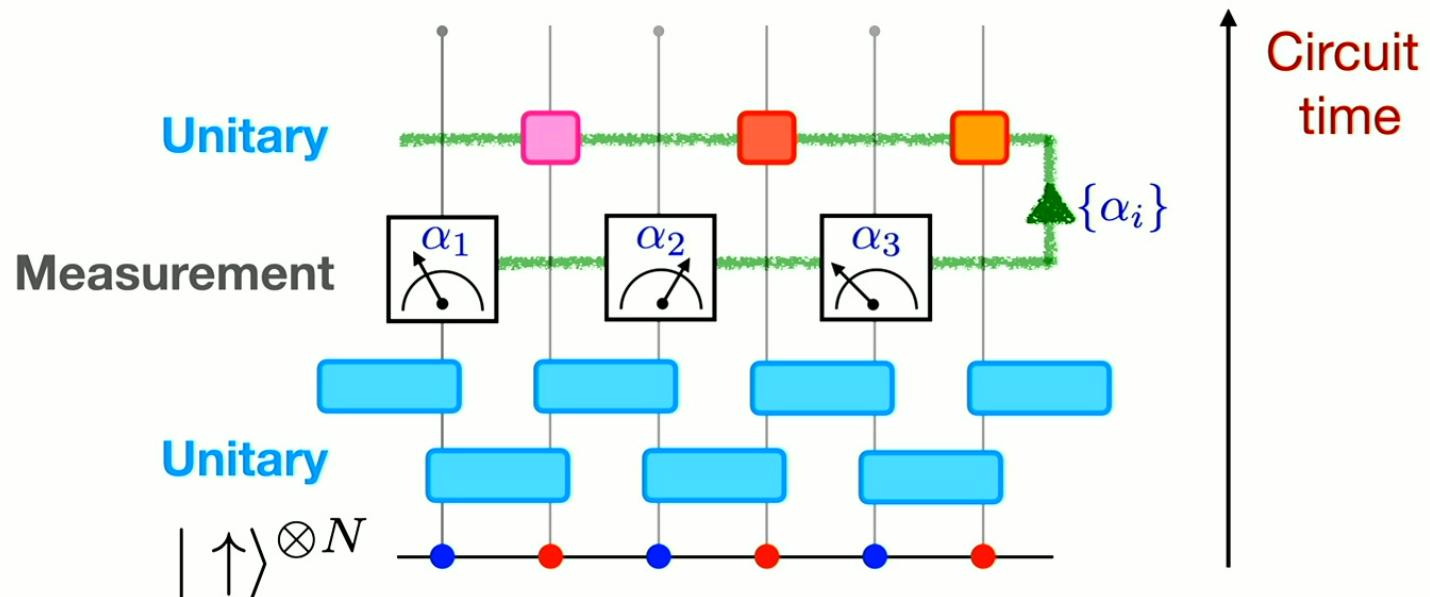


Measurement - LRE possible in $O(1)$ time!

e.g. topological orders

Measurement-based approach

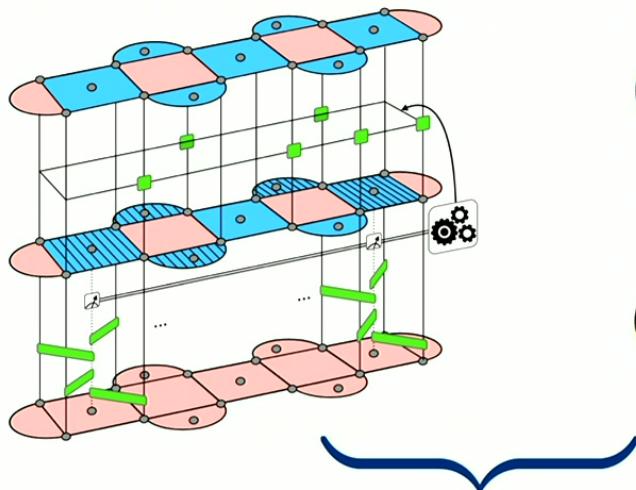
Local quantum operation & **Non-local classical** communication
(unitary gates & measurement)



Briegel, Raussendorf (2001); Raussendorf, Bravyi, Harrington (2005); Piroli, Styliaris, Cirac (2021);
Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021) ; Bravyi, Kim, Kliesch, Koenig (2022); TCL, Lessa, Kim, Hsieh (2022),

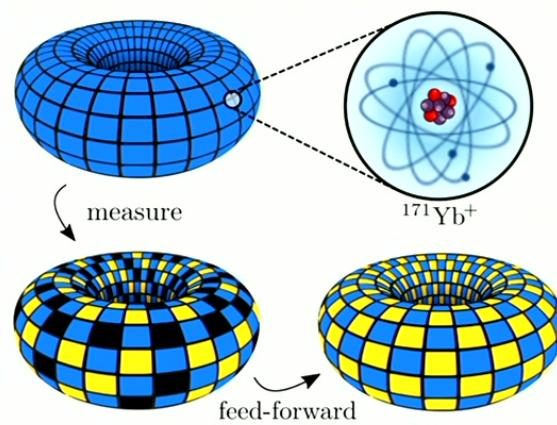
Experimental realization

Quantinuum, UC Davis, Perimeter
arXiv:2302.03029 (2023)



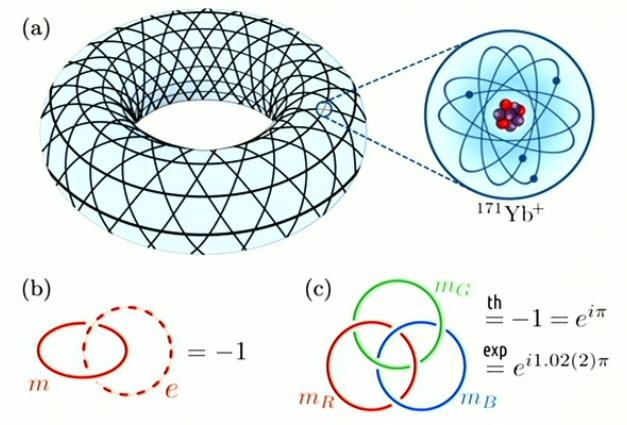
Abelian topological orders

Quantinuum, Caltech, Harvard
arXiv:2302.01917 (2023)



Non-abelian topological orders

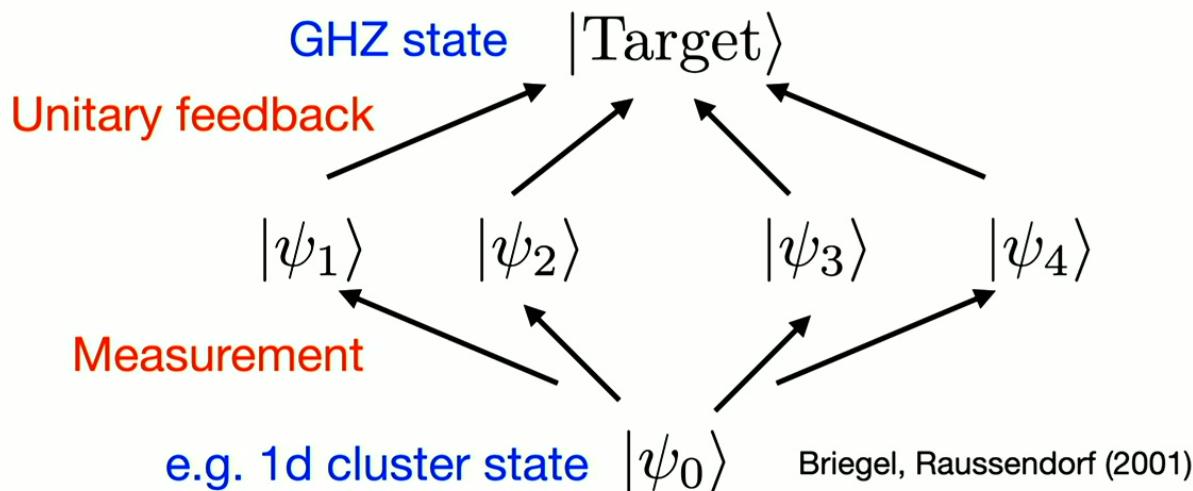
Quantinuum, Caltech, Harvard,
arXiv:2305.03766 (Nature 2023)



New directions with measurement

- What LRE states can be efficiently prepared via measurement & feedback
- Are the measurement-feedback protocols stable?
(e.g. imperfect unitary or measurement)

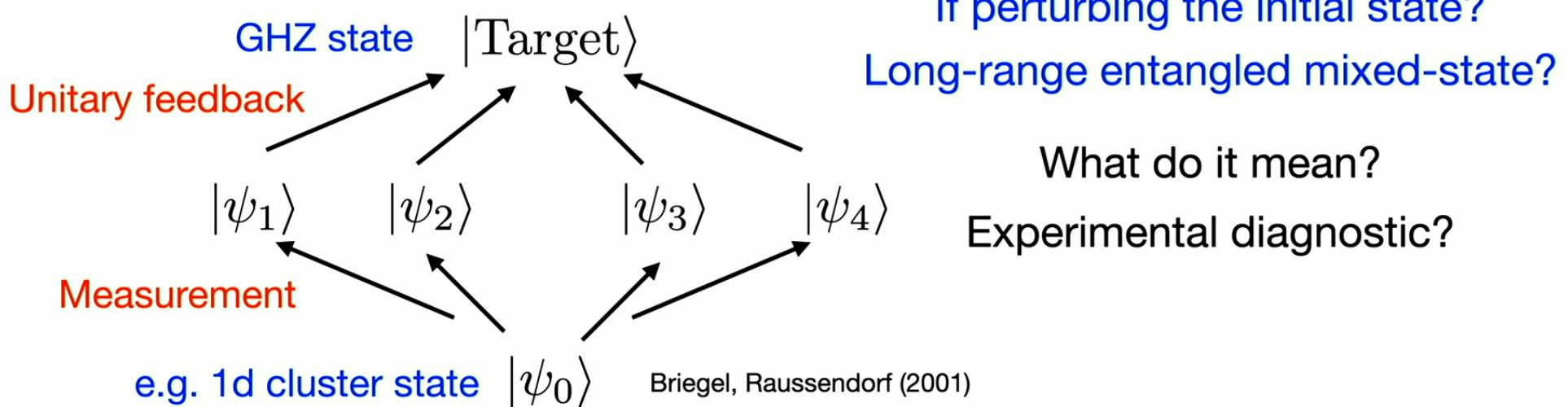
Motivating example



New directions with measurement

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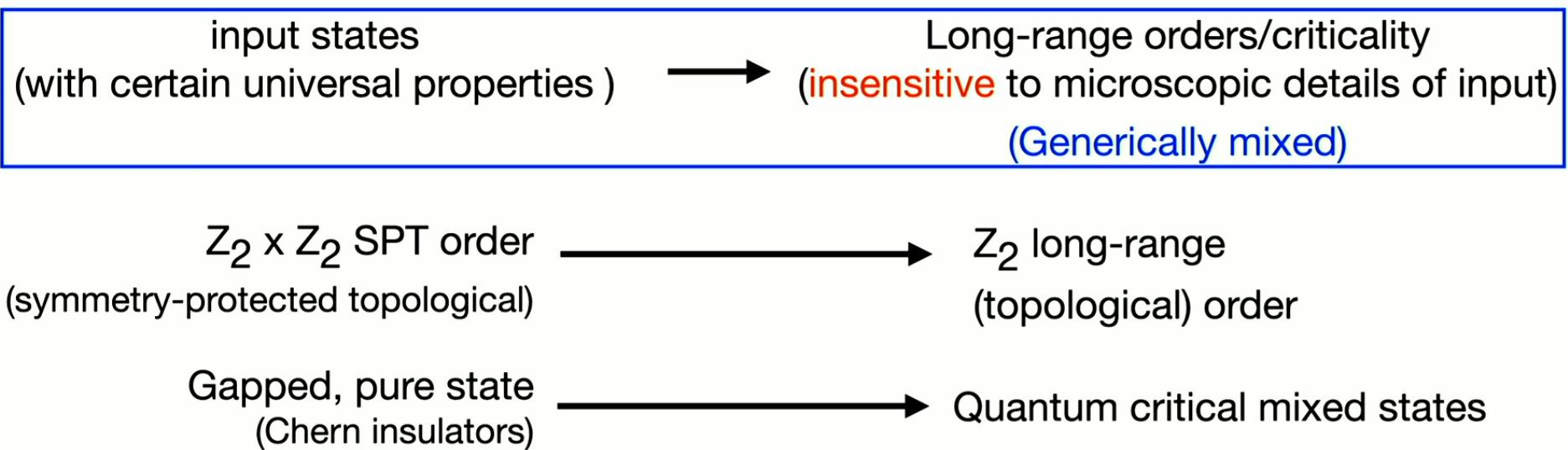
Motivating example



(Deterministic) non-local quantum channels

via measurement + feedback

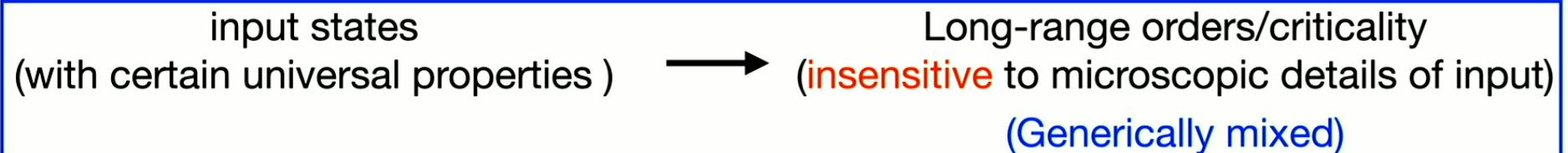
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(Deterministic) non-local quantum channels

via measurement + feedback

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Quan. Info.

Robust protocols for generating long-range entanglement

Quan. Matter

Non-trivial mixed-state matter in $O(1)$ depth
In contrast, thermal Gibbs (mixed) states are generically trivial
(Exception - 4d toric code)

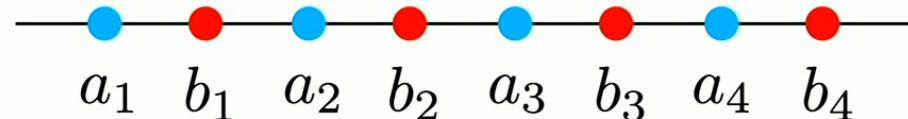
Application with SPT

(Symmetry Protected Topological)

Inspired by Briegel, Raussendorf (2001);
Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021)

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2$$

Cluster-state SPT

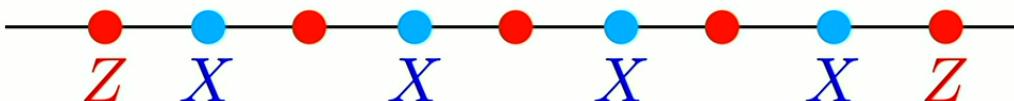


$$H = - \sum_i Z_{a,i} X_{b,i} Z_{a,i+1} - \sum_i Z_{b,i-1} X_{a,i} Z_{b,i}$$

Symmetry $\prod_i X_{a,i} \quad \prod_i X_{b,i}$

String order
See e.g. Pollmann, Turner (2012)

$$\langle Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \rangle = 1$$



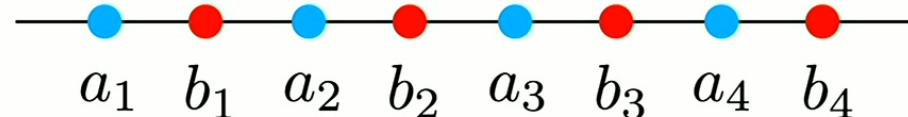
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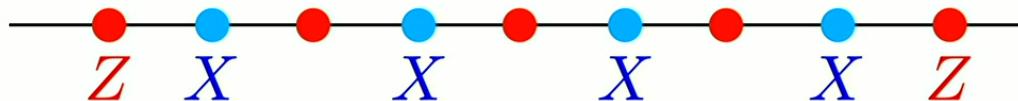


$$H = - \sum_i Z_{a,i} X_{b,i} Z_{a,i+1} - \sum_i Z_{b,i-1} X_{a,i} Z_{b,i} \quad \text{Symmetry} \quad \prod_i X_{a,i} \quad \prod_i X_{b,i}$$

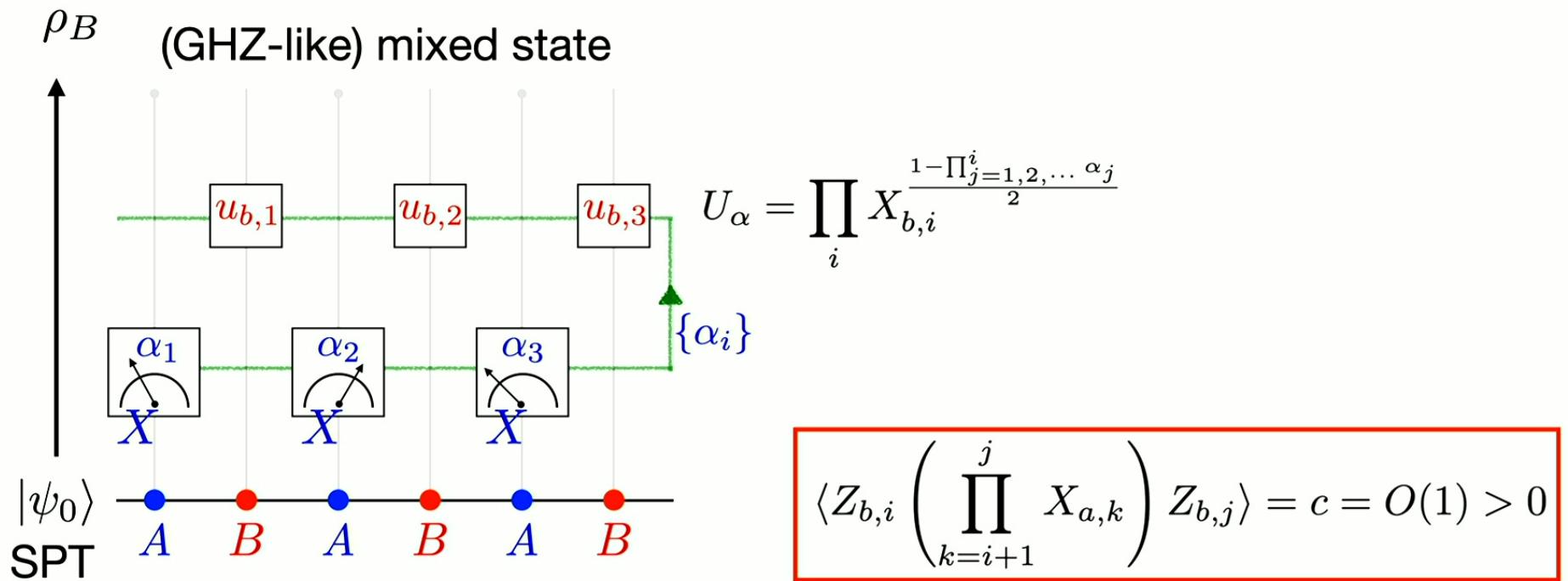
$$-g \sum_i O_i \quad \text{Symmetry-preserving perturbation}$$

String order
See e.g. Pollmann, Turner (2012)

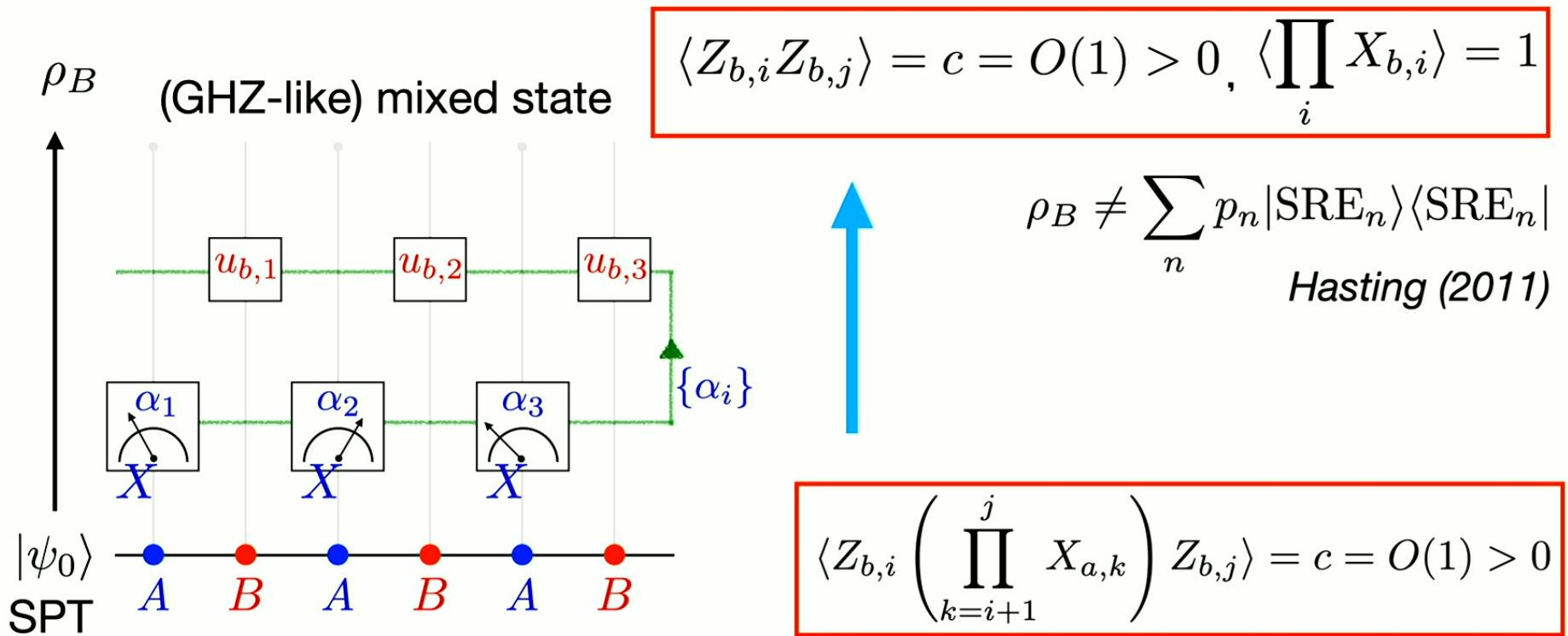
$$\langle Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \rangle = c = O(1)$$



SPT to mixed-state long-rang entanglement

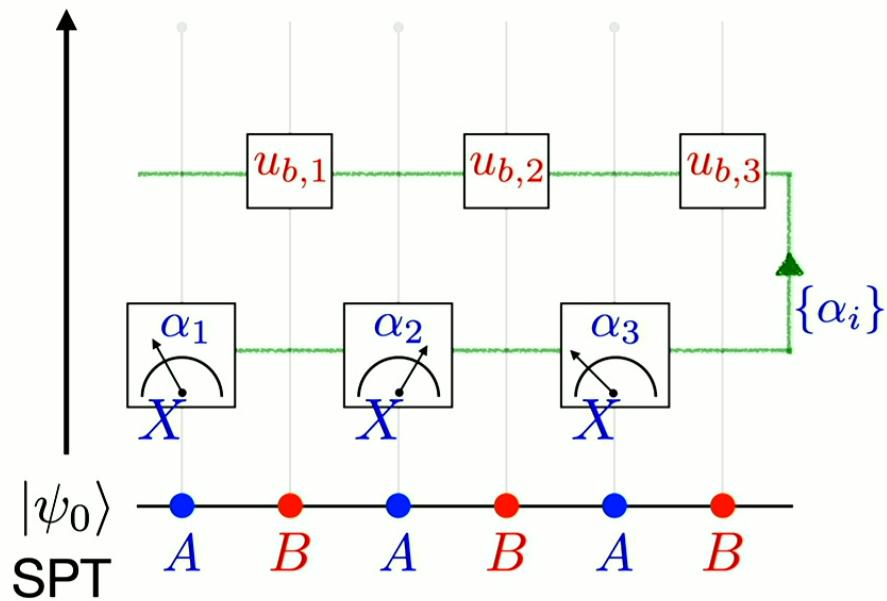


SPT to mixed-state long-rang entanglement



SPT to mixed-state long-rang entanglement

$$\rho_B = \text{tr}_A (|\psi\rangle\langle\psi|)$$



$$|\psi\rangle = U|\psi_0\rangle$$

$$U = \sum_{\alpha} (U_{\alpha})_B \otimes (|\alpha\rangle\langle\alpha|)_A$$

$$U_{\alpha} = \prod_i X_{b,i}^{\frac{1-\prod_{j=1,2,\dots,\alpha_j}^i}{2}}$$

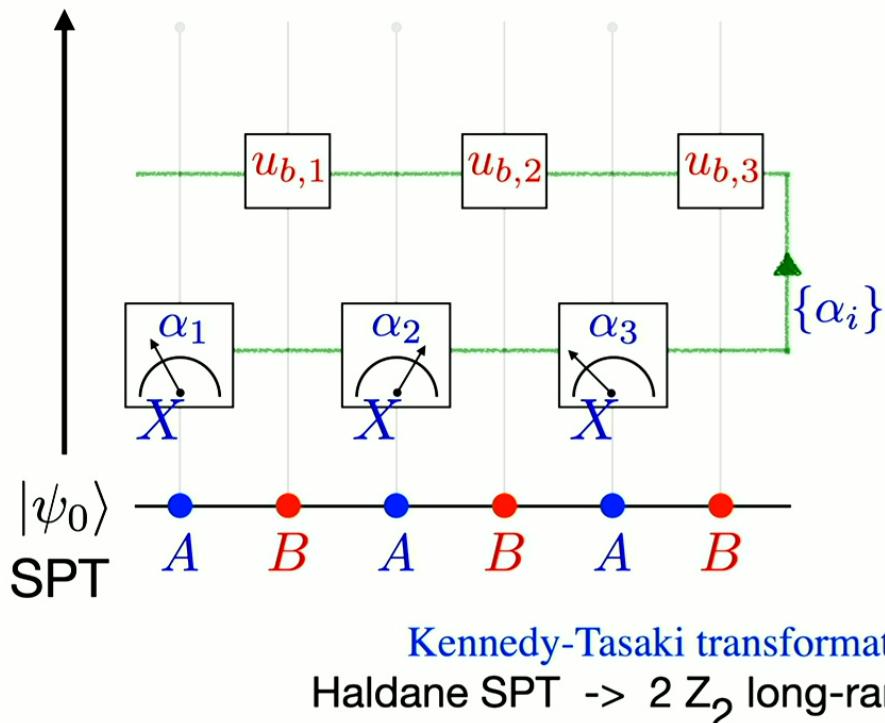
A controlled unitary
implementing non-local transformation

$$U \left[Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \right] U^\dagger = Z_{b,i} Z_{b,j}$$

Hidden string order -> long-range order

SPT to mixed-state long-rang entanglement

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$$|\psi\rangle = U|\psi_0\rangle$$

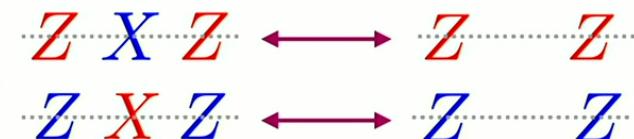
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Hidden string order → long-range order



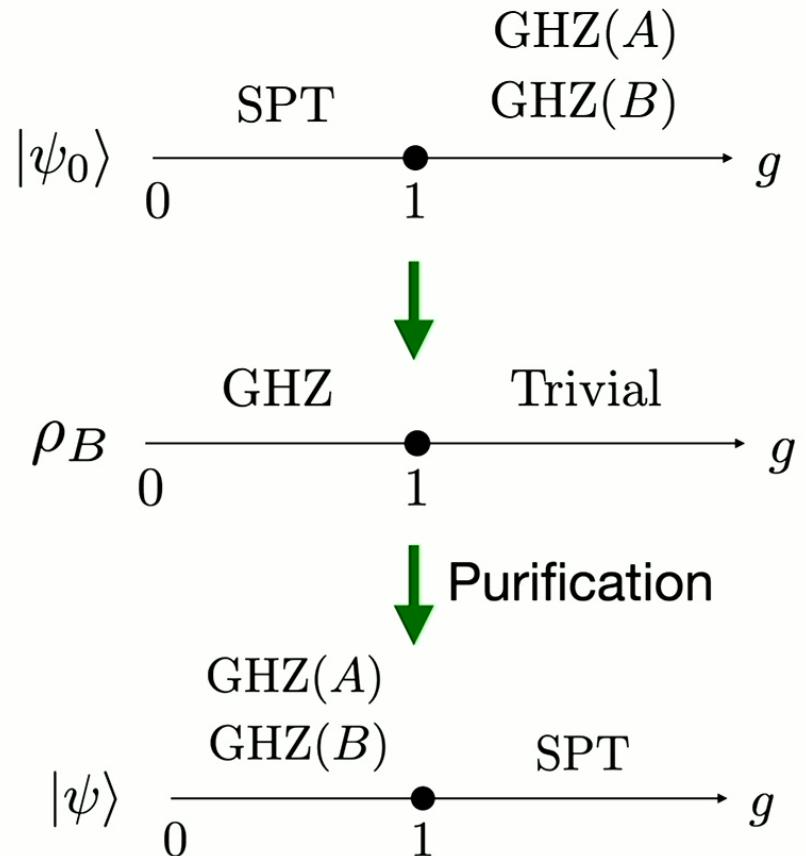
Example

$$H_0 = - \sum \textcolor{blue}{Z} \textcolor{red}{X} \textcolor{blue}{Z} - \sum \textcolor{red}{Z} \textcolor{blue}{X} \textcolor{red}{Z} \\ -g \sum \textcolor{blue}{Z} \textcolor{blue}{Z} - g \sum \textcolor{red}{Z} \textcolor{red}{Z}$$

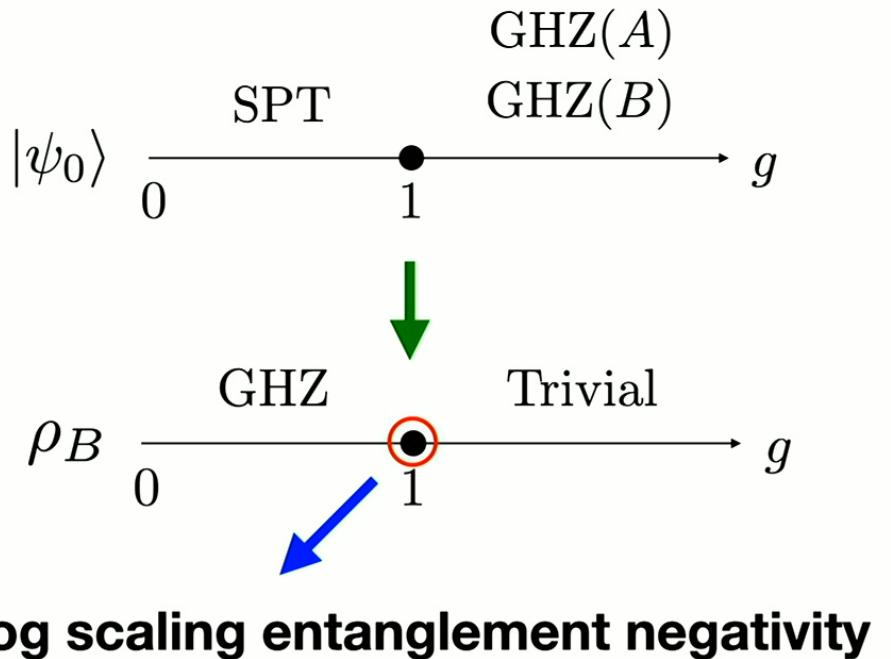
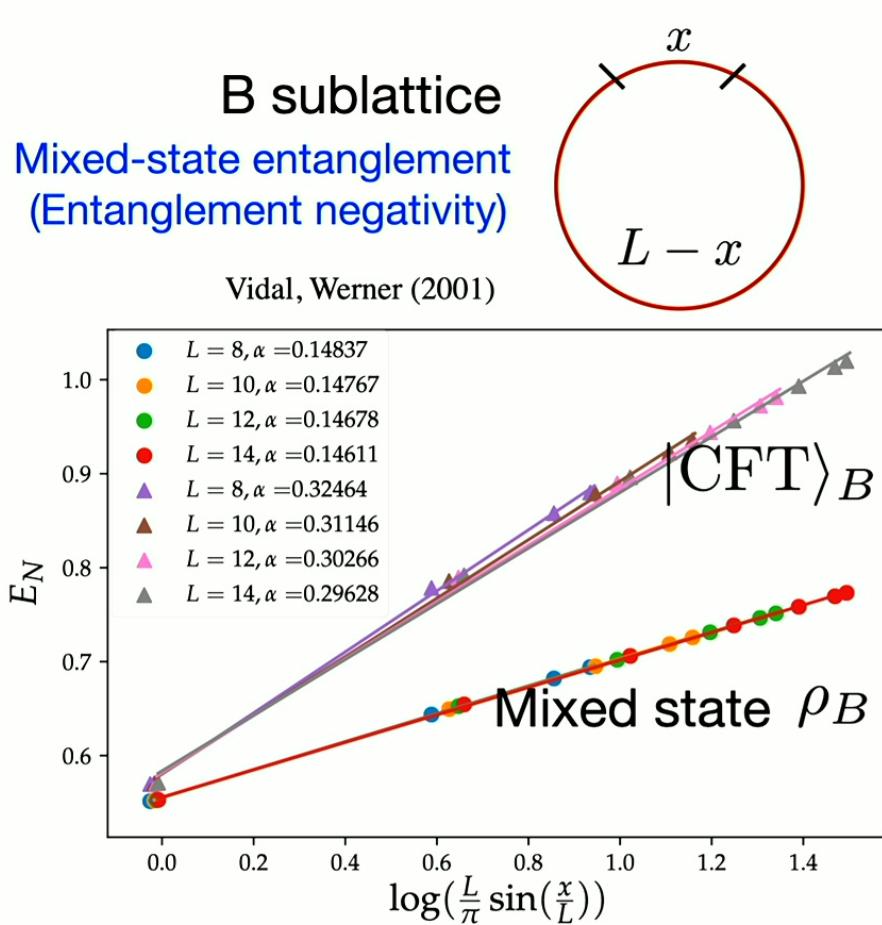
Controlled unitary

$$U = \sum_{\alpha} U_{\alpha} P_{\alpha}$$

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Mixed-state quantum criticality

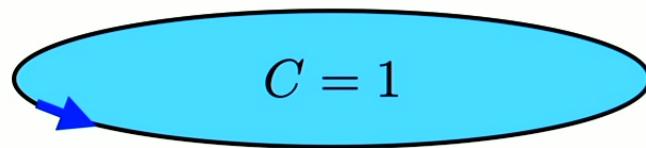


in contrast, area-law negativity in Gibbs state of local Hamiltonians

Sherman et.al. (2016)

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators



Chiral current

Trivial bulk correlation

$$\langle \psi_0 | c(\mathbf{x}) c^\dagger(\mathbf{x}') | \psi_0 \rangle \sim e^{-\frac{|\mathbf{x}-\mathbf{x}'|}{\xi}}$$

Fermion correlation

$|\psi_0\rangle$

Off-diagonal long-range order

Girvin, MacDonald (1987);
Kvorning, Spånslått, Chan, Ryu (2019)

$$b^\dagger(\mathbf{x}) = \eta(\mathbf{x}) c^\dagger(\mathbf{x})$$

boson

Fermion

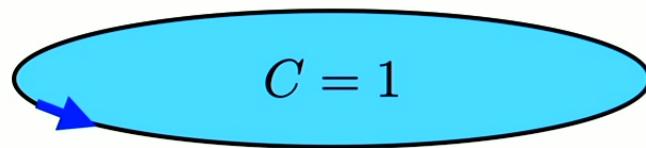
$$\eta(\mathbf{x}) = e^{i \int d^2 x' \hat{n}(\mathbf{x}') \arg(\mathbf{x}-\mathbf{x}')}$$

Flux attachment

$$\langle \psi_0 | b(\mathbf{x}) b^\dagger(\mathbf{x}') | \psi_0 \rangle \sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^\alpha}$$

Mixed-state quantum criticality from gapped states of matter

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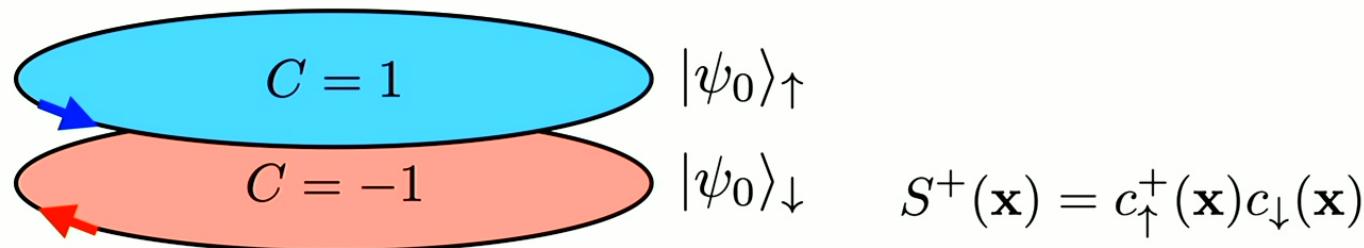
Flux attachment

$$\langle \psi_0 | b(\mathbf{x}) b^\dagger(\mathbf{x}') | \psi_0 \rangle \sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^\alpha}$$

Flux attachment via measurement + feedback !

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators $|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$



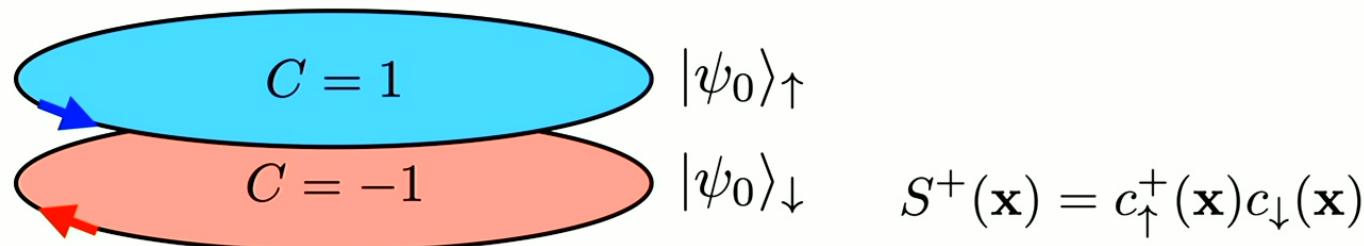
$$|\psi_0\rangle \quad \left\langle e^{i[\hat{\phi}_\uparrow(\mathbf{x}) - \hat{\phi}_\downarrow(\mathbf{x})]} S^+(\mathbf{x}) S^-(\mathbf{x}') e^{-i[\hat{\phi}_\uparrow(\mathbf{x}') - \hat{\phi}_\downarrow(\mathbf{x}')]}\right\rangle_0 \sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^{2\alpha}}$$

$$\hat{\phi}_\uparrow(\mathbf{x}) = \int d^2x' \hat{n}_\uparrow(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')$$

$$\hat{\phi}_\downarrow(\mathbf{x}) = - \int d^2x' \hat{n}_\downarrow(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')$$

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators $|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$



$$|\psi_0\rangle \quad \left\langle e^{i[\hat{\phi}_\uparrow(\mathbf{x})-\hat{\phi}_\downarrow(\mathbf{x})]} S^+(\mathbf{x}) S^-(\mathbf{x}') e^{-i[\hat{\phi}_\uparrow(\mathbf{x}')-\hat{\phi}_\downarrow(\mathbf{x}')]}\right\rangle_0 \sim \frac{1}{|\mathbf{x}-\mathbf{x}'|^{2\alpha}}$$

$$\downarrow \qquad \qquad \parallel$$

$$\rho$$

$$\langle S^+(\mathbf{x}) S^-(\mathbf{x}') \rangle_\rho$$

Measuring fermion numbers
+ feedback unitary

Occupation number measurement
via quantum gas microscope

e.g. review by Gross, Bakr
Nature Phys (2021)

Non-local channel using measurements

arXiv: 2303.15507 (PRX Quantum), **TCL**, Zhang, Vijay, Hsieh

Long-range entangled mixed-states

Kennedy-Tasaki transformation	SPT -> long-range (topological) order (generalized to higher dimension and more symmetries)
Flux attachment	Gapped states -> critical states

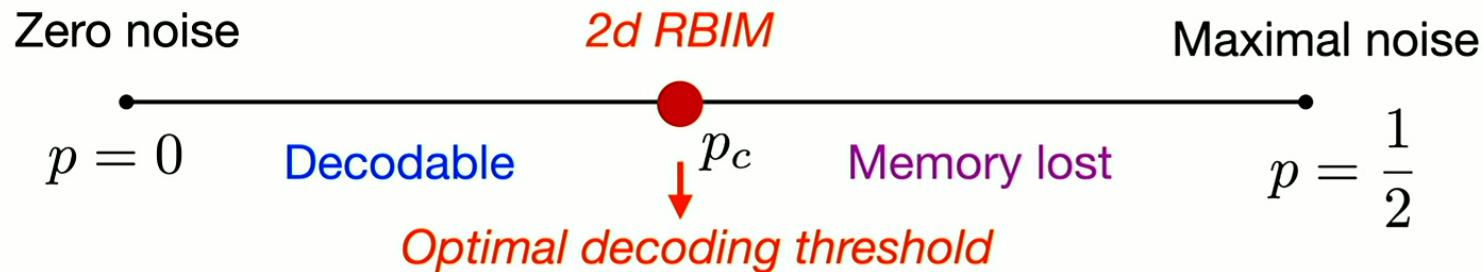
Classification and entanglement structures remains to be better understood

See Ralph Jason Costales's poster for partial progress

Part II: Stability of LRE states

arXiv:2404.06514, **TCL**

Topological codes (e.g. toric code) protect information up to a critical noise rate



Intrinsic property of the noisy mixed state? Fan, Bao, Altman, Vishwanath (2023)

Mixed-state topological order survives up to p_c ?

also, Lee, Jian, Xu (2023)

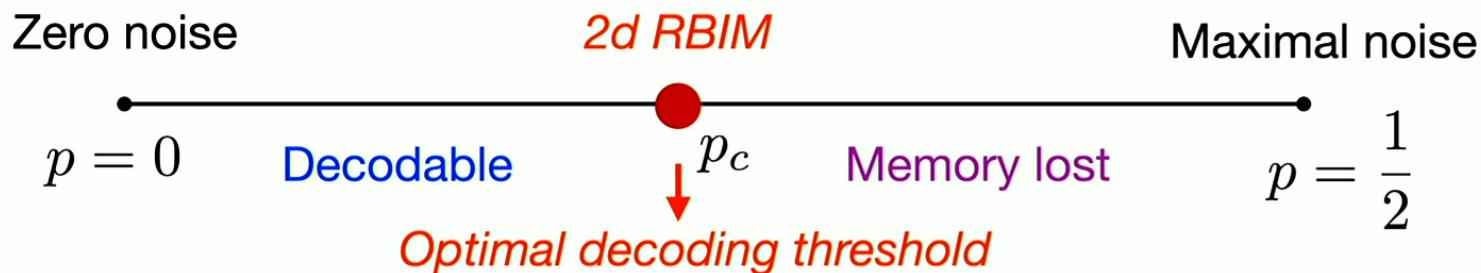
Mixed-state phases of matter?

$\text{Tr}[\rho O]$ smooth function of p

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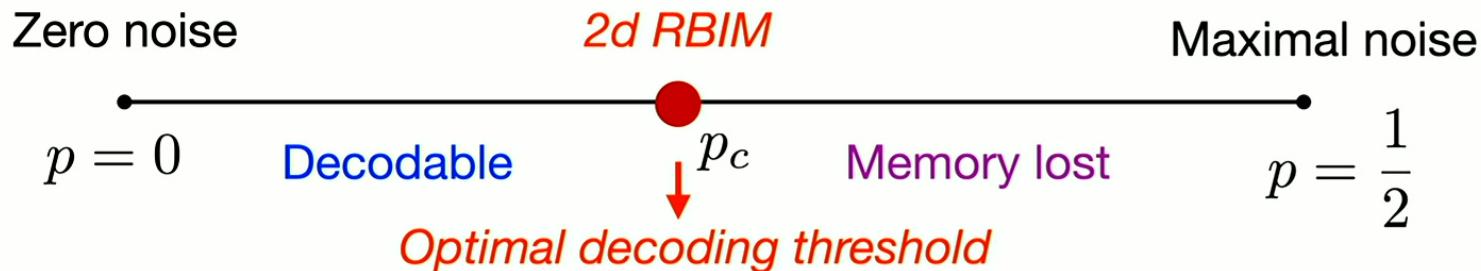
Intrinsic property of the noisy mixed state? Fan, Bao, Altman, Vishwanath (2023)
also, Lee, Jian, Xu (2023)

- Mixed-state phase and channel (Sang, Zou, Hsieh, 2023) (Sang, Hsieh, 2024)
- Separability transition (Chen, Grover, 2023, 2024)

Part II: Stability of LRE states

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Intrinsic property of the noisy mixed state? Fan, Bao, Altman, Vishwanath (2023)
also, Lee, Jian, Xu (2023)

- Direct diagnosis via **mixed-state entanglement** remains missing

Fan et.al. - **Renyi negativity** different critical p with different universality

$$R_{2n} = \frac{1}{2-2n} \log \frac{\text{tr}(\rho^\Gamma)^{2n}}{\text{tr}\rho^{2n}}$$

$$\lim_{n \rightarrow \frac{1}{2}} R_{2n} = E_N = \log |\rho^\Gamma|_1 \quad \text{Entanglement negativity ??}$$

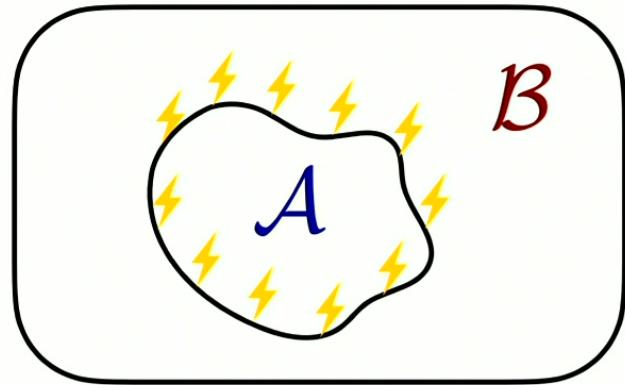
Boundary decoherence

arXiv:2404.06514, **TCL**

Exact results of mixed-state entanglement
& universality of the entanglement transition

Boundary decoherence

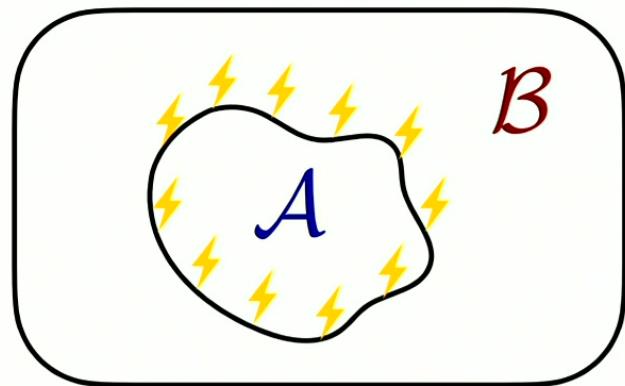
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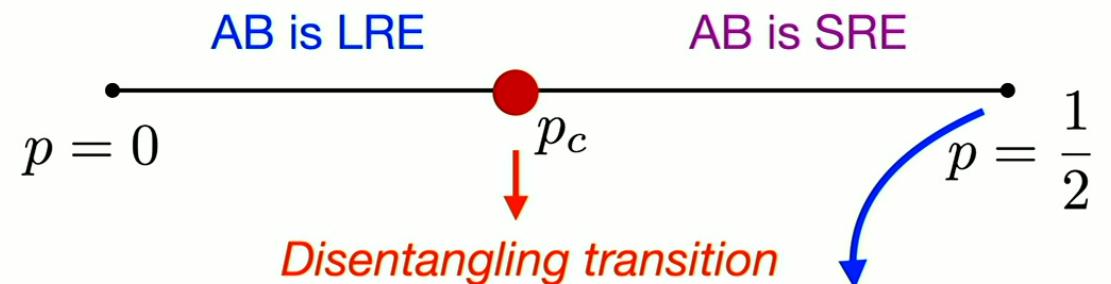
Toric code under boundary decoherence
(Proliferates boundary anyons)

Boundary decoherence

arXiv:2404.06514, **TCL**



Toric code under boundary decoherence
(Proliferates boundary anyons)

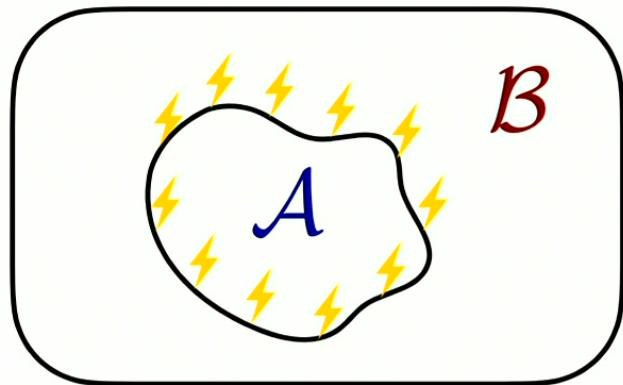


$$\rho = \sum_i p_i \rho_{A,i} \otimes \rho_{B,i}$$

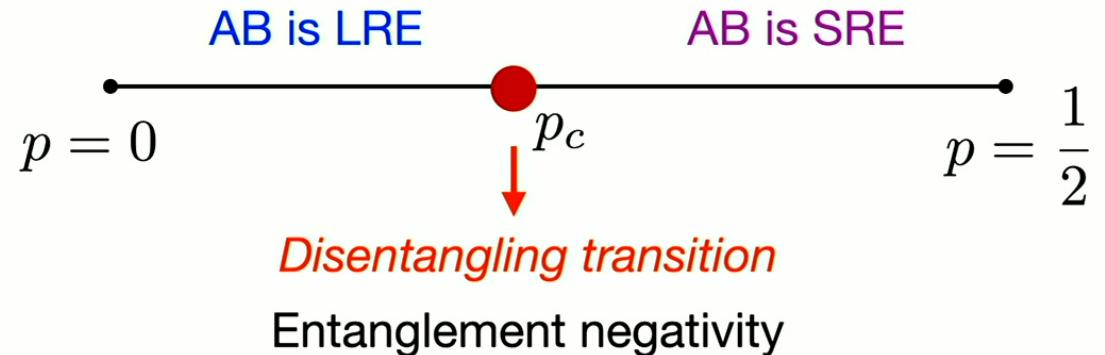
Bulk topo. order in A,B

Boundary decoherence

arXiv:2404.06514, **TCL**



Toric code under boundary decoherence
(Proliferates boundary anyons)



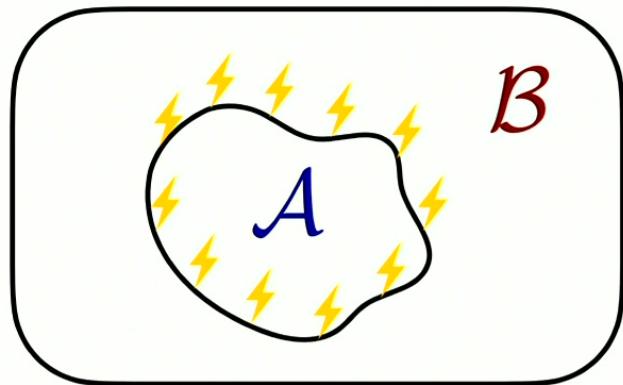
Peres (1996), Horodecki (1996), Vidal, Werner (2001)

$$E_N = \log |\rho^\Gamma|_1 = \log \left(\sum_i |\lambda_i| \right)$$

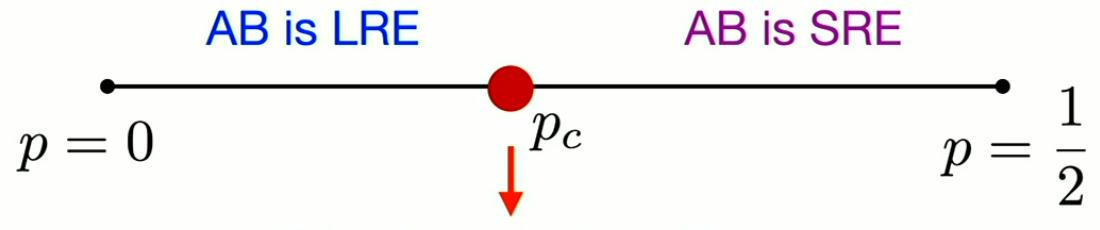
$\{\lambda_i\}$ Negativity spectrum

Boundary decoherence

arXiv:2404.06514, **TCL**



Toric code under boundary decoherence
(Proliferates boundary anyons)



Disentangling transition

Entanglement negativity

Peres (1996), Horodecki (1996), Vidal, Werner (2001)

$$E_N = \alpha L - E_{\text{topo}}$$

Topological negativity: LRE diagnostic

Diagnostic for Finite-T Topo. order

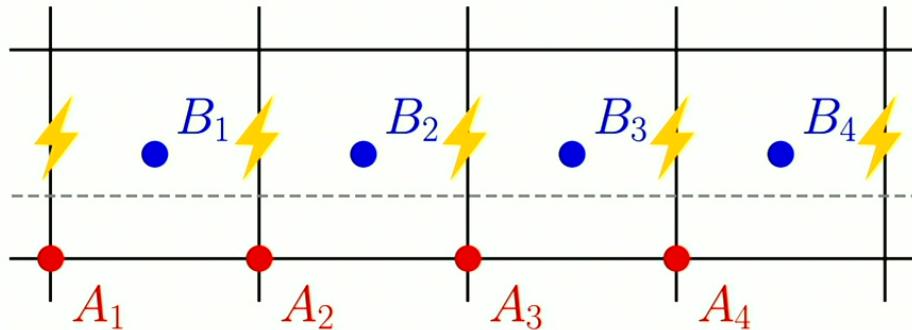
TCL, Hsieh, Grover, PRL (2020)

TCL, Vijay (PRR, 2022)

2d toric code

$$A_s = \begin{array}{|c|c|} \hline & Z \\ \hline Z & Z \\ \hline \end{array}$$

$$B_p = \begin{array}{|c|c|} \hline X & X \\ \hline X & X \\ \hline \end{array}$$



Pauli-Z noise;
Pauli-Z with prob. p

$$E_N = \log Z_{1\text{d}, \text{Ising}} - \log \tilde{Z}_{1\text{d}, \text{Ising}}$$



$$\tanh \beta = \frac{p}{1-p}$$

$$\sum_{\{\sigma_i\}} e^{\beta \sum_{i=1}^L \sigma_i \sigma_{i+1}}$$

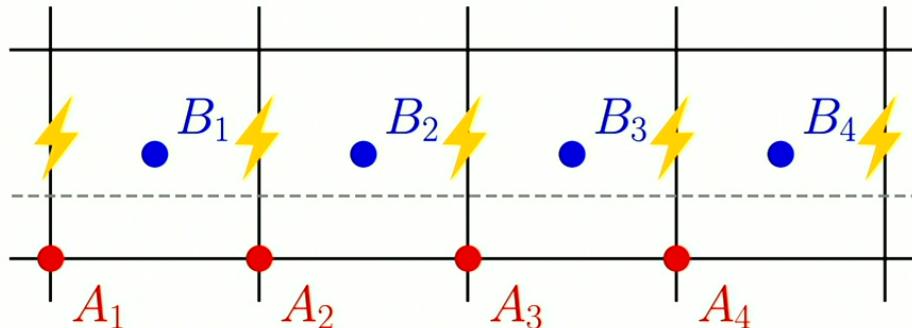
$$\sum_{\{\sigma_i\}} e^{\beta \sum_{i=1}^L \sigma_i \sigma_{i+1}} \delta(\sigma_i \sigma_{i+1} = 1) = 2e^{\beta L}$$



2d toric code

$$A_s = \begin{array}{|c|c|} \hline & Z \\ \hline Z & Z \\ \hline \end{array}$$

$$B_p = \begin{array}{|c|c|} \hline X & X \\ \hline X & X \\ \hline \end{array}$$



Pauli-Z noise;
Pauli-Z with prob. p

$$E_N = \log Z_{1d, \text{Ising}} - \log \tilde{Z}_{1d, \text{Ising}}$$

$$\tanh \beta = \frac{p}{1-p}$$

$$\sum_{\{\sigma_i\}} e^{\beta \sum_{i=1}^L \sigma_i \sigma_{i+1}}$$

$$\sum_{\{\sigma_i\}} e^{\beta \sum_{i=1}^L \sigma_i \sigma_{i+1}} \delta(\sigma_i \sigma_{i+1} = 1) = 2e^{\beta L}$$

$$E_N = \frac{\alpha L}{2} - E_{\text{topo}}$$

$$E_{\text{topo}} = \log \left(\frac{2}{1 + (\frac{p}{1-p})^L} \right)$$

$$E_{\text{topo}} = 0$$

$$E_{\text{topo}} = \log 2$$

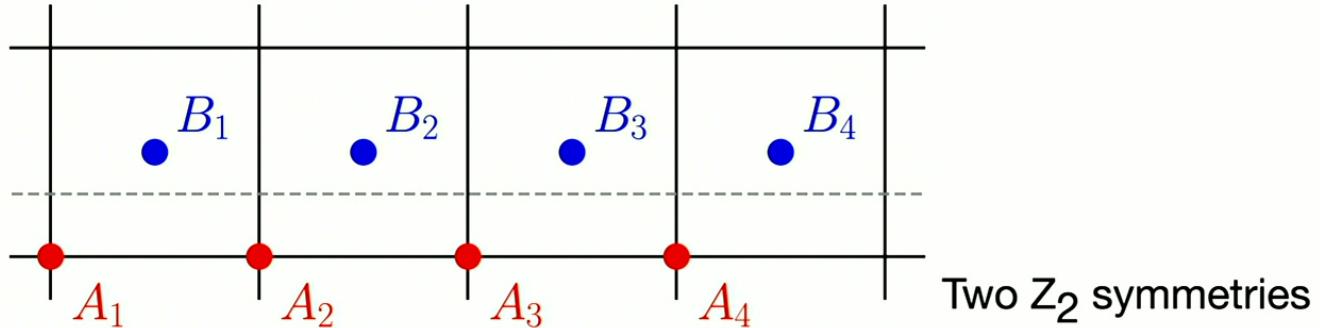
$$\begin{array}{ll} p = 0 & p = \frac{1}{2} \\ \beta = 0 & \beta = \infty \end{array}$$

Negativity spectrum (pure-state toric code)

TCL, Vijay (PRR 2023)

$$A_s = \begin{array}{c} Z \\ \diagdown \\ Z \\ \diagup \\ Z \end{array}$$

$$B_p = \begin{array}{c} X \\ \diagup \\ X \\ \diagdown \\ X \end{array}$$



$$\rho^\Gamma(\{A_i, B_i\}) \propto \chi(\{A_i, B_i\}) = \pm 1$$

$$\boxed{\prod_i A_i = \prod_i B_i = 1}$$

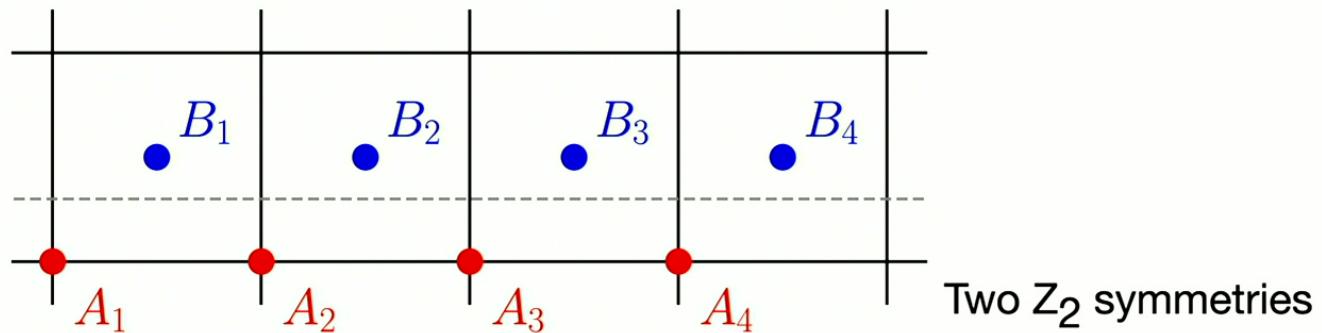
$$A_i = -1 \quad B_i = -1 \\ \text{e anyons} \quad \text{m anyons}$$

Negativity spectrum (pure-state toric code)

TCL, Vijay (PRR 2023)

$$A_s = \begin{array}{c} Z \\ \diagdown \\ Z \\ \diagup \\ Z \end{array}$$

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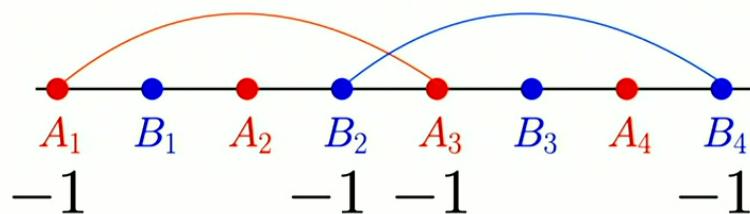


$$\rho^\Gamma(\{A_i, B_i\}) \propto \chi(\{A_i, B_i\}) = \pm 1$$

$$\prod_i A_i = \prod_i B_i = 1$$

$$A_i = -1 \quad B_i = -1 \\ \text{e anyons} \quad \text{m anyons}$$

$$\chi = -1$$

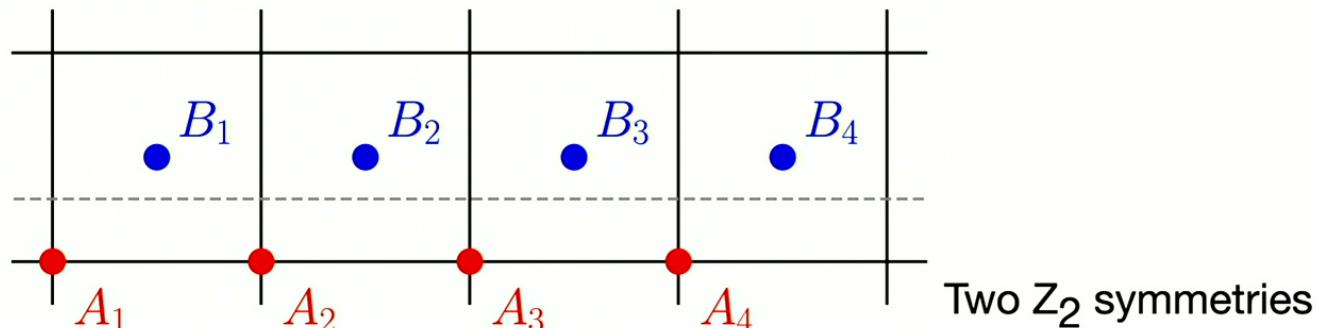


Negativity spectrum (pure-state toric code)

TCL, Vijay (PRR 2023)

$$A_s = \begin{array}{|c|c|} \hline & Z \\ \hline Z & Z \\ \hline \end{array}$$

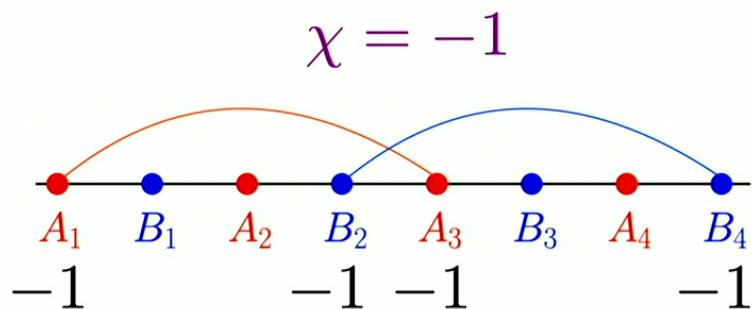
$$B_p = \begin{array}{|c|c|} \hline X & X \\ \hline & X \\ \hline \end{array}$$



$$\rho^\Gamma(\{A_i, B_i\}) \propto \chi(\{A_i, B_i\}) \\ = \pm 1$$

$$A_i, B_i = \pm 1$$

$$\prod_i A_i = \prod_i B_i = 1$$



Partial transpose proliferates the boundary anyons and reveals the braiding phase between **e**, **m** anyons

$$|1d \text{ cluster}\rangle = \sum_{\{A_i, B_i\}} \chi(\{A_i, B_i\}) |A_1, B_1, A_2, B_2, \dots\rangle_X$$

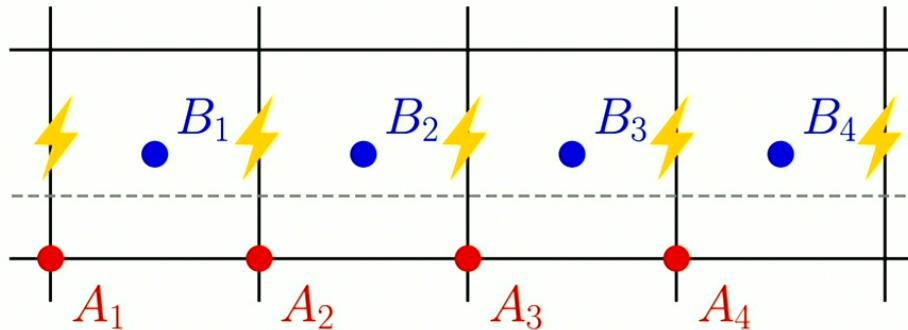
Z₂ × Z₂ SPT ↓

Negativity spectrum

Negativity spectrum (boundary decohered)

$$A_s = \begin{array}{c} Z \\ | \\ Z \\ | \\ Z \end{array}$$

$$B_p = \begin{array}{c} X \\ | \\ X \\ | \\ X \end{array}$$



Pauli-Z noise;
Pauli-Z with prob. p

$$\rho^\Gamma(\{A_i, B_i\}) \propto \chi(\{A_i, B_i\}) e^{\beta \sum_{i=1}^L A_i}$$

$$\prod_{i=1}^L A_i = 1$$

$$A_i = \sigma_i \sigma_{i+1}$$

Domain-wall variable

$$\tanh \beta = \frac{p}{1-p}$$

Gibbs weight of
finite-T 1d Ising

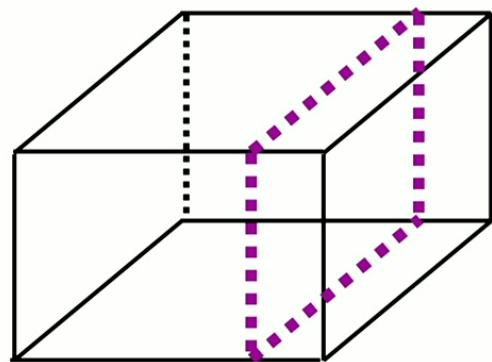


Negativity spectrum computable for any stabilizer models

3d toric code with boundary decoherence

$$A_s = \begin{array}{c} Z \\ \diagup \quad \diagdown \\ Z \quad Z \\ \diagdown \quad \diagup \\ Z \end{array}$$

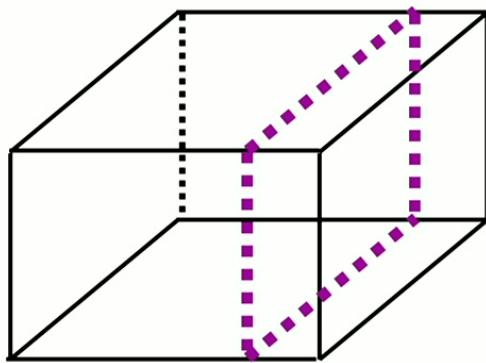
$$B_p = \begin{array}{|c|c|} \hline X & X \\ \hline X & \\ \hline \end{array}$$



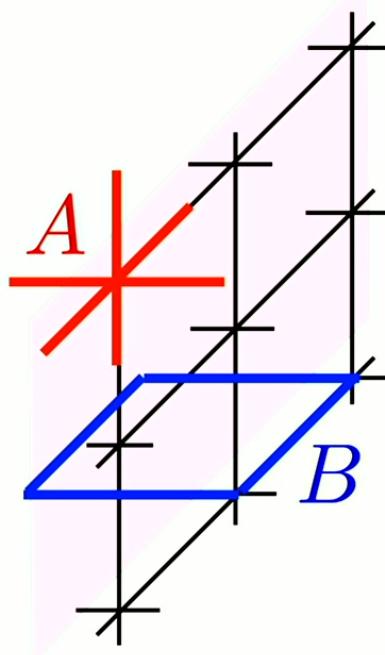
3d toric code with boundary decoherence

$$A_s = \begin{array}{c} z \\ z \\ z \\ z \end{array}$$

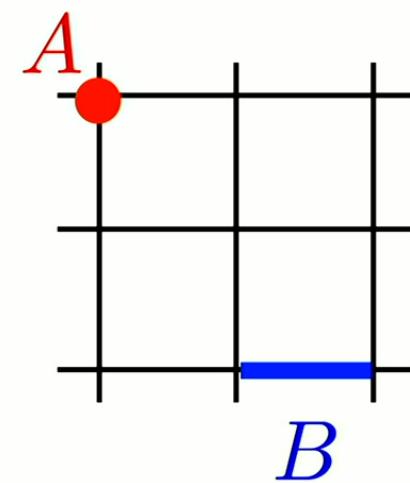
$$B_p = \begin{array}{c} x \\ x \\ x \end{array}$$



2d boundary



2d cross-section

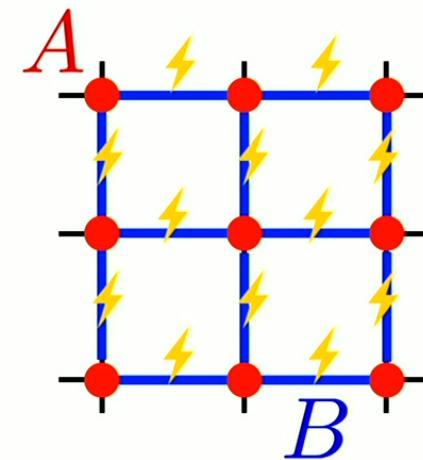


3d toric code with boundary decoherence

$$A_s = \begin{array}{c} Z \\ \diagup \quad \diagdown \\ Z \quad Z \\ \diagdown \quad \diagup \\ Z \end{array}$$

$$B_p = \begin{array}{c} X \\ \diagup \quad \diagdown \\ X \quad X \\ \diagdown \quad \diagup \\ X \end{array}$$

2d cross-section



X noise; Pauli-X with prob. p
Excite A_s vertex charges

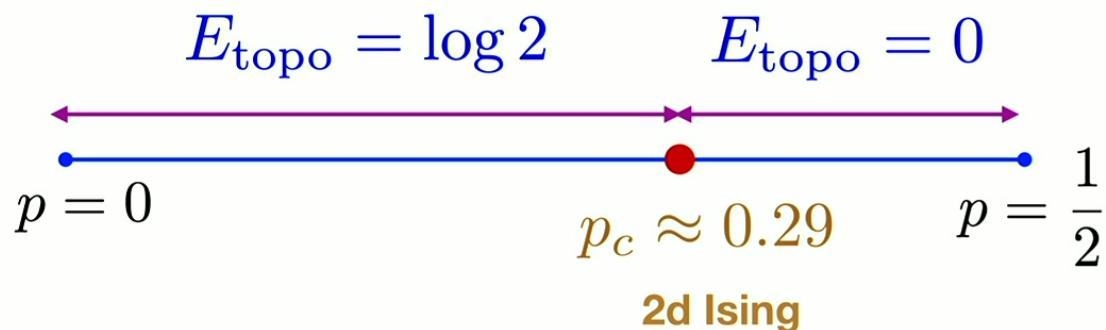
3d toric code with boundary decoherence

$$A_S = \begin{array}{c} Z \\ \diagup \\ Z \\ \diagdown \\ Z \\ \diagup \\ Z \end{array}$$

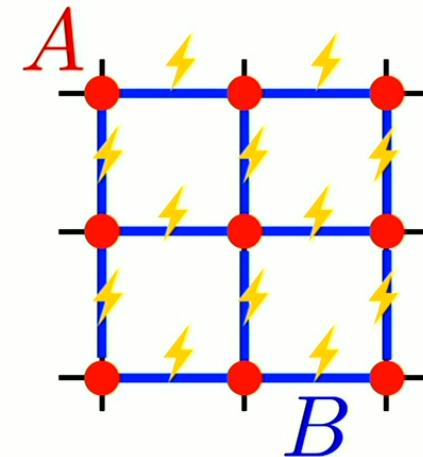
$$B_p = \begin{array}{c} X \\ \square \\ X \\ X \end{array}$$

$$E_N = \log \frac{Z_{\text{2d,Ising}}}{2e^{2\beta L^2}} - \log \frac{\tilde{Z}_{\text{2d,Ising}}}{\sum_{\sigma} e^{\beta \sum_{\langle ij \rangle} \sigma_i \sigma_j}}$$

$$\tanh \beta = \frac{p}{1-p}$$



2d cross-section



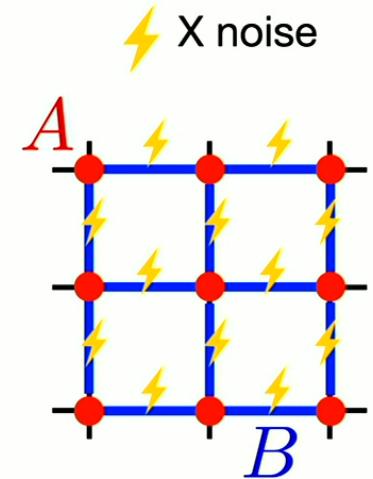
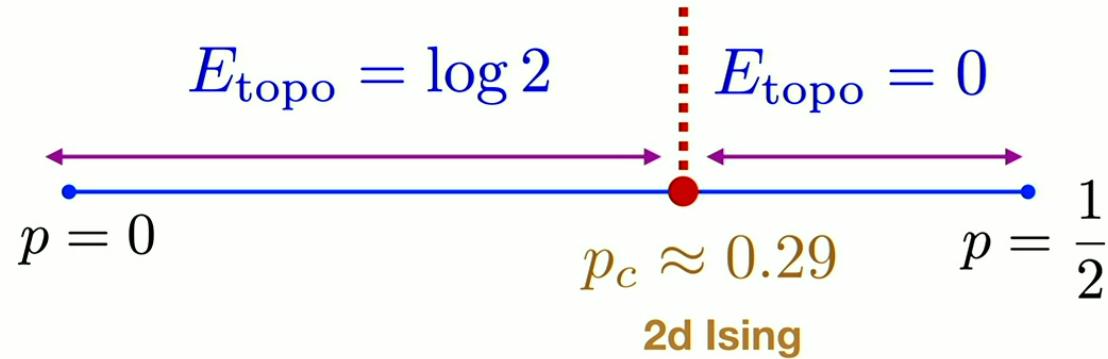
⚡ X noise; Pauli-X with prob. p
Excite A_S vertex charges

Boundary decoherence

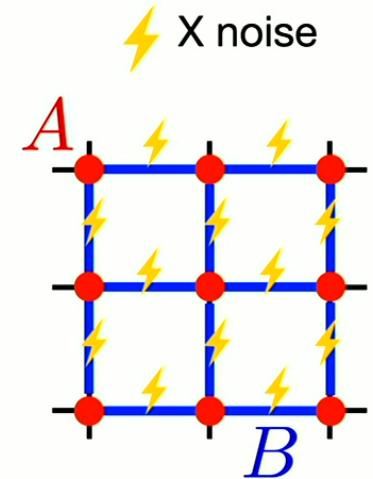
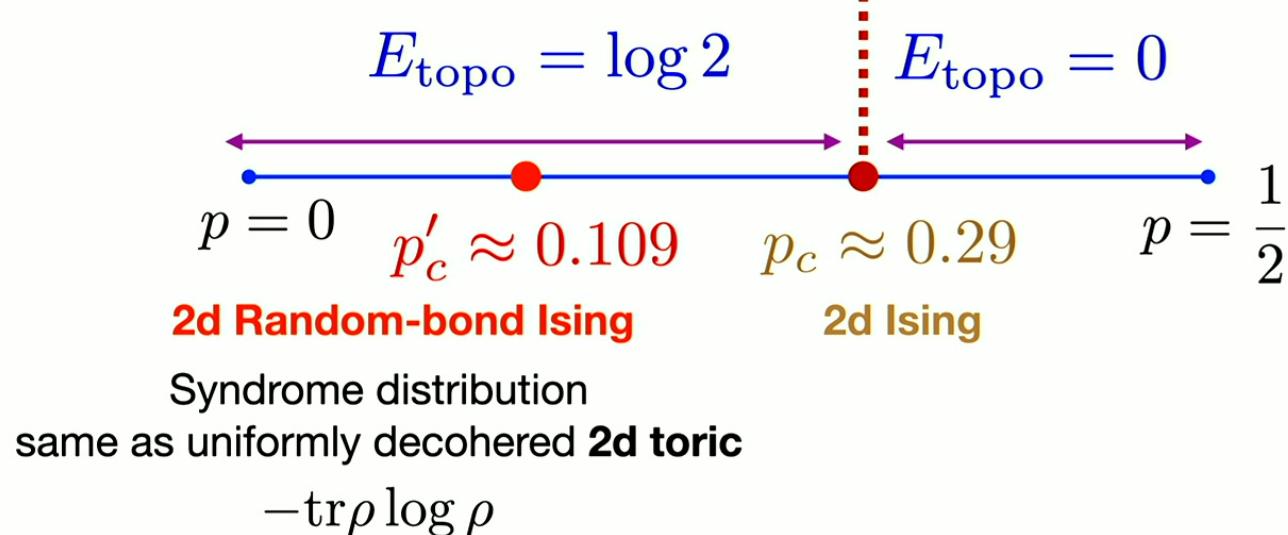
Formalism applied to any qubit stabilizer models
e.g. 4d toric code

These exact results of entanglement negativity lead to **new puzzles...**

3d toric code revisited

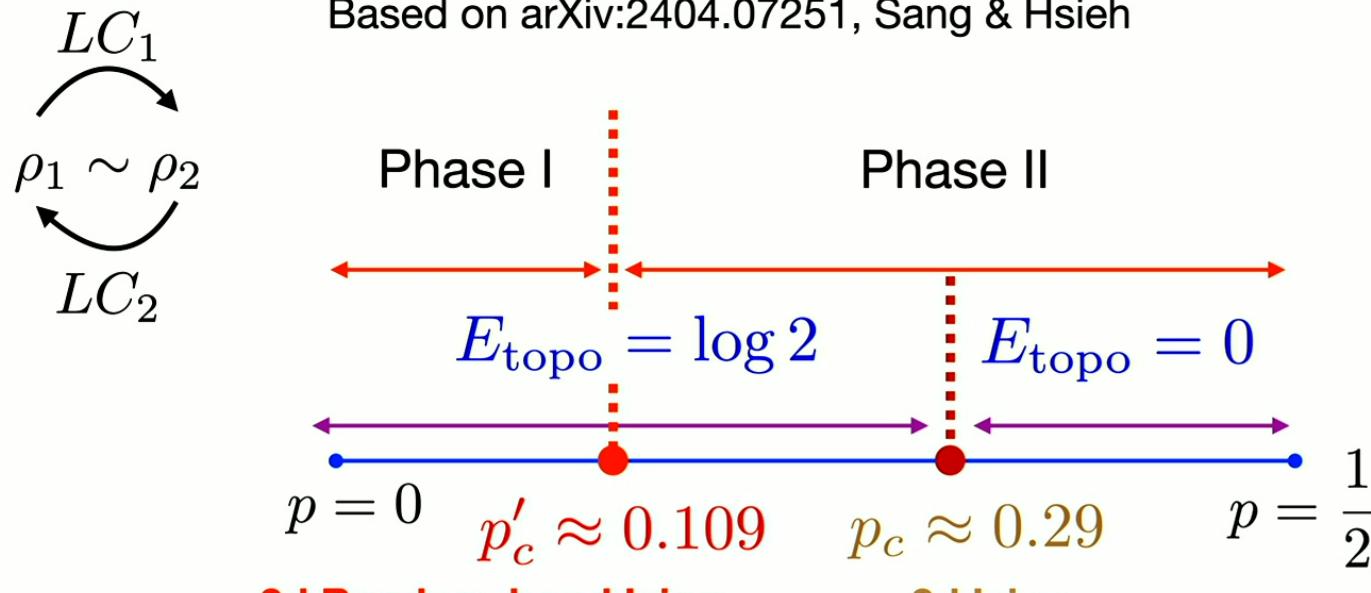


3d toric code revisited



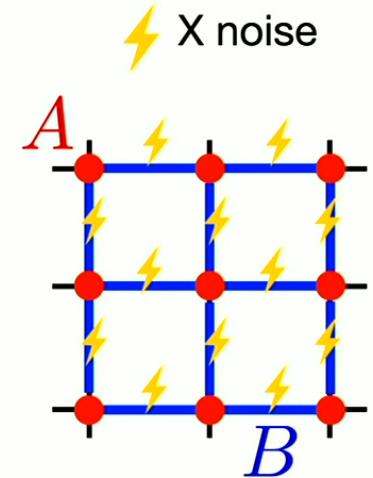
3d toric code revisited

Based on arXiv:2404.07251, Sang & Hsieh



Syndrome distribution
same as uniformly decohered **2d toric**

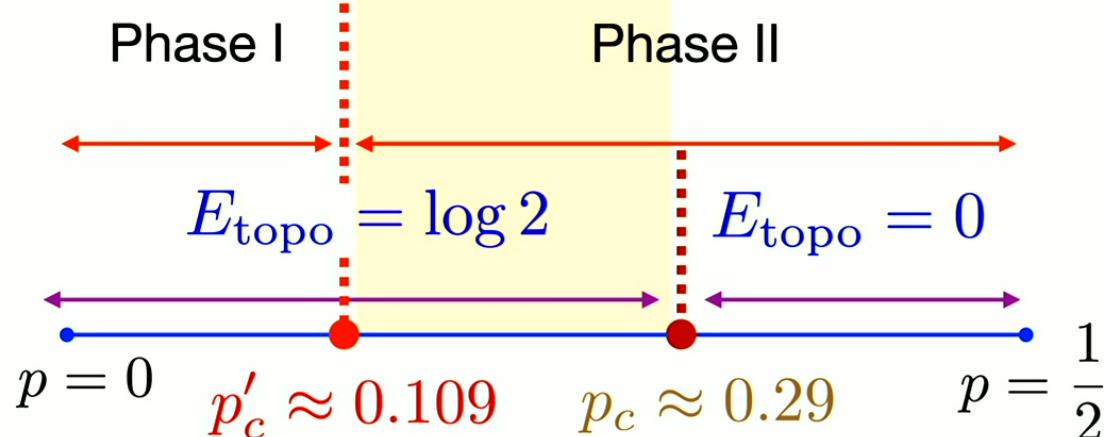
$$-\text{tr} \rho \log \rho$$



3d toric code revisited

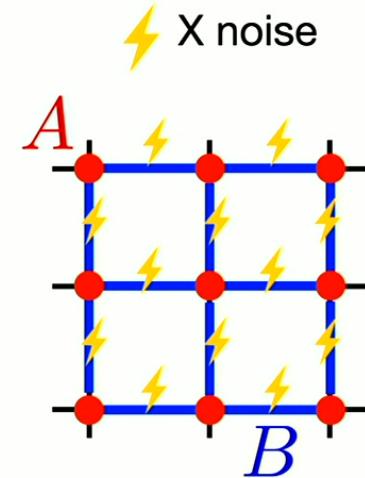
$$\begin{array}{c} LC_1 \\ \curvearrowright \\ \rho_1 \sim \rho_2 \\ \curvearrowleft \\ LC_2 \end{array}$$

Based on arXiv:2404.07251, Sang & Hsieh

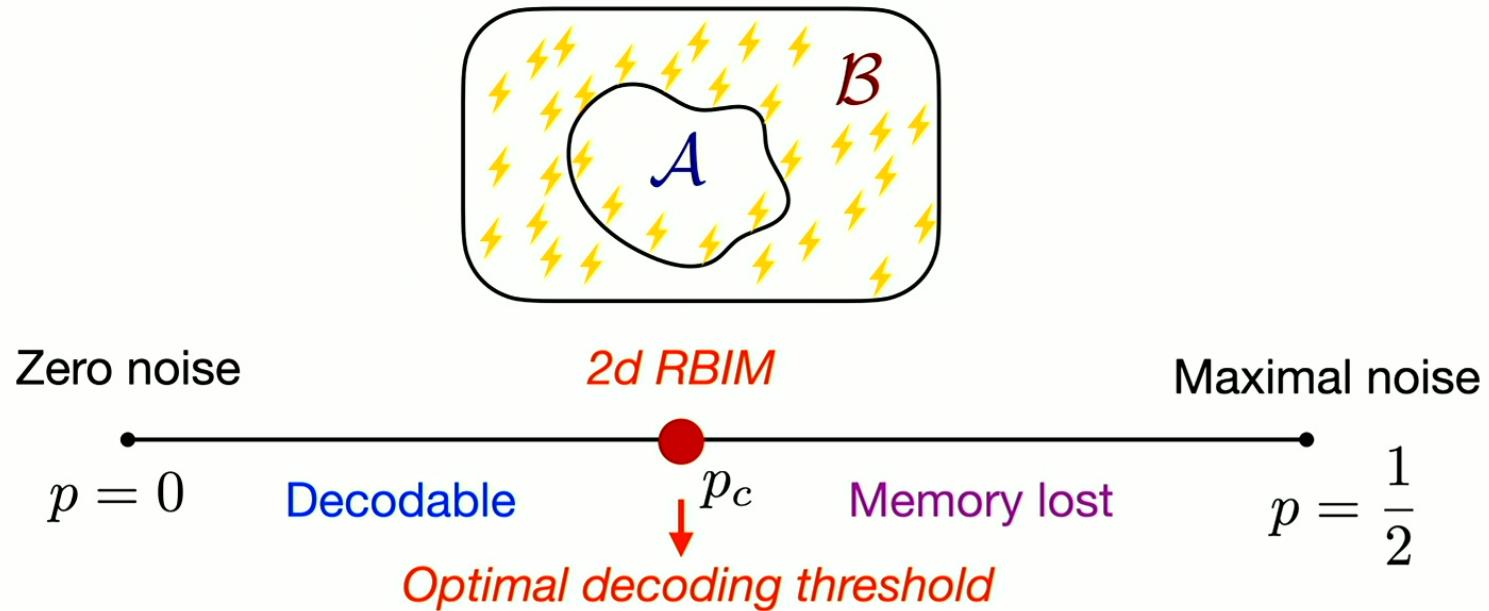


Syndrome distribution
same as uniformly decohered **2d toric**
 $-\text{tr} \rho \log \rho$

$p'_c < p < p_c$ Spurious entanglement?
(quasi) local channel creates
non-zero topological negativity?



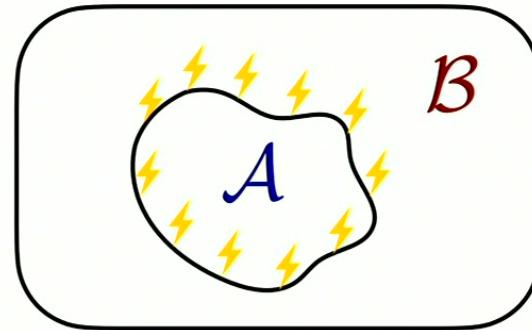
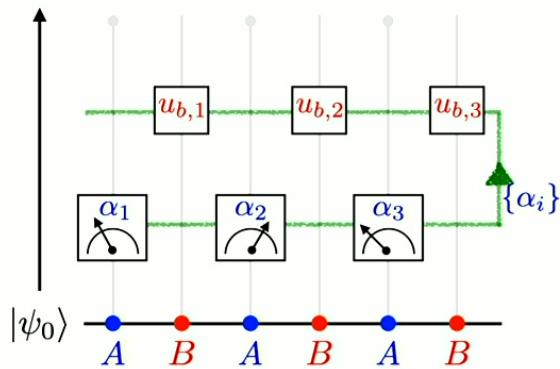
Decohered toric code (noise everywhere)



Does negativity witness the decoding transition?

Perhaps require other (better) entanglement measure?

Summary



Creating LRE via measurement-feedback

TCL, Zhang, Vijay, Hsieh (PRX Quantum, 2023)

- Limitation of measurements & feedback?
Gapped pure state -> critical pure states?
- Our prepared mixed states -
classification & entanglement structure?

Destroying LRE via decoherence

TCL, arXiv:2404.06514 (2024)

- Boundary separability perspective
- Negativity transition in the 2d toric code
(noise everywhere)
- Boundary decoherence in various quantum matter