Title: Repetition Code Revisited

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Collection: Physics of Quantum Information

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Abstract: "Optimal fault tolerant error correction thresholds for CCS codes are traditionally obtained via mappings to classical statistical mechanics models, for example the 2d random bond Ising model for the 1d repetition code subject to bit-flip noise and faulty measurements. Here, we revisit the 1d repetition code, and develop an exact "stabilizer expansion" of the full time evolving density matrix under repeated rounds of (incoherent and coherent) noise and faulty stabilizer measurements. This
expansion enables computation of the coherent information, indicating whether encoded information is retained under the noisy dynamics, and generates a dual representation of the (replicated) 2d random bond Ising model. However, in the fully generic case with both coherent noise and weak measurements, the stabilizer expansion breaks down (as does the canonical 2d random bond Ising model mapping). If the measurement results are thrown away all encoded information is lost
at long times, but the evolution towards the trivial steady state reveals a signature of a quantum transition between an over and under damped regime. Implications for generic noisy dynamics in other CCS codes will be mentioned, including open issues."

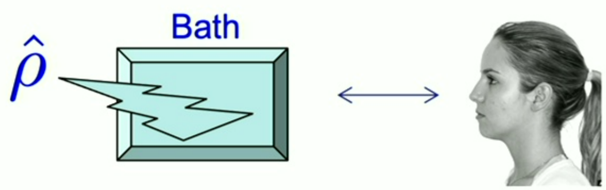
Repetition Code Revisited

Perimeter
5/27/24

MPA Fisher

Noisy (open) quantum system dynamics: All quantum mechanics lost at long times (exception: 4d toric code)

General Goal: Measurements to control open system (noisy) dynamics



Results:

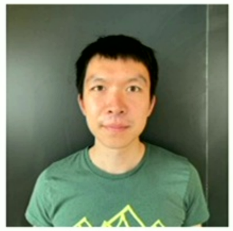
- Explore dynamics of quantum information under monitored/noisy dynamics; $\hat{\rho}(T)$
  "Stabilizer expansion" for evolving density matrix

- Unmonitored noisy quantum dynamics: Non-trivial quantum mechanics approaching trivial dynamical steady-state

Collaborators

- Stabilizer expansion dynamics: To appear…

Jake Hauser



Yimu Bao

Shengqi Sang

Ali Lavasani

Utkarsh Agrawal

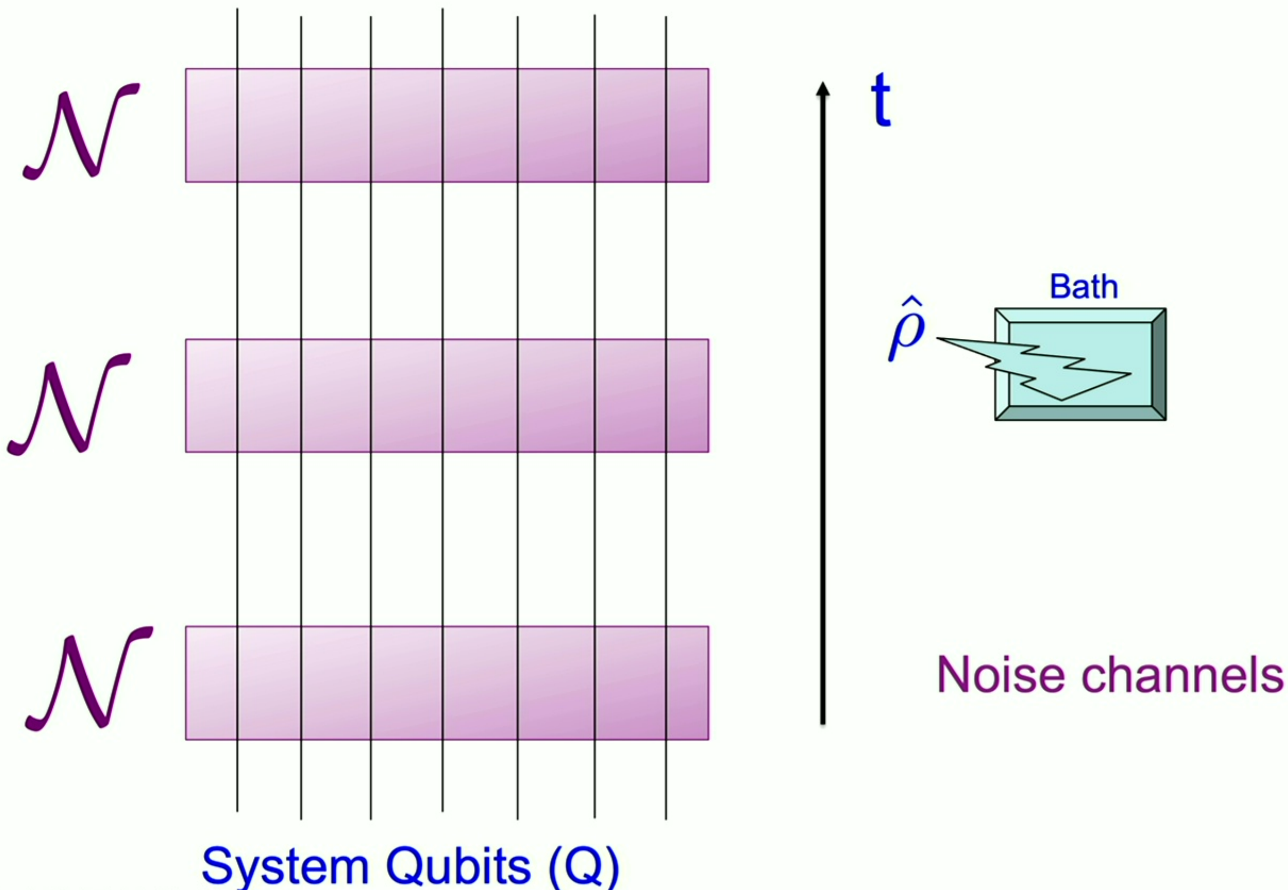
- Unmonitored noisy quantum dynamics: To appear…

Stephen Yan

Sagar Vijay

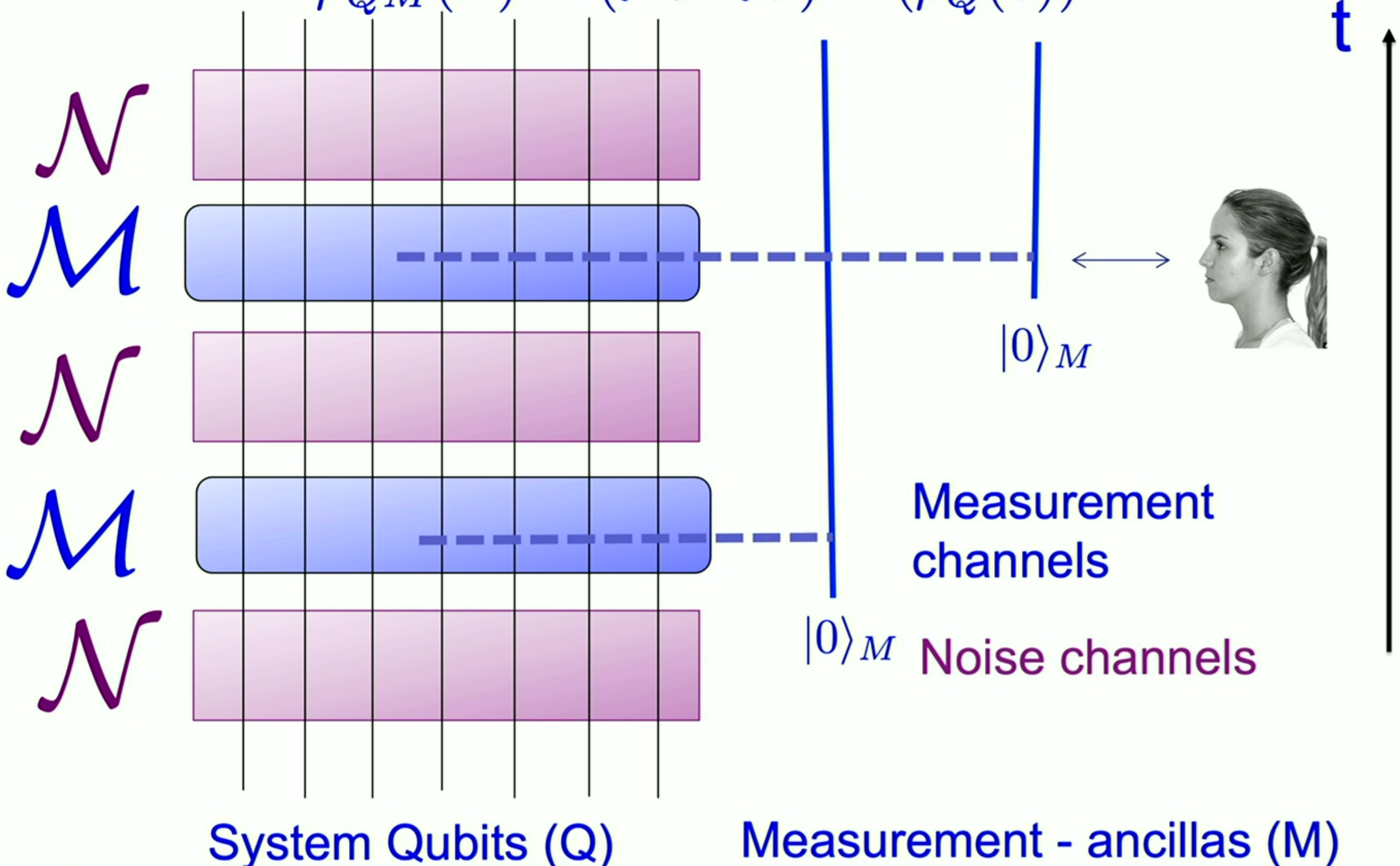
Open system (noisy) "Dynamics"

$$\rho_Q(T) = \mathcal{N}^{\otimes T}(\rho_Q(0))$$



“Monitor” (measure) to “control”

$$\rho_{QM}(T) = (\mathcal{M} \circ \mathcal{N})^{\otimes T}(\rho_Q(0))$$



Repetition Code w/ 3 qubits

3 Qubit Bit flip code

Redundant encoding

$$|\bar{0}\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$|\bar{1}\rangle = |1\rangle \otimes |1\rangle \otimes |1\rangle$$

[L,k,d] = [3,1,1] Stabilizer Code

L = 3 Physical Qubits
k = 1 Logical Qubit
2 stabilizers

$$g_1 = Z_1 Z_2$$
$$g_2 = Z_2 Z_3$$

Code space (logical qubit) $g_j|\Psi\rangle = |\Psi\rangle$ $\quad |\Psi\rangle = a|\bar{0}\rangle + b|\bar{1}\rangle$

Logical (Pauli) operators: $\overline{Z} = Z_1$ $\quad \overline{Z}|\overline{x}\rangle = (-1)^x|\overline{x}\rangle$

$$\overline{X} = X_1 X_2 X_3 \qquad \overline{X}|\overline{x}\rangle = |\overline{x} \oplus 1\rangle$$

Correcting Bit-flip errors

Single qubit flipped, corrupting state:

$$|\Psi\rangle \rightarrow |\Psi_j\rangle = X_j|\Psi\rangle \qquad j = 1, 2, 3$$

Measure stabilizers in "corrupted state"
– one of four possible outcomes (syndrome)

$$(g_1, g_2) = (Z_1 Z_2, Z_2 Z_3) = (\pm, \pm)$$

syndrome		Z_1Z_2	Z_2Z_3
$\vert\psi\rangle$		1	1
$\vert\psi_1\rangle$	$X_1\vert\psi\rangle$	-1	1
$\vert\psi_2\rangle$	$X_2\vert\psi\rangle$	-1	-1
$\vert\psi_3\rangle$	$X_3\vert\psi\rangle$	1	-1

Syndrome determines which qubit was flipped

Flip that bit back, recovering the original state

$$|\Psi_j\rangle \rightarrow X_j|\Psi_j\rangle \equiv |\Psi\rangle$$

Error correction (recovery) stat mech models

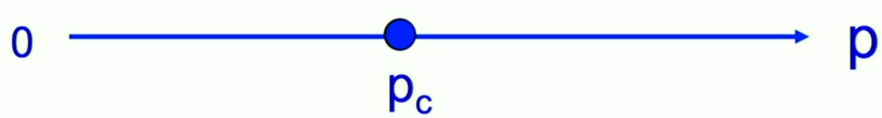
Topological quantum memory*

Eric Dennis,[(1)†] Alexei Kitaev,[(2)‡] Andrew Landahl,[(2)§] and John Preskill[(2)**] arXiv:01.10143

Topological stabilizer quantum codes

Example: 1d repetition code; repeated bit-flip noise and faulty "syndrome" measurements, with error rate p

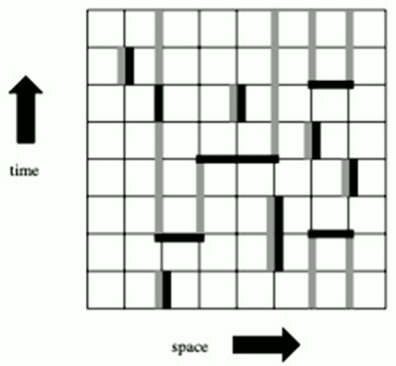
Goal – compute error threshold, p_c



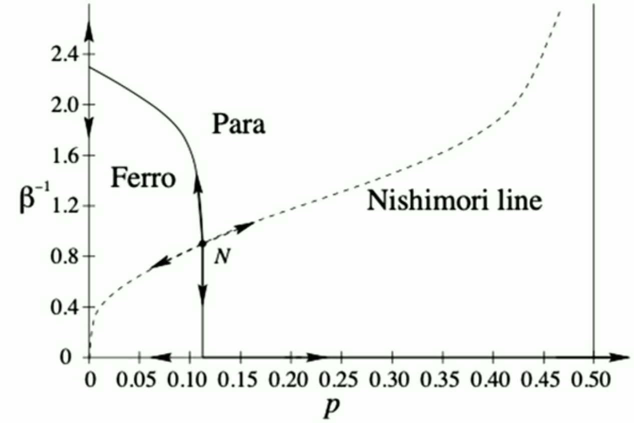
Encoded (initial) logical qubit (quantum state) is recoverable

Quantum information lost to environment

Error threshold stat mech models



Syndromes and errors



2d Random Bond Ising Model on Nishimori line (RBIM)

p= probability of negative bonds

$$e^{-2\beta} = p/(1-p)$$

0 → p

FM p_c PM

Information is recoverable

Information is lost

Goal: Quantum information dynamics

Explore time-evolving density matrix under bit-flip/measurement dynamics

$$\rho(0) \to \rho(T)$$

Derive "stabilizer" expansion for evolving density matrix

Generates a "classical" description of quantum info dynamics

$$\rho_{QM}(T) = \sum_{\sigma} e^{-H(\sigma)} \prod_{j} g_j^{(1-\sigma_{j,T+1})/2} \prod_{t=1}^{T} Z_{Mj,t}^{(1-\sigma_{j,t}\sigma_{j,t+1})/2}$$

Compute (coherent) quantum information; determines information loss/retention

Result: "Stabilizer" stat mech model is dual to RBIM

Builds on and generalizes important work $\rho(0) \to \rho(1)$

Diagnostics of mixed-state topological order and breakdown of quantum memory

Ruihua Fan,[1,*] Yimu Bao,[2,*] Ehud Altman,[2,3] and Ashvin Vishwanath[1] arXiv:2301.05689

1d Repetition Code Revisited

L Qubits in 1d w/ periodic boundary conditions

Code space: two dimensional spanned by FM states;

$$|\overline{0}\rangle = |0\rangle^{\otimes L} \qquad |\overline{1}\rangle = |1\rangle^{\otimes L}$$

One Logical Qubit: $|\psi\rangle = \alpha|\overline{0}\rangle + \beta|\overline{1}\rangle$

Stabilizer Code:

Check operators $g_i = Z_i Z_{i+1}; \quad i = 1, 2, ...L \qquad g_i|\psi\rangle = |\psi\rangle$

$L-1$ Independent Stabilizers $\langle Z_1 Z_2, Z_2 Z_3, ..., Z_{L-1} Z_L \rangle$

Initialize Density Matrix

Initial Density Matrix maximally mixed in code space

$$\rho_Q(0) = |\overline{0}\rangle\langle\overline{0}| + |\overline{1}\rangle\langle\overline{1}| = \prod_{j=1}^{L}(1+g_j),$$

Re-express as ***stabilizer expansion*** (summation over stabilizers)

$$\rho_Q(0) = \sum_{\{\sigma_j=\pm 1\}} \prod_{j=1}^{L} g_j^{(1-\sigma_j)/2},$$

Ising symmetry since $\prod_{j=1}^{L} g_j = 1$

Dynamics:

- Noise (Incoherent bit-flip or coherent unitary)
- Measure check operators and store syndrome in ancillas
- Repeat…

Z_2 Symmetric dynamics; Krauss operators commuting with $\overline{X} = \prod_{j=1}^{L} X_j$

Incoherent (Bit-flip) noise channel

$$\mathcal{N}(\rho) = \prod_i \mathcal{N}_i(\rho)$$

$$\mathcal{N}_i = (1-p)\rho + pX_i\rho X_i$$

Act w/ Noise channel: Weighted **"Stabilizer expansion"**

$$\mathcal{N}(\rho_Q(0)) = \sum_{\{\sigma_j\}=\pm 1} e^{-H_0(\sigma)} \prod_{j=1}^{L} g_j^{(1-\sigma_j)/2}$$

$$H_0 = -J_p \sum_{j=1}^{L} \sigma_j \sigma_{j+1}$$

1d Classical Ising Model $2J_p = -\log(1-2p)$

Phases of Ising model:

PM Phase $p < 1/2; \quad J_p < \infty$ This will be the "recovery" phase

FM Phase $p = 1/2; \quad J_p = \infty$ $\mathcal{N}(\rho_Q(0)) \sim \hat{1}_Q$

Maximally mixed - "info lost" phase

Measure Check Operators (stabilizers)

"Weak" Measurement operators

$$M_m^\lambda = \frac{1}{\sqrt{2(1+\lambda^2}}[1+(-1)^m \lambda Z_i Z_{i+1}] \qquad 0 \le \lambda \le 1$$

$\lambda = 0$ No measurement

$\lambda = 1$ Projective measurement

$$\sum_{m=0,1} M_m^\lambda M_m^\lambda = \hat{1}$$

"Weak" Measurement Channel

$$\mathcal{M}_\lambda(\rho) = \sum_{\mathbf{m}} M_{\mathbf{m}}^\lambda \rho M_{\mathbf{m}}^\lambda \otimes |\mathbf{m}\rangle_{\mathbf{M}}\langle \mathbf{m}|$$

$\mathbf{m} = m_1, m_2, ..., m_L$

measurement results stored in ancilla's, one for each check operator

"Faulty" Measurement Channel

Projective measurements w/ subsequent bit-flip on measurement ancillas

$$\mathcal{M}_F(\rho) = \sum_{\mathbf{m}} P_{\mathbf{m}} \rho P_{\mathbf{m}} \otimes \mathcal{N}_{\mathcal{M}}(|\mathbf{m}\rangle_{\mathbf{M}}\langle \mathbf{m}|) \qquad P_m = M_m^{\lambda=1}$$

$$\mathcal{N}_M(\rho_M) = (1-q)\rho_M + q X_M \rho_M X_M.$$

"Monitored" Dynamics: Repeating Noise/Measurements

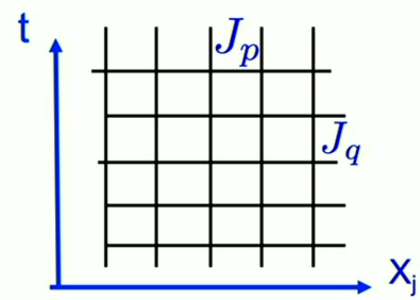
$$\rho_{QM}(T) = \mathcal{E}_T(\rho_Q(0)) \qquad \mathcal{E}_T = (\mathcal{M} \circ \mathcal{N})^{\otimes T}$$

$$\rho_{QM}(T) = \sum_{\sigma} e^{-H(\sigma)} \prod_j g_j^{(1-\sigma_{j,T+1})/2} \prod_{t=1}^{T} Z_{Mj,t}^{(1-\sigma_{j,t}\sigma_{j,t+1})/2}$$

$Z_{M,j,t}$ Pauli Z-operator for ancilla qubit corresponding to stabilizer measurement j at time t.

Dynamics: "Stabilizer Expansion", weights 2d Classical Ising model

$$H(\sigma) = -\sum_{i=1}^{L}\sum_{t=1}^{T}[J_p \sigma_{i,t}\sigma_{i+1,t} + J_q \sigma_{i,t}\sigma_{i,t+1}]$$



Weak/Faulty measurements generate Ising coupling in time-direction, favoring FM

$$2J_q = -\log(2\lambda/(1+\lambda^2)) = -\log(1-2q)$$

Decoherence (vs Monitored Dynamics)

Throw away (trace out) measurement outcomes

Trace out ancilla's - this gives "unconditional" dynamics

$$\rho_Q = \mathrm{Tr}_M \rho_{QM}$$

Sets $\sigma_{j,t} = \sigma_{j,t+1} \equiv \sigma_j$

$$\rho_Q(T) = \sum_{\sigma} e^{-TH_0(\sigma)} \prod_{j=1}^{L} g_j^{(1-\sigma_j)/2}$$

$$H_0 = -J_p \sum_{j=1}^{L} \sigma_j \sigma_{j+1}$$

At long times (1d Ising model orders) and density matrix becomes maximally mixed

All info lost into the "environment"

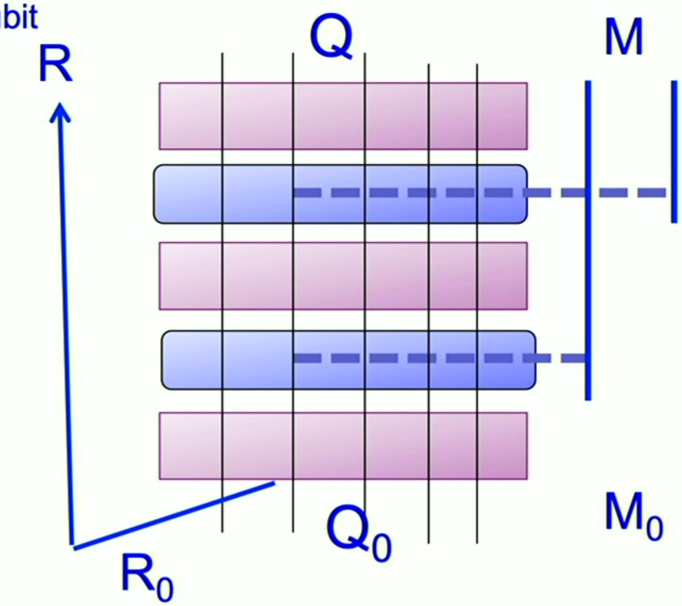
$$\rho_Q(T \to \infty) \sim \hat{1}_Q$$

Quantum Info Dynamics: Coherent Information

Form Bell pair between reference (R) and logical qubit

$$\rho_{QMR}(0) = |QR_0\rangle\langle QR_0| \otimes |0\rangle_M\langle 0|$$

$$|QR_0\rangle = |\bar{0}\rangle \otimes |0\rangle_R + |\bar{1}\rangle \otimes |1\rangle_R$$



Coherent Information:

Quantifies "transmission" of quantum information

$$I_C(R\rangle QM) = S_{QM} - S_{QMR}$$

Initially: $I_C(t=0) = \log(2)$

After channel dynamics:

- If $I_C(T) = 0$ All quantum info lost into "environment"
- If $I_C(T \to \infty) = \log 2$ All (one qubit of) quantum info is transmitted (retained)

Renyi Coherent Information

n^{th} Renyi Coherent Information $\quad I_C^{(n)}(R\rangle QM) = \frac{1}{1-n} \log \mathrm{Tr}\rho_{QM}^n / \mathrm{Tr}\rho_{QMR}^n$

$$lim_{n\to 1} I_C^{(n)} = I_C$$

Need to compute n-copy "partition function":

$$\mathcal{Z} \equiv Tr\rho_{QM}^n$$

Use "Stabilizer Expansion"

Replicate $\quad \sigma_{j,t} \to \sigma_{j,t}^{\alpha} \qquad \alpha = 1, 2, ...n$

Perform Trace: $\quad \mathcal{Z}_n = \mathrm{Tr}\rho_{QM}^n$

Gives constraint: $\quad \prod_{\alpha=1}^{n} \sigma_{j,t}^{\alpha} = 1$ $\qquad$ Solve constraint $\quad \sigma_{j,t}^{n} = \prod_{\alpha=1}^{n-1} \sigma_{j,t}^{\alpha}$

Derived n-1 Flavor 2d Ising model

$$\mathcal{Z}_n = \sum_{\{\sigma^\alpha\}} e^{-H_n}$$

$$H_n = \sum_{\alpha=1}^{n-1} H(\sigma^\alpha) + H\left(\prod_{\alpha=1}^{n-1} \sigma^\alpha\right)$$

$$H(\sigma) = -\sum_{i=1}^{L}\sum_{t=1}^{T}[J_p \sigma_{i,t}\sigma_{i+1,t} + J_q \sigma_{i,t}\sigma_{i,t+1}]$$

Symmetry $(Z_2^n \rtimes S_n)/Z_2$

S_n n-fold permutation symmetry, trace removed one Z_2

for $n = 2; \quad H_2 = 2H$

Phase Diagram

p=noise strength
q="weakness" of measurement

Low-noise/strong-measurements

Noisy/weak-measurements

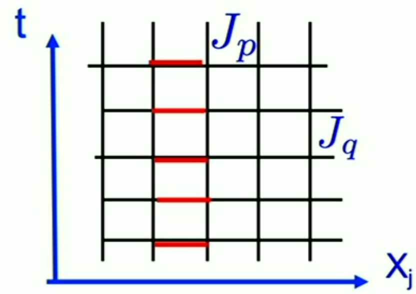
Coherent Information via Stabilizer Expansion

Express Renyi coherent information as

$$I_C^{(n)}(R\rangle QM) = \frac{1}{n-1}\log\Big(\sum_{\{d_\alpha=0,1\}} \exp(-\Delta F(d_\alpha))\Big)$$

$\Delta F(d_\alpha = 1) =$ free energy cost w/ column of negative J_p bonds in replica, α

$\Delta F(d_\alpha = 0) = 0$



$\Delta F(d_\alpha = 1) \sim 0$ $\Delta F(d_\alpha = 1) \sim T$

0 — PM — ● — FM → $J_p = J_q$ (p=q)

Noisy/weak-measurements

$$I_C(T \to \infty) = \log 2$$

$$I_C(T \to \infty) = 0$$

Coherent Information after dynamics

$$\mathcal{E}_T = (\mathcal{M} \circ \mathcal{N})^{\otimes T}$$

0 — PM — FM → $J_p = J_q$ (p=q)

"Low" noise, "strong" measurements

$$I_C(T \to \infty) = \log 2$$

All (one qubit) of Quantum info transmitted thru the channel (retained)

Reversal channel (***R***) can (in principle) recover initial quantum information

$$(\mathcal{R} \circ \mathcal{E})(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi|$$

$\forall |\psi\rangle$ in code space

Noisy, weak measurements

$$I_C(T \to \infty) = 0$$

No quantum info remains after channel

Mutual information between QM and R is log2

One classical bit of info survives, the conserved parity of initial state $\overline{X} = \prod_{j=1}^{L} X_j = \pm 1$

(n→1) flavor Ising model dual to (disorder averaged) 2d Random Bond Ising Model (RBIM) on Nishimori Line

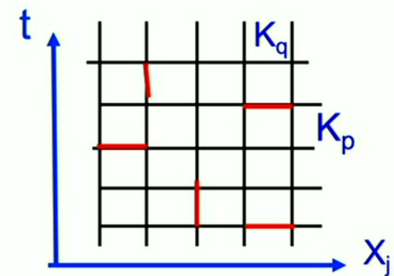
$$Z_{RBIM} = \sum_{s_{j,t}=\pm 1} e^{-H_{RBIM}} \qquad H_{RBIM} = -\sum_{j,t}(K_p \eta_p^{j,t} s_{j,t}s_{j,t+1} + K_q \eta_q^{j,t} s_{j,t}s_{j+1,t})$$

$$\eta_p, \eta_q = \pm 1 \qquad \eta_p = -1;\ \text{w/ prob } p$$
$$\eta_q = -1;\ \text{w/ prob } q$$

Nishimori condition

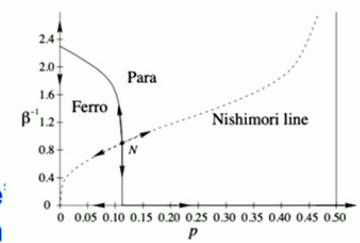
$$e^{-2K_p} = p/(1-p) \qquad e^{-2K_q} = q/(1-q)$$

(gives enhanced symmetry) $(Z_2^n \rtimes S_n)/Z_2$



$$\lim_{n\to 1} \mathcal{Z}_n = \lim_{m \to 0} \overline{\mathcal{Z}^m_{RBIM}}$$

Duality: Low-T expansion for RBIM equals high-T expansion for (n-1) flavor Ising model.



FM of 2d RBIM is PM phase (n-1)-flavor Ising model – "recovery is possible"
PM phase of 2d RBIM is FM phase of (n-1)-flavor Ising model – "information

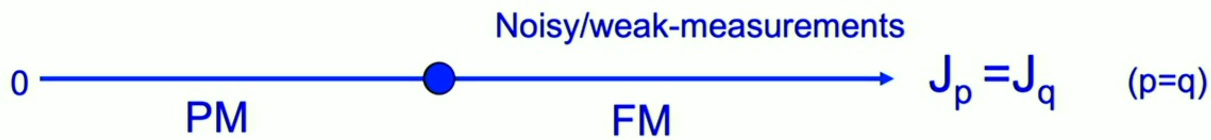
Another example: 2d Toric Code w/ X-noise and faulty/weak measurements via "stabilizer expansion"

n-1 Flavor 3d Ising model

$$\mathcal{Z}_n = Tr\rho_{QM}^n = \sum_{\{\sigma^\alpha\}} e^{-H_n} \qquad H_n = \sum_{\alpha=1}^{n-1} H(\sigma^\alpha) + H(\prod_{\alpha=1}^{n-1}\sigma^\alpha) \qquad H(\sigma) = \text{ 3d Ising}$$

Phase Diagram

p=noise strength
q="weakness" of measurement



$$\lim_{n\to 1} \mathcal{Z}_n = \lim_{m\to 0} \overline{\mathcal{Z}_{3dRG}^m}$$

Duality: 3d Random Plaquette Ising-Gauge theory dual to (n-1) flavor Ising model.

(Dennis et. al. 2001)

Perimeter-law phase of 3dRG is PM phase of (n-1)-flavor Ising model – "recovery is possible"
Area-law phase of 3dRG is FM phase of (n-1)-flavor Ising model – "information is lost"

“Stabilizer Expansion” general for stabilizer codes

Other stabilizer codes w/ noise and faulty/weak measurements in stabilizer expansion:

(1) 2d Repetition Code w/ X-errors w/ faulty/weak measurements:
Maps to 3d (n-1) Flavor Z_2 Gauge Theory (dual to the 3d RBIM)

(2) Toric code w/ Y-errors and faulty/weak measurements:
Maps to (n-1) Flavor (2+1)d quantum Xu-Moore model – has a transition

(3) XZZX model…

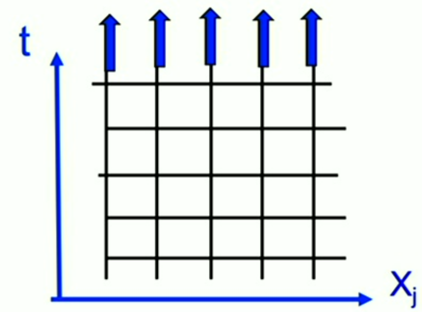
Retain only measurement record

$$\rho_M = \mathrm{Tr}_Q \rho_{QM} = \sum_{\mathbf{m}} p_{\mathbf{m}} |\mathbf{m}\rangle\langle\mathbf{m}|$$

$$\rho_M(T) = \sum_{\sigma} e^{-H_+(\sigma)} \prod_{j=1}^{L} \prod_{t=1}^{T-1} Z_{Mj,t}^{(1-\sigma_{j,t}\sigma_{j,t+1})/2} Z_{Mj,T}^{(1-\sigma_{j,T})/2}$$

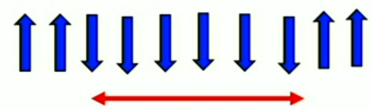
$$H_+(\sigma) = H(\sigma) - J_q \sum_{j=1}^{L} \sigma_{j,T}$$

2d Ising model w/ field on final time slice



Distinguish flipped domain of spins, from measurement record?

Add domain to initial state



$$\rho_{Q,r}(0) = \overline{X}_r \rho_Q(0) \overline{X}_r \qquad \overline{X}_r = \prod_{j=1}^{r} X_j$$

Run dynamics for time T; $\rho_{M,r}(T)$

Define relative entropy (KL divergence) $D(\rho_M || \rho_{M,r}) = \mathrm{Tr}\rho_M \log(\rho_M / \rho_{M,r})$

(and Renyi version) $D_r^{(n)} = \frac{1}{n-1} \log \frac{\mathrm{Tr}\rho_M^n}{\mathrm{Tr}\rho_M \rho_{M,r}^{n-1}}$

$$\mathcal{Z}_{n,+} = \mathrm{Tr}\rho_M^n = \sum_{\{\sigma^\alpha\}} e^{-H_{n+}} \qquad H_{n+} = \sum_{\alpha=1}^{n-1} H_+(\sigma^\alpha) + H_+(\prod_{\alpha=1}^{n-1} \sigma^\alpha)$$

Relative entropy for flipped domain

Spin-spin correlator in first time slice (and one replica)

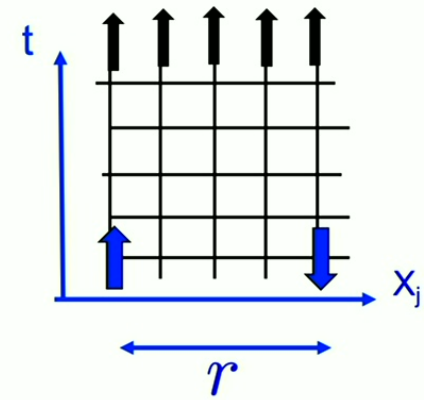
$$D_r^{(n)} = \frac{1}{1-n} \log\langle \sigma_{0,1}^{(1)} \sigma_{r,1}^{(1)} \rangle_+ \qquad \langle \mathcal{O} \rangle_+ = \frac{1}{\mathcal{Z}_{n,+}} \sum_{\{\sigma^\alpha\}} \mathcal{O} e^{-H_{n+}}$$

PM Phase; Measurements "detect" flipped domain

$$D_r^{(n)} \sim |r|/\xi$$ Grows with r,

FM Phase; Measurements cannot "detect" flipped domain

$$D_r^{(n)} \sim const$$ Constant at large r



Full Real time Quantum Dynamics expressed as classical stat models in space time!!!

Classical stat mech models have positive Boltzmann weights
(2d RBIM, say, or (n-1) flavor Ising model in stabilizer expansion)

This Huge simplification is very surprising

Too good to be true…

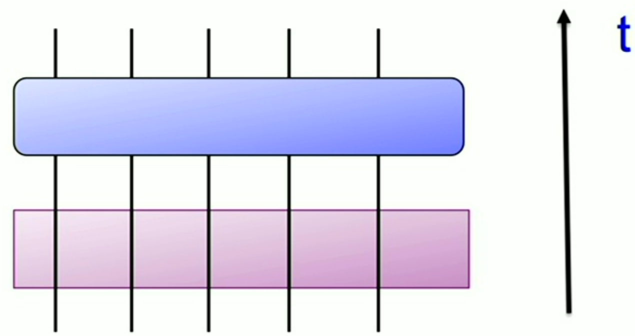
For generic noisy dynamics (w/ coherent errors, say) "classical" simplification can (generally will) break down

Coherent dynamics plus **weak** measurements

$\rho_Q \to U \rho_Q U^\dagger$ with $U = e^{i\theta \sum_{j=1}^{L} X_j}$

$\mathcal{M}_\lambda(\rho)$

$\mathcal{E}_X(\rho) = U \rho U^\dagger$



Weak measurements of stabilizers **do not** convert coherent into incoherent noise,

$$\mathcal{M}_\lambda(U \rho_Q U^\dagger) \neq (\mathcal{M}_\lambda \circ \mathcal{N})(\rho_Q)$$

Implications: weak measurement and coherent noise

"Stabilizer expansion" of density matrix breaks down

Dynamics takes density matrix out of stabilizer expansion space,

$$\rho_{QM}(T=1) \neq \sum_{\{\sigma_j\}=\pm 1} W(\sigma) \prod_{j=1}^{L} g_j^{(1-\sigma_j)/2}$$ (even after one round)

Under generic dynamics density matrix wanders into operator neverlands…

Dynamics not described by (simple) (n-1) flavor Ising model, nor 2d RBIM

No classical stat mech description: "full" real time Quantum dynamics needed

Generic coherent dynamics plus weak measurement

Questions:

(1) How to generalize stabilizer expansion?

(2) How to access full non-trivial quantum dynamics?

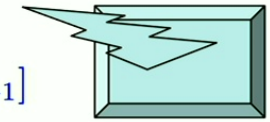
(3) How to show/calculate an error threshold?

(4) Can measurements distinguish initial flipped domain?

Question 2: For very special measurement dynamics (w/ coherent noise)
Y. Suzuki et. al. arXiv:1703.036712) mapped "error-threshold"
stat mech model into RBIM w/ complex coupling constants,
used Majorana mapping to obtain a phase transition

Any QM left if discard measurement results?
(**unmonitored** noisy/coherent dynamics)

Bath + coherence



"Measure" ZZ weakly, don't store results

$$\mathcal{E}_{ZZ}(\rho) = \prod_j [(1-p)\rho + pZ_j Z_{j+1} \rho Z_j Z_{j+1}]$$

Unitary rotation around X-axis

$$\mathcal{E}_X(\rho) = \mathcal{U}_X \rho \mathcal{U}_X^\dagger \quad \mathcal{U}_X = e^{i\theta \sum_j X_j}$$

Maximally mixed state is "fixed" point of channel $\mathcal{E}_{ZZ} \cdot \mathcal{E}_X(\hat{1}_Q) = \hat{1}_Q$

For any initial state: Maximally mixed at long times $\rho_Q(T \to \infty) = \hat{1}_Q$

Goal: Explore competition between noisy and coherent dynamics giving non-trivial (quantum) approach to trivial steady state

(S. Yan, S. Vijay, MPAF in preparation)

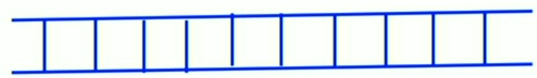
Finite time dynamics, starting with all spins up, say

Use Double state formalism $\hat{\rho} \longrightarrow |\rho\rangle$

Take time-continuum limit

$$|\rho(T)\rangle = e^{-pHT}|\rho(0)\rangle$$

Non-Hermitian "Hamiltonian" on 2-leg ladder



$$H = -\sum_j Z_j Z_{j+1} \otimes Z_j Z_{j+1} + ig\sum_j (X_j \otimes 1 - 1 \otimes X_j) \qquad g = \theta/p$$

Locally Conserved quantity: $[X_j \otimes X_j, H] = 0$

Define: $\mu_j^x = X_j \otimes X_j \qquad \tau_j^z = Z_j \otimes Z_j$

$\mu_j^z = I \otimes Z_j \qquad \tau_j^x = X_j \otimes I$

Re-express Hamiltonian

$$H = -\sum_j \tau_j^z \tau_{j+1}^z + ig\sum_j \tau_j^x (1 - \mu_j^x)$$

Ground state $\tau_j^z = \mu_j^x = 1$ Maximally mixed

Correlator Linear in Density Matrix

Define "correlator" linear in density matrix, initial state all spins up

$$G(T) = \langle \prod_j Z_j \rangle = Tr(\prod_j Z_j \rho(T))$$

Re-express Correlator in Double Hilbert space representation

$$G(T) = \langle \uparrow_\tau^{\otimes L} | e^{-pH_0 T} | \uparrow_\tau^{\otimes L} \rangle$$

Quantum complex-transverse field Ising model in 1d

$$H_0 = -\sum_j \tau_j^z \tau_{j+1}^z + ig \sum_j \tau_j^x$$

Majorana Fermions: Model has a phase transition at $g_c = 1$

Temporal Behavior in the two phases?

Universal Behavior of Underdamped oscillations

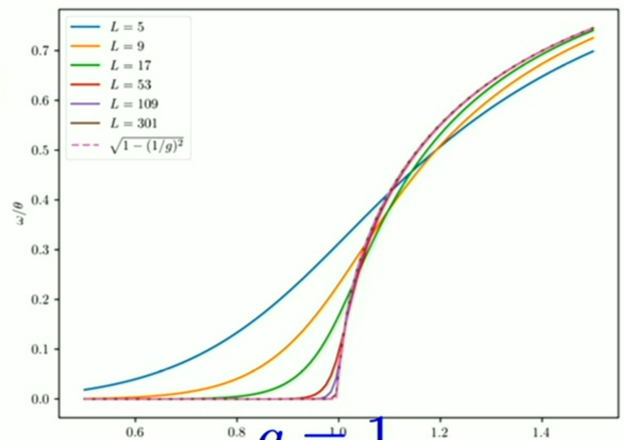
Focus on L odd and find

$$G(T) \sim e^{-\Gamma_g L T} \cos(\omega_g T)$$

g>1: "Underdamped oscillations" survive L to infinity $\omega_g \sim \theta\sqrt{1-\frac{1}{g^2}}$

g<1; $\omega_g \to 0$ as L to infinity, overdamped

ω_g



g

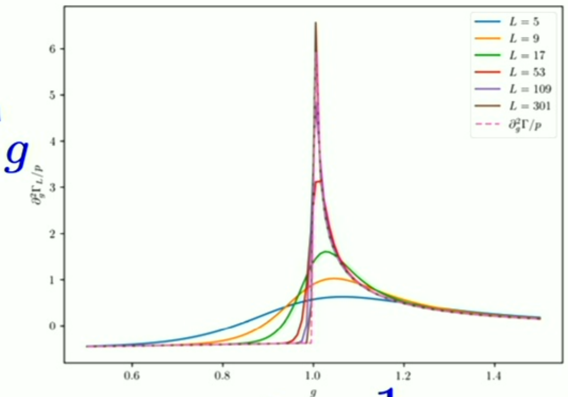
$g = 1$

Singularity in Decay rate

Decay rate: singular as g → 1⁺ (L to infinity) $\partial_g^2 \Gamma_g \sim (g-1)^{-1/2} + \ldots \quad g \to 1^+$

Decay rate regular for g → 1⁻

$\partial_g^2 \Gamma_g$



$g = 1$ $\quad g$

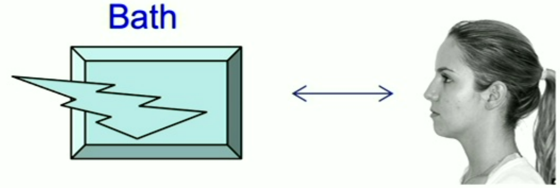
Conclusion: Correlator linear in density matrix retains non-trivial QM upon approaching a trivial steady state, (competition between coherent and incoherent dynamics)

Questions:

- Behavior in higher dimension?
- Generality of result?

Conclusions

Bath

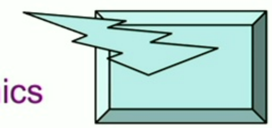


Noisy monitored dynamics in stabilizer codes;
Derived “stabilizer expansion”

- Explored information retention/loss employing coherent information
- Stabilizer expansion dual to error configuration expansions (RBIM)
- Generically, stabilizer expansion breaks down, no classical state mech model

Noisy unmonitored system with coherent dynamics

Bath + coherence



- Quantum Phase transition in dynamical approach to trivial steady state, competition between noise and coherent dynamics

Future

- Stabilizer expansion for floquet codes, subsystem codes,…?
- Dynamics of Toric code w/ coherent noise and weak measurements?
- Coherent dynamics + noise w/ non-trivial approach to trivial steady state; other/higher d examples?