Title: Contextuality, entanglement, magic: many qubits, many questions

Speakers: Ravi Kunjwal

Collection: Foundations of Quantum Computational Advantage

Date: May 02, 2024 - 11:15 AM

URL: https://pirsa.org/24050011

Abstract: I will present some recent work on the interplay between contextuality, entanglement, and magic in multiqubit systems. Taking a foundational inquiry into entanglement in the Kochen-Specker theorem as our point of departure, I will proceed to outline some questions this raises about the role of these resources in models of multiqubit quantum computation. The purpose of this talk is to raise questions that can hopefully feed into the discussion sessions.

Contextuality, entanglement, magic: many qubits, many questions

Ravi Kunjwal
Laboratoire d'Informatique et des Systèmes (LIS)
Aix-Marseille Université

FoQaCiA Conference, Perimeter Institute, Waterloo, Canada
May 2, 2024

Based on V.J. Wright and R. Kunjwal, Quantum 7, 900 (2023)

Aix Marseille université Socialement engagée

FRANCE 2030

A*Midex Initiative d'excellence Aix-Marseille

Annonces

Quantum @Marseille

Master/PhD, postdoc opportunities in Marseille!

CANA
Calcul Naturel ⊆ LIS

LIS
LABORATOIRE D'INFORMATIQUE & SYSTÈMES

Aix*Marseille université
Socialement engagée

FRANCE 2030

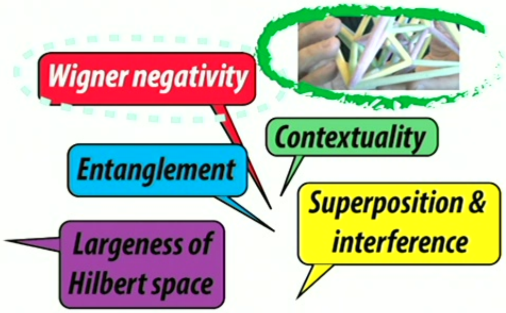
A*Midex
Initiative d'excellence Aix-Marseille


F Q C 2 0 2 4
where foundations and computing meet
3-5 September, RHUL, UK
Farid Shahandeh
Ravi Kunjwal
Matty Hoban
LOCATION
Royal Holloway, University of London
Egham TW20 0EX, UK

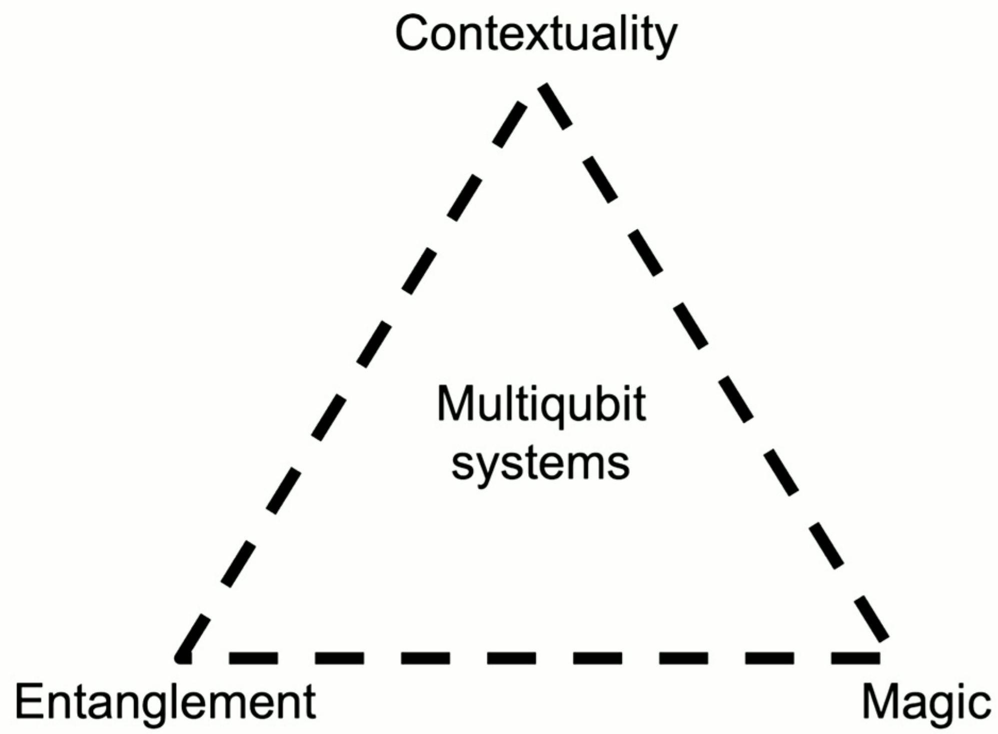
FQC2023

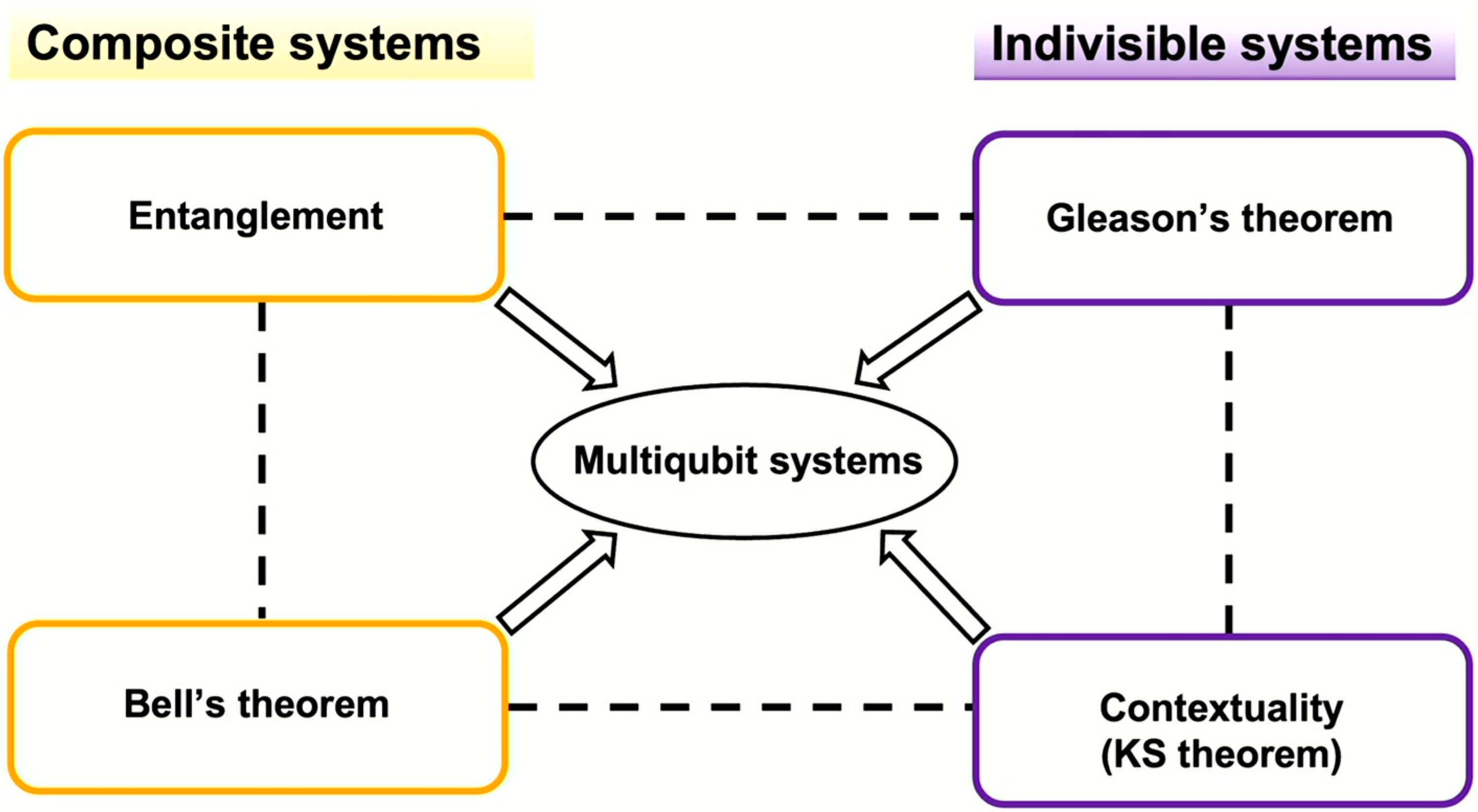
where foundations and computing meet

September 5-6, 2023



What makes quantum computing work?


Contextuality
Multiqubit
systems
Entanglement
Magic



When is entanglement 'magical'?

Concepts

Entanglement

- Entangled state: not a convex mixture of product states

$$\rho \neq \sum_i p_i \rho_{\mathrm{A}}^i \otimes \rho_{\mathrm{B}}^i$$

- Entangled measurement: at least one of the eigenstates is entangled

- Unentangled measurement:

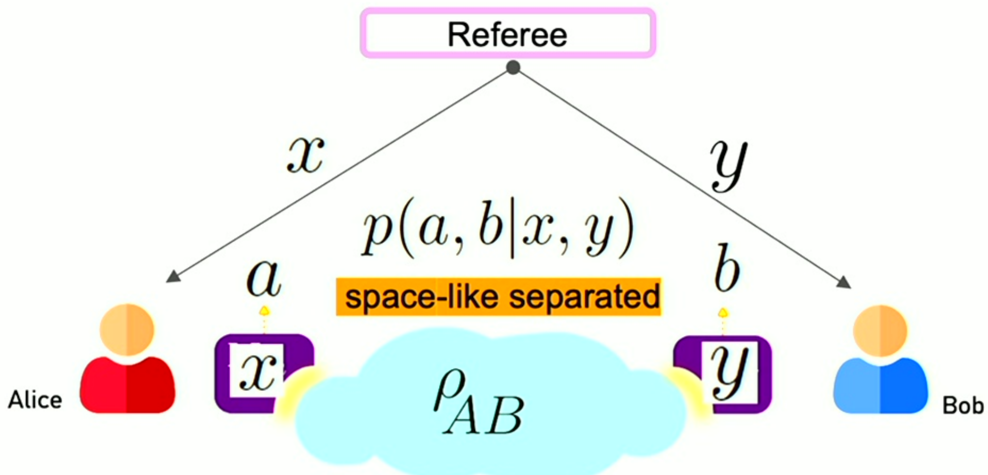
LOCC: $\{|0+\rangle, |0-\rangle, |1+\rangle, |1,-\rangle\}$ (product: LO, *e.g.*, Bell) $\{|00\rangle, |01\rangle, |1+\rangle, |1,-\rangle\}$ (adaptive: LOCC)

Non-LOCC: $\{|000\rangle, |{+10}\rangle, |0+1\rangle, |10+\rangle, |111\rangle, |{-10}\rangle, |0-1\rangle, |10-\rangle\}$ (non-adaptive)

[LOCC='Local Operations and Classical Communication']

Bell's theorem

Bell's theorem: product measurements on an entangled state can violate Bell inequalities, i.e., achieve correlations beyond those from classical shared randomness



$$S_{\text{Bell}} \leq S_{\text{local}}^{\text{max}}$$

Gleason's theorem

Structure of measurements → Structure of states (Born rule)

Formal statement

Let $\mathcal{H}$ be a separable Hilbert space of dimension at least three. Any map $f : \mathcal{P}(\mathcal{H}) \to [0,1]$ satisfying

$$f(\Pi_1) + f(\Pi_2) + \cdots = f(\Pi_1 + \Pi_2 + \cdots),$$

for any set of mutually orthogonal projections $\{\Pi_1, \Pi_2, \ldots\}$, and $f(\mathrm{I}_{\mathcal{H}}) = 1$ where $\mathrm{I}_{\mathcal{H}}$ is the identity operator on $\mathcal{H}$, admits an expression

$$f(\Pi) = \mathrm{Tr}(\Pi\rho),$$

for some density operator ρ on $\mathcal{H}$.

Kochen-Specker theorem

Structure of measurements → Constraint on the structure of states

Formal statement

Let $\mathcal{H}$ be a separable Hilbert space of dimension at least three. There does not exist a map $c : \mathcal{P}(\mathcal{H}) \to \{0, 1\}$ satisfying

$$c(\Pi_1) + c(\Pi_2) + \cdots = c(\Pi_1 + \Pi_2 + \cdots)\,,$$

for any set of mutually orthogonal projections $\{\Pi_1, \Pi_2, \ldots\}$, and $c(\mathrm{I}_\mathcal{H}) = 1$ where $\mathrm{I}_\mathcal{H}$ is the identity operator on $\mathcal{H}$.

Gleason vs. KS

Gleason → Kochen-Specker

However: KS only requires a finite set of projectors (KS set)

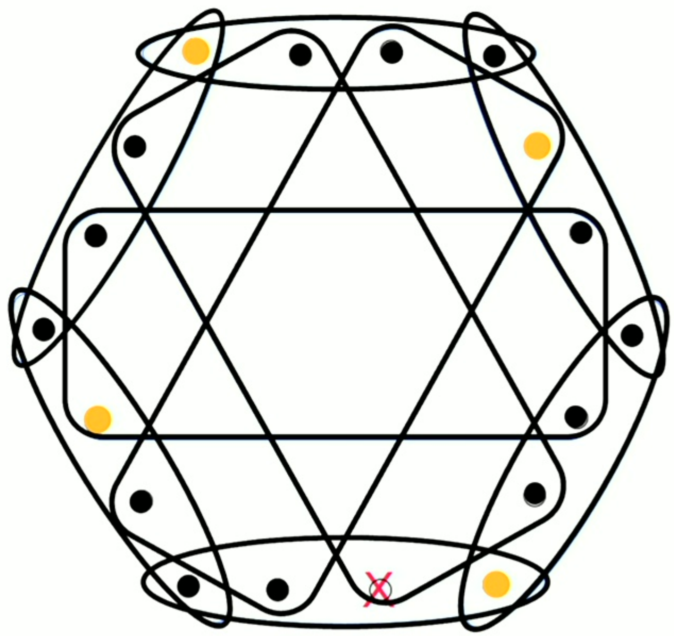
Kochen-Specker theorem

Two kinds of proof of KS:

- Logical proofs: proceed from logical contradiction, structure of measurements enough

- Statistical proofs: proceed from statistical violations of inequalities, require structure of states

Example of a logical proof in d=4



18 ray KS set

[*cf.* Cabello *et al.*, Phys. Lett. A 212 (1996) 183-187]

Probability 1

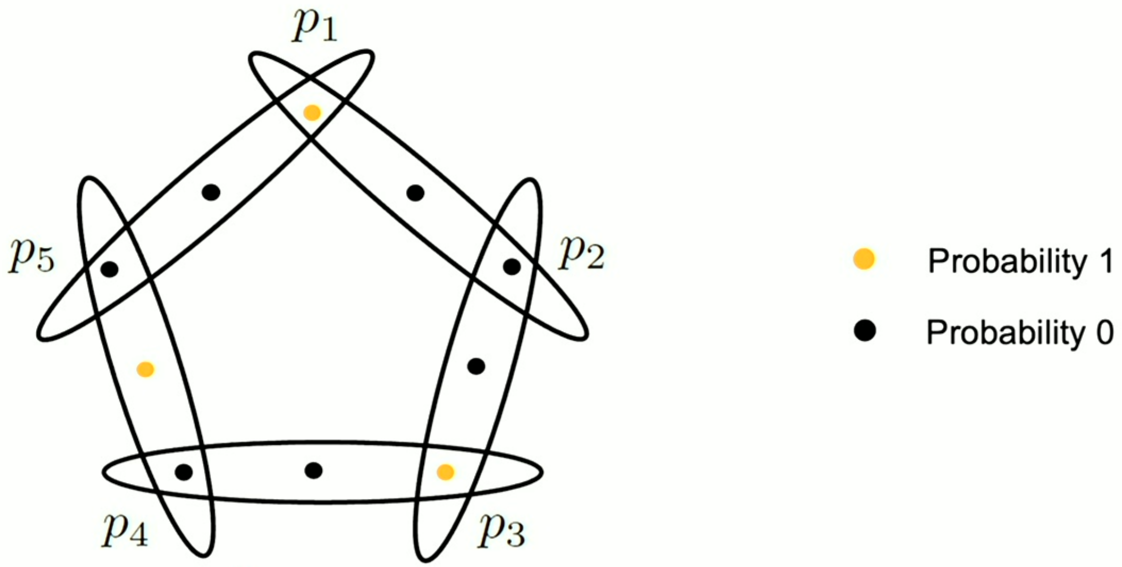
Probability 0

9 bases: odd number of ●

every ● appears in two bases: even number of ●

contradiction!

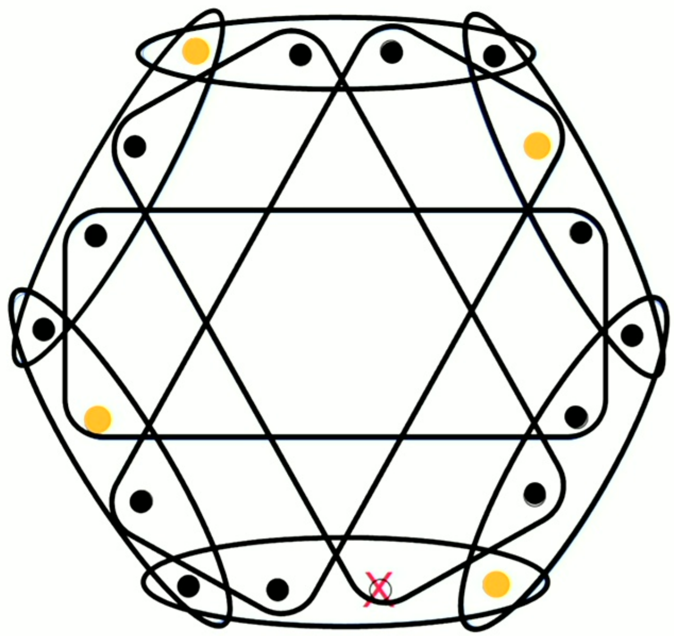
Example of a statistical proof in d=3



$$\sum_{i=1}^{5} p_i \leq 2$$

[*cf.* Klyachko *et al.*, Phys. Rev. Lett. 101, 020403 (2008)]

Example of a logical proof in d=4



18 ray KS set

[*cf.* Cabello *et al.*, Phys. Lett. A 212 (1996) 183-187]

● Probability 1

● Probability 0

9 bases: odd number of ●

every ● appears in two bases:
even number of ●

contradiction!

Peres-Mermin square (logical proof, d=4)

$X \otimes I$	$I \otimes X$	$X \otimes X$	$I \otimes I$
$I \otimes Y$	$Y \otimes I$	$Y \otimes Y$	$I \otimes I$
$X \otimes Y$	$Y \otimes X$	$Z \otimes Z$	$I \otimes I$
$I \otimes I$	$I \otimes I$	$-I \otimes I$	

$$v(X \otimes I)v(I \otimes X)v(X \otimes X) = 1$$
$$v(I \otimes Y)v(Y \otimes I)v(Y \otimes Y) = 1$$
$$v(X \otimes Y)v(Y \otimes X)v(Z \otimes Z) = 1$$
$$v(X \otimes I)v(I \otimes Y)v(X \otimes Y) = 1$$
$$v(I \otimes X)v(Y \otimes I)v(Y \otimes X) = 1$$
$$v(X \otimes X)v(Y \otimes Y)v(Z \otimes Z) = -1$$

A simultaneous solution in {+1,-1} doesn't exist!

Peres-Mermin requires entanglement:
e.g., the third column requires measurement in an entangled basis (Bell basis)

[*cf.* N.D. Mermin, Rev. Mod. Phys. 65, 803 (1993)]

Necessity of entangled measurements

Any logical proof of the KS theorem on a multiqubit system necessarily requires entangled measurements

- Non-LOCC unentangled measurements offer no advantage over LOCC measurements in logical proofs of KS
- Presence of entanglement in Peres-Mermin is not accidental: it is necessary in any such proof

Necessity of entangled states

Any statistical proof of the KS theorem on a multiqubit system with unentangled measurements necessarily requires an entangled state

Are entangled states sufficient for such a proof?

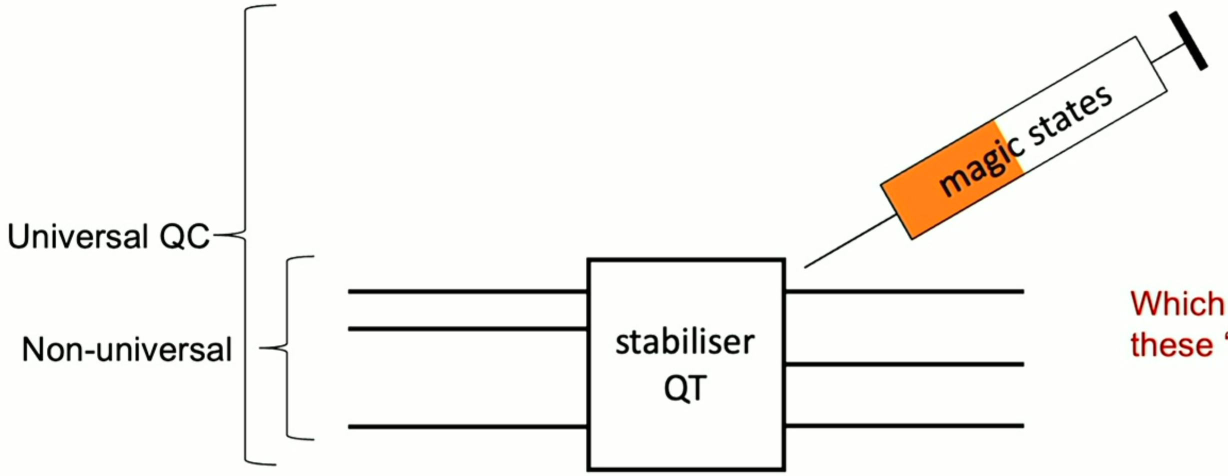
Gleason vs. KS for multiqudit systems

	when dim($\mathcal{H}_j$):	≥ 3 for all j	2 for j and ≥ 3 for j'	2 for all j
Unentangled	Gleason	yes	no	no
	K-S	yes **+direct**	**yes** **+direct**	**no**

- +direct: result also holds for the subset of unentangled measurements given by direct product bases

- bold and purple: our results

Quantum computation with state-injection (QCSI)



Which properties pick out these 'magic' states?

S. Bravyi and A. Kitaev, Phys. Rev. A 71, 022316 (2005)

Contextuality as a resource in QCSI

ARTICLE

doi:10.1038/nature13460

Contextuality supplies the 'magic' for quantum computation

Mark Howard[1,2], Joel Wallman[2], Victor Veitch[2,3] & Joseph Emerson[2]

Quantum computers promise dramatic advantages over their classical counterparts, but the source of the power in quantum computing has remained elusive. Here we prove a remarkable equivalence between the onset of contextuality and the possibility of universal quantum computation via 'magic state' distillation, which is the leading model for experimentally realizing a fault-tolerant quantum computer. This is a conceptually satisfying link, because contextuality, which precludes a simple 'hidden variable' model of quantum mechanics, provides one of the fundamental characterizations of uniquely quantum phenomena. Furthermore, this connection suggests a unifying paradigm for the resources of quantum information: the non-locality of quantum theory is a particular kind of contextuality, and non-locality is already known to be a critical resource for achieving advantages with quantum communication. In addition to clarifying these fundamental issues, this work advances the resource framework for quantum computation, which has a number of practical applications, such as characterizing the efficiency and trade-offs between distinct theoretical and experimental schemes for achieving robust quantum computation, and putting bounds on the overhead cost for the classical simulation of quantum algorithms.

Works for "quopits",
fails for qubits because of
logical KS

PRL 119, 120505 (2017) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2017

Contextuality as a Resource for Models of Quantum Computation with Qubits

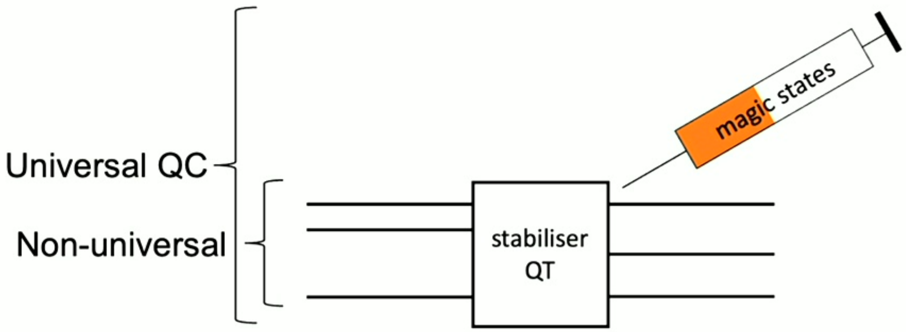
Juan Bermejo-Vega,[1,2] Nicolas Delfosse,[3,4] Dan E. Browne,[5] Cihan Okay,[6] and Robert Raussendorf[7]
[1]*Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany*
[2]*Max-Planck Institut für Quantum Optics, Theory Division, 85748 Garching, Germany*
[3]*Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA*
[4]*Department of Physics and Astronomy, University of California, Riverside, California 92521, USA*
[5]*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*
[6]*Department of Mathematics, University of Western Ontario, London, Ontario N6A 5B7, Canada*
[7]*Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada*
(Received 16 November 2016; published 21 September 2017)

A central question in quantum computation is to identify the resources that are responsible for quantum speed-up. Quantum contextuality has been recently shown to be a resource for quantum computation with magic states for odd-prime dimensional qudits and two-dimensional systems with real wave functions. The phenomenon of state-independent contextuality poses *a priori* an obstruction to characterizing the case of regular qubits, the fundamental building block of quantum computation. Here, we establish contextuality of magic states as a necessary resource for a large class of quantum computation schemes on qubits. We illustrate our result with a concrete scheme related to measurement-based quantum computation.

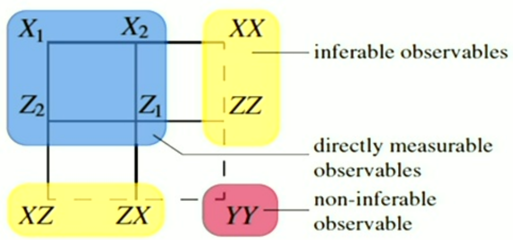
DOI: 10.1103/PhysRevLett.119.120505

Works for qubits, requires 'resource
character' restriction to
avoid logical KS

Restrictions on QCSI schemes



Introduce restrictions for qubit circuits



[J. Bermejo-Vega *et al.*, PRL 2017]

Restriction on 'resource character': there exists a quantum state w/o contextuality w.r.t. available mmts in the scheme.

Amounts to, effectively, no entangled measurements in the proposed schemes

All entanglement comes from the magic states

Which entangled states are magical?

- Non-stabilizer, can lift restricted stabilizer circuits to universality
- A necessary condition: Bell inequality violation
- Other necessary/sufficient conditions?

PRX QUANTUM **3,** 020333 (2022)

Many-Body Quantum Magic

Zi-Wen Liu[1,*] and Andreas Winter[2,3,†]

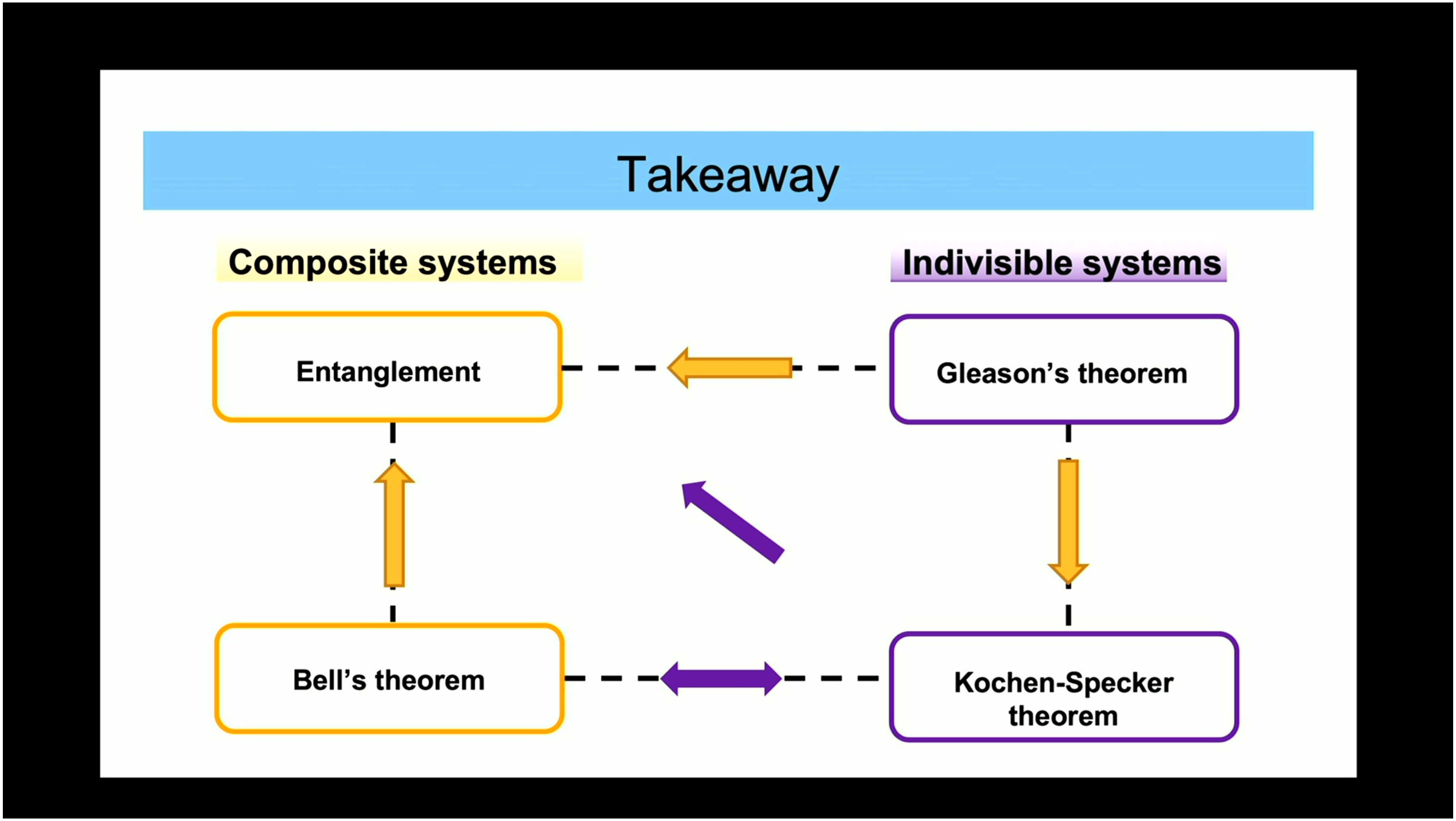
[1] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[2] *ICREA—Institució Catalana de Recerca i Estudis Avançats, Pg. Lluis Companys, 23, Barcelona ES-08001, Spain*
[3] *Física Teòrica: Informació i Fenòmens Quàntics, Departament de Física, Universitat Autònoma de Barcelona, Bellaterra (Barcelona) ES-08193, Spain*

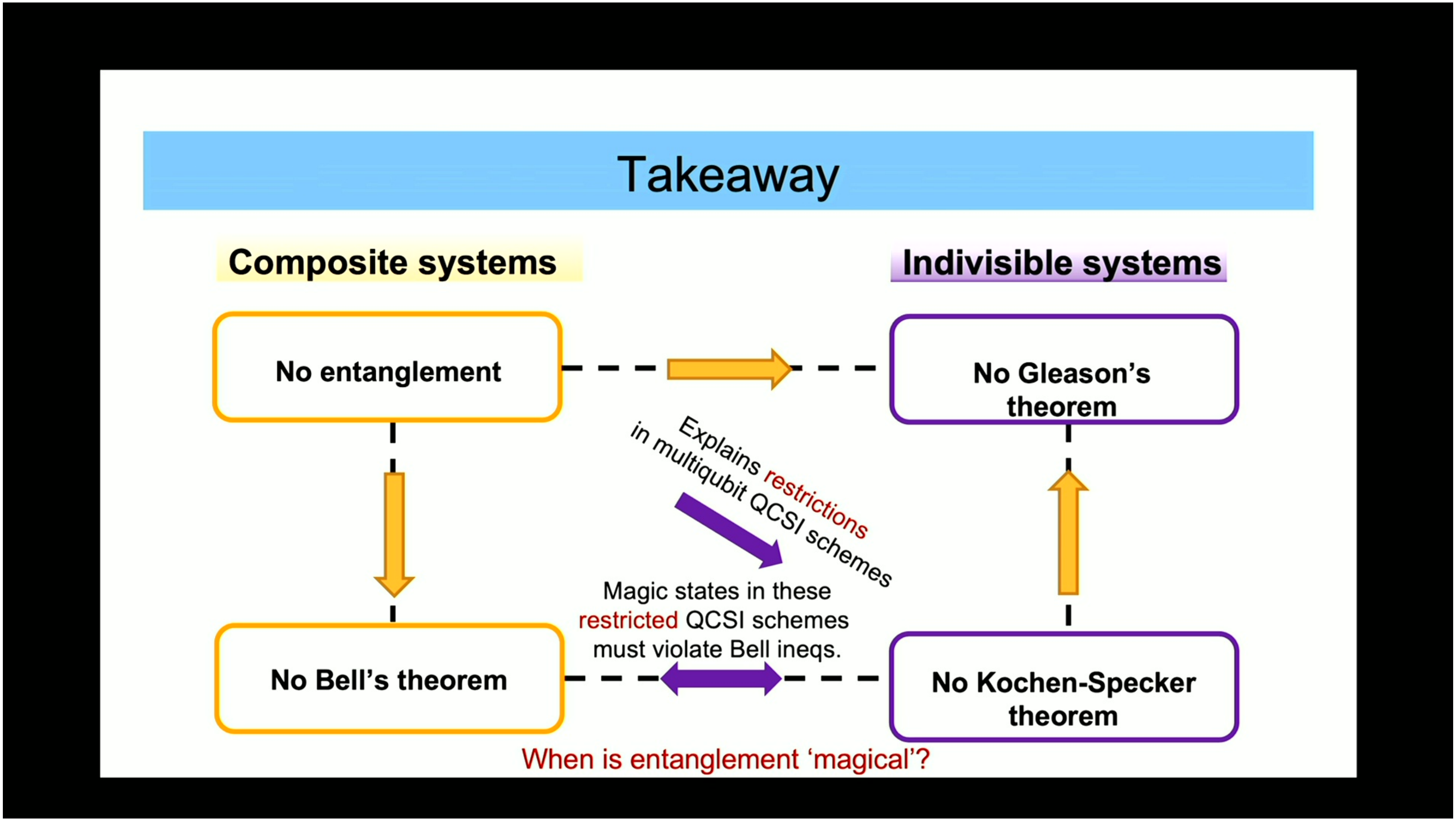
(Received 21 November 2020; revised 28 February 2022; accepted 11 April 2022; published 12 May 2022)

Magic (nonstabilizerness) is a necessary but "expensive" kind of "fuel" to drive universal fault-tolerant quantum computation. To properly study and characterize the origin of quantum "complexity" in computation as well as physics, it is crucial to develop a rigorous understanding of the quantification of magic. Previous studies of magic mostly focused on small systems and largely relied on the discrete Wigner formalism (which is only well behaved in odd prime power dimensions). Here we present an initiatory study of the magic of genuinely many-body quantum states that may be strongly entangled, with focus on the important case of many qubits, at a quantitative level. We first address the basic question of how "magical" a many-body state can be, and show that the maximum magic of an n-qubit state is essentially n, simultaneously for a range of "good" magic measures. As a corollary, the resource theory of magic has

Which entangled states are magical?

From a foundational perspective, restricted QCSI models are a potentially fruitful avenue to understand the role of entanglement/magic vis-à-vis contextuality in universal quantum computation


Takeaway
Composite systems
Indivisible systems
Entanglement
Gleason's theorem
Bell's theorem
Kochen-Specker theorem


Takeaway
Composite systems
Indivisible systems
No entanglement
No Gleason's theorem
Explains restrictions in multiqubit QCSI schemes
Magic states in these restricted QCSI schemes must violate Bell ineqs.
No Bell's theorem
No Kochen-Specker theorem
When is entanglement 'magical'?

Thanks!

Check out this paper for more:

V.J. Wright and R. Kunjwal
Quantum 7, 900 (2023)