Title: Gong Show

Speakers:

Collection: Foundations of Quantum Computational Advantage

Date: May 01, 2024 - 1:45 PM

URL: https://pirsa.org/24050007

Abstract: IN PERSON: Kim Vallée, Thomas Vinet

VIRTUAL: Farid Shahandeh, Nitica Sakharwade, Shashank Virmani, Rafael Wagner, Roberto Dobal Baldijao

Resource-theoretic hierarchy of generalized contextuality

FoQaCia conference, Waterloo - 30/04/2024

Lorenzo Catani

Joint work with Thomas Galley and Tomáš Gonda

INTERNATIONAL IBERIAN NANOTECHNOLOGY LABORATORY

FoQaCiA

- Resource theory of generalized contextuality in the framework of general probabilistic theories (GPT).

Objects ⟹ GPT systems.

Free operations ⟹ Embeddings with free access to classical systems.

D. Schmid, J. H. Selby, E. Wolfe, R. Kunjwal, and R. W. Spekkens, PRX Quantum 2, 010331 (2021).

M.P. Müller and A. J. P. Garner, PRX 13, 041001 (2023).

- Monotone is the *classical excess* $\varepsilon(\cdot, \Delta_{\mathbb{N}})$.

- Can contextuality be associated with information erasure?

Corrected inequalities for realistic experiments

Pierre-Emmanuel Émeriau

Boris Bourdoncle

Adel Sohbi

Shane Mansfield

Damian Markham

arxiv : 2310.19383   *Published version DOI : 10.1098/rsta.2023.0011*

Kim Vallée

Sorbonne Université, LIP6

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1

Some useful notions...

arxiv.org/abs/2310.19383

Sheaf theoretic framework (Abramsky & Brandenburger)

- Mathematical framework
- Kochen Specker & Bell inspired
- Non-locality & contextuality embedded

Contextual Fraction (CF)

- Quantitative measure of contextuality
- Between 0 and 1
- Convex quantity

Measurement scenario in

Representation of an experiment

- X : A set of labels for the observables
- M : A set of compatible observables
- O : A set of outcomes

Empirical behavior in

Realization of a measurement scenario

	00	01	10	11
AB	$1/2$	0	0	$1/2$
AB’	$1/2$	0	0	$1/2$
A’B	$1/2$	0	0	$1/2$
A’B’	0	$1/2$	$1/2$	0

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... Some useful results ...

Hidden variable assumptions
Outcome Determinism & Parameter Independence

Relax...
...the assumptions

η_λ outcome non-determinism

σ_λ parameter dependence

Main statement
For any measurement scenario
If $2\eta + \sigma < 1$

$$CF(empirical\ behaviour) \leq \eta$$

$(\sigma_\lambda \leq \sigma, \eta_\lambda \leq \eta)$

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... Some useful followups ?

Does η have a relation with generalized contextuality ?

How to get η and σ experimentally ?

How to treat experiments with vanishing η ?

Thank you

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Higher-order blind quantum computation

Thomas Vinet[1,2], Hlér Kristjánsson[1]

1. Perimeter Institute for Theoretical Physics, Waterloo, Ontario

2. Télécom Paris, Palaiseau

Foundations of Quantum Computational Advantage (FoQaCia)

Introduction

- In the future: few people will be able to do big computations
- Can a client, with no computational power, gain advantage of those servers without compromising its data?
- **Blind Quantum Computation**: Multiple protocols developed (Childs[1], Universal Blind Quantum Computation[2])
- Works for computation on states, can we extend this to computation of subroutines?

[1] Childs, *Quantum Information and Computation* **5**, ISSN: 1533-7146, (http://dx.doi.org/10.26421/QIC5.6) (Sept. 2005).

[2] A. Broadbent *et al.*, presented at the 2009 50th Annual IEEE Symposium on Foundations of Computer Science, (http://dx.doi.org/10.1109/FOCS.2009.36).

Contributions

- Transformations of unitaries are described by higher-order maps: $f(U) = S(U^{\otimes n})$

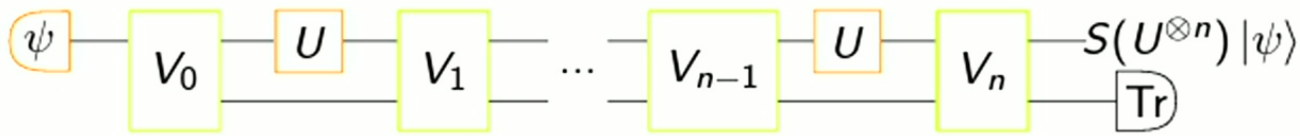
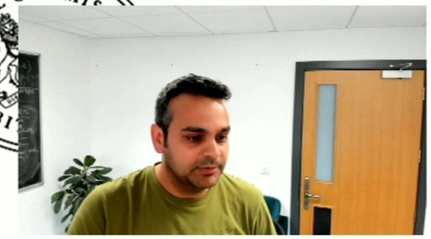


Figure: For each function causally defined on unitaries f, there exists a decomposition[3] as above S such that $S(U^{\otimes n}) = f(U)$

- First protocol: Alice knows the implementation of the higher-order map, protocol uses multiple rounds of communication
- Second protocol: Alice knows the function, Bob knows the implementation $\rightarrow$ Bob wants to keep its implementation private !
  - We search for commuting matrices that do not give away the unitary, while creating a fully mixed state from Bob's point of view

[3] G. Chiribella *et al.*, *EPL (Europhysics Letters)* **83**, 30004, ISSN: 1286-4854, (http://dx.doi.org/10.1209/0295-5075/83/30004) (July 2008).



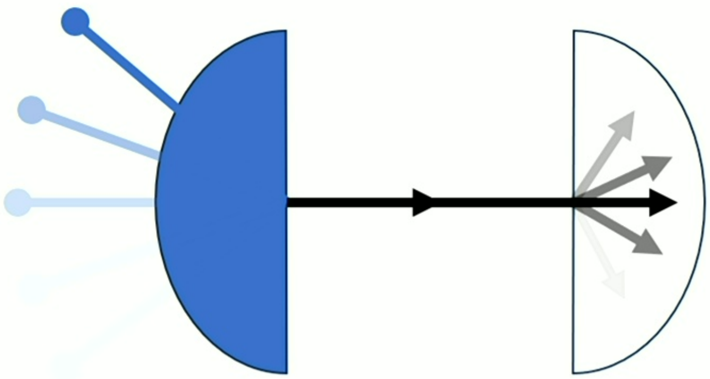
Quantum Computing & Simulation Hub

Contextuality via Dimension Witnesses

Theo Yianni[1], Mina Doosti[2], Farid Shahandeh[1]

[1]Royal Holloway University of London

[2]University of Edinburgh



preparations

$$T := \begin{bmatrix} p_{11} & \cdots & \\ \vdots & \ddots & \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}$$

Conditional
Outcome
Probability
Evaluation matrix

How is quantum manifested in T?
Quantum can tell us the rank of T.

What is the cardinality of the smallest ontic space?
It's NP-hard to tell!

Barrett, PRA (2007), **FS**, PRXQuantum (2021)

We can lower bound the size of the smallest ontic space:

Theorem 1. *Suppose T is a COPE for n preparations and n measurement outcomes. S*
(columns) of the matrix have at least one zero entry that differs from all other rows (co
the impossibility of a specific outcome given a specific preparation. Then,

(a) *the dimension of the ontic space $\mathcal{V}_{\text{ont}}$ for any ontological model reproducing A is lower bounded by the smallest integer m such that $n \leq \binom{m}{\lfloor m/2 \rfloor}$, and,*

(b) *the ontic distributions or the indicator functions span an l-dimensional subspace of $\mathcal{V}_{\text{ont}}$ for any integer l for which $\binom{l}{\lfloor l/2 \rfloor} \leq n$.*

$$T := \begin{bmatrix} 1 & 0 & \cdots & p_{1,251} & p_{1,252} \\ 0 & 1 & \cdots & p_{2,251} & p_{2,252} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{251,1} & p_{25212} & \cdots & 1 & 0 \\ p_{252,1} & p_{252,2} & \cdots & 0 & 1 \end{bmatrix}_{252\times 252}$$

rank of T no more than 4!

$m \geq 10$

$\boldsymbol{m = \Omega(\log(n))}$

Hardy, Studies in History and Philosophy of Modern Physics (2004)

Corollary 1. *An operational theory is ontologically noncontextual if and only if the dimensionalities of the vector spaces spanned by its corresponding ontic distributions and indicator fun*

$$T := \begin{bmatrix} 1 & 0 & \cdots & p_{1,251} & p_{1,252} \\ 0 & 1 & \cdots & p_{2,251} & p_{2,252} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{251,1} & p_{25212} & \cdots & 1 & 0 \\ p_{252,1} & p_{252,2} & \cdots & 0 & 1 \end{bmatrix} = \begin{bmatrix} & \end{bmatrix} \begin{bmatrix} & \end{bmatrix}$$

$\text{rank} \leq 4$

ontic indicator functions $\text{rank} \geq 10$

ontic probability distributions $\text{rank} \leq 4$

A single qubit is contextual.

Contextuality of many protocols is easy to prove now:
MESD[1], all clonings (state-dependent[2], **phase covariant, universal**)

[1]Schmid, PRX (2018), [2]Lostaglio, Quantum (2020)

Unbounded Quantum Advantage in Strong Communication Complexity of Relations

Nitica Sakharwade
Postdoctoral researcher
University of Naples Federico II
Università degli Studi di Napoli Federico II

Work with:
Sumit Rout
Some Sankar Bhattacharya
Ravishankar Ramanathan
Paweł Horodecki

arXiv:2305.10372

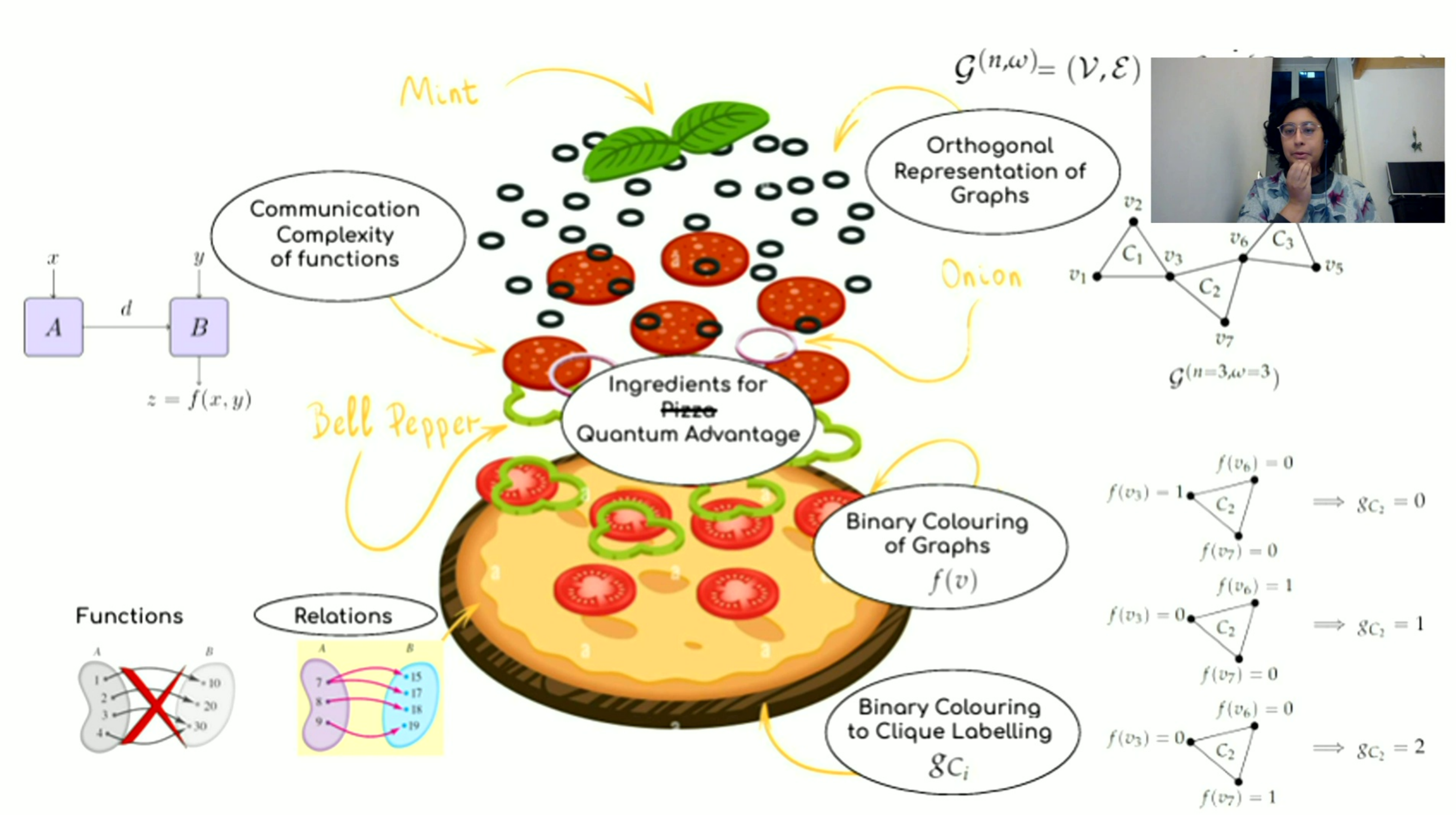
ICTQT

30 April - 3 May 2024

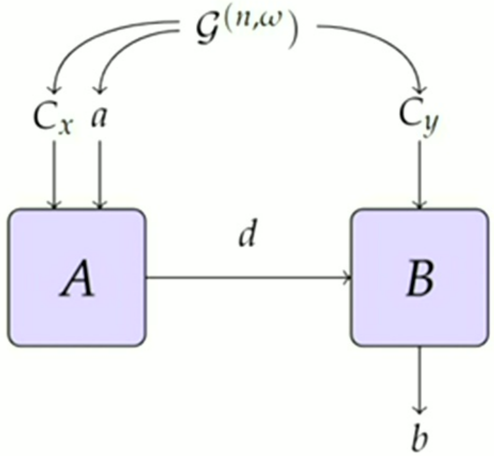
Foundations of
Quantum Computational
Advantage Conference

FoQaCiA

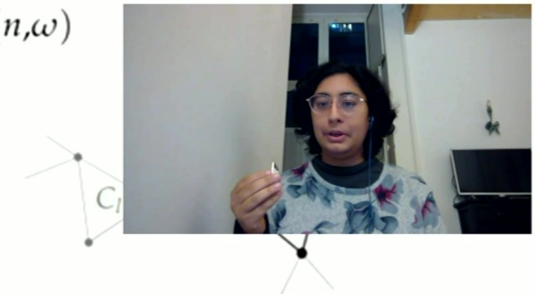
PI Perimeter Institute


$\mathcal{G}^{(n,\omega)} = (\mathcal{V}, \mathcal{E})$
Mint
Orthogonal Representation of Graphs
Communication Complexity of functions
Onion
x
y
A
d
B
$z = f(x, y)$
Ingredients for Pizza Quantum Advantage
Bell Pepper
Binary Colouring of Graphs $f(v)$
Binary Colouring to Clique Labelling g_{C_i}
Functions
Relations
v_1 v_2 v_3 v_5 v_6 v_7 C_1 C_2 C_3
$\mathcal{G}^{(n=3,\omega=3)}$
$f(v_6) = 0$, $f(v_3) = 1$, $f(v_7) = 0$ $\implies g_{C_2} = 0$
$f(v_6) = 1$, $f(v_3) = 0$, $f(v_7) = 0$ $\implies g_{C_2} = 1$
$f(v_6) = 0$, $f(v_3) = 0$, $f(v_7) = 1$ $\implies g_{C_2} = 2$

Communication Complexity of relations



$\mathcal{G}^{(n,\omega)}$



Communication Complexity of relations

(C_x, a, C_y, b)
$\in \mathcal{R}_{CLP}(\mathcal{G}^{(n,\omega)})$.

No Quantum Advantage!

Strong - Communication Complexity of relations

$\{(C_x^i, a^i, C_y^i, b^i) | i \in$ runs of the experiments$\}$

Reconstructor

$\mathcal{R}_{CLP}(\mathcal{G}^{(n,\omega)})$

Unbounded Quantum Advantage!

Efficient classical simulation of variants of clu
computation

Shashank Virmani
Brunel University London

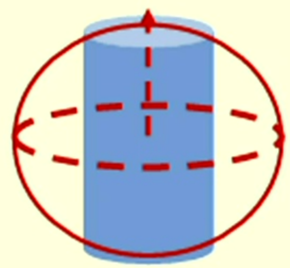
Sahar Atallah (Brunel), Michael Garn (STFC), Sania Jevtic (Phytoform Labs), Yukuan Tao (Dartmouth), SV (Brunel), **Quantum 8, 1243 (2024)**

&

Sahar Atallah (Brunel), Michael Garn (STFC), Sania Jevtic (Phytoform Labs), Yukuan Tao (Dartmouth), SV (Brunel), **arXiv:2307.01800**

Sometimes it is fun (e.g. discrete Wigner, quasi-probabilities) to think about spaces of nor

Here we will consider a strange universe in which computers are built from non-physical
of arbitrary radius:



Cylinders stick out of Bloch sphere.

WHY DO THIS? **It will give us efficient classical simulations of systems previously unknown to be classical**

Cylindrical Computers:

1) *Place cylinder states (of some arbitrary radius) on nodes of a graph of finite degree D.*
2) *Interact with CZs on edges, or in fact any diagonal gate.*
3) *Measure destructively using Z or XY plane measurements – probabilities given by usual Born rule.* **Yes, we will have to be careful to avoid negative probabilities** – later.

Just like cluster state computation, but with cylinder extrema replacing $|+\rangle$ state inputs.

All based on the following entanglement/separability related fact:

Diagonal Gate$(\rho_A \otimes \rho_B) = \sum_i p_i \, \rho_A^i \otimes \rho_B^i$ ← $\lambda >$

E.g. for CZ gate, $\lambda = \left(\sqrt{5} - 2\right)^{-1/2} \approx 2.06$

As long as you keep growing your cylinders with every gate applied, there is no "cylindrical" entanglement.

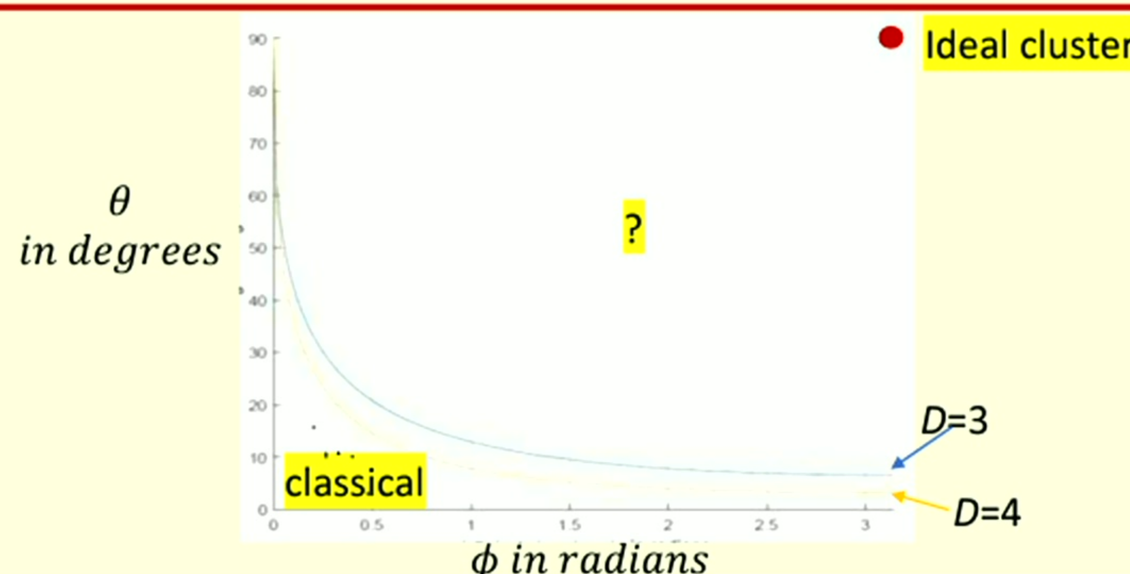
⇒ **NEW CLASSICAL SIMULATION METHOD!** As long as the grown cylinders have $r \leq 1$, this gives an **efficiently simulatable (sampling to additive error) LHV model**. Helps populate part of computational "phase diagram":

E.g. Contained in our $r \leq 1$ universe:

Qubits: $\cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle$
for $\theta \leq sin^{-1}(\lambda^{-D})$

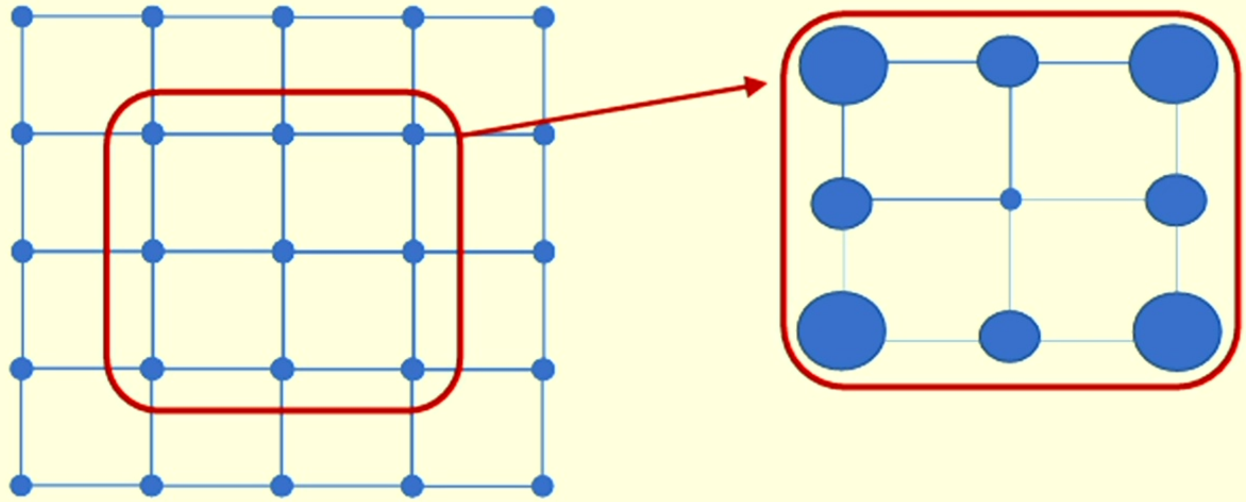
Interactions: Control-ϕ gate := $|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + e^{i\phi}|11\rangle\langle 11|$

Measurements: destructive Z or XY plane
(like cluster state quantum computers)



Further results + generalisation + coarse graining + conjecture:

1) Cylinders may be "slowest" growing state spaces that maintain separable decomp –
2) Analogue of cylinders + Z/XY measurements exists for any diagonal qudit gates (in fa spaces" to maintain separability can be applied to any gate).
3) Coarse graining: cut into blocks, and only grow boundary cylinders:



If initial *r* is such that block gives valid probabilities, then classically simulatable efficiently. Increases efficiently simulatable inputs, e.g. for CZ on square lattice may (?) double region

A conjecture: for some lattices our weird cylindrical computers either give negative probabilities (i.e. the sampling problem is ill-defined), or can be efficiently simulated with this coarse graining approach.

Fundação para a Ciência e a Tecnologia

Funding

CERTIFYING NONSTABILIZERNESS IN QUANTUM PROCESSORS

Rafael Wagner

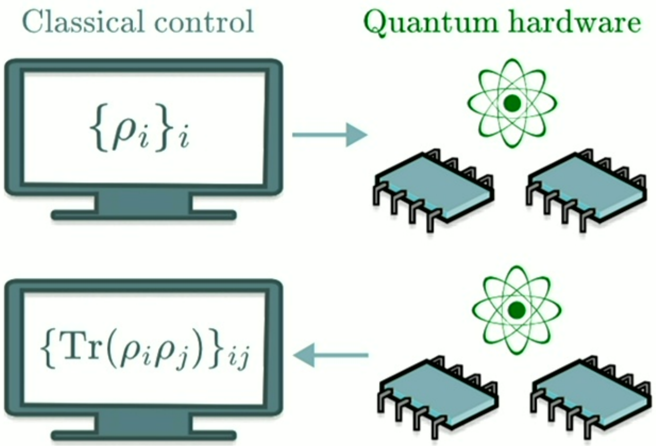
International Iberian Nanotechnology Laboratory (INL)

University of Minho (UM)

Filipa C. R. Peres
(University of Porto/INL)

Emmanuel Zambrini Cruzeiro
(IST/Instituto de Telecomunicações)

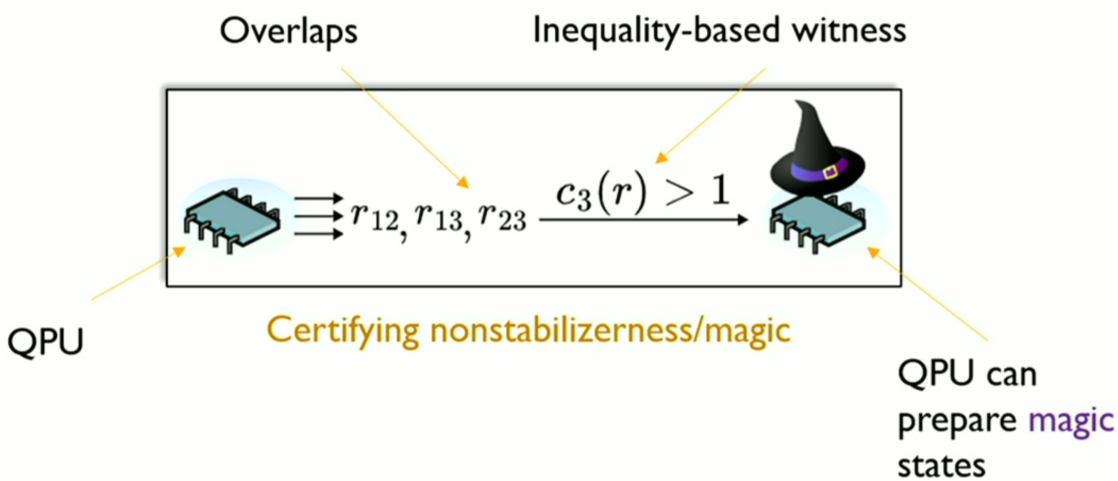
Ernesto F. Galvão
(UFF/INL)

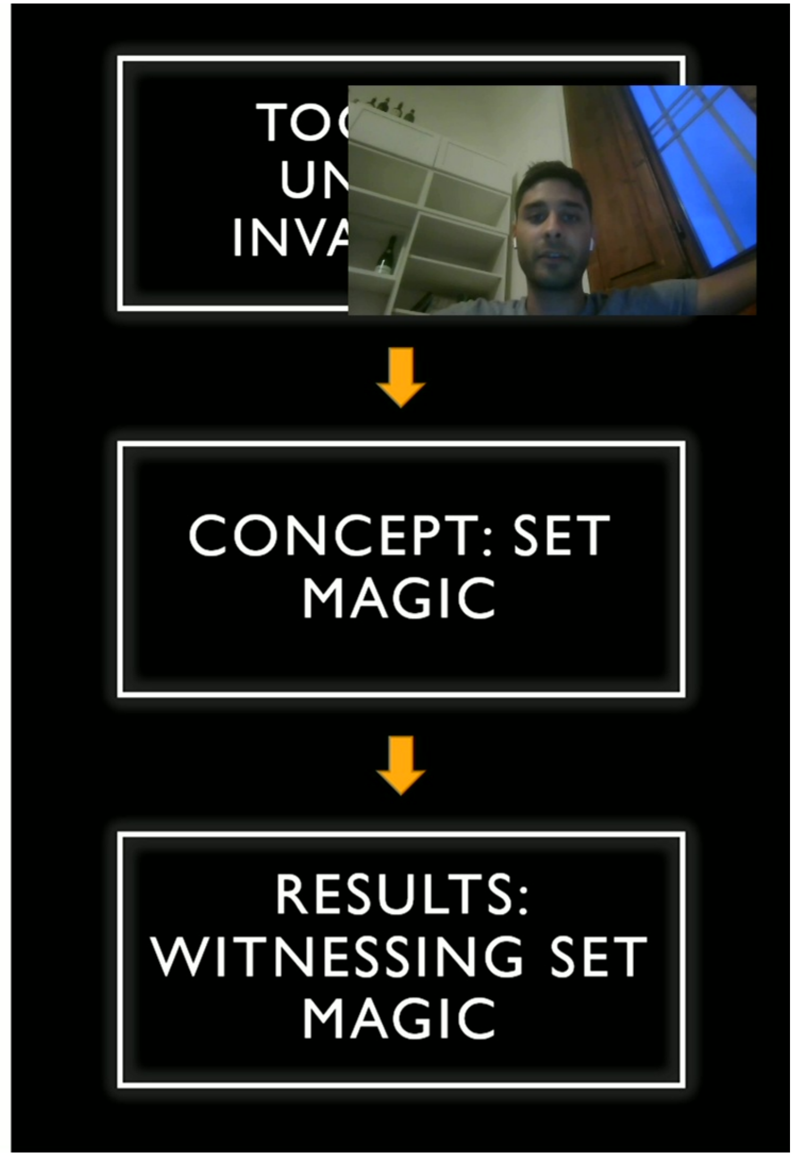


- Efficient to estimate
- Robust to noise
- Semi-device independent
- Unitary-invariant

$$\mathrm{Tr}(\rho\sigma) = \mathrm{Tr}(U\rho U^\dagger U\sigma U^\dagger)$$

- No method exists that certify magic in a robust, efficiently measurable, and semi-device independent way





Set magic

$\exists i \;\; \rho_i \notin \mathrm{STAB}$

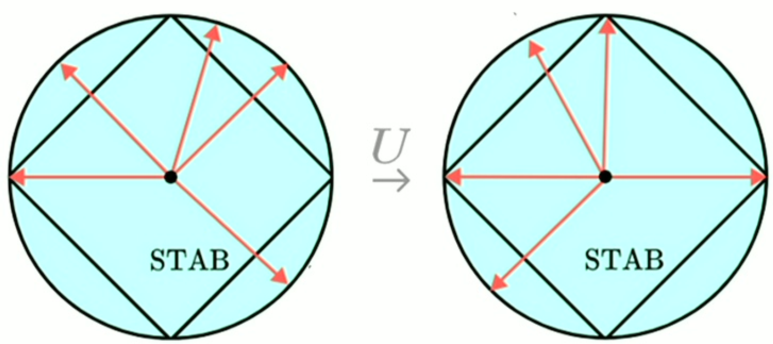
$\forall U \uparrow$

$\{ \rho_1 \quad \rho_2 \quad \rho_3 \quad \rho_4 \}$

$\exists U \downarrow$

$\forall i \, \rho_i \in \mathrm{STAB}$

Set stabilizer



4 magic states 2 magic states

TOO
UN
INVA

CONCEPT: SET MAGIC

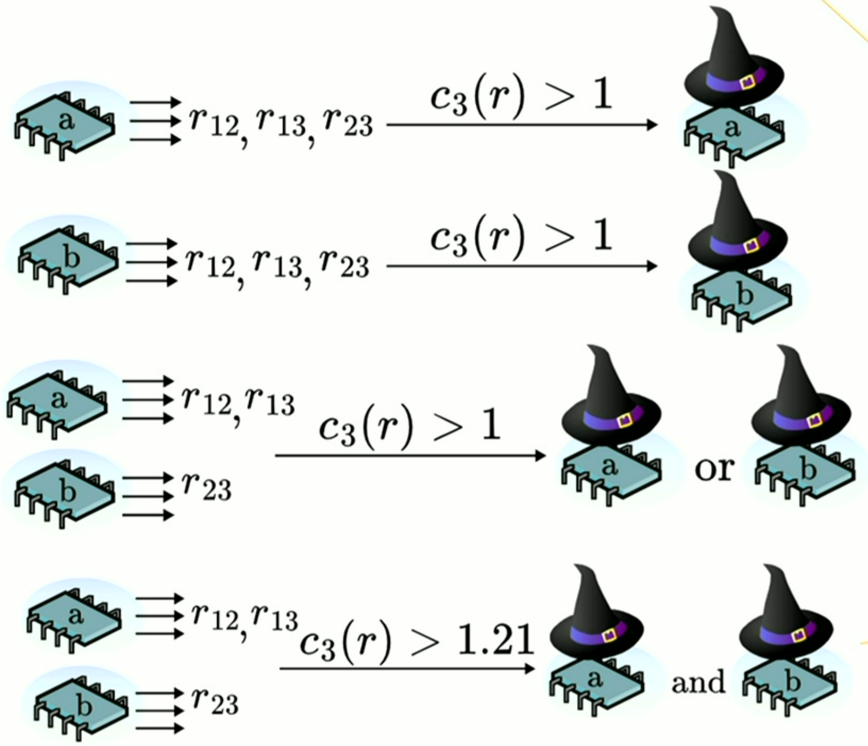
RESULTS: WITNESSING SET MAGIC

These inequalities are witnesses of set magic

$$c_m(r) = -r_{1,2} + r_{2,3} + \cdots + r_{m,m-1} + r_{1,m} \leq m - 2$$

$$h_4(r) := r_{1,2} + r_{1,3} + r_{1,4} - r_{2,3} - r_{2,4} - r_{3,4} \leq 1$$

$$r_{i,j} := \mathrm{Tr}(\rho_i \rho_j)$$

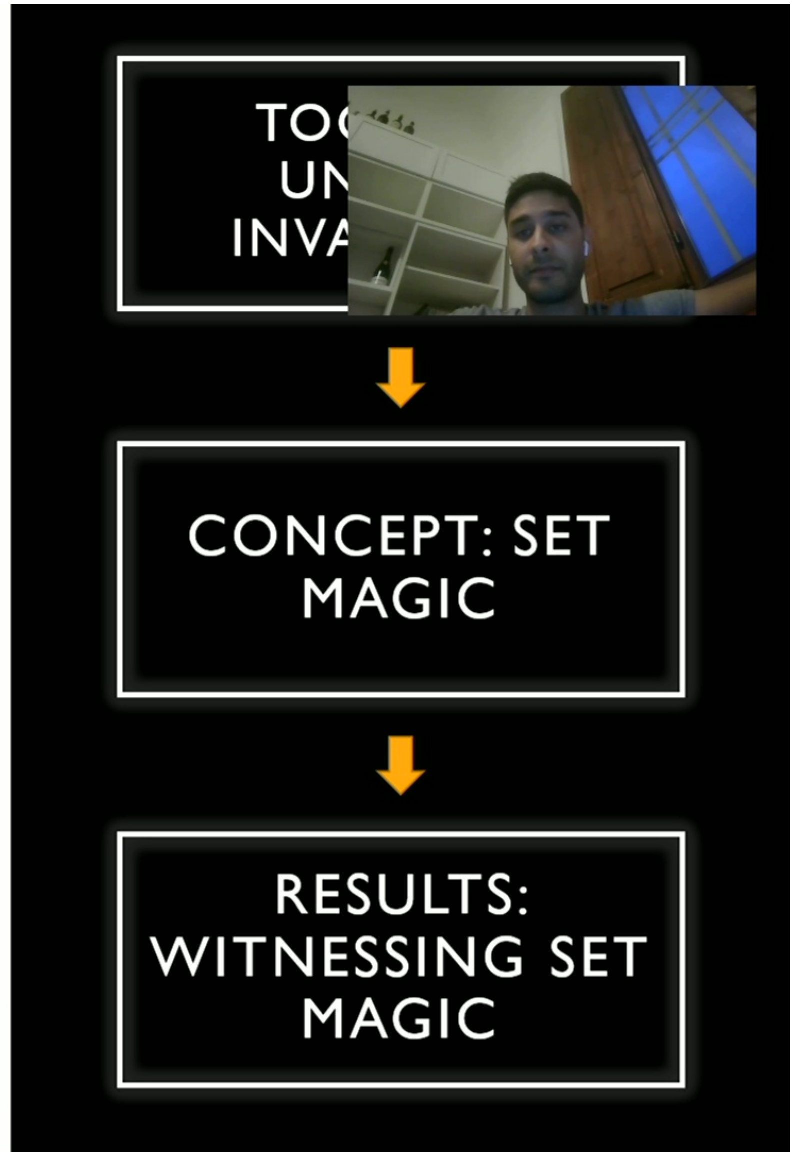


This inequality also certify Hilbert space dimension

Distribute the certification

All states (but one) must be magic

Thank you for your attention ☺



Transformation Noncontextuality Inequalities in prepare-transform-measure fragments

David Schmid, **Roberto D Baldijao**, John Selby, Ana Belen-Sainz

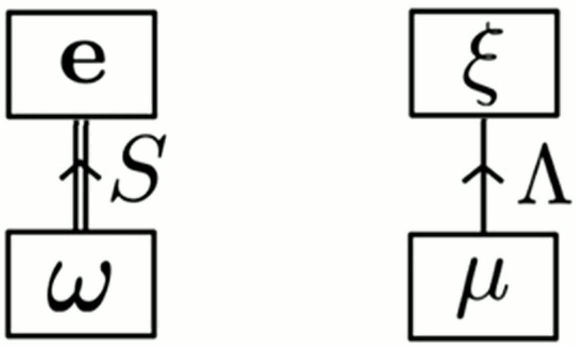
FoQaCiA Conference, April 2024

Funded by
the European Union

From PM to PTM and ontological mod

Noncontextuality ⟺ Ontological Model for GPT fragmen

PM fragment



Operational identities

$$\left\{\sum_s \alpha_s^{(a)} \boldsymbol{\omega}_s = 0\right\}_a$$

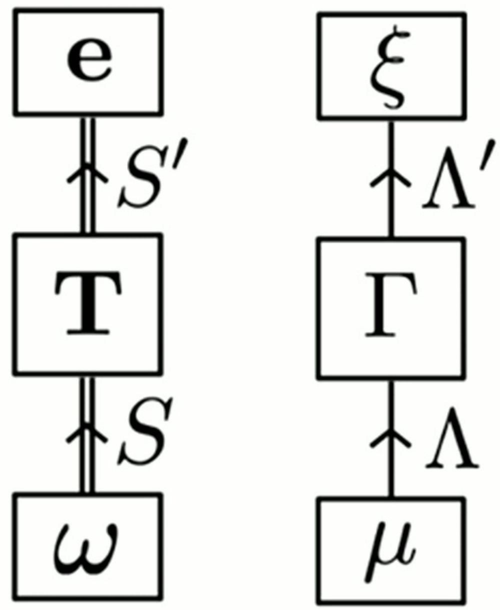
Constraints in the OM

$$\forall \lambda, (a): \quad \sum_s \alpha_s^{(a)} \mu_s(\lambda) = 0.$$

David Schmid, Robert W. Spekkens, and Elie Wolfe, Phys. Rev. A **97**, 062103

Anirudh Krishna, Robert W Spekkens, and Elie Wolfe, *New J. Phys.* **19** 123031

PTM fragment



Compositions may imply different kinds of operational identities!

Operational identities*
(can be assessed!)

$$\sum_t \alpha_t^{(c)} \mathbf{T}_t = 0,$$

Constraints in the OM

$$\forall \lambda, (c): \quad \sum_t \alpha_t^{(c)} \Gamma_t(\lambda'|\lambda) = 0.$$

➡ **Linear constraints!**

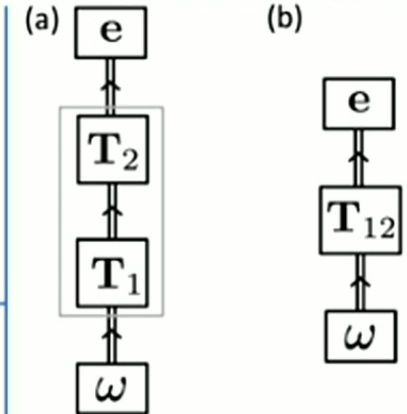
Results

We get a **Linear Program**!

- All the inequalities (necessary AND sufficient for classicality) in PTM
- Witnesses of classicality for general scenarios (necessary, but not sufficient)

PT…TM fragment: c something?

- Closure under sequential composition*
- Lumping procedure



Application: Single quantum stabilizer

- All stabilizer states and effects
- 4 stabilizer transformations: (Id, Z, S, ZS)

$$\begin{aligned}
&3\left[p_{112}-p_{221}+p_{464}+p_{423}+p_{533}-p_{544}-p_{614}+p_{632}+p_{641}\right]\\
&+2\left[-p_{131}+p_{232}-p_{333}+p_{434}-p_{511}-p_{512}-p_{523}\right]\\
&+p_{143}+p_{152}-p_{262}+p_{341}+p_{351}+p_{364}-p_{452}\\
&-p_{463}+p_{524}+p_{563}-p_{651}-p_{652}+p_{654}\\
&+5\left[p_{244}-p_{432}\right]\geq -6,
\end{aligned}$$