Title: Quantum Gravity Lecture

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Collection: Quantum Gravity 2023/24

Date: May 01, 2024 - 9:00 AM

URL: https://pirsa.org/24050001

GR

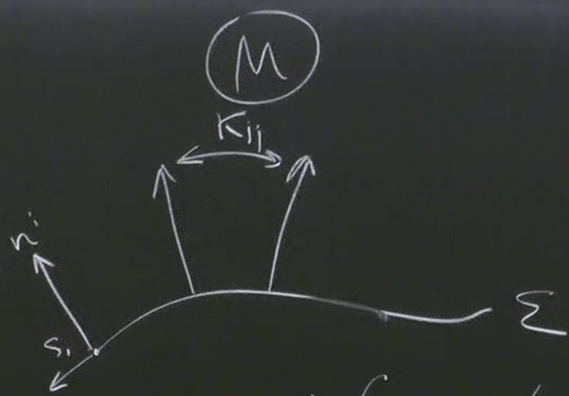
$$\tilde{\Theta}^a = -\frac{1}{2}\left(\nabla_b \,\delta g^{ab} - \nabla^a \delta g\right)$$

rmk: $\delta(\nabla g) \equiv 0 \Rightarrow \nabla \delta g \sim \delta\Gamma\, g$

$$\tilde{\Theta}^a = -\frac{1}{2}\left(g^{bc}\,\delta\Gamma^a_{bc} - g^{ab}\,\delta\Gamma^c_{bc}\right)$$

$$\Theta_\Sigma = \int_\Sigma \sqrt{g}\, n_a \tilde{\Theta}^a \sim \int_\Sigma \underbrace{n\cdot\delta\Gamma}_{\delta(\nabla n) \pm \nabla \delta n}$$

recall: $K_{ij} = h^a_i h^b_j \nabla_a n_b$



$$\beta^a = \delta n^a + g^{ab}\,\delta n_b$$

$$n_a \beta^a \equiv 0$$

$$\Theta_\Sigma = -\frac{1}{2}\int_\Sigma \sqrt{h}\left(K^{ij} - K h^{ij}\right)\delta h_{ij} + \delta\int_\Sigma \sqrt{h}\,(2K) + \underbrace{\oint_{\partial\Sigma}\sqrt{\gamma}\, s_i \beta^i}_{\equiv 0,\ \boxed{\partial\Sigma = \emptyset}}$$

extrinsic curvature

induced metric on Σ

$$K_{ij} = \frac{1}{2} L_n h_{ij} \sim \text{"}\frac{1}{2}\dot{h}_{ij}\text{"}$$

$$K = L_n \sqrt{h}$$

$= \phi$

(Off shell) canonical phase space:

$$\mathcal{F} \longrightarrow \mathcal{P}_{can}$$

$$g_{ab} \longmapsto \left(h_{ij} , \Pi^{ij} \right) = \left(\imath_\Sigma^* g_{ab} , \sqrt{h} \left(K^{ij} - K h^{ij} \right) \right)$$

$$\text{“} (q , p) = \left(\gamma|_\Sigma , \dot{\gamma}|_\Sigma \right) \text{”}$$

$$\omega_{can} = -\frac{1}{2} \int_\Sigma \mathbb{d} \Pi^{ij} \wedge \mathbb{d} h_{ij}$$

<u>Rmk</u>: h_{ij} is a 3d tensor

Π^{ij} is a — " — <u>density</u>

induced (3d) connection

if X is a 3d diffeo
$$L_X h_{ij} = \bar{\nabla}_i X_j + \bar{\nabla}_j X_i$$
$$L_X \Pi^{ij} = X^k \bar{\nabla}_k \Pi^{ij} - 2 \Pi^{k(i} \bar{\nabla}_k X^{j)} (\bar{\nabla} X) \Pi^{ij}$$

$(P_{can}, \omega_{can}) \equiv$ ADM ph. sp.

$$P_{can} = T^* \mathrm{Riem}(\Sigma)$$

SYMMETRIES

$$\mathrm{diff}(M) \simeq \mathfrak{X}^1(M) \ni \xi^a$$

$$\xi^a|_\Sigma = X^a + N n^a$$

$N \equiv - n \cdot \xi|_\Sigma$

$X^a \equiv h^a_b \xi^b \in T\Sigma$

Rmk. This decomposition requires knowledge of entire g_{ab} (i.e. h_{ij}, k_{ij} are not enough)

Noeth

ι^*_Σ

$Q_\Sigma($

Noether current

$$\iota^*_\Sigma \underline{J} = \underline{C}_{\Sigma a}\,\xi^a + d\underline{j}(\xi)$$

ignore b.c. $\partial\Sigma = \phi$

constraints

$$Q_\Sigma(\xi) \equiv \int_\Sigma \underline{J} = \int_\Sigma \underline{C}_{\Sigma b}\,\xi^b$$

$$= \int_\Sigma \underbrace{n_a\left(G^a_{\ b} + \Lambda\,\delta^a_{\ b}\right)}_{\text{constraints}}\xi^b$$

vector constr. Hamiltonian constr.

$$V(X) + H(N) = \int_\Sigma n_a\,G^a_{\ b}\,X^b + \int_\Sigma n_a n_b\left(G^{ab} + \Lambda g^{ab}\right)N$$

knowledge

are not energy

__Fact__: although G_{ab} contains 2nd "time" derivatives of h_{ij} ($\sim$ 1st "time" der. of K) which are __not__ ph. sp. variables, the constraints can be fully written in terms of (h, π)

$$V(X) = \int_\Sigma (\bar{\nabla}_i \pi^i{}_j) X^j$$

$$H(N) = \int_\Sigma \left[\frac{1}{2\sqrt{h}}\left(\pi : \pi - \frac{1}{2}\pi^2\right) - \frac{1}{2}\sqrt{h}\left({}^{(3)}\bar{R} - 2\Lambda\right)\right] N$$

__Rmk.__ Vector constr looks like Gauss!

$\rightarrow$ Tangential diffeos behave very much like YM gauge transf.

of entire job (i.e. h_{ij}, K_{ij} are not enough)

Rmk

$$\mathcal{H} = \frac{1}{2}\mathbb{G}(h)^{ijkl}\,\Pi_{ij}\Pi_{kl} - U(h)$$

$$U(h) = \frac{1}{2\ell_{pl}^2}\sqrt{h}\,\left(\bar{R} - 2\Lambda\right)$$

$$\mathbb{G}^{ijkl} = \ell_{pl}^2\left(h^{i(k}h^{l)j} - \frac{1}{2}h^{ij}h^{kl}\right)$$

DeWitt supermetric

$\hookrightarrow$ metric in Riem(Σ)

must be 1/2
to get a good diffeo
inv. theory, at the end
of the day.

Lik

$\varepsilon)$

Like for the parametrized particle
$\mathcal{H}$ is no "real" Hamiltonian, but a
constraint!

⇝ Formally physical states in QG
must satisfy

$$2\hat{\mathcal{H}}\Psi = 0$$ WHEELER-DEWITT

↑ wave f. of the universe

$$2\hat{\mathcal{H}}\Psi(h) = -G^{ijkl}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}}\Psi + \sqrt{h}\,(R-2\Lambda)\Psi = 0$$

$\mathcal{H}$

$\Rightarrow ($

$\mathcal{H}$ quadratic in π

$\Rightarrow$ no natural "physical time" to deparametrize the WdW eq.

Q1: how do the constraints relate to 4d diffeos?

Q2: does their algebra close?

$)\underline{\Psi} = 0$

Q1

in $\mathcal{F}$ $\quad \mathring{I}_{\rho(\xi)} \Omega_\Sigma = - d\, Q_\Sigma(\xi) - \int_\Sigma i_\xi \underline{E}^{ab} \, d g_{ab} + \underbrace{\oint_{\partial\Sigma} i_\xi \Theta}_{\equiv 0,\ \partial\Sigma = \phi}$

normal diffeos

Can we "project" this eq down to $\mathcal{P}_{can}$?

1) tangential diffeos : $\xi = X$

- $\rho(X)$ descends to $\mathcal{P}$,
- $\mathring{I}_{\rho(X)} \omega_{can} = - d\, V(X) + \text{zero}$

Everything goes well, like for YM!

2) normal diffeos

• $\mathbb{L}_{\rho(n^a)} h_{ij} = L_n h_{ij} = 2K_{ij} \sim 2\frac{\Pi_{ij}}{\sqrt{h}}$

• $\mathbb{L}_{\rho(n)} \Pi^{ij} \sim \dot{\Pi} \sim \ddot{h}$ not a variable on P_{can}!

$\Rightarrow$ $\rho(n^a)$ does not descend to P_{can}!

2') the term $\int_\Sigma i_n E$ involves 2nd derivatives of $h \to \ddot{h}$

$\rightsquigarrow$ same problem!

$\Longrightarrow$ 4d diffeos can't be represented on $P^{can} = T^* Riem(\Sigma)$!

Q2. but what about my 4-constraints?

- they do close:

$$\{V(X), V(Y)\} = V([X,Y]) \quad \leftarrow \text{(like YM)}$$

$$L_{\rho(X)} H \sim \{V(X), H(N)\} = H(L_X N)$$

$$\{H(N), H(M)\} = V\left(N \boxed{h^{ij}} \partial_j M - (N \leftrightarrow M)\right)$$

structure "constants" depend on h_{ij}!

Dirac's Hypersurface deformation "algebra"