Title: Strong Gravity Lecture

Speakers: William East

Collection: Strong Gravity 2023/24

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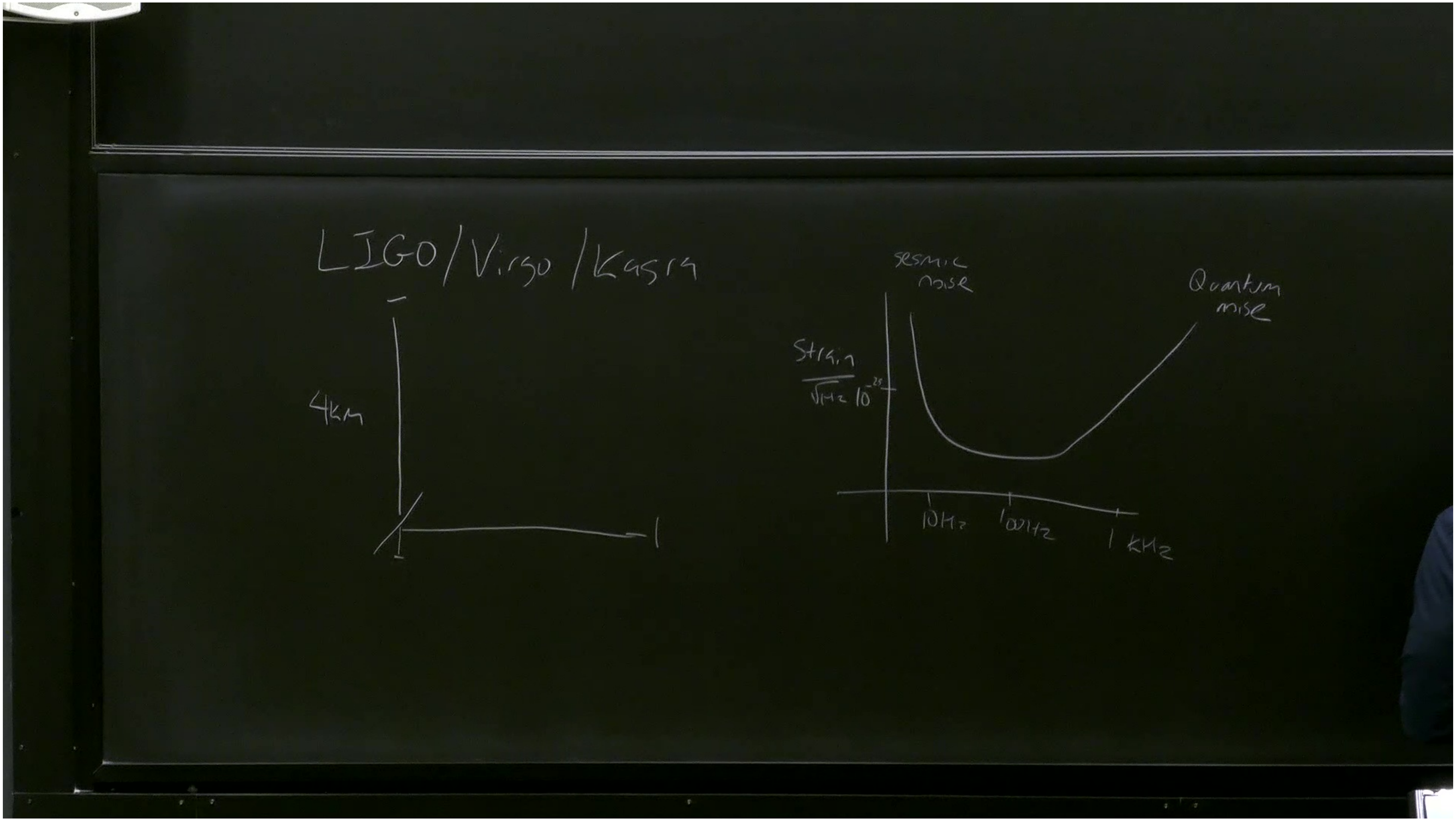
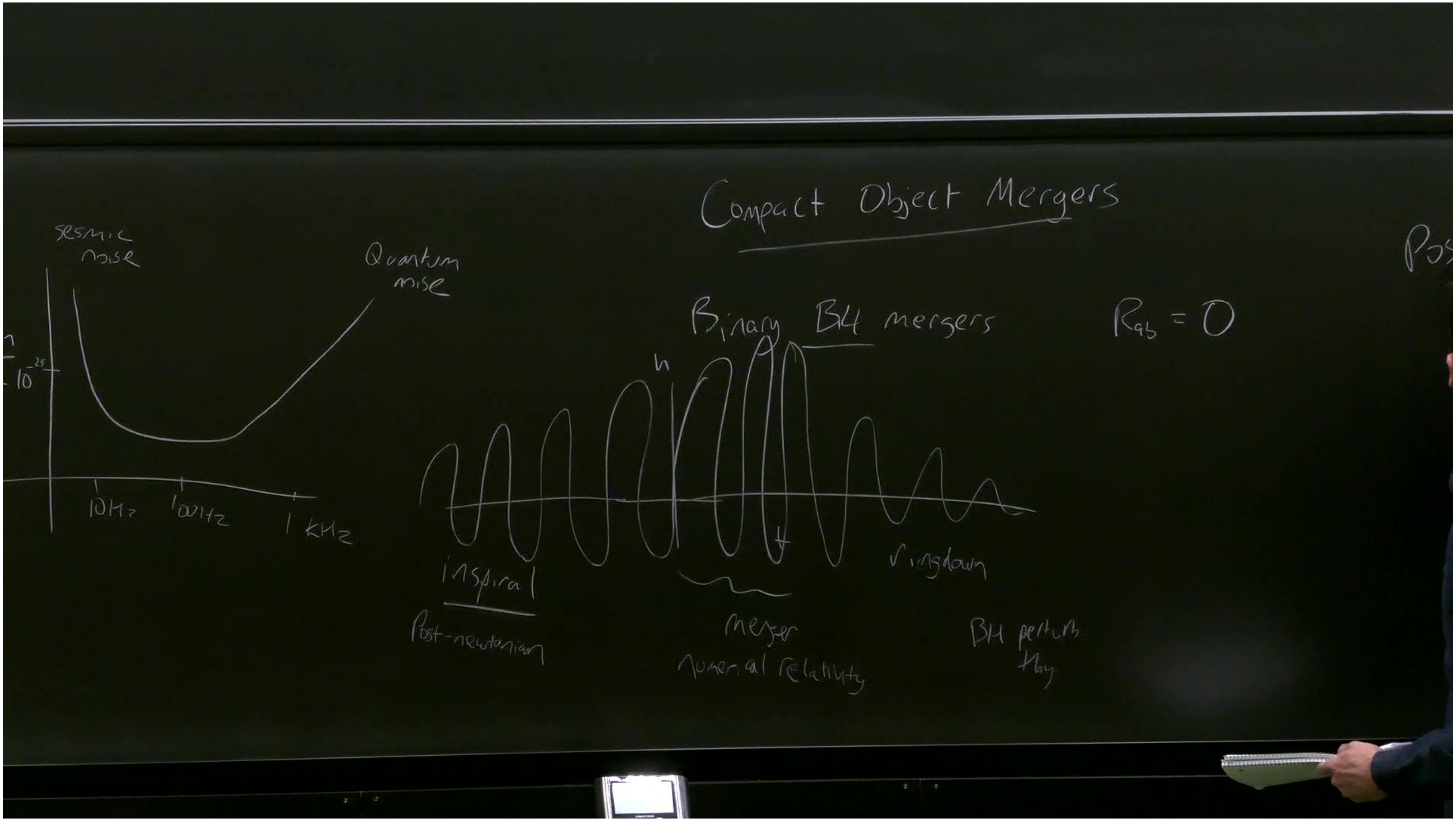
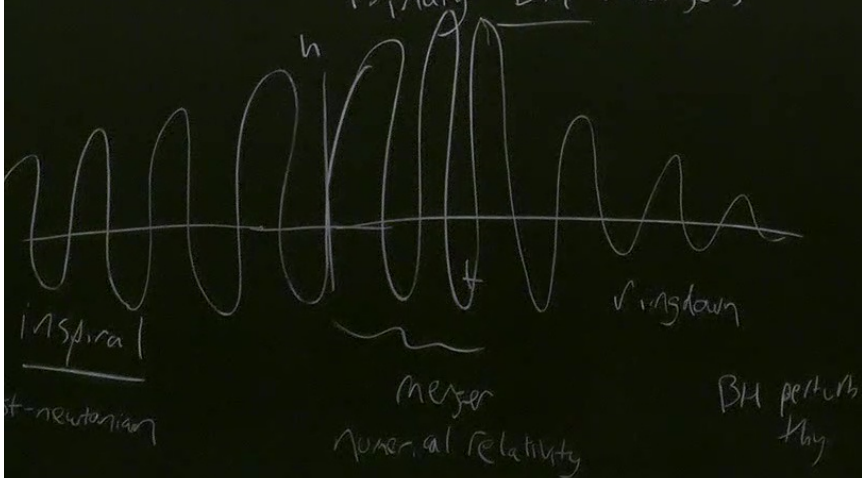

LIGO/Virgo/Kagra
4km
seismic noise
Quantum noise
Strain
√Hz
10Hz
100Hz
1 kHz


Compact Object Mergers
sesmic noise
Quantum noise
10^-25
10Hz
100Hz
1 kHz
Binary BH mergers
R_ab = 0
h
inspiral
Post-newtonian
merger
numerical relativity
ringdown
BH perturb. thy

Compact Object Mergers

Binary BH mergers $\qquad R_{ab} = 0$



Post-Newtonian

$$\frac{M}{r}, \quad \frac{v}{c}$$

Viral Thm $\left(\frac{v}{c}\right)^2 \sim \frac{M}{r} \sim \epsilon$

PN means up $\epsilon^{\#}$

2 body motion

$$\frac{d\vec{v}}{dt} = \frac{GM}{r^2}\left(-\hat{n} + \frac{1}{c^2}\vec{A}_{1PN} + \frac{1}{c^4}A_{2PN} + \frac{1}{c^5}A_{2.5PN} + \dots\right)$$

$$A_{1PN} = \left[(4+2\tilde{\mu})\frac{GM}{r} - (1+3\tilde{\mu})v^2 + \frac{3}{2}\tilde{\mu}\dot{r}^2 \right]\hat{n} + (4-2\tilde{\mu})\dot{r}\vec{v} \qquad \tilde{\mu} = \frac{M_1 M_2}{M^2}$$

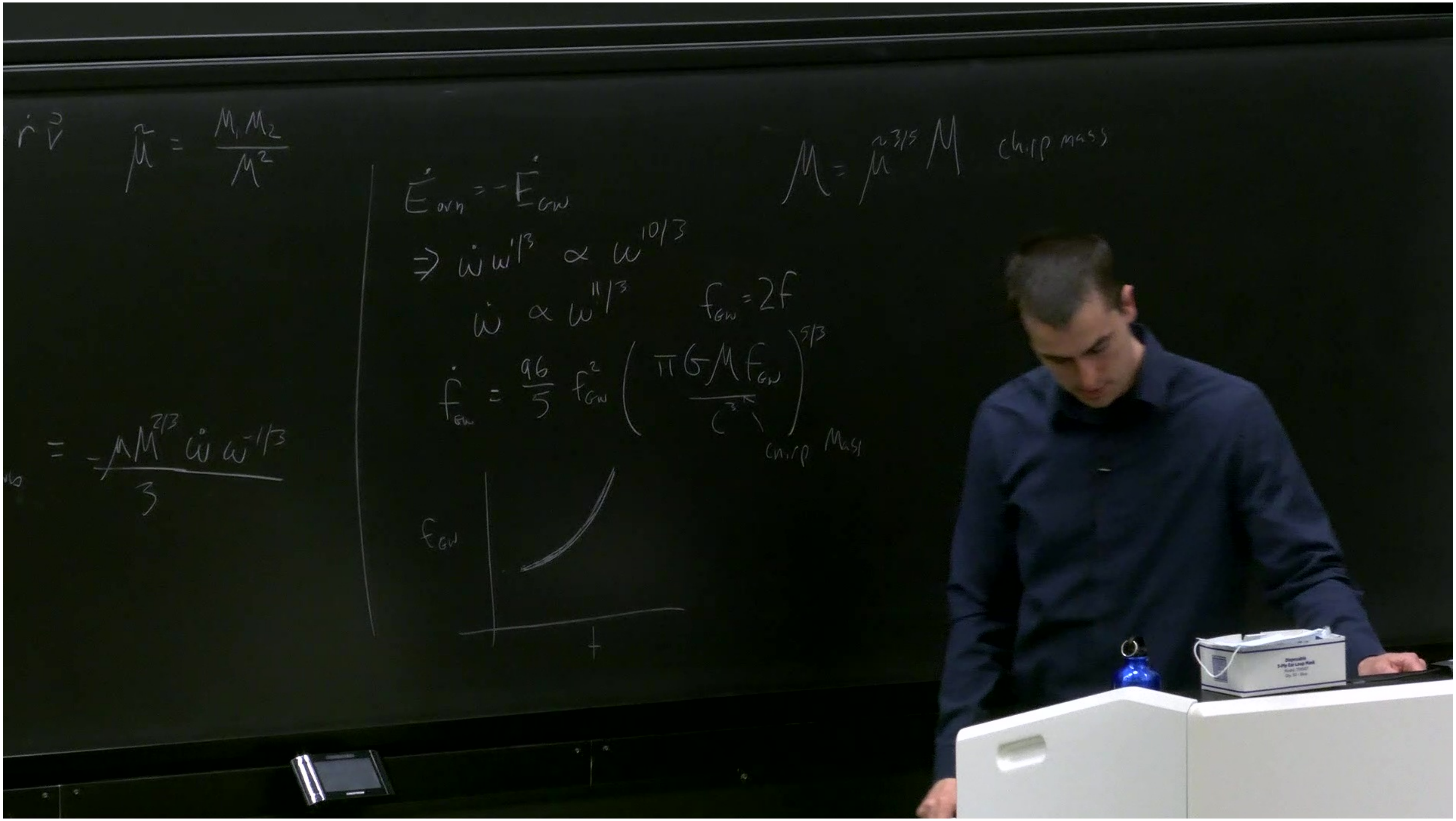
$A_{2.5PN}$ - leading order radiation reaction

For circular orbit, from Newtonian mech.

$$\omega^2 = \frac{M}{r} \quad , \quad E_{orb} = \frac{-\mu M}{2r} = \frac{-\mu M^{2/3}\omega^{2/3}}{2} \quad , \quad \dot{E}_{orb} = -\frac{\mu M^{2/3}\dot{\omega}\,\omega^{-1/3}}{3}$$

$$\dot{E}_{GW} = \frac{1}{5}\left\langle \dddot{I}_{ij}\,\dddot{I}^{ij} \right\rangle \qquad \tilde{I}_{ij} \sim \mu r^2 \cos(2\omega t)$$

$$\sim (\mu r^2 \omega^3)^2 = \mu^2 r^4 \omega^6 \qquad \dddot{\tilde{I}}_{ij} \sim \mu r^2 \omega^3 \cos(2\omega t)$$


$\tilde{\mu} = \frac{M_1 M_2}{M^2}$
$\mathcal{M} = \tilde{\mu}^{3/5} M$ chirp mass
$\dot{E}_{orb} = -\dot{E}_{GW}$
$\Rightarrow \dot{\omega}\omega^{-1/3} \propto \omega^{10/3}$
$\dot{\omega} \propto \omega^{11/3}$
$f_{GW} = 2f$
$\dot{f}_{GW} = \frac{96}{5} f_{GW}^2 \left(\frac{\pi G \mathcal{M} f_{GW}}{c^3}\right)^{5/3}$
chirp Mass
$= -\frac{\mu M^{2/3} \dot{\omega} \omega^{-1/3}}{3}$
f_{GW}
t

$$\sim (\mu r^2 \omega^3)^2 = \mu^2 r^4 \omega^6 \qquad I_{ij} \sim \mu r^2 \omega \cos(2\omega t)$$

$$\sim \mu^2 M^{2/3} \omega^{10/3}$$

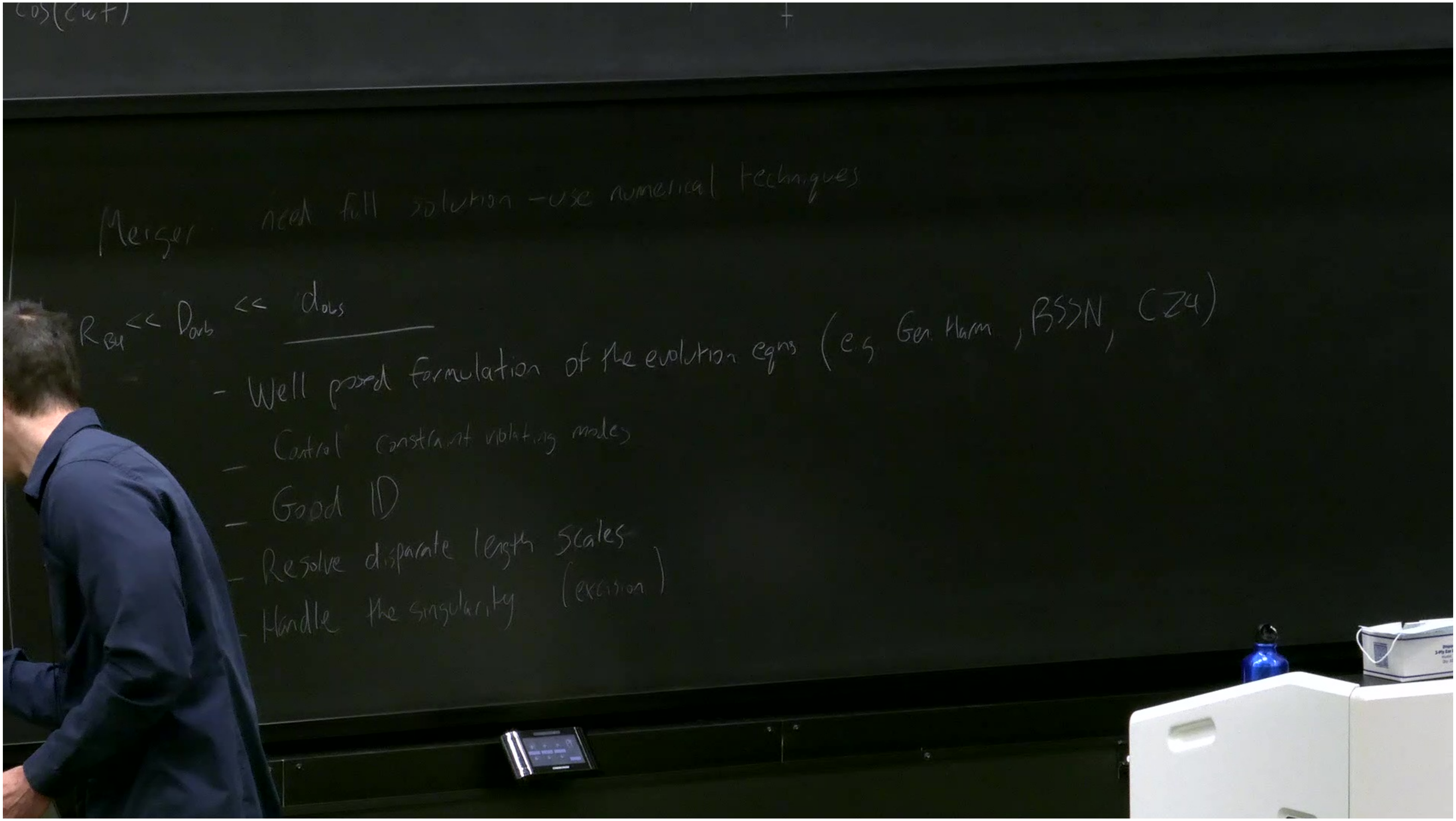
$$N = \int_{f_0}^{f_1} \frac{f}{\dot{f}}\, df \propto \int_{f_0}^{f_1} f^{-8/3}\, df$$

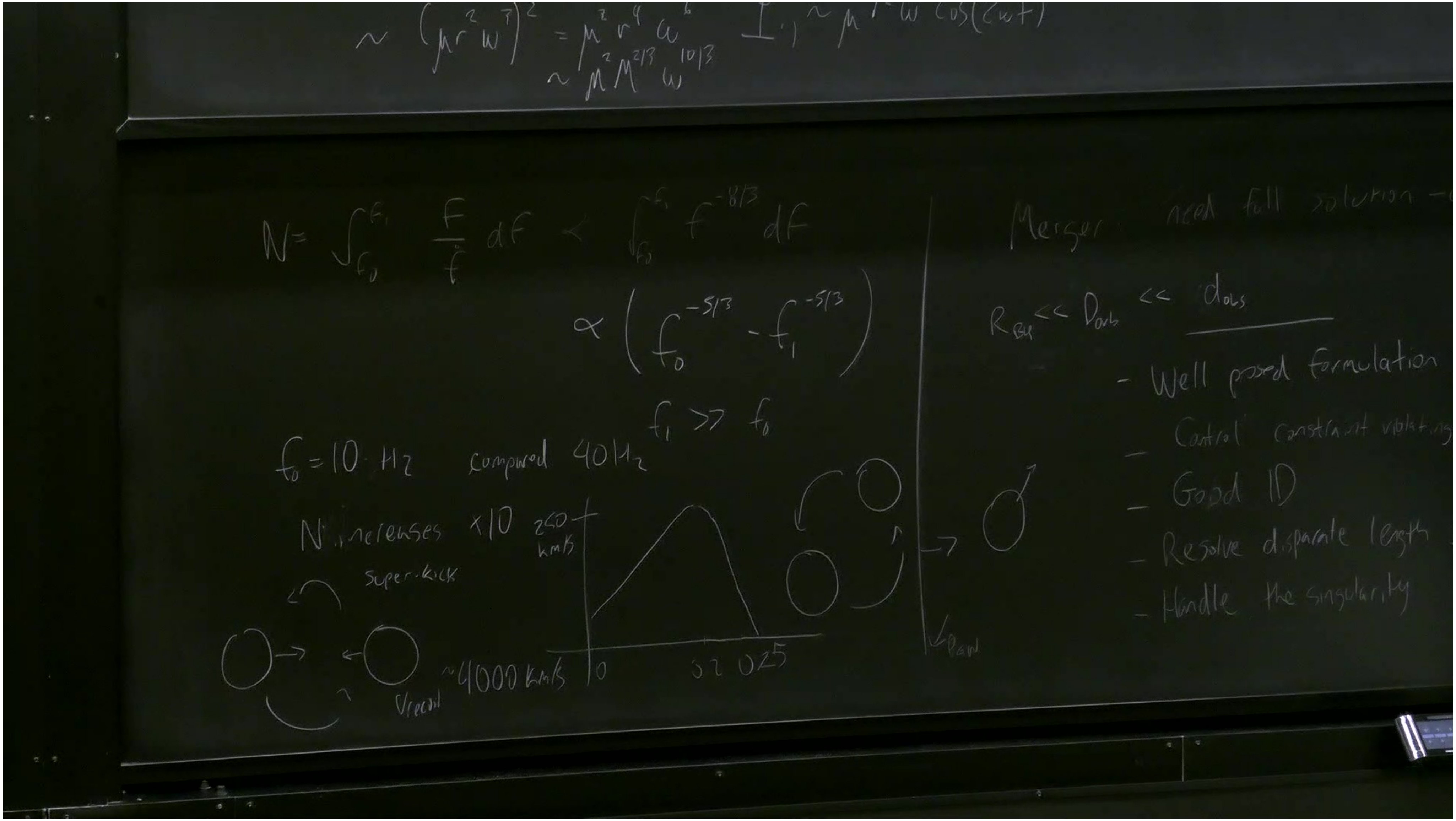
$$\propto \left(f_0^{-5/3} - f_1^{-5/3} \right)$$

$$f_1 \gg f_0$$

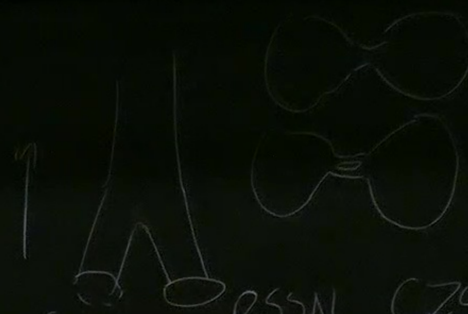
$f_0 = 10$ Hz compared 40 Hz

N increases $\times 10$


cos(2ωt)
Merger need full solution - use numerical techniques
R_BH << D_orb << d_obs
- Well posed formulation of the evolution eqns (e.g. Gen. Harm, BSSN, CZ4)
- Control constraint violating modes
- Good ID
- Resolve disparate length scales
- Handle the singularity (excision)

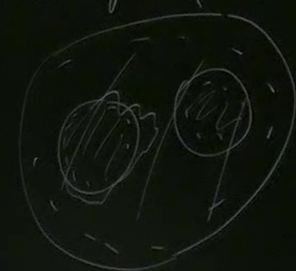

$\sim (\mu r^2 \omega^3)^2 = \mu^2 r^4 \omega^6$
$\sim \mu^2 M^{2/3} \omega^{10/3}$
$N = \int_{f_0}^{f_1} \frac{f}{\dot f} df \propto \int_{f_0}^{f_1} f^{-8/3} df$
$\propto \left(f_0^{-5/3} - f_1^{-5/3} \right)$
$f_1 \gg f_0$
$f_0 = 10\ Hz$ compared $40\ Hz$
N increases x10
super-kick
V_{recoil}
~4000 km/s
250 km/s
0
Merger need full solution
$R_{BH} \ll D_{orb} \ll d_{obs}$
- Well posed formulation
- Control constraint violating
- Good ID
- Resolve disparate length
- Handle the singularity

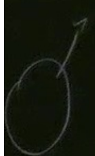
Merger — need full solution - use numerical techniques

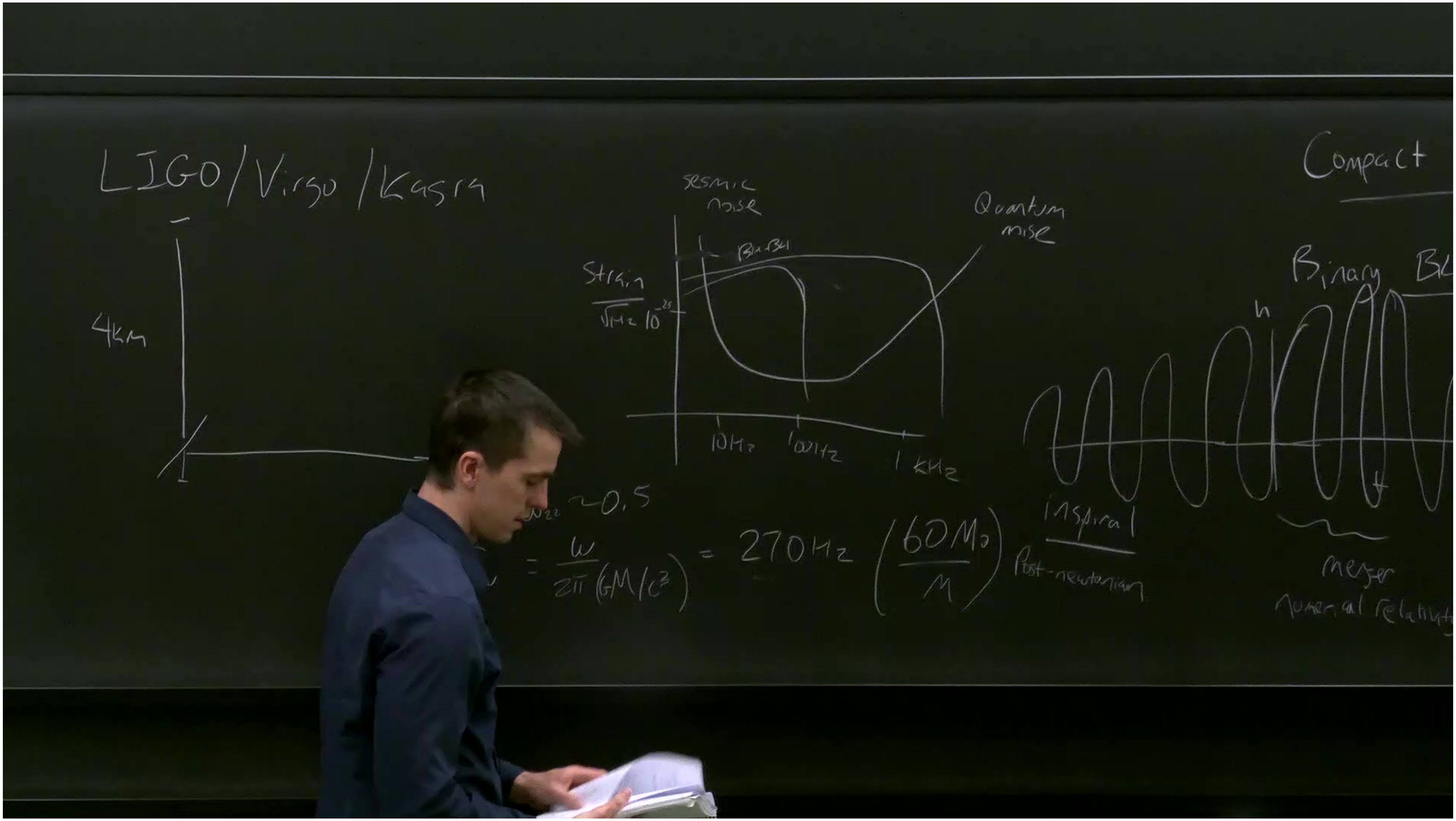


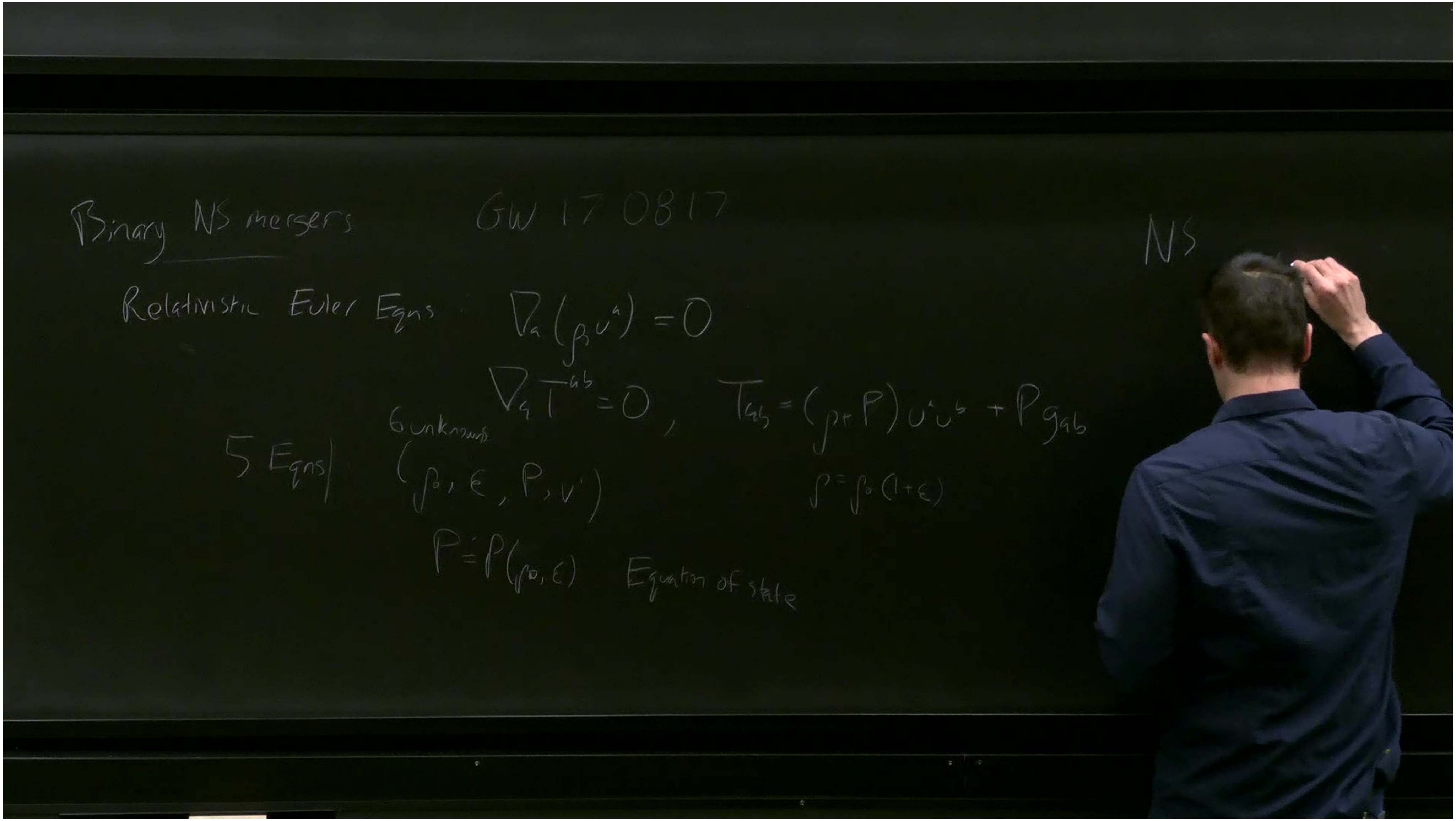
$R_{BH} << D_{orb} << d_{obs}$

- Well posed formulation of the evolution eqns (e.g. Gen. Harm., BSSN, CZ4)
- Control constraint violating modes
- Good ID
- Resolve disparate length scales
- Handle the singularity (excision)






LIGO/Virgo/Kagra
4km
seismic noise
Quantum noise
Strain
BH-BH
10Hz
100Hz
1kHz
~0.5
270Hz
inspiral
Post-newtonian
Compact
Binary
h
merger
numerical relativity


Binary NS mergers
GW 170817
NS
Relativistic Euler Eqns: $\nabla_a(\rho_0 u^a) = 0$
$\nabla_a T^{ab} = 0$, $T_{ab} = (\rho + P) u_a u_b + P g_{ab}$
6 unknowns
5 Eqns
$(\rho_0, \epsilon, P, v^i)$
$\rho = \rho_0(1+\epsilon)$
$P = P(\rho_0, \epsilon)$ Equation of state

numerical relativity

thy

$\frac{d\omega}{dt} = \frac{\quad}{r^2}\Big(\quad + \ldots \Big)$

NS , $\rho \sim 10^{15}\, g/cm^3$

Leading order effect in inspiral

Phase $\Psi(f) = \frac{3}{128(\pi M f)^{5/3}}\Big[1 + \alpha_{1PN}\, x + \quad + (\alpha_{5PN} + \alpha_{Tide})\, x^5 + \ldots \Big]$

$x = (\pi M f)^{2/3}$

g_{ab}

Quadrupole $\qquad$ tidal tensor

$Q_{ab} = -\Lambda M^5 \mathcal{E}_{ab}$

$\alpha_{Tide} = -24\Big[\Big(1 + 12\frac{M_2}{M_1}\Big)\frac{M_1^5}{M^5}\Lambda_1 + (1 \leftrightarrow 2) \Big]$

$\Lambda_1 = \frac{2}{3} k_2 \Big(\frac{R_1}{M_1} \Big)^5$