Title: Multi-loop Null Polygons from Fishnet theory to N=4 SYM

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Series: Quantum Fields and Strings

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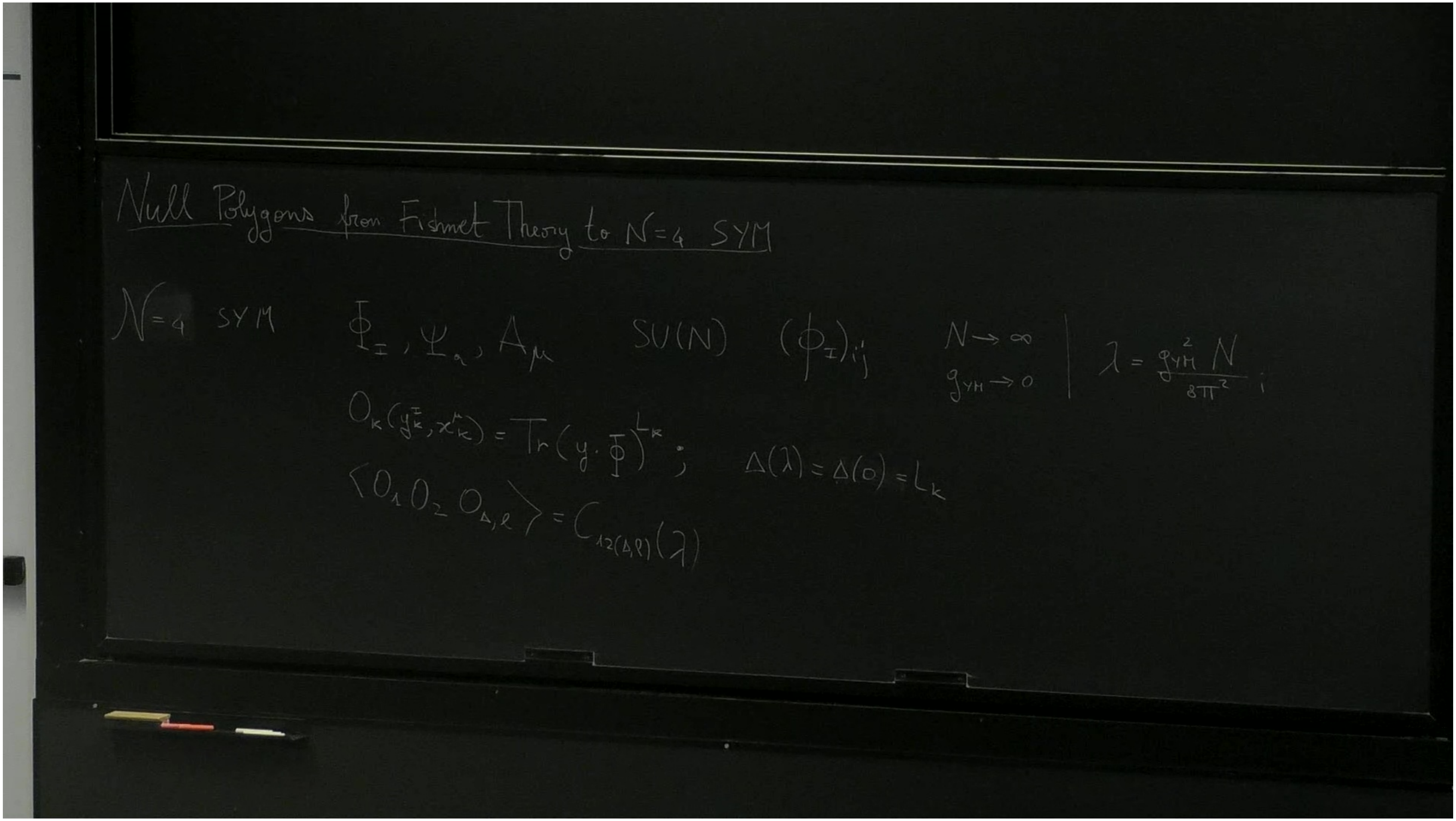
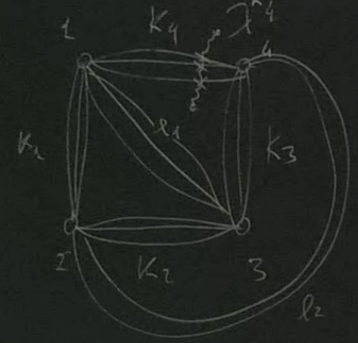
Abstract: "Null Polygons" in N=4 SYM theory describe the multi-point correlators of 1/2-BPS local operators with large R-charge, when they approach the vertices of a light-like polygon. The leading UV divergences of null polygons is conjectured to satisfy a hierarchy of coupled Toda field theory equations [E.O., Vieira '22]. I will present some progress towards the prediction of Null Polygons beyond leading logarithms via the hexagons technique, appropriately truncated in the light-cone regime. The method, still conjectural, relies on a series of weak-coupling derivations performed in the Fishnet limit of the theory, where the hexagon representation is derived in the basis of eigenfunctions of a conformal Heisenberg magnet in the principal series. I will present a number of worked-out examples for multi-point multi-loop Fishnet Feynman integrals and Null Polygons.

Zoom link


Null Polygons from Fishnet Theory to $\mathcal{N}=4$ SYM
$\mathcal{N}=4$ SYM
$\Phi_I , \Psi_\alpha , A_\mu$
$SU(N)$
$(\phi_I)_{ij}$
$N \to \infty$
$g_{YM} \to 0$
$\lambda = \frac{g_{YM}^2 N}{8\pi^2}$;
$O_k(y_k^I, x_k^\mu) = \mathrm{Tr}(y \cdot \Phi)^{L_k}$;
$\Delta(\lambda) = \Delta(0) = L_k$
$\langle O_1 O_2 O_{\Delta,\ell} \rangle = C_{12(\Delta,\ell)}(\lambda)$

$$\langle O_1 O_2 O_3 O_4\rangle\Big|_{\lambda=0} \sim \sum_{\underline{k},\underline{\ell}} \left(\frac{y_1\cdot y_2}{x_{12}^2}\right)^{k_1}\left(\frac{y_2\cdot y_3}{x_{23}^2}\right)^{k_2}\left(\frac{y_3\cdot y_4}{x_{34}^2}\right)^{k_3}\left(\frac{y_1\cdot y_4}{x_{14}^2}\right)^{k_4}\left(\frac{y_1\cdot y_3}{x_{13}^2}\right)^{\ell_1}\left(\frac{y_2\cdot y_4}{x_{24}^2}\right)^{\ell_2};$$

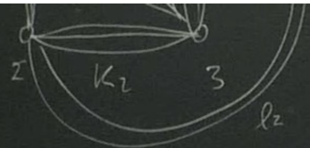


$$+\lambda\, G^{(1)}_{\underline{k},\underline{\ell}} + \lambda^2\, G^{(2)}_{\underline{k},\underline{\ell}} + \dots =$$

$k_i \gg 1$

$(z,\bar z)$

$$= P_{\ell_1}(z,\bar z)\times P_{\ell_2}(z,\bar z)$$



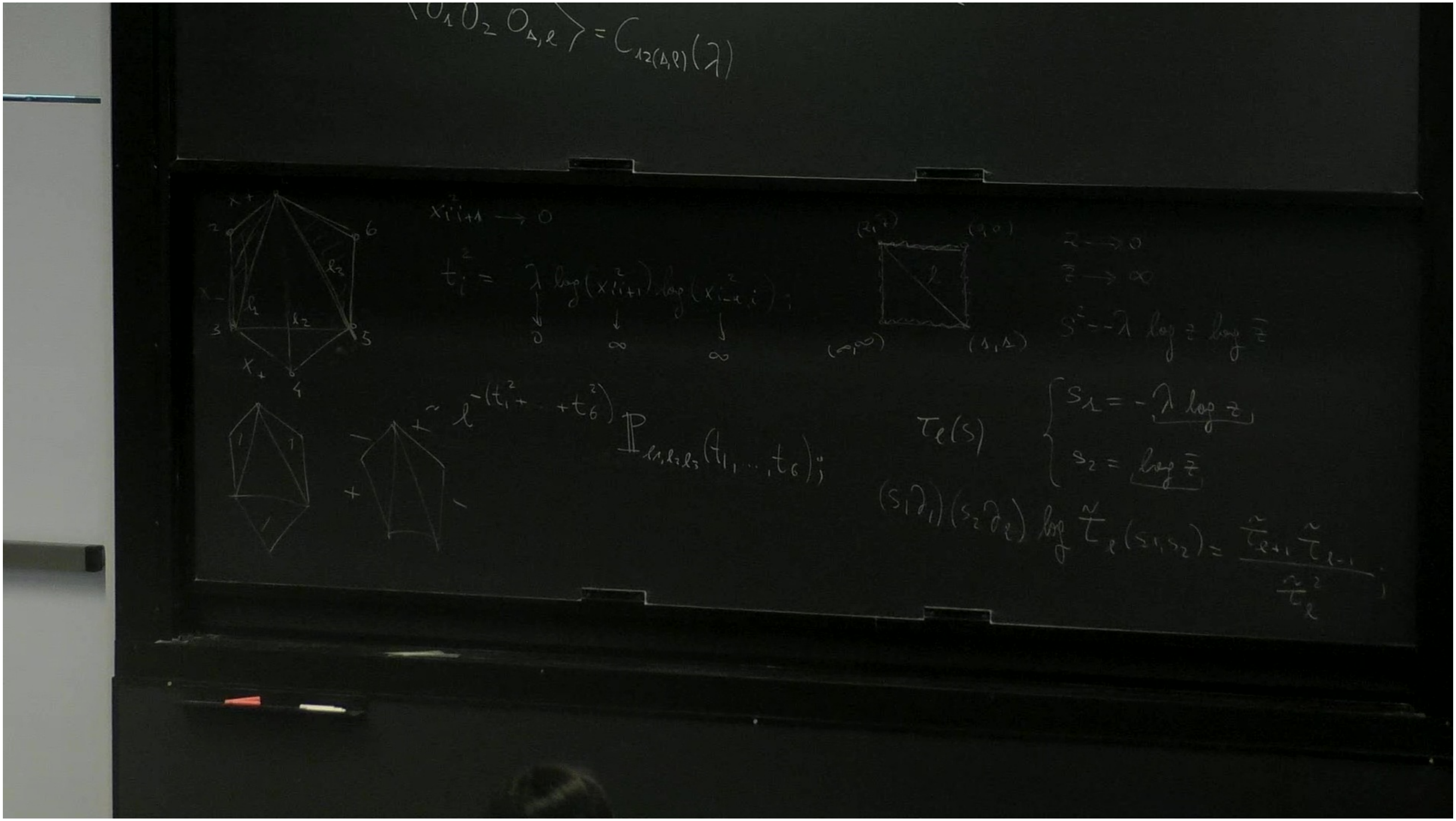
$(z,\bar z)$

$$= \mathbb{P}_{\ell_1}(z,\bar z)\times \mathbb{P}_{\ell_2}(z,\bar z)$$

- $$(t_1\partial_{t_1}+t_6\partial_{t_6})(t_5\partial_{t_5}+t_6\partial_{t_6})\log \mathbb{P}_{\ell_1\ell_2\ell_3} = t_6^2\,\frac{\mathbb{P}_{\ell_1\ell_2\ell_3+1}\,\mathbb{P}_{\ell_1\ell_2\ell_3-1}}{\mathbb{P}^2_{\ell_1\ell_2\ell_3}};$$

- $t_2=t_6=0$ $$\mathbb{P}_{\ell_1\ell_2\ell_3} = \tau_{\ell_1}(t_3)\,\tau_{\ell_1+\ell_2+\ell_3}(t_1)\,\tau_{\ell_3}(t_5)$$

$$\tau_\ell(s) = \det_{1\le i,j\le \ell} I_{|i-j|}(2s);$$ $\mathbb{P}_{\ell_1,\ell_2,\ell_3}$ for $\ell_1,\ell_3\in\{0,1\}$


⟨O_1 O_2 O_{Δ,ℓ}⟩ = C_{12(Δ,ℓ)}(λ)
x²_{i i+1} → 0
t_i² = λ log(x²_{i i+1}) log(x²_{i-1 i});
~ e^{-(t_1² + ... + t_6²)} P_{ℓ1,ℓ2,ℓ3}(t_1, ..., t_6);
z → 0
z̄ → ∞
S² = −λ log z log z̄
τ_ℓ(s)
s_1 = −λ log z
s_2 = log z̄
(s_1∂_1)(s_2∂_2) log τ̃_ℓ(s_1,s_2) = τ̃_{ℓ+1} τ̃_{ℓ-1} / τ̃_ℓ²

Fishnet Theory

γ-deformation $\mathcal{N}=4$ SYM

$$[\varphi_a, \varphi_b] = \kappa\, \varphi_a \varphi_b - \frac{1}{\kappa} \varphi_b \varphi_a \; ; \qquad q_{A_\mu} = (0,0,0)$$

$$\varphi_a \to q_a = (q_{a,1}, q_{a,2}, q_{a,3}) \qquad SU(4)$$

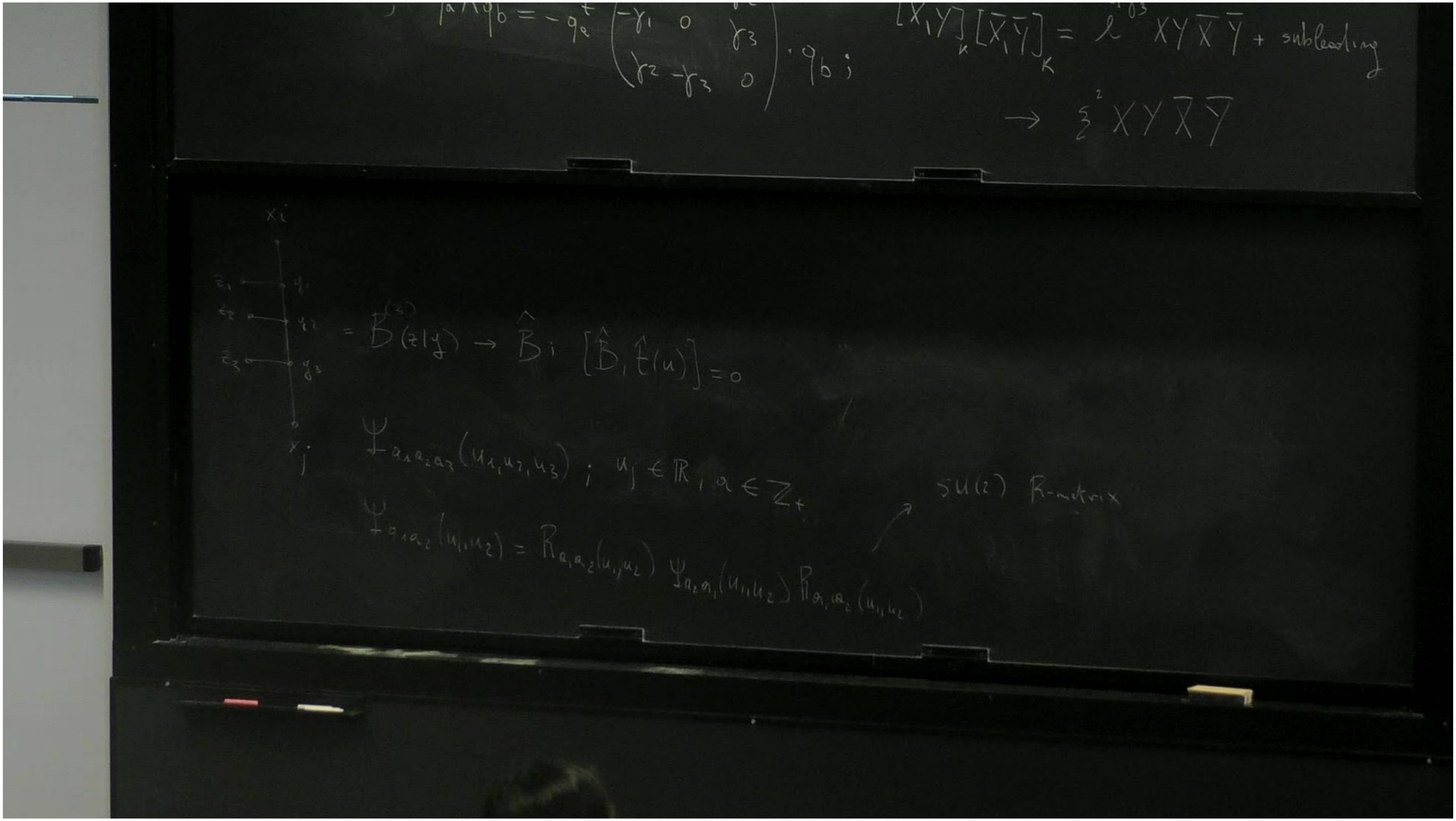
X, Y, Z

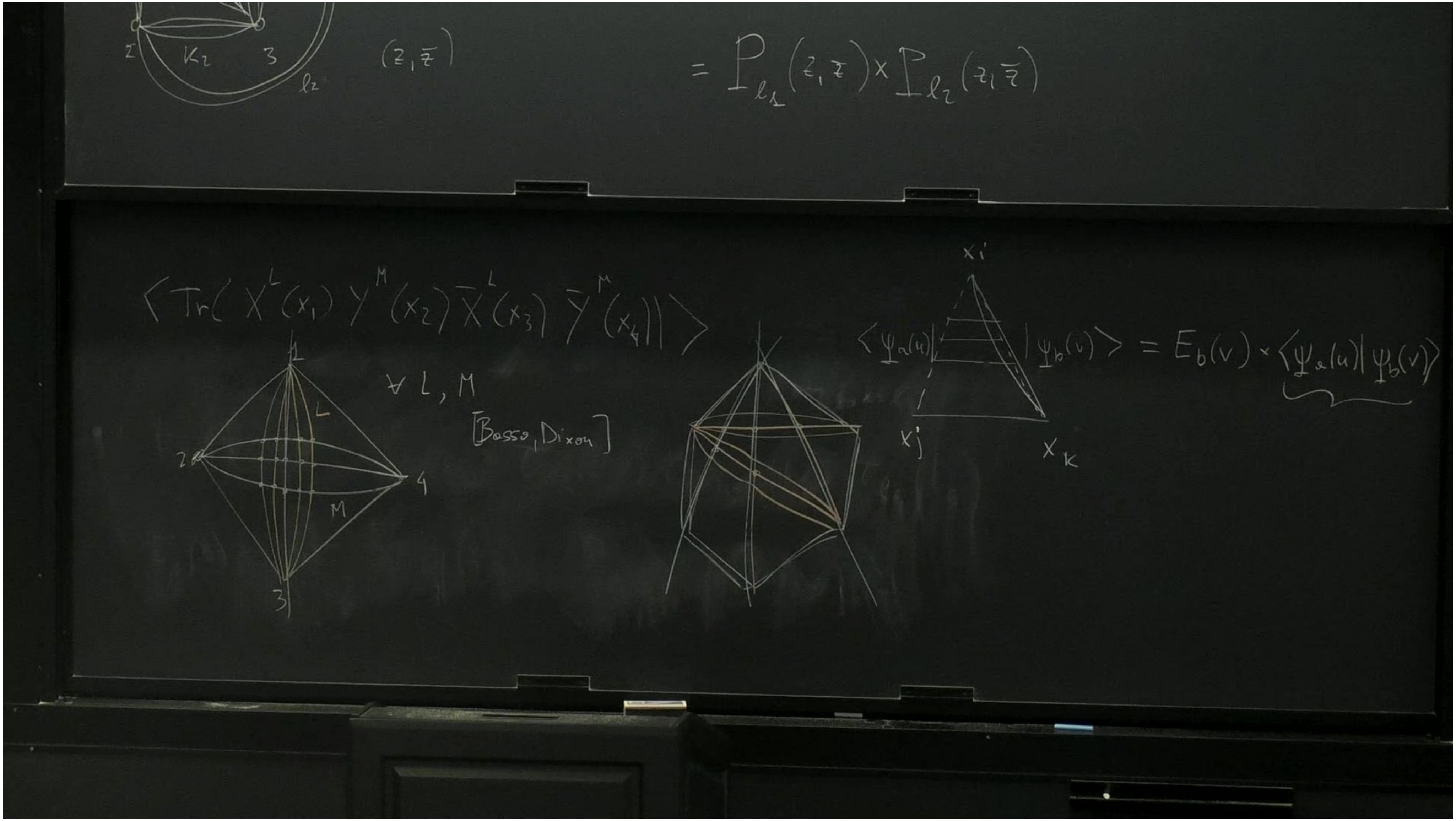
$$\kappa = e^{i q_a \wedge q_b} \; ; \quad q_a \wedge q_b = -q_a^T \begin{pmatrix} 0 & \gamma_1 & -\gamma_2 \\ -\gamma_1 & 0 & \gamma_3 \\ \gamma_2 & -\gamma_3 & 0 \end{pmatrix} \cdot q_b \; ;$$

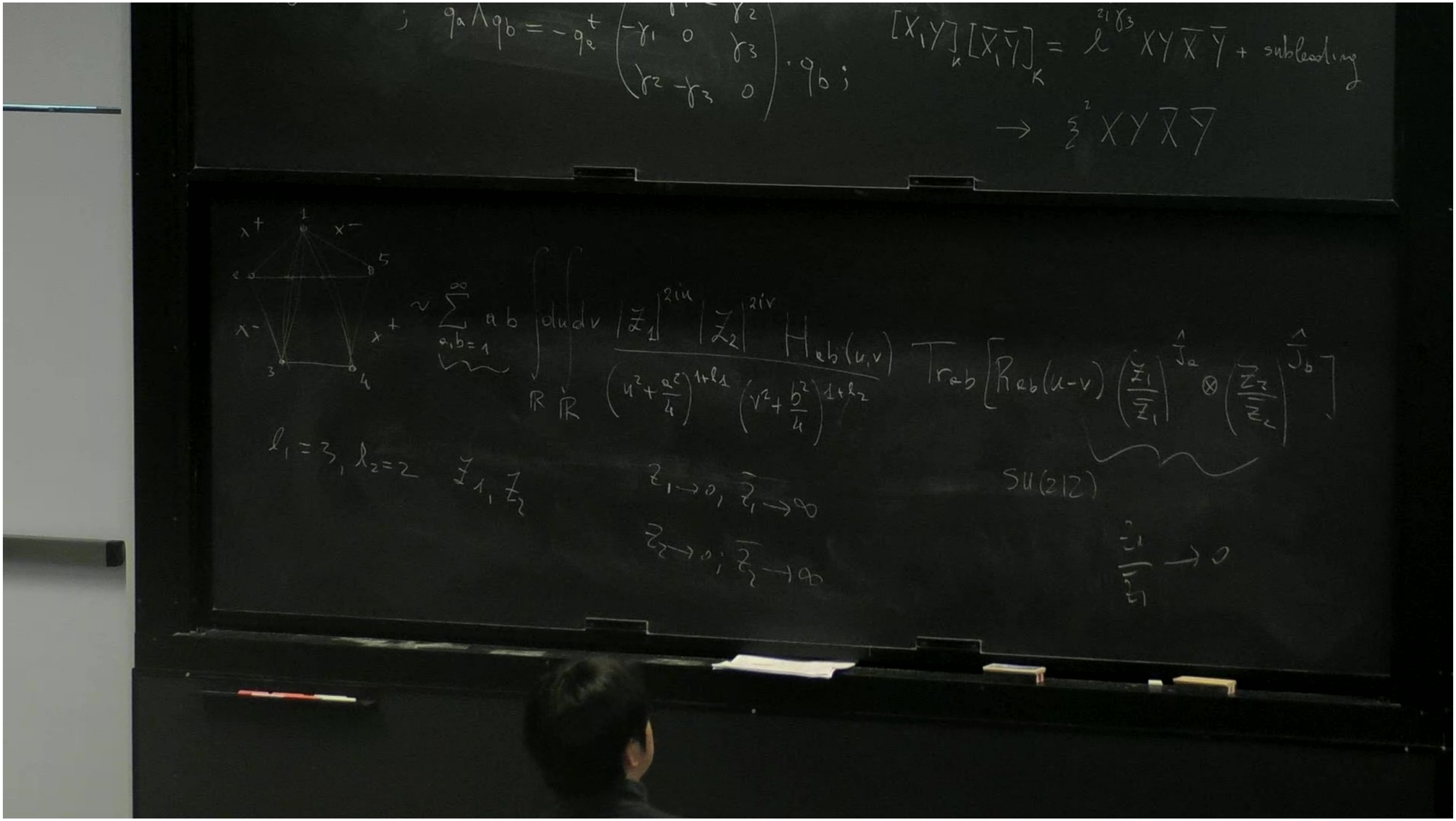
$$\gamma_3 \to -i\infty \; ; \; \lambda \to 0 \;\Big|\; \xi_3^2 = \lambda e^{i\gamma_3} \text{ finite}$$

$$[X,Y]_\kappa [\bar{X},\bar{Y}]_\kappa = e^{2i\gamma_3} X Y \bar{X} \bar{Y} + \text{subleading}$$

$$\to \xi^2 X Y \bar{X} \bar{Y}$$


+ subleading
SU(2) R-matrix


(z, z̄)
= P_{ℓ1}(z, z̄) × P_{ℓ2}(z, z̄)
⟨Tr(X^L(x1) Y^M(x2) X̄^L(x3) Ȳ^M(x4))⟩
∀ L, M
[Basso, Dixon]
xi
⟨ψa(u)| |ψb(v)⟩ = Eb(v) × ⟨ψa(u)|ψb(v)⟩
xj
xk


[X,Y]_κ [X̄,Ȳ]_κ = e^{2iγ3} X Y X̄ Ȳ + subleading
→ ξ² X Y X̄ Ȳ
Tr_ab [R_ab(u−v)
SU(2|2)
l1 = 3, l2 = 2
Z1, Z2
Z1 → 0, Z̄1 → ∞
Z2 → 0, Z̄2 → ∞