Title: Quantum Gravity Seminar Series - TBA

Speakers: Bianca Dittrich

Series: Quantum Gravity

Date: April 25, 2024 - 2:30 PM

URL: https://pirsa.org/24040119

Abstract: Abstract TBA

Zoom link

Surprising aspects of the Lorentzian gravitational path integral

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Perimeter Institute
April 2024

Plan

Will discuss an eclectic selection of issues arising in Lorentzian dynamics and path integrals, showcasing differences to Euclidean dynamics and path integrals, and many open questions.

Will consider various models for the Lorentzian path integrals.

(Will not discuss CDT, which is a prime example how different Lorentzian and Euclidean path integrals can be.)

Lorentzian path integrals - models

Spin foams

Gauge theory →
quantum geometry

Semi-classical/
Continuum
limit ???

The Flatness
Problem!

Effective Spin Foams

Higher gauge theory→
more accessible quantum geometry

Allowed for
(perturbative)
continuum limit:

(Linearized) GR
+ correction
resulting from
area metrics

Effective action
for the
continuum
theory: based
on area metrics

Effective spin foam cosmology

Lorentzian Regge calculus

↓

Lorentzian continuum path integral

Area vs Length metric

Length metric $g_{\mu\nu}$:

- Measuring length of tangent vectors and angles between vectors
- 10 independent components

Area metric $A_{\mu\nu\rho\sigma}$: [Schuller et al 05+: Area metrics]

- Measuring areas of parallelograms and angles between parallelograms
- require cyclicity (as for Riemann tensor): $A_{[\mu\nu\rho\sigma]} = 0$ (for mathematical and physical reasons)
- 20 independent components - same symmetries as for Riemann tensor

Each length metric induces a (cyclic) area metric:

$$A_{\mu\nu\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}$$

Area metric configuration space (quantum) extends length configuration space.

In spin foams: due to an anomaly in the simplicity constraint algebra, parametrized by Barbero-Immirzi parameter. [BD, Ryan '09,'12]

Area metrics in spin foams: three different ways

[BD, Ryan '08+, Freidel, Speziale '10]

[BD, Padua-Arguelles '23]

Micro-scopic:

4-simplex quantum geometry specified by 20 quantities	→	20 quantities define area metric for 4-simplex

Meso-scopic:

Macroscopic:

Area metrics in spin foams: three different ways

Micro-scopic:

[BD, Ryan '08+, Freidel, Speziale '10]
4-simplex quantum geometry specified by 20 quantities
→
[BD, Padua-Arguelles '23]
20 quantities define area metric for 4-simplex

Meso-scopic:

[Asante, BD, Haggard '20, Asante, BD, Padua-Arguelles '21]
Effective spin foams: allowing for lattice perturbation theory (50 variables per vertex)
→
[BD '21, BD, Kogios '22]
Continuum limit: Leading order dynamics described by area metrics associated to hypercubes

Macroscopic:

[Krasnov '07]
Modified Plebanski framework
→
[Borissova, BD '22]
Allows derivation of area metric action

Second order area metric actions

[Borissova, BD, Krasnov 2023]

Linearized area metric perturbations: $a_{\mu\nu\rho\sigma} \to h_{\mu\nu},\ \chi^{\pm}_{\mu\nu}$

Most general diff-invariant Lagrangian
(with massless gravitons):

$$\mathscr{L}_{\text{diff}} = \mathscr{L}_{EH} + \sum_{\pm} \frac{\rho_\pm}{2} h_{\mu\nu}\chi^{\pm\mu\nu} p^2 + \frac{p^2 + M_\pm^2}{4} \chi^{\pm}_{\mu\nu}\chi^{\pm\mu\nu}$$

Special case arising from modified Plebanski: $\rho_+^2 + \rho_-^2 = 2\,,\ M_+ = M_- = M$

[Borissova, BD 2022]

Remaining coupling is the Barbero-Immirzi-parameter:
parametrizes parity violating terms

$$\rho^+ - \rho_- \sim \gamma$$

It does affect classical EOM in area metric gravity.
Mixing of cross and plus polarization of TT-modes.

[Borissova, BD, Krasnov 2023]

Effective length metric action:

$$\mathscr{L}_{\text{diff}}(h) = \mathscr{L}_{EH}(h) - {}^{(1)}C_{\mu\nu\rho\sigma}(h) \frac{1}{p^2 + M^2} {}^{(1)}C^{\mu\nu\rho\sigma}(h)$$

Non-local Weyl^2 term

Modified gravity without additional poles:

$$(\text{Prop})^{\text{spin-2}} = 2\left(\frac{1}{p^2} + \frac{1}{M^2}\right)$$

just leads to a shift in propagator.

[chiral Plebanski: Freidel '08, Krasnov '08,
Area-metric: Borissova, BD, 2022]

Second order area metric actions

[Borissova, BD, Krasnov 2023]

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Area-metric: Borissova, BD, 2022]

Euclidean vs Lorentzian signature

Euclidean	Lorentzian
Positive definite terms for χ	Indefinite terms for χ

Nevertheless stable dynamics at second order.

Higher order?

Implications for spin foams / loop quantum gravity Hamiltonian?

Euclidean vs Lorentzian signature

$$a_{\mu\nu\rho\sigma} \to h_{\mu\nu}, \chi^{\pm}_{\mu\nu}$$

Euclidean		Lorentzian	
$\chi^{\pm}_{\mu\nu}$ are real		$\chi^{\pm}_{\mu\nu}$ are complex conjugated	
		$\chi^{1,2}_{\mu\nu}$ real and imaginary parts	
$(\chi^+)^2 + (\chi^-)^2$	$\longrightarrow$	$(\chi^1)^2 - (\chi^2)^2$	Indefinite terms →Instabilities?

happens also in electro-dynamics:

$H = E^2 + B^2$	$\longrightarrow$	$H = E^2 - B^2$	But E, B are not independent fields.

[Borissova, BD, Krasnov 2023]

- Canonical Analysis of special case Lagrangian (with additional shift symmetry)
- Propagating degrees of freedom: graviton + 5 χ fields
- Hamiltonian has indeed indefinite signature
- Couples graviton and χ fields

$$\begin{aligned}\mathcal{H}'_{full} &= \frac{1}{2}(P_{ab} + 2\cosh(2\xi)D\phi_{ab})^2 - \frac{1}{4}(\delta^{ab}P_{ab})^2 + h_{00}\partial^a\partial^b(H_{ab} - \delta_{ab}(\delta^{cd}H_{cd})) + 2P^{ab}\partial_a h_{b0} \\ &+ \frac{1}{2}(\partial_a H_{bc})^2 - (\partial^b H_{ab})^2 - \delta^{ab}H_{ab}\partial^c\partial^d H_{cd} - \frac{1}{2}(\partial^a\delta^{cd}H_{cd})^2 \\ &- \frac{1}{2}(\Phi_{ab})^2 - \frac{1}{2}(\partial_a\phi_{bc})^2 - \frac{1}{2m^2}(F_{ab})^2 - \frac{1}{2}m^2(\phi_{ab})^2 \quad .\end{aligned}$$

$$F_{ab} = 2\sinh(2\xi)DP_{ab} + 2\cosh(2\xi)D\Phi_{ab} + 2\mathbb{D}H_{ab} + 4\cosh(2\xi)\sinh(2\xi)D^2\phi_{ab}$$

Nevertheless:

Stable dynamics with two massless (dressed graviton) and two massive modes.
Massive modes have negative energy, but decouple from massless modes.

Euclidean vs Lorentzian signature

Euclidean

Positive definite terms for χ

Lorentzian

Indefinite terms for χ

Nevertheless stable dynamics at second order.

Higher order?
Implications for spin foams / loop quantum gravity Hamiltonian?

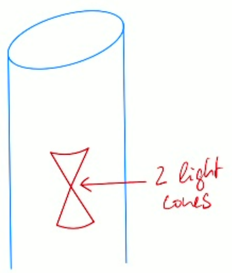
Loop Quantum Gravity: real vs complex self-dual formulation.
Similar sign issue for the Kodama state: Euclidean-normalizable, Lorentzian- not normalizable. [Freidel, Smolin 2003]

Continuum limit of effective spin foams in Lorentzian signature. [Asante, BD wip]

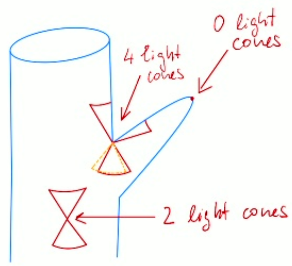
Light cone structure

in Lorentzian path integrals

Light cone structure

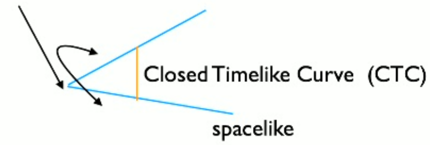


Lorentzian space time with
regular light cone structure.



Lorentzian space time with
irregular light cone structure.

Point without light cone



[Marolf 2022]
[Jacobson, Visser 2022]

Such conical singularities essential for calculation of entropy from Lorentzian (semi-classical) path integral.

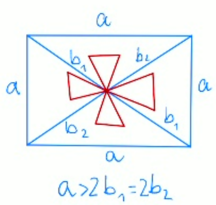
[BD, Jacobson, Padua-Arguelles 2024]

In Regge calculus:

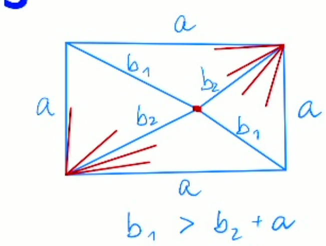
- Appear even in configurations describing mini-superspace cosmology - for small lapse.

[BD Gielen, Schander, Asante 21, BD, Padua-Arguelles 2022] Discretization artifact?

Light cone structure in Regge calculus



- Model spacetime with piecewise (Minkowski-) flat building blocs
- Can easily produce light cone irregular configurations

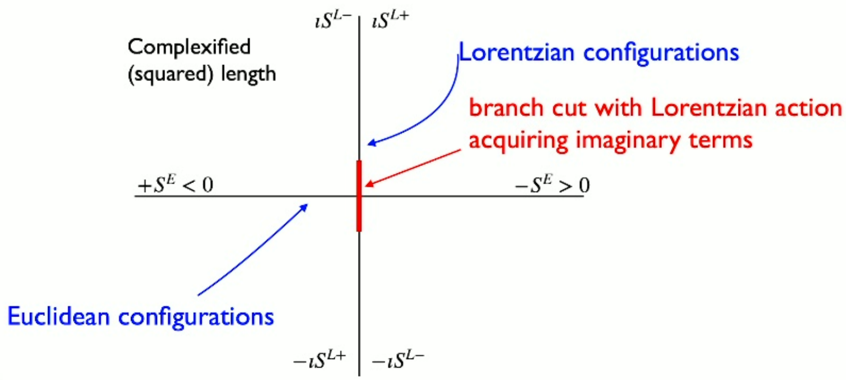


Light cone irregular configurations lead to imaginary terms in the Regge action, with an ambigious sign.
Light cone irregular configurations lead to branch cuts for the complex Regge action: ambiguity in integration contour.

Regge calculus provides most natural way to derive imaginary terms in the action for conical singularities.

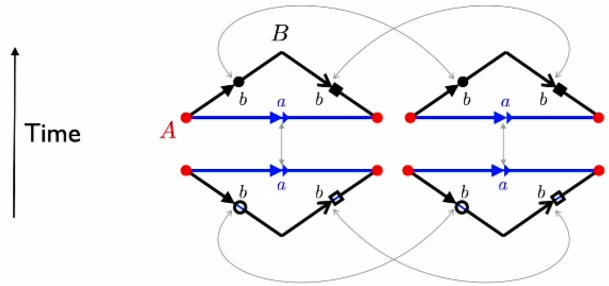
[Sorkin 1974, 2019]
[Asante, BD, Padua-Arguelles 2021]
[compare with Marolf 2022, O'Connel 2024]

Example describing (de Sitter) cosmology:

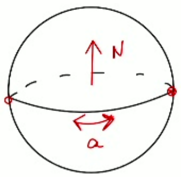


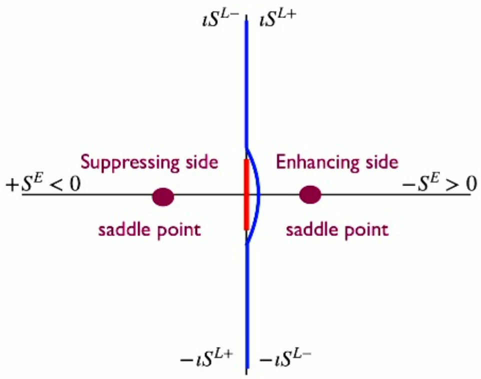
De Sitter horizon entropy from Lorentzian path integral

[BD, Jacobson, Padua-Arguelles 2024]



2D analogue of our 3D (or 4D) triangulation
- Sufficient small height: CTC singularity
- Larger heights: big bang-big crunch cosmology





We (have to) choose the enhancing side of the branch cut:

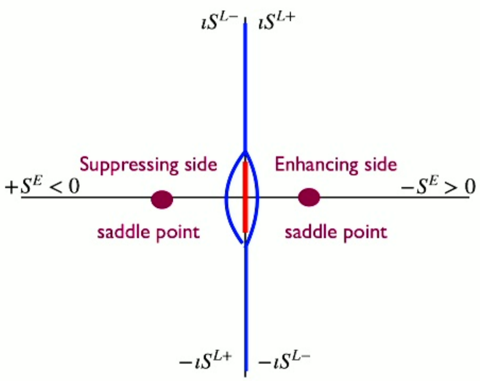
- in order to obtain de Sitter horizon entropy
- diverges in classical limit
- choice consistent with fluctuation convergence criterion

[Halliwell-Hartle, Louko-Sorkin, Kontsevich-Segal, Witten, …]

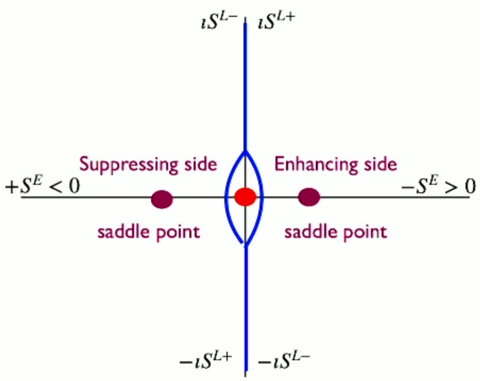
Why does a mini-superspace path integral compute entropy?
See path integral as an effective description for unknown UV physics.

De Sitter mini-superspace: continuum and discrete

Discrete: Light cone irregularities for small lapse



Continuum: Essential singularity for vanishing lapse



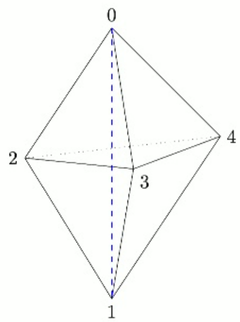
Ambiguities for Lorentzian path integrals in the discrete and continuum.

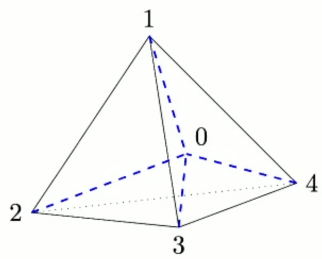
Debate about choice of contour for mini-superspace path integral.
A growing number of papers: [Feldbrugge, Lehners, Turok] vs [Diaz Dorronsoro, Halliwell, Hartle, Hertog, Janssen] and others.

Discrete framework offers an interpretation of (complex) configurations around N=0 singularity.

Irregular spike configurations

[Borissova, BD, Qu, Schiffer to appear]





3D: all configurations with bounded boundary edges
and large space-like bulk edges are light cone irregular (of yarmulke type).

What to do with such configurations?

Physical interpretation?

A time step in
Effective spin foam cosmology

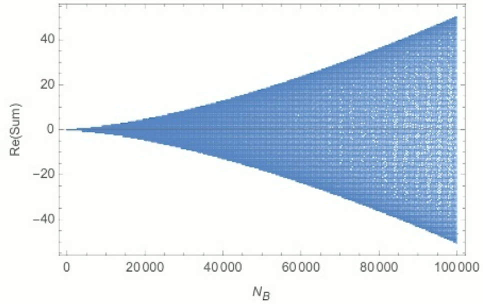
Effective spin foam cosmology

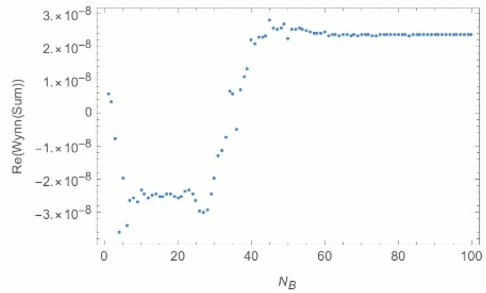
Areas have discrete spectrum. [Rovelli, Smolin 94]

- How to compute oscillating path sums? Deformation of contour not available anymore.
- What is the effect of discrete spectra for mini-superspace path integrals→sums?
- Future: What is the effect of extension from length to area metrics?

Shanks transform/ Wynn algorithm: Works very well for quantum mechanical problems / asymptotic regime in spin foams. [BD, Padua-Arguelles 23]
Contour deformation not necessary. But reproduces results for integrals treated via contour deformation.

For computation of expectation value:

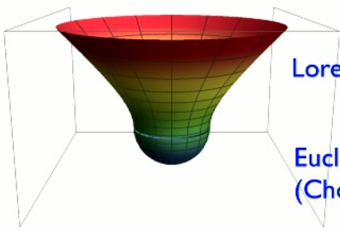




Rel. Error~10^{-8}

De Sitter cosmology - discrete spectrum effects [BD, Padua-Arguelles 23]

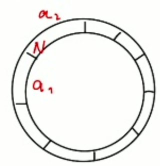
Quantum deSitter

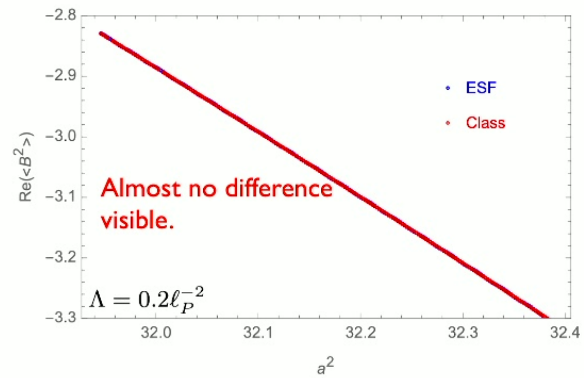


Lorentzian solution

Euclidean solution
(Choice: Tunneling)

Lorentzian phase:
modelled with Regge calculus
vs Effective Spin Foams





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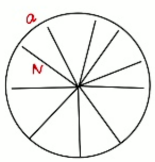

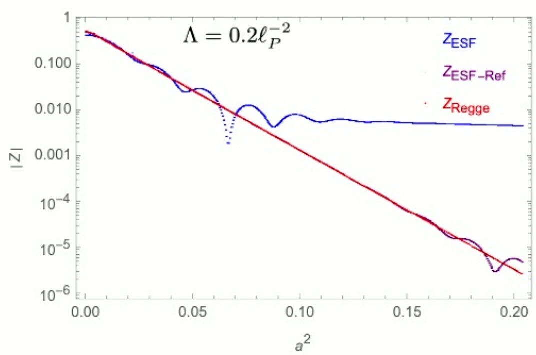

Tunneling amplitudes:
Significant differences between integral vs sum

Do discrete spectra make
tunnelling amplitudes less suppressed?

Should be confirmed by using more time steps.

Resonance effects between oscillations and discrete spectra?
(pseudo-saddle points, asymptotic regimes)

Summary

There are many features in Lorentzian path integrals, that one does not see in Euclidean signature/ Euclidean path integrals.

- Spin foams/ Area metric dynamics: Area metric actions as effective continuum actions for spin foams.
  Negative energy for massive chi-modes in Lorentzian signature.
  Stability of the theory? Selection criterium for area metric actions?

- Light cone irregular configurations: Capturing UV physics?
  Seem to be generic. Lead to ambiguities for Lorentzian path integral.
  Needed for (mini-superspace) entropy calculation.
  Correspondence to complexified continuum metrics?
  Appearance/ role in spin foams?

- Discrete spectra and oscillating amplitudes: Beyond semi-classical/saddle point approximation.
  Tunneling amplitudes more enhanced?
  Resonance effects?