Title: Double Scaled SYK and de Sitter Holography

Speakers: Herman Verlinde

Series: Quantum Fields and Strings

Date: April 23, 2024 - 2:00 PM

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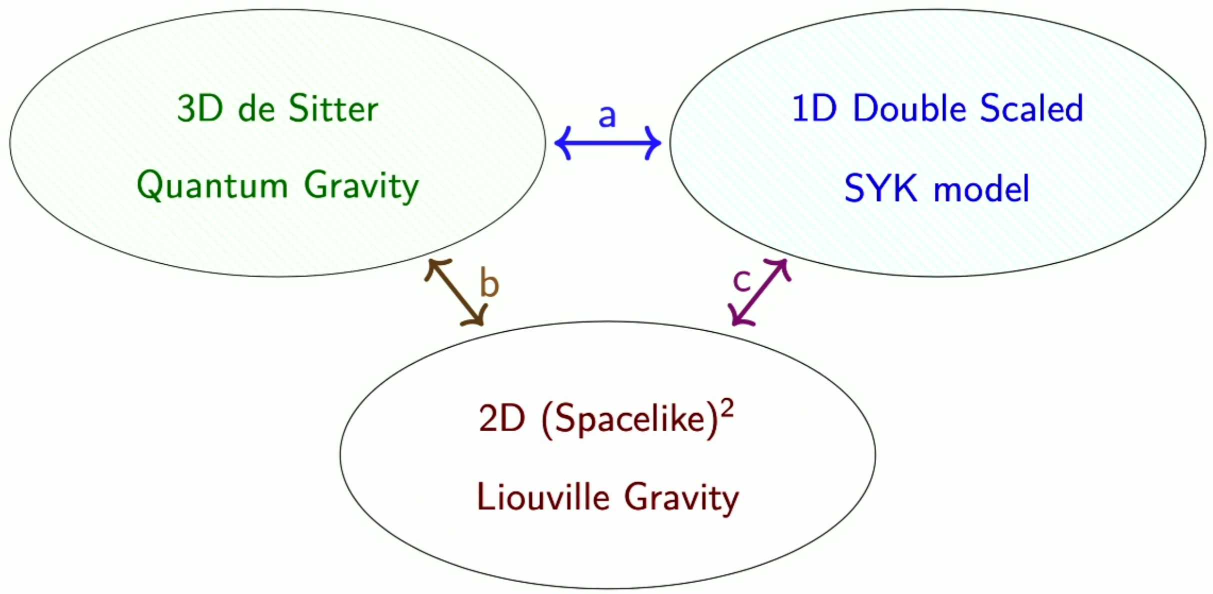
Abstract: In this talk I will describe the correspondence between the double scaling limit of the SYK model and infinite temperature and de Sitter gravity in 2 and 3 dimensions. The dictionary involves a precise match between the chord rules that govern the SYK correlation functions and the braid interactions between line operators in the gravity theory.

Zoom link

Double scaled SYK and de Sitter gravity

Herman Verlinde

Perimeter Institute, 04/23/24



3D de Sitter
Quantum Gravity
a
1D Double Scaled
SYK model
b
c
2D (Spacelike)^2
Liouville Gravity

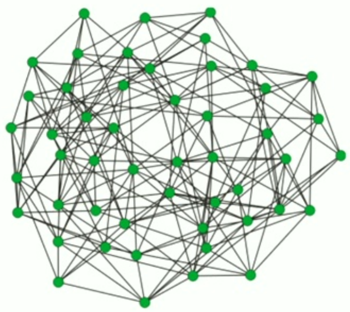
SYK model = 1D many body QM with maximal chaos

$$H_{\mathrm{SYK}} = i^{p/2} \sum_{i_1 \ldots i_p} J_{i_1 \ldots i_p} \psi_{i_1} \ldots \psi_{i_p}$$

random couplings

$$\{\psi^i, \psi^j\} = \delta^{ij}$$

N majorana variables



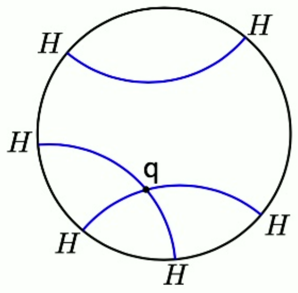
$$\left\langle J^2_{i_1 \ldots i_p} \right\rangle = \mathcal{J}^2 \frac{2^{p-1} p!}{p^2 N^{p-1}}.$$

$$G(\tau_1, \tau_2) = \frac{1}{N} \sum_i \psi_i(\tau_1) \psi_i(\tau_2)$$

$$S_{\mathrm{eff}} = \frac{N}{8p^2} \int d\tau_1 d\tau_2 \left[\partial_{\tau_1} g \partial_{\tau_2} g - 4\mathcal{J}^2 \exp g(\tau_1, \tau_2) \right]$$

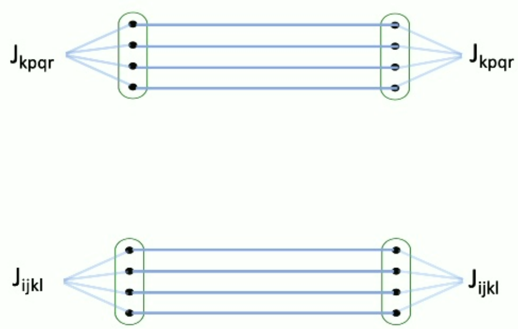
SYK Chord Rules

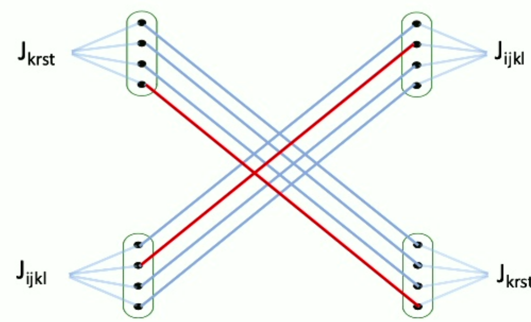
Berkooz et al



Every chord intersection comes with a factor of

$$q = \sum_m e^{-\frac{p^2}{N}} \frac{\left(\frac{p^2}{N}\right)^m}{m!} (-1)^m = e^{-\frac{2p^2}{N}}$$



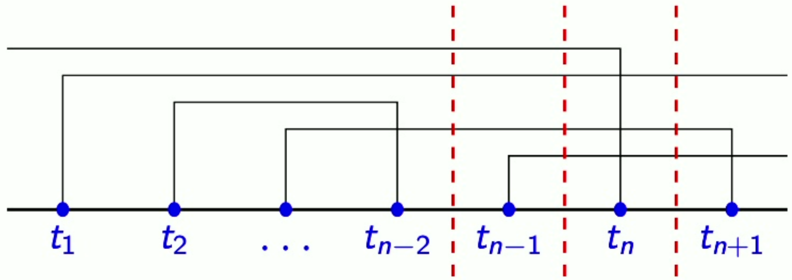


Chord Rules

Introduce an auxiliary Hilbert basis labeled by chord number n

On this basis, the Hamiltonian acts via

$$\mathbf{H}|n\rangle = |n+1\rangle + [n]_q\,|n-1\rangle, \qquad [n]_q \equiv \frac{1-q^n}{1-q}.$$



Introduce the Hamiltonian and chord number operator

$$\mathbf{H} = \mathfrak{a}^\dagger + \mathfrak{a}$$

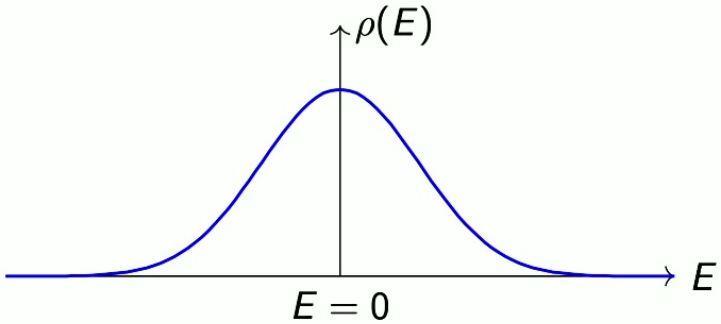
$$\mathfrak{a}^\dagger|n\rangle = |n+1\rangle, \qquad \mathfrak{a}|n\rangle = [n]_q|n-1\rangle, \qquad \mathfrak{n}\,|n\rangle = n\,|n\rangle.$$

The operators $\mathfrak{a}$, $\mathfrak{a}^\dagger$ and $\mathfrak{n}$ satisfy the q-deformed oscillator algebra

$$\mathfrak{a}\,\mathfrak{a}^\dagger = \frac{1-q^{\mathfrak{n}+1}}{1-q}, \qquad \mathfrak{a}^\dagger\mathfrak{a} = \frac{1-q^{\mathfrak{n}}}{1-q}, \qquad [\mathfrak{a}, \mathfrak{a}^\dagger]_q = 1.$$

Energy spectrum of DSSYK

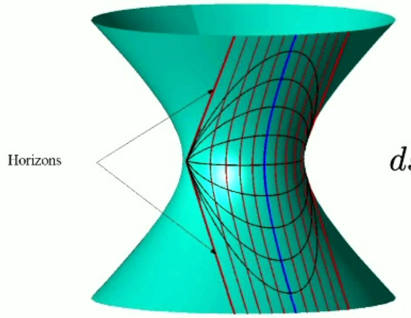
$$\mathbf{H}|\theta\rangle = \frac{\cos\theta}{\sqrt{1-\mathrm{q}}}|\theta\rangle, \qquad \rho(E) = \rho(0)\,\vartheta_1(2\theta, q)$$



Can we explain this spectrum from the gravity side?

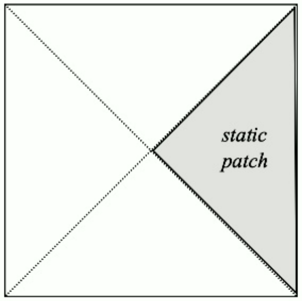
Pure Einstein gravity in 2+1-D de Sitter space:

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\,(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{\gamma}K$$



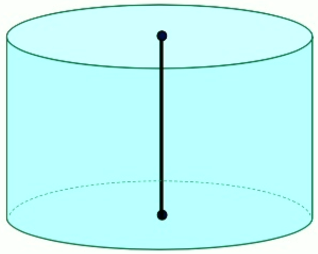
Static patch:

$$ds^2 = -(1-\Lambda r^2)dt^2 + \frac{dr^2}{1-\Lambda r^2} + r^2 d\theta^2$$



Schwarzschild de Sitter thermodynamics:

$$ds^2 = -(1 - 8GE - r^2)dt^2 + \frac{dr^2}{(1 - 8GE - r^2)} + r^2 d\phi^2$$



$$T_{\rm SdS} = \frac{\sqrt{1 - 8GE}}{2\pi}.$$

$$S_{\rm SdS} = \frac{A_H}{4G} = \frac{\pi}{2G}\sqrt{1 - 8GE}.$$

$$\frac{dS_{\rm SdS}}{d(-E_{\rm dS})} = \frac{1}{T_{\rm SdS}}$$

Can we reproduce this from a microscopic theory?

2+1 de Sitter gravity can be reformulated as an $SL(2,\mathbb{C})$ CS theory

$$S_E = i\kappa \int (AdA + \tfrac{2}{3}A^2) - i\kappa \int (\bar{A}d\bar{A} + \tfrac{2}{3}\bar{A}^3)$$

$$A = e + \omega \qquad\qquad \bar{A} = e - \omega$$

Quantum states of SL(2,C) CS theory are Virasoro conformal blocks:

$$S = \frac{i\kappa}{2\pi}\int d^2z\Big(\frac{1}{2}\partial\phi_+\bar{\partial}\phi_+ + 2e^{\phi_+}\Big) - \frac{i\kappa}{2\pi}\int d^2z\Big(\frac{1}{2}\partial\phi_-\bar{\partial}\phi_- + 2e^{\phi_-}\Big)$$

$$b_\pm = e^{\pm i\pi/4}\beta \qquad ; \qquad c_\pm = 13 \pm i\Big(\beta^2 - \frac{1}{\beta^2}\Big)$$

2D gravity theory: $c_+ + c_- = 26$ + bc ghost

Doubled SYK model

Motivated by the triality between SYK, LCFT and dS_3 gravity, we consider a **pair** of double scaled SYK models, coupled via physical state condition

$$(H_L - H_R)|\mathrm{phys}\rangle = 0.$$

These exist if H_L and H_R have the same random couplings. We will denote

$$|E\rangle = |E\rangle_L\,|E\rangle_R$$

These satisfy

$$H|E\rangle = E|E\rangle, \qquad \text{with} \qquad H = \frac{1}{2}(H_L + H_R)$$

We interpret the time evolution generated by H as the physical time.

Effectively treat one SYK model as the clock for the other SYK model.

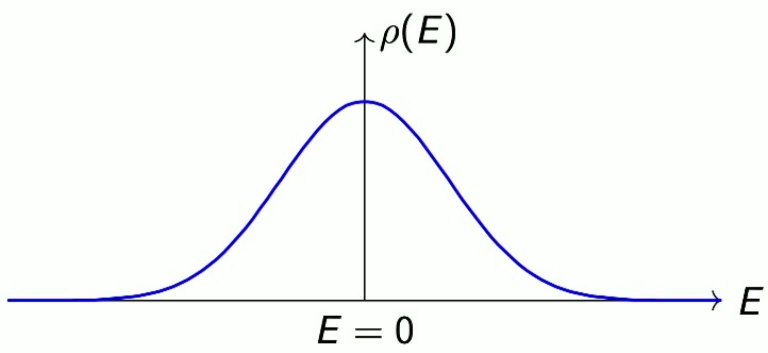
Energy spectrum of Doubled SYK Model

$$E = \frac{-2\cos\lambda s}{\sqrt{\lambda(1-q)}}, \qquad \rho(E) = \rho(0)\,\vartheta_1(2\lambda s, q)$$

We identify the de Sitter vacuum state with the special energy eigenstate

$$|\Psi_{\mathrm{dS}}\rangle \; = \; |E_0\rangle, \qquad E_0 = 0$$

This is a maximal entropy state with infinite temperature.



Physical operators must commute with the equal energy constraint

$$[H_L - H_R, \mathcal{O}^{\rm phys}] = 0$$

A natural subclass of physical operators is given by the time-integral of a product of two local SYK operators with total scale dimension 1

$$\boxed{\mathcal{O}^{\rm phys}_{\Delta}(\tau) = \int dt\, \mathcal{O}^{L}_{1-\Delta}(t)\, \mathcal{O}^{R}_{\Delta}(\tau - t)}$$

We can view these as gravitationally dressed operators.

We define two types of physical operators $\mathcal{O}^{\pm}_{\Delta}(\tau)$ time shifted by $\pm i\beta_{\rm dS}/4$

Two-point function of the physical operators

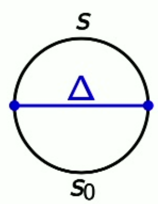
$$\boxed{G_\Delta^{ab}(\tau) = \langle \Psi_{\rm dS} | \mathcal{O}_\Delta^a(\tau)\, \mathcal{O}_\Delta^b(0) | \Psi_{\rm dS} \rangle} \qquad a, b = \pm$$

One finds

$$G_\Delta^{++}(\tau) \;=\; \int dE\, G_\Delta(E)\, e^{-i(E-E_0)\tau}$$

with

$$G_\Delta(E) \;=\; \rho(E)$$



$$= \rho(0)\, \vartheta_1(2\lambda s, q)\, \frac{\vartheta_1(2i\lambda\Delta, q)}{\vartheta_1(\lambda(i\Delta \pm s_0 \pm s), q)}.$$

Writing $s = s_0 - \omega/2$ and $\mu = 2\Delta - 1$

$$G_\Delta(E)_{|\lambda\to 0} = \frac{\sin \pi\mu}{\cosh \frac{\pi}{2}(\omega + i\mu) \cosh \frac{\pi}{2}(\omega - i\mu)}$$

Fourier transforming gives

$$G_\Delta^{++}(\tau)_{|\lambda\to 0} = \int d\omega\, G_\Delta(E)_{|\lambda\to 0}\, e^{i\omega t} = \frac{2\sinh \mu\tau}{\pi \sinh \tau}$$

$$G_\Delta^{+-}(\tau)_{|\lambda\to 0} = \frac{2\sinh(\mu(i\pi + \tau))}{\pi \sinh \tau}$$

Candidate Holographic Dual: JT/de Sitter Gravity

$$S_{\text{gravity}} = \frac{1}{2\lambda} \int_{\mathcal{M}} \sqrt{-g}\, (\Phi R + \cosh(\Phi)) + \text{b.t.}$$

Introducing $d\hat{s}^2 = e^{\Phi}\, ds^2$ the vacuum classical solution assumes the form

$$d\hat{s}^2 = \frac{-4dUdV}{(1-UV)^2}, \qquad e^{\Phi} = \frac{1+UV}{1-UV}$$

This is the dimensional reduction of the 3D de Sitter metric

$$ds^2_{3D} = d\hat{s}^2_{2D} + e^{2\Phi} d\varphi^2, \qquad e^{\Phi} = \frac{1+UV}{1-UV}$$

Scalar Green's Function

2D matter action obtained by dimensional reduction from 3D

$$S_{\rm matter} = -\int \sqrt{-\hat{g}_2}\, e^{\Phi} (\hat{\nabla}\varphi_+ \hat{\nabla}\varphi_- + m^2 \varphi_+ \varphi_-)$$

In terms of the proper time τ, the Green function solves

$$\left(\partial_\tau^2 - \mu^2\right) \frac{G(\tau)}{\sinh \tau} = 0$$

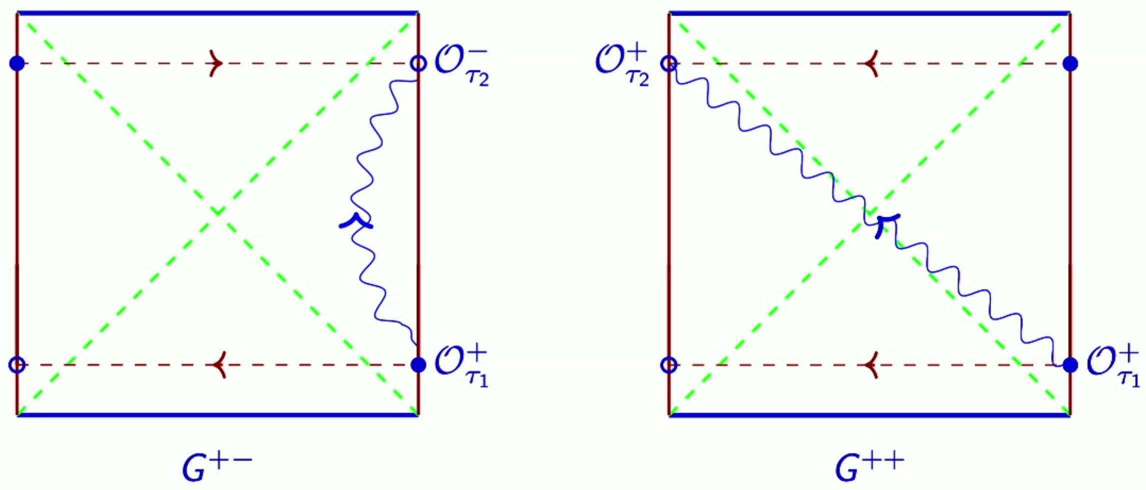
with $\mu^2 = 1 - m^2$. This has two independent solutions

$$G(\tau) = \frac{\sinh \mu(i\pi - \tau)}{\sin \tau}, \qquad G_A(\tau) = \frac{\sin \mu\tau}{\sin \tau}$$

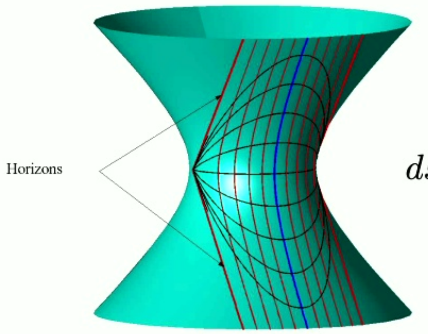
This can be writen as a sum over quasinormal mode frequencies

$$G(\tau) = \sum_n c_+ \, e^{i\omega_n^+ \tau} + \sum_n c_- \, e^{i\omega_n^- \tau}, \qquad \omega_n^\pm = i(\Delta_\pm + n)$$

⇒ precise match between the SYK 2-pt function and Green's function of a massive scalar field on dimensional reduction of 3D de Sitter to 2D.

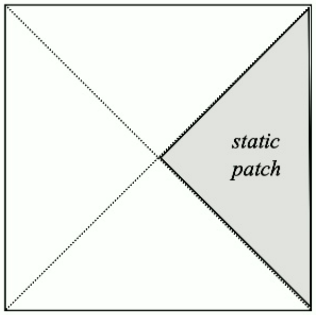


Observing the de Sitter temperature



Static patch:

$$ds^2 = -(1 - \Lambda r^2)dt^2 + \frac{dr^2}{1 - \Lambda r^2} + r^2 d\theta^2$$



Consider a detector coupling to the SYK model via

$$\int d\tau \Big(X^+(\tau)\mathcal{O}^-_\Delta(\tau) + X^-(\tau)\mathcal{O}^+(\tau) \Big)$$

Transition rate is given by the detector response function

$$\dot{P}_{E_i \to E_j} = |X_{ij}|^2 G_\Delta(E_j - E_i) \qquad G_\Delta(E) = \int d\tau \, e^{-iE\tau} G_\Delta(\tau - i\epsilon)$$

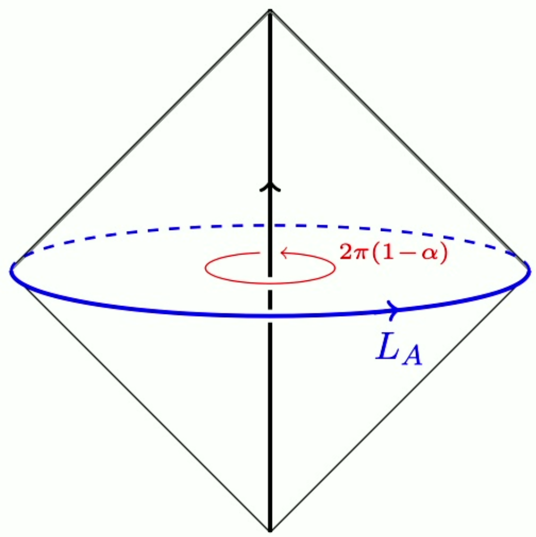
From the explicit form of the Green's function, one derives that

$$\frac{\dot{P}_{E_i \to E_j}}{\dot{P}_{E_j \to E_i}} = e^{-2\pi(E_j - E_i)}$$

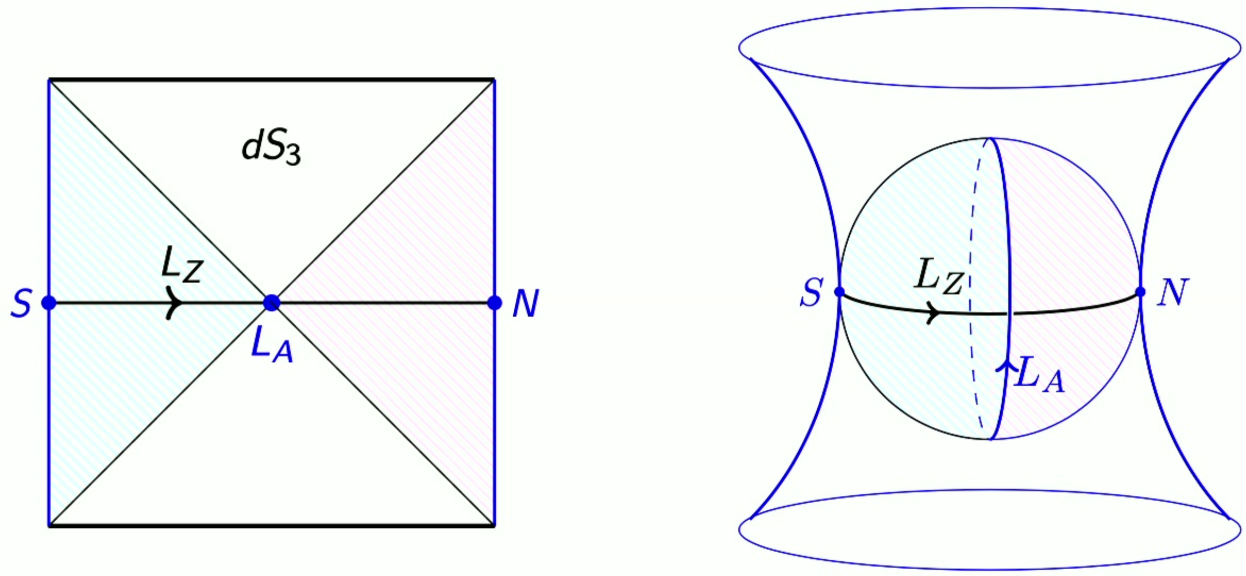
The observer sees an environment with spectral entropy $S(E)$ with

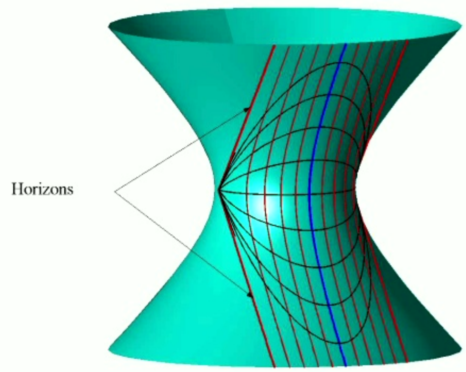
$$e^{-2\pi(E_j - E_i)} = e^{S_{\rm dS}(E_i) - S_{\rm dS}(E_j)}$$

Can we explain the SYK spectrum from Gravity?



$$2\pi\alpha = 8\pi G_N M = \theta \quad ?$$


dS_3
L_Z
S
N
L_A
L_Z
S
N
L_A



$$\mathbf{X} = \begin{pmatrix} X_0 + X_3 & X_1 + iX_2 \\ X_1 - iX_2 & X_0 - X_3 \end{pmatrix}, \qquad \det \mathbf{X} = -1.$$

$$\mathbf{X} \rightarrow gXg^\dagger \qquad g \in SL(2, \mathbb{C})$$

Let $J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. We impose $\mathcal{CPT}$ symmetry with

$$\mathcal{PT}: \qquad \mathbf{X} \to J\mathbf{X}^{-1}J$$

Breaks $SL(2,C)$ to $SU(1,1)$ matrices that satisfy

$$g^\dagger J g = J.$$

$SU(1,1)$ admits two invariant traces, the usual one and

$$\mathrm{tr}_J(g) \equiv -i\,\mathrm{tr}(gJ)$$

$$\mathbf{X} \rightarrow g_A \mathbf{X} g_A^{-1}, \qquad g_A = \begin{pmatrix} e^{i\pi\alpha} & 0 \\ 0 & e^{-i\pi\alpha} \end{pmatrix}$$

deficit angle

$$\mathbf{X} \rightarrow h_Z \mathbf{X} h_Z, \qquad h_Z = e^{i\pi/2} \begin{pmatrix} e^{z/2} & 0 \\ 0 & e^{-z/2} \end{pmatrix}$$

north south holonomy
$t_N = t_S + z$

$$\boxed{L_A \equiv \mathrm{tr}_J(g_A) = 2\sin(\pi\alpha)}$$

$$\boxed{L_Z \equiv e^{z/2}}$$

Static coordinate system that covers that both static patches

$$(X_0, X_3, X_1, X_2) = (\cos\rho \sinh t, \cos\rho \cosh t, \sin\rho \cos\varphi, \sin\rho \sin\varphi)$$

$$t \in \mathbb{R}\ ,\ \ 0 \leq \varphi \leq 2\pi(1-\alpha)\ ;\ \ 0 \leq \rho \leq \pi.$$

The metric in these coordinates the metric becomes

$$ds^2 = -\cos^2\rho\, dt^2 + \sin^2\rho\, d\varphi^2 + d\rho^2$$

$$\omega\ = \omega^a \sigma_a\ = i \sin\rho\, \sigma_3\, dt + \cos\rho\, \sigma_2\, d\varphi$$

$$e\ =\ e^a \sigma_a\ = \sigma_1\, d\rho + \cos\rho\, \sigma_2\, dt + i\sin\rho\, \sigma_3\, d\varphi$$

At the equator

$$A_\pm = A^a_\pm \sigma_a = \pm i\sigma_1 d\rho \pm i\sigma_3(dt \pm id\varphi)$$

$$g^\pm_A \;\equiv\; \mathrm{P}\exp\oint_A A_\pm = \cos(\pi\alpha) + i\sin(\pi\alpha)\sigma_3$$

$$g^\pm_Z \;\equiv\; \mathrm{P}\exp\int_S^N A_\pm = \begin{pmatrix} 0 & e^{z/2} \\ -e^{-z/2} & 0 \end{pmatrix}$$

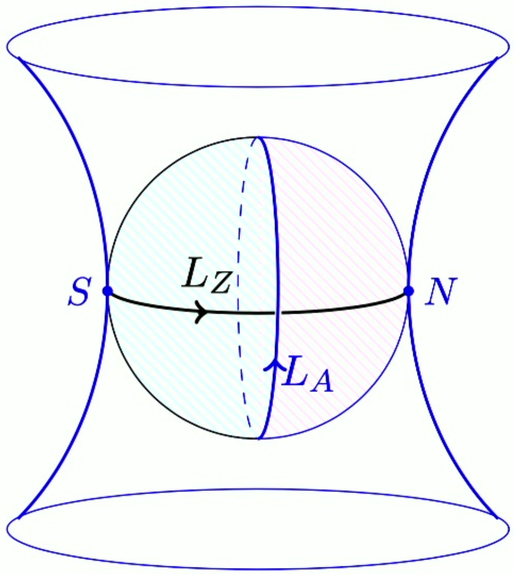
$$L_A \equiv \mathrm{tr}_J(g_A) \;=\; 2\sin(\pi\alpha)$$

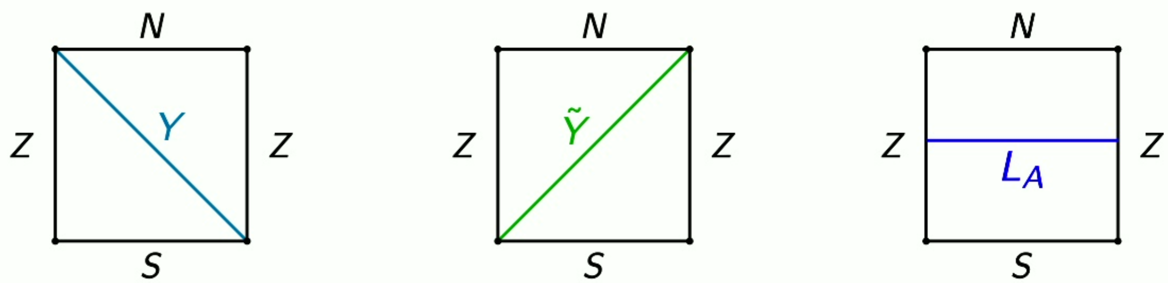
$$L_Z \equiv e^{z/2}$$

$$\mathbf{H}|\theta\rangle = \frac{\cos\theta}{\sqrt{1-\mathfrak{q}}}|\theta\rangle$$

$$L_A|\alpha\rangle = 2\sin(\pi\alpha)|\alpha\rangle$$

$$\mathbf{H} = \frac{L_A}{\sqrt{1-\mathfrak{q}}}$$

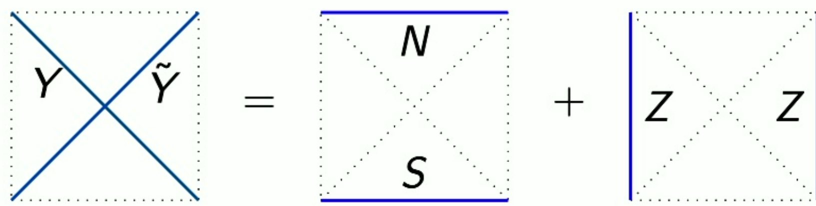




$$ZL_A = Y + \tilde{Y}$$

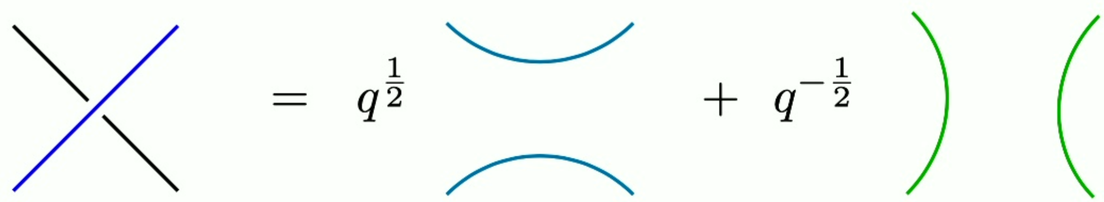
$$\tilde{Y}Y = NS + Z^2$$

Ptolemy theorem



$$[A_a^{\pm}(x), A_b^{\pm}(y)] = \pm\hbar\epsilon_{ab}\,\delta(x-y), \qquad \hbar \equiv \frac{2\pi i}{\kappa}$$

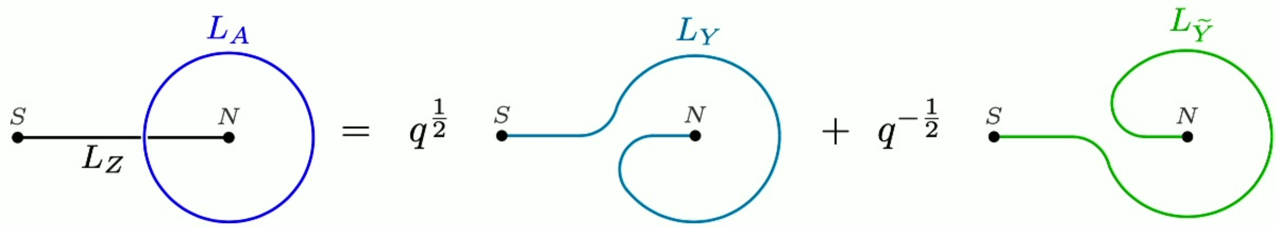
Skein relation



$$q = e^{-\frac{2\pi}{\kappa}} = e^{-4\pi G_N}$$

q-Deformed Ptolemy theorem

$$L_Z L_A \;=\; q^{\frac{1}{2}} L_Y \;+\; q^{-\frac{1}{2}} L_{\widetilde{Y}} \qquad \left(\begin{matrix}\text{skein}\\ \text{relation}\end{matrix}\right)$$



$$ZY = qYZ, \qquad Z\tilde{Y} = q^{-1}\tilde{Y}Z$$

$$\tilde{Y}Y = 1 + qZ^2, \qquad Y\tilde{Y} = 1 + q^{-1}Z^2$$

Introduce the gravitational Hamiltonian and chord number operator

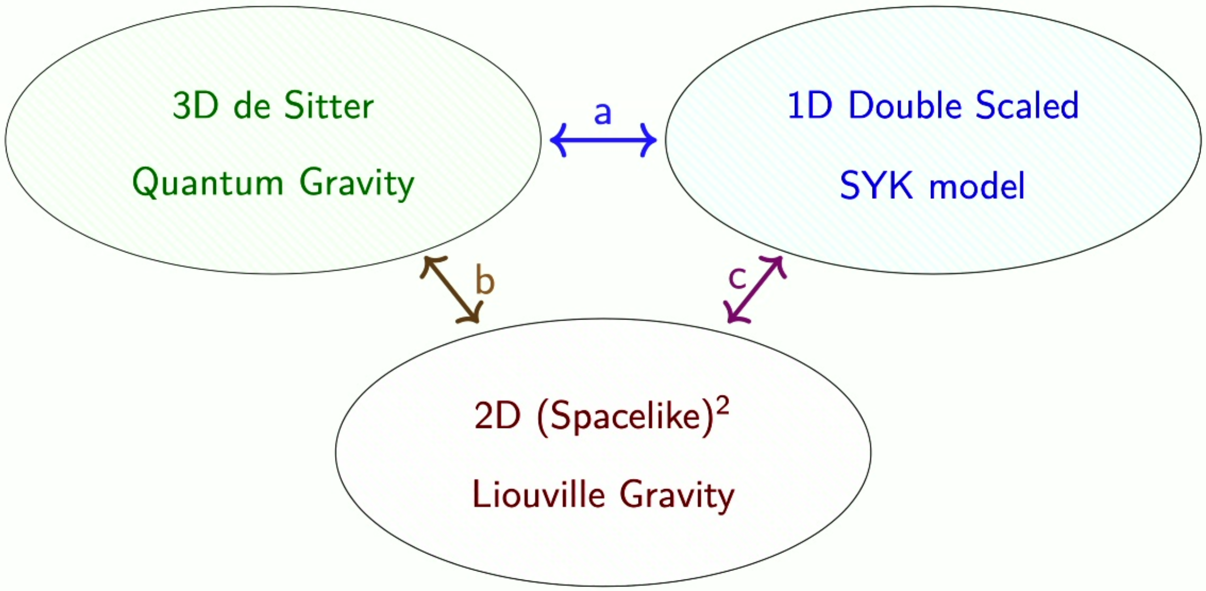
$$\mathbf{H} = \frac{L_A}{\sqrt{1-q}} = a^\dagger + a \qquad\qquad q^n = q^{1/2} Z^{-2}$$

Here $a^\dagger$ and a are defined as

$$\sqrt{1-q}\, a^\dagger = q^{\frac{1}{4}} Z^{-1} Y, \qquad \sqrt{1-q}\, a = q^{\frac{1}{4}} \tilde{Y} Z^{-1}$$

The operators a, $a^\dagger$ and n satisfy the q-deformed oscillator algebra

$$a\, a^\dagger = \frac{1-q^{n+1}}{1-q}, \qquad a^\dagger a = \frac{1-q^n}{1-q}, \qquad [a, a^\dagger]_q = 1.$$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