Title: Holographic Phase Transitions in the early Universe

Speakers: Rashmish Mishra

Series: Particle Physics

Date: April 26, 2024 - 1:00 PM

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Abstract: Strongly coupled confining theories are well-motivated in many BSM frameworks. The early universe cosmological history of these theories provides possibilities for observable signals. These theories undergo confinement deconfinement phase transition in the early universe, which can result in gravitational wave signals, observable in upcoming experiments. Using AdS/CFT, these theories have been studied in the Randall-Sundrum framework, and various quantitative aspects of the phase transition have been calculated. In the models that have been considered, the rate of transition from the deconfined phase to the confined phase is very small and leads to a period of supercooling. This enhances the gravitational wave signal, but presents a tension between a low confinement scale and fitting to the standard picture of BBN. In this talk, I will briefly review the calculations leading to these conclusions, and argue that some of the issues are specific to the simplified models that have been studied. I will present two modifications that are expected on general grounds, motivated by including strong IR effects systematically. Such effects change the results significantly. In particular, new qualitative features appear which have been missed in previous investigations. I will briefly comment on the phenomenological implications and open questions. The talk will be based on 2309.10090 and 2401.09633.

Zoom link

Perimeter HEP Seminar

Holographic Phase Transitions in the Early Universe

Rashmish K. Mishra
Harvard University

With L. Randall,
Based on JHEP 2023, 36 (2309.10090) + 2401.09633 + in progress

Strongly coupled theories appear in several BSM directions

Naturalness, Dark Matter/Dark Sectors, Composite Axions…

- Phenomenologically relevant theories are confining theories with a small confinement scale, generated via dimensional transmutation like in QCD.

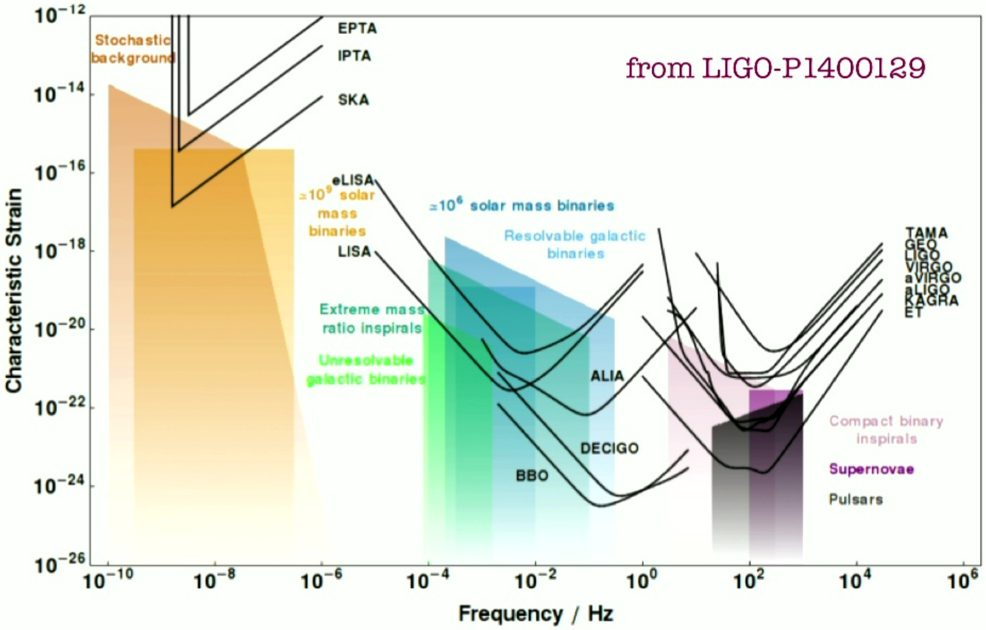
- Randall-Sundrum framework provides calculability via AdS/CFT to make predictions about strong dynamics.

Early Universe Cosmology

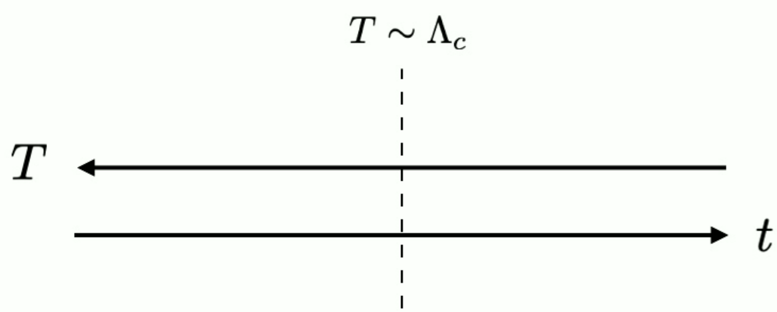
Strong dynamics can generate many interesting phenomena

- Deconfinement/Confinement transition in the early universe
- Non-standard cosmologies with changing scale factors

Phenomenological relevance: Can lead to stochastic gravitational waves visible in present and upcoming detectors.



Early Universe Cosmology



Generic prediction is a period of supercooling.

Statics — High Temperature: Deconfined Phase | Low Temperature: Confined Phase

Dynamics — Starting with (meta-stable) phase, the system settles to the stable phase.

This is due to a small rate of transition from the deconfined to the confined phase.

Supercooling: feature or bug?

Λ_c ~ TeV is close to BBN ~ MeV.

A small rate/supercooling presents a challenge for embedding into the standard picture of cosmology

Λ_c is a free parameter.

Alternatively, for phase transition in decoupled dark sectors, small rate/supercooling helps in interesting dynamics (enhanced gravitational waves, PBH, bubble wall dynamics)

How generic are these features?

Randall-Sundrum framework

A slice of 5D AdS space, between a “UV” and an “IR” brane.

AdS boundary

$$\mathrm{d}s^2 = e^{-2r}\mathrm{d}x^2 + \mathrm{d}r^2$$
$$0 \le r \le r_{\mathrm{ir}}$$
$$\ell_{\mathrm{AdS}} = 1$$

UV Brane

$r = 0$

IR Brane

$r = r_{\mathrm{ir}}$

By AdS/CFT duality, RS is dual to a strongly coupled CFT (coupled to 4D Einstein gravity) that confines in the IR.

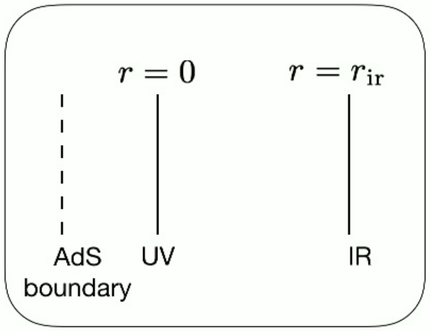
RS geometry needs a stabilization mechanism, e.g. provided by a 5D bulk scalar. Dual to a scalar deformation of the CFT.

Two phases of the theory

Creminelli, Nicolis, Rattazzi, hep-th/0107141

There are *two* solutions to the Einstein equations with a constant negative cosmological constant:

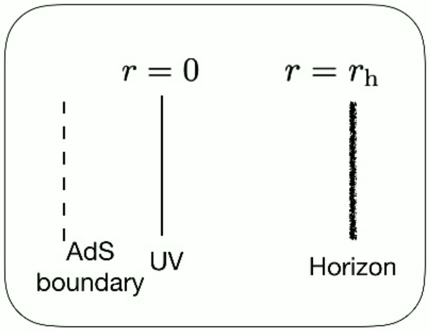
Confined Phase



RS $$\mathrm{d}s^2 = -e^{-2r}\mathrm{d}t^2 + e^{-2r}\mathrm{d}x^2 + \mathrm{d}r^2$$

$$0 \le r \le r_{\rm ir}$$

Deconfined Phase



BB $$\mathrm{d}s^2 = -e^{-2r}\left(1 - e^{4(r-r_h)}\right)\mathrm{d}t^2 + e^{-2r}\mathrm{d}x^2 + \left(1 - e^{4(r-r_h)}\right)^{-1}\mathrm{d}r^2$$

$$0 \le r \le r_{\rm h}$$

An approximate solution, exact in the limit of UV brane sent to boundary.

Temperature: $T = \frac{1}{\pi}e^{-r_h}$

Free Energy of the two phases

Creminelli, Nicolis, Rattazzi, hep-th/0107141

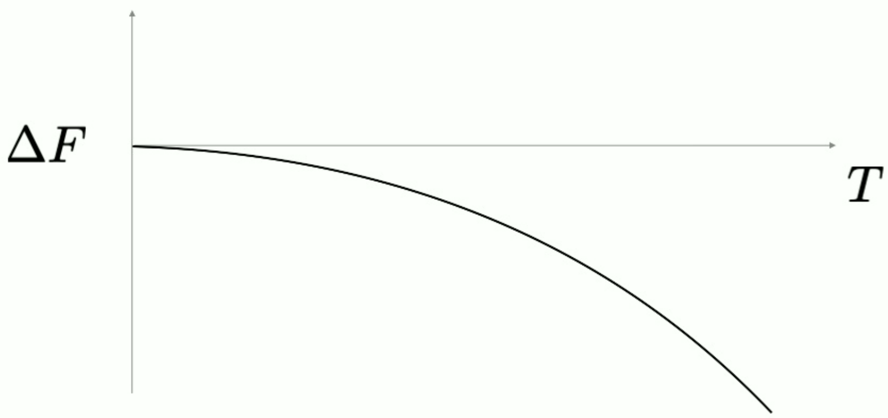
Without a stabilization mechanism:

Free Energy is UV divergent. Difference in the free energy is a physical quantity and is finite.

$$F_{BB} - F_{RS} = -2\pi^4 M_5^3 T^4$$

$$e^{-\beta F} = \int \mathcal{D}\phi \, e^{-S_E} \approx e^{-S_E^0}$$

$$F = T S_E^0$$



$\Delta F < 0$ at all temperatures.

The BB phase is always the thermodynamically preferred phase.

Two phases of the theory

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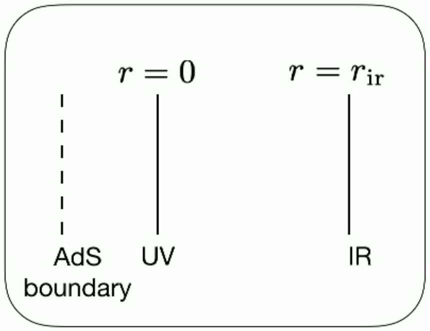

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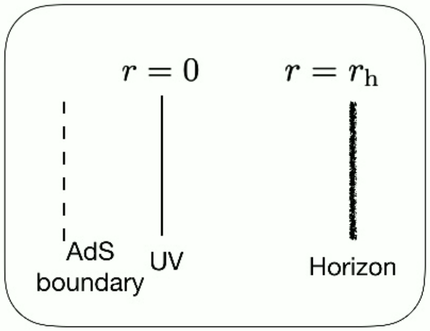

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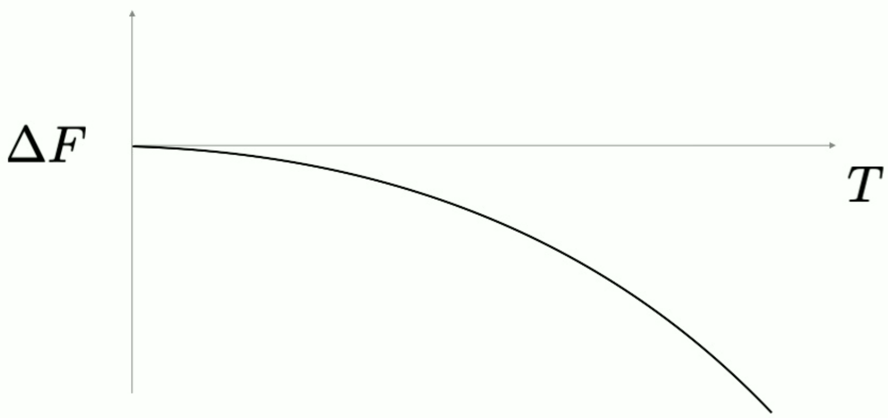

$\Delta F < 0$ at all temperatures.

The BB phase is always the thermodynamically preferred phase.

Free Energy of the two phases

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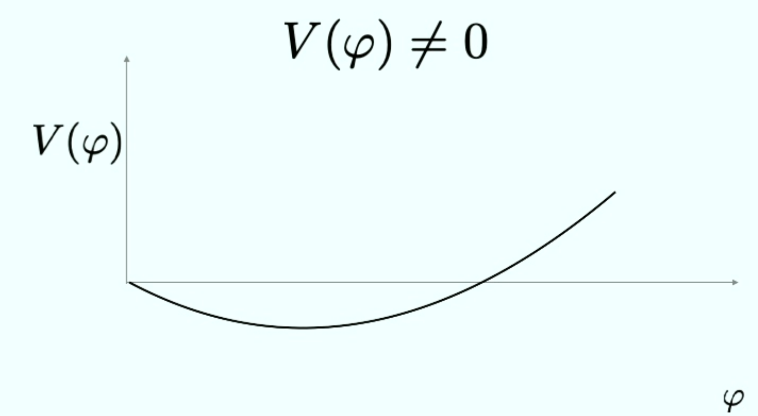
With a stabilization mechanism (e.g. a 5D bulk scalar)

Without a stabilization, one can identify the modulus which has a flat potential: the **radion** φ

The stabilization mechanism generates a potential for the radion.



Bulk scalar action

$$S_\chi = \int d^5x\sqrt{g}\left(-\frac{1}{2}(\partial\chi)^2 - V_B(\chi)\right) - \sum_i \int d^4x\,\sqrt{g_i}\,V_i(\chi)$$

$V_B(\chi)$: bulk potential

$V_i(\chi)$: i = UV, IR, fixes the boundary condition

Free Energy of the two phases

Creminelli, Nicolis, Rattazzi, hep-th/0107141

With a stabilization mechanism

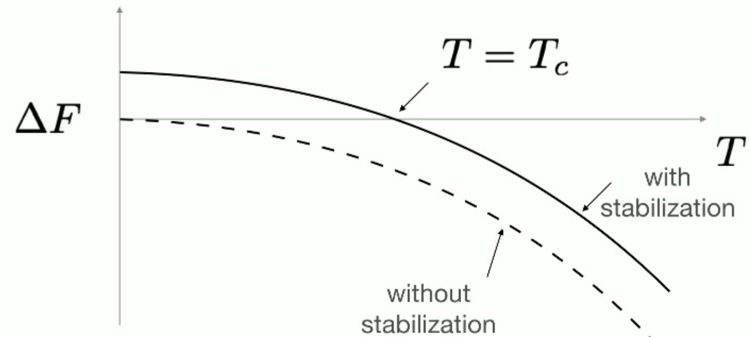
$$F_{RS} \approx V(\varphi_{\rm min}) + \mathcal{O}(T^4)\ ,\ V(\varphi_{\rm min}) < 0$$

$$\Delta F = F_{BB} - F_{RS} = -2\pi^4 M_5^3 T^4 - V(\varphi_{\rm min}) \equiv \quad 2\pi^4 M_5^3 (T_c^4 - T^4)$$

$\Delta F = 0$ at $T = T_c$

$T > T_c$: BB geometry (deconfined phase) favored.

$T < T_c$: RS geometry (confined phase) favored.



Free Energy of the two phases

Creminelli, Nicolis, Rattazzi, hep-th/0107141
Agashe, Du, Ekhterachian, Kumar, Sundrum, 2010.04083

Explicit expression for T_c depends on the details of the stabilization.

Examples with small back-reaction (small breaking of CFT in the IR)

A

$$V_B(\chi) = 2\epsilon\chi^2$$

$$\chi_{\rm UV} = v_{\rm uv}, \chi_{\rm IR} = v_{\rm IR},$$

$$V(\varphi) \sim \varphi^4 \left(1 - \left(\frac{\varphi}{\varphi_{\rm min}}\right)^{\epsilon}\right)^2 - \epsilon\varphi^4$$

$$T_c \sim \epsilon^{3/8}\varphi_{\rm min}$$

B

$$V_B(\chi) = 2\epsilon\chi^2$$

$$\chi_{\rm UV} = v_{\rm uv}, \chi'_{\rm IR} = -\alpha$$

$$V(\varphi) \sim \varphi^4 \left(1 - \frac{1}{1+\epsilon/4}\left(\frac{\varphi}{\varphi_{\rm min}}\right)^{\epsilon}\right)$$

$$T_c \sim \epsilon^{1/4}\varphi_{\rm min}$$

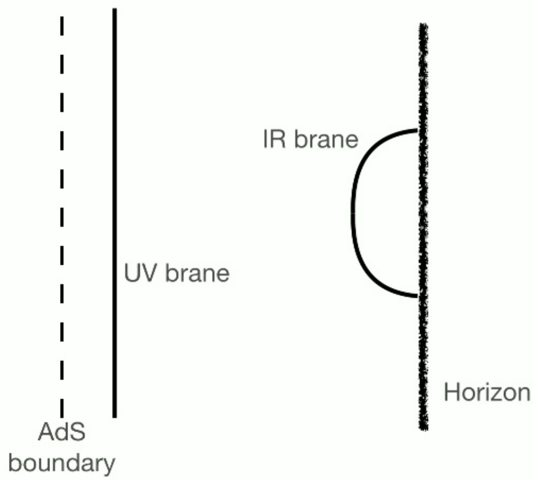
$$T_c \ll \varphi_{\rm min}$$

Dynamics of phase transition

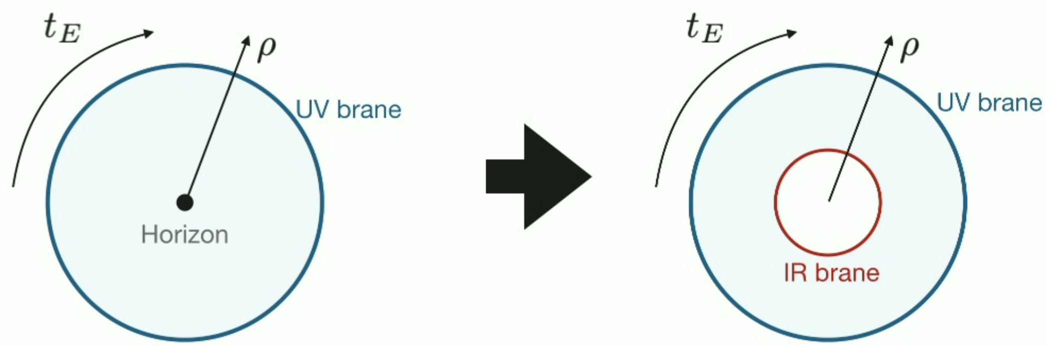
Creminelli, Nicolis, Rattazzi,
hep-th/0107141

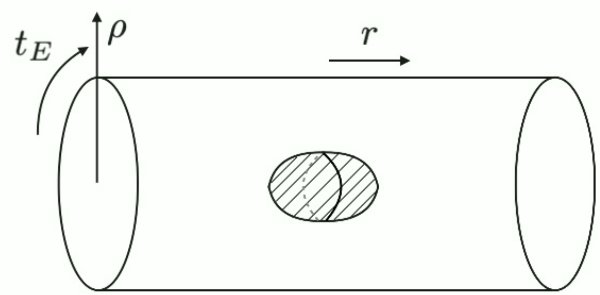
5D picture

A non-perturbative, topology changing, 5D quantum gravity process, in which bubbles of IR brane nucleate from the horizon, expand and collide.



$\sim e^{-1/G_{N,5}}$





Dynamics of phase transition

Creminelli, Nicolis, Rattazzi, hep-th/0107141

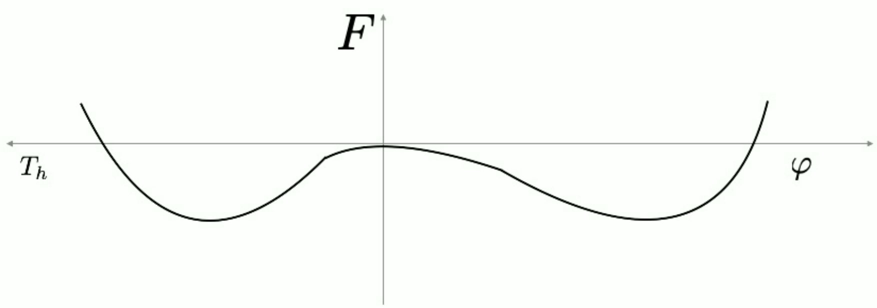
Addressed in the 4D radion EFT

Off-shell free energy

$$F_{RS} = V(\varphi) + \mathcal{O}(T^4)\ , \qquad F_{BB} = 6\pi^4 M_5^3 T_h^4 - 8\pi^4 M_5^3 T T_h^3 + \mathcal{O}(\epsilon)$$

$$T_h = \frac{1}{\pi} e^{-r_h}$$

$T_h = T$ on-shell



In the radion EFT, the rate for phase transition becomes a false vacuum decay problem in field theory.

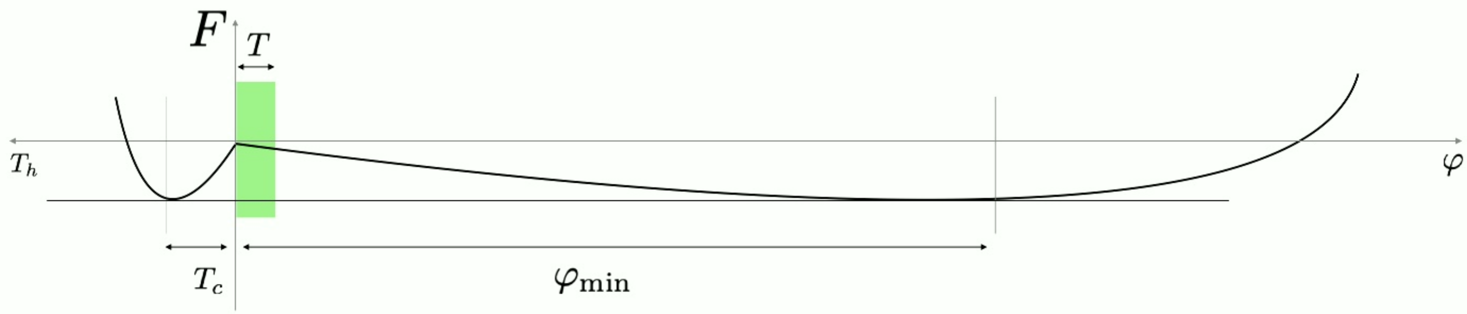
But with important features.

Dynamics of phase transition

Creminelli, Nicolis, Rattazzi,
hep-th/0107141

Addressed in the radion EFT

There are temperature corrections to the picture

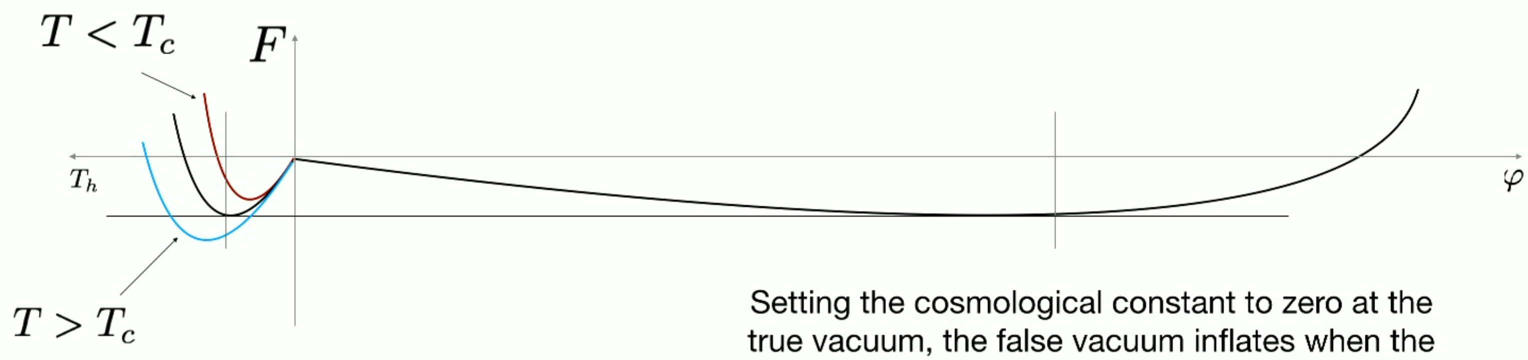


Dynamics of phase transition

Creminelli, Nicolis, Rattazzi, hep-th/0107141

Addressed in the radion EFT

The left side changes with temperature



Setting the cosmological constant to zero at the true vacuum, the false vacuum inflates when the phase transition can take place (i.e. $T < T_c$)

Phase transition completes when the rate $\Gamma > H^4$, $H^2 \sim \rho_{\rm vac}/M_{\rm pl}^2$, $\rho_{\rm vac} \sim 2\pi^4 M_5^3 T_c^4$

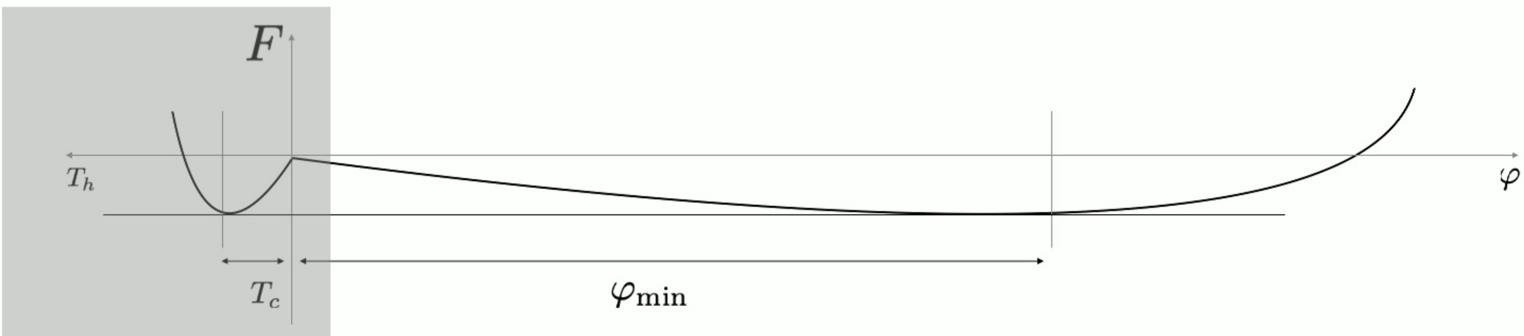
Depending on the rate, the phase transition from the hot phase to the RS phase will occur at $T = T_n < T_c$

Dynamics of phase transition

Creminelli, Nicolis, Rattazzi,
hep-th/0107141

Addressed in the radion EFT

An estimate for the bounce action is possible



Subleading contribution

Dynamics of phase transition

Addressed in the radion EFT

Thin wall limit: $T \lesssim T_c$

A $$V(\varphi) \sim \varphi^4 \left(1 - \left(\frac{\varphi}{\varphi_{\min}}\right)^{\epsilon}\right)^2 - \epsilon\varphi^4 \qquad S_3/T \approx 0.13 \frac{N^2}{\epsilon^{9/8}(v_{\rm ir}/N)^{3/2}} \frac{T_c/T}{(1-(T/T_c)^4)^2}$$

B $$V(\varphi) \sim \varphi^4 \left(1 - \frac{1}{1+\epsilon/4}\left(\frac{\varphi}{\varphi_{\min}}\right)^{\epsilon}\right) \qquad S_3/T \approx 8 \frac{N^2}{(\epsilon\lambda)^{3/4}} \frac{T_c/T}{(1-(T/T_c)^4)^2}$$

Recall

$M_5^3 \sim N^2$

Dynamics of phase transition

Addressed in the radion EFT

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Recall

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General structure: $\Gamma \sim \exp\left(-\frac{N^2}{\delta} f(T/T_c)\right)$

$N \gg 1$: theoretical control

$\delta \ll 1$: small CFT breaking

$f(T/T_c) \sim 1$: thin wall

All these features make the rate small.

Rate in 4D EFT calculation

$$\Gamma \sim \exp\left(-\frac{N^2}{\delta} f(T/T_c)\right)$$

A measure of CFT breaking in the IR

The CFT breaking in IR can be different (and larger) than in UV. It can come entirely from CFT dynamics or from interplay with fields that couple to the CFT.

Csaki et al (2301.10247): Explicit CFT breaking with a relevant operator.

Agashe et al (2010.04083): Flow from a UV CFT to an IR CFT

Harling, Servant (1711.11554),
Baratella, Pomarol, Rompineve (1812.06996): Contribution from QCD condensate.

Rate in 4D EFT calculation

A measure of the CFT degrees of freedom

$$\Gamma \sim \exp\left(-\frac{N^2}{\delta} f(T/T_c)\right)$$

N in the IR can be different than in the UV, as in some examples of confining gauge theories.

Hassanain, March-Russell, Schvellinger (0708.2060): RS like geometry to model Klebanov-Tseytlin solution (dual to a CFT with reducing N).

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Out of radion EFT?

Most approaches run out of theoretical control quickly, and out of radion EFT anyway.

CFT breaking in the IR is typically different (and larger) than in the UV

4d picture: higher order terms in the beta function of the deformation

$$\mathcal{L}_{\rm CFT} + \lambda \mu^{4-\Delta} \mathcal{O}$$

$$\frac{{\rm d}\lambda}{{\rm d}\log\mu} = (\Delta - 4)\lambda + c_1\lambda^2 + c_2\lambda^3 + \cdots$$

5d picture: higher order terms in the bulk potential for the 5D scalar

$$S_\chi = \int d^5x\sqrt{g}\left(-\frac{1}{2}(\partial\chi)^2 - V_B(\chi)\right) - \sum_i \int d^4x\,\sqrt{g_i}\,V_i(\chi)$$

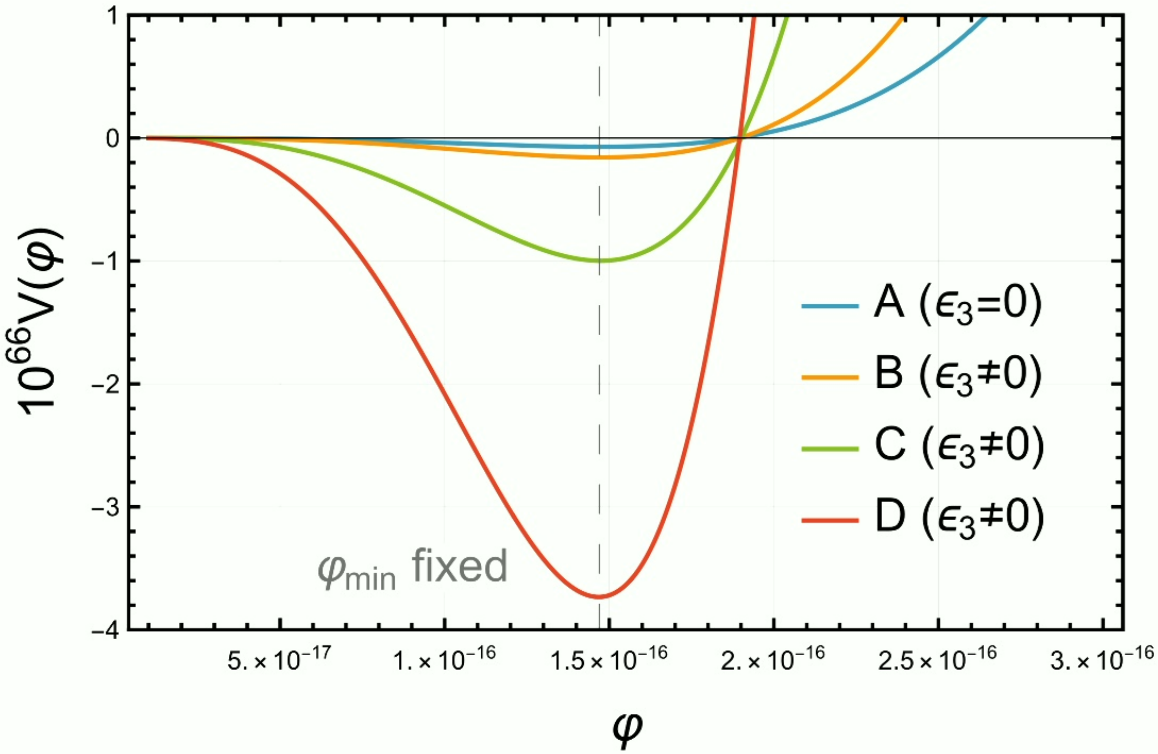
$$\frac{{\rm d}^2\chi}{{\rm d}r^2} - 4\frac{{\rm d}\chi}{{\rm d}r} - \partial_\chi V_B(\chi) = 0$$

$$V_B(\chi) = \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{6}\eta\chi^3 + \cdots \equiv 2\epsilon_2\chi^2 + \frac{4}{3}\epsilon_3\chi^3 + \cdots$$

$$\frac{{\rm d}\chi}{{\rm d}r} \approx -\epsilon_2\chi - \epsilon_3\chi^2 + \cdots$$

Radion Potential from self-interaction

$$S = \int d^4x \sqrt{g} \left(-12 M_5^3 (\partial \varphi)^2 - V(\varphi) \right) \quad \varphi = e^{-r_{\rm ir}}$$



Other parameters fixed.

A: $\epsilon_3 = 0$

B: $\epsilon_3 = -1/100$

C: $\epsilon_3 = -1/90$

D: $\epsilon_3 = -1/80$

Potential becomes deeper from a non-zero self-interaction, without changing $\varphi_{\rm min}$ (i.e. the hierarchy between UV and IR).

Phase Transition: Free Energy and Rate

Follow the approach advocated in the literature so far

- Stay in radion EFT
- Small back-reaction

Bounce Action

$$S_b = \int \mathrm{d}^4x \left(12M_5^2(\partial\varphi)^2 + V(\varphi) + 2\pi^4 M_5^3 T^4\right)$$

Recall: $M_5^3 \sim N^2$

Rescaling $\frac{1}{24M_5^3}\widetilde{V}(\varphi) \equiv \kappa^4\,\varphi^4\,v(\varphi)\,,\; y = \kappa\,\varphi\,T^{-1}\,,\;\; x = \kappa\,r\,T$

$$S_b = \frac{96\pi M_5^3}{\kappa^3}\int \mathrm{d}x\, x^2 \left(\frac{1}{2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + y^4\, v(Ty/\kappa) + \frac{\pi^4}{12}\right)$$

$O(3)$ dominates

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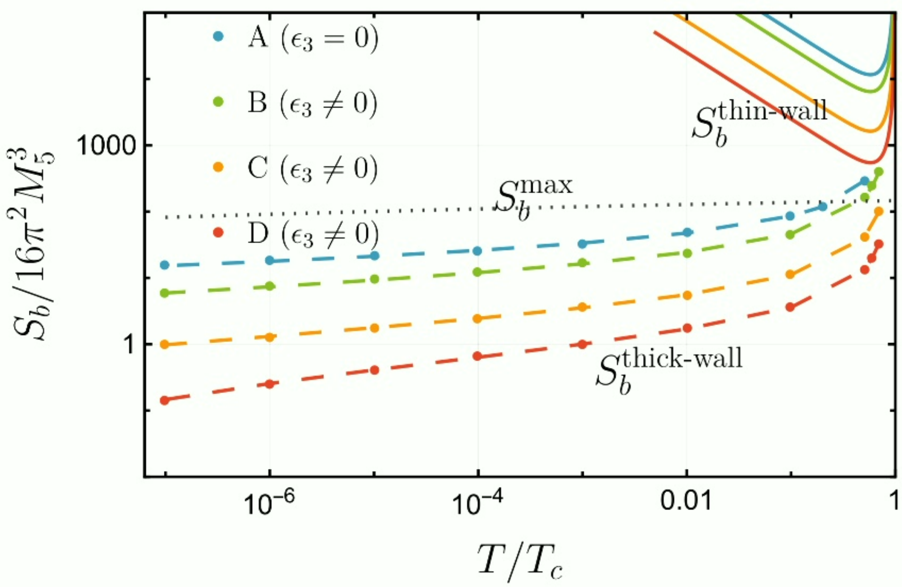
Phase Transition: Free Energy and Rate

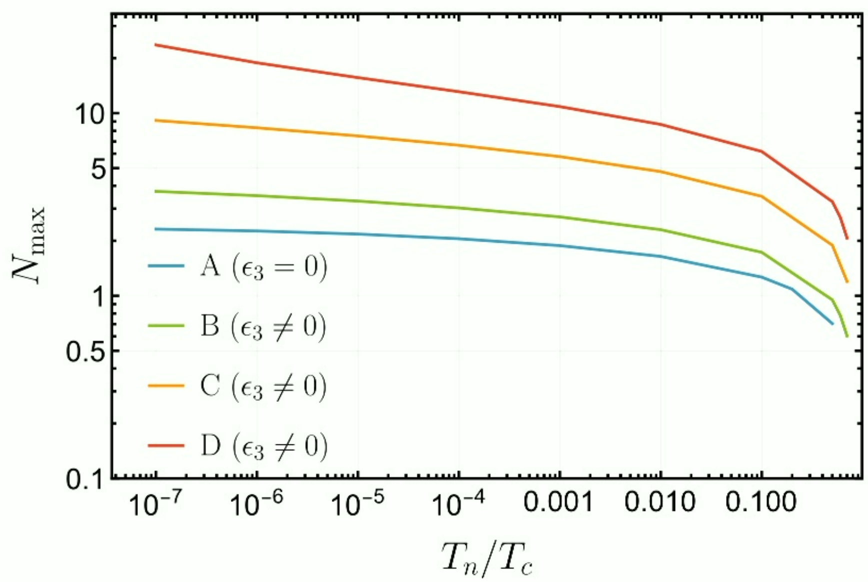
Phase transition completes: $S_b(T) < -\log\left(\frac{4\pi^8}{9}\frac{M_5^6 T_c^8}{T^4 M_{\rm Pl}^4}\right) \equiv S_b^{\rm max}(T/T_c)$

At nucleation Temperature: T_n , $S_b(T_n) = S_b^{\rm max}(T_n/T_c)$

Maximum N for which the phase transition completes: $N_{\rm max}(T_n/T_c) = \sqrt{\frac{S_b^{\rm max}(T_n/T_c)}{S_{b,N_c=1}(T_n)}}$

Phase Transition: Reduction in bounce action

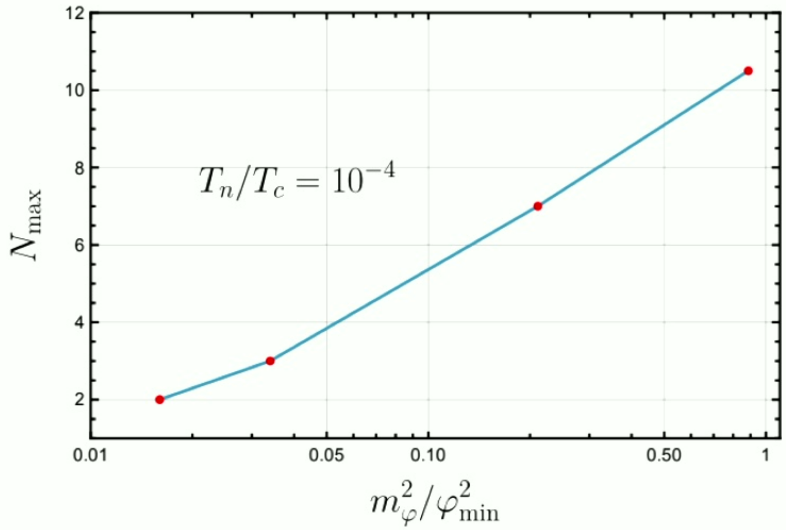


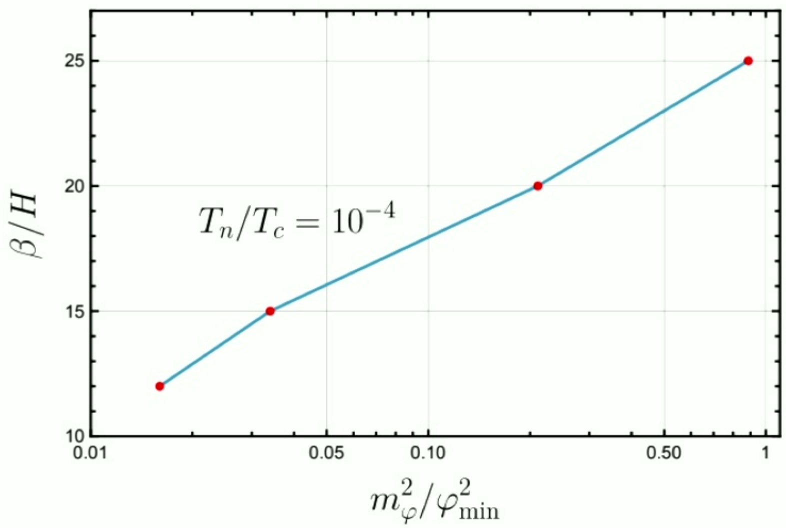


Reduced bounce action from self-interaction means reduced supercooling.

Summary

Larger $|\epsilon_3|$ means deeper radion potential, and therefore a higher (physical) radion mass.





Going out of radion EFT suggests a smaller rate, but needs a 5D analysis

Motivation

Known results in realistic examples are qualitatively different.

In examples of confining gauge theories with a super gravity dual, the geometrical description have additional compact spatial dimensions that shrinks to zero size in the IR.

- S^2 in type IIB supergravity (KS solution)
- S^1 in type IIA supergravity (Witten Model)

- The metric deviates from AdS significantly in the IR.
- The phase diagram is very different.

Phenomenological Model

From UV to IR, the effective 5D Planck scale decreases.
Model this with a scalar, such that as the scalar evolves from UV to IR, terms in the gravitational action change.

$$S = S_{\rm GR} + S_\phi + S_{\rm bdy} \,,$$

$$S_{\rm GR} = 2M_5^3 \int \mathrm{d}^5x\sqrt{-g}\Big((1-\phi/\phi_c)^n \; R - 2\,(1-\phi/\phi_c)^m \; \Lambda\Big)$$

$$S_\phi = 2M_5^3 \int \mathrm{d}^5x\sqrt{-g}\Big(- a(\partial\phi)^2 - v(\phi)\Big) \,.$$

Choose $v(\phi) = 2\epsilon\phi^2, \epsilon < 0$

The scalar starts with a small value $\phi_{\rm uv} \ll 1$ and grows in the IR where the effect of $1-\phi/\phi_c$ starts to show up.

Action in the Einstein Frame

After Weyl rescaling to go to a frame with constant 5D Planck scale.

$$S_\phi = 2M_5^3 \int \mathrm{d}^5x\sqrt{-g}\left(-\frac{1}{2}\, g^{MN} G(\phi)\, \partial_M\phi\, \partial_N\phi - V(\phi)\right)$$

$$G(\phi) = 2a\left(1-\frac{\phi}{\phi_c}\right)^{-n} + \frac{8n^2}{3\phi_c^2}\left(1-\frac{\phi}{\phi_c}\right)^{-2} \quad = \quad \left(2+\frac{32}{3\phi_c^2}\right)\left(1-\frac{\phi}{\phi_c}\right)^{-2}$$

$$V(\phi) = 2\epsilon\phi^2\left(1-\frac{\phi}{\phi_c}\right)^{-\frac{5n}{3}} + 2\Lambda\left(1-\frac{\phi}{\phi_c}\right)^{-\frac{5n}{3}+m} \quad = \quad 2\Lambda + 2\epsilon\phi^2\left(1-\frac{\phi}{\phi_c}\right)^{-10/3}$$

Choices from physics/simplicity

$n = 2, a = 1$: simpler kinetic term

$m = 5n/3$: only cosmological constant survives for $\epsilon = 0$

Phase Diagram with large back-reaction

What is the expectation?

Confined Phase

Free energy of the confined phase is set by the confinement scale, no large or small parameters.

$$T_c \sim \Lambda_c$$

no small "ϵ"

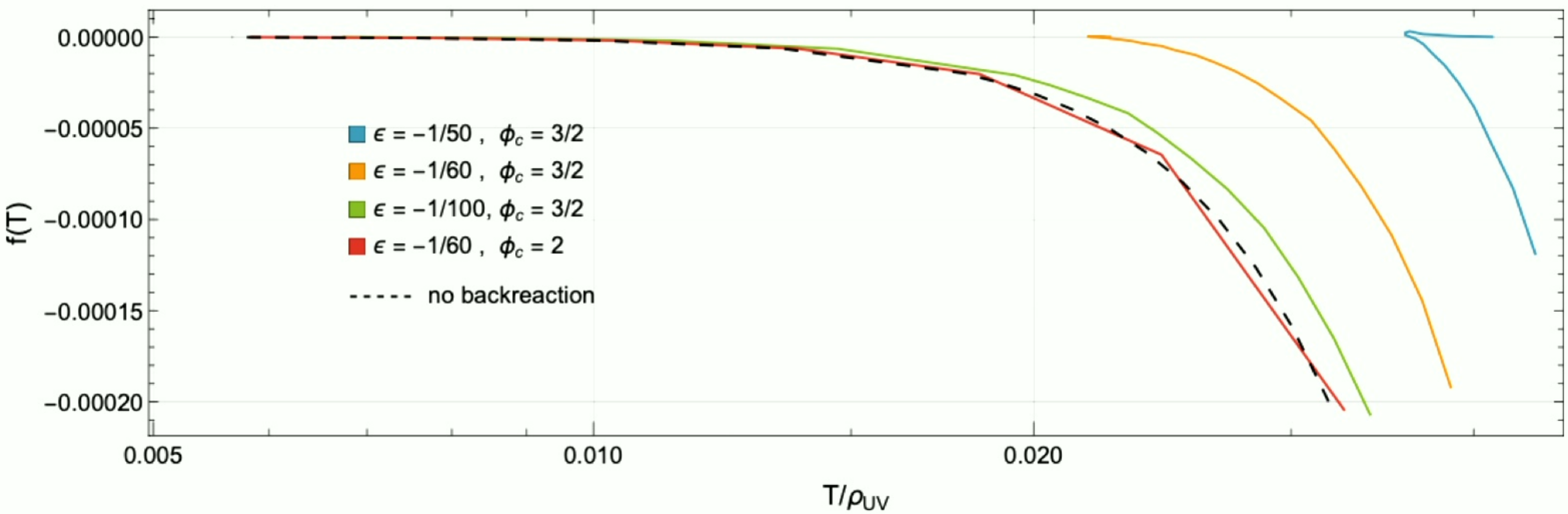
Deconfined Phase

Free energy of the deconfined phase at low temperatures, (i.e. when the deformation is large) can change significantly.

Focus on this

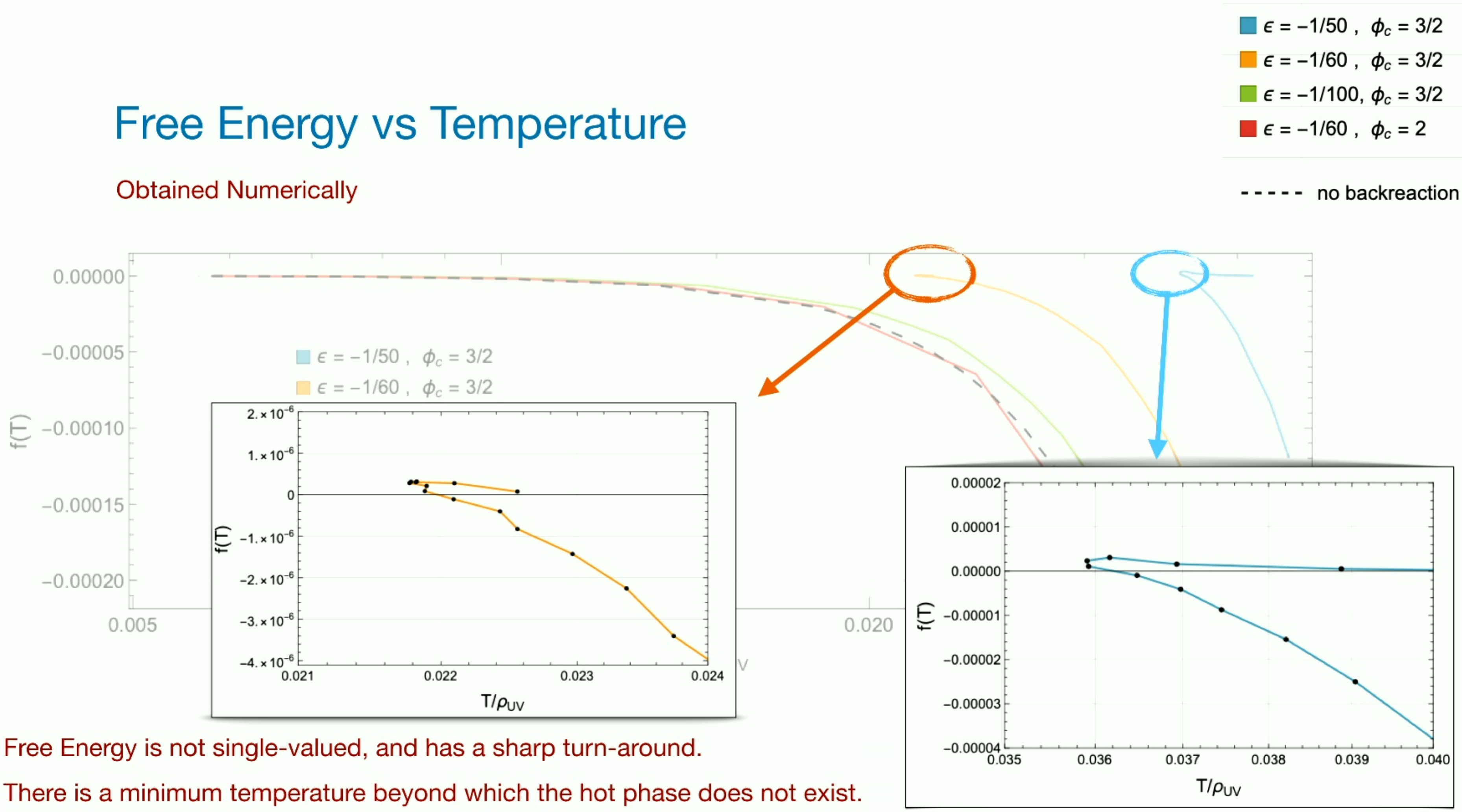
Free Energy vs Temperature

Obtained Numerically



Free Energy vs Temperature

Obtained Numerically



Free Energy is not single-valued, and has a sharp turn-around.

There is a minimum temperature beyond which the hot phase does not exist.

Action in the Einstein Frame

After Weyl rescaling to go to a frame with constant 5D Planck scale.

$$S_\phi = 2M_5^3 \int \mathrm{d}^5x\sqrt{-g}\left(-\frac{1}{2}\,g^{MN}G(\phi)\,\partial_M\phi\,\partial_N\phi - V(\phi)\right)$$

$$G(\phi) = 2a\left(1-\frac{\phi}{\phi_c}\right)^{-n} + \frac{8n^2}{3\phi_c^2}\left(1-\frac{\phi}{\phi_c}\right)^{-2} \quad = \quad \left(2+\frac{32}{3\phi_c^2}\right)\left(1-\frac{\phi}{\phi_c}\right)^{-2}$$

$$V(\phi) = 2\epsilon\phi^2\left(1-\frac{\phi}{\phi_c}\right)^{-\frac{5n}{3}} + 2\Lambda\left(1-\frac{\phi}{\phi_c}\right)^{-\frac{5n}{3}+m} \quad = \quad 2\Lambda + 2\epsilon\phi^2\left(1-\frac{\phi}{\phi_c}\right)^{-10/3}$$

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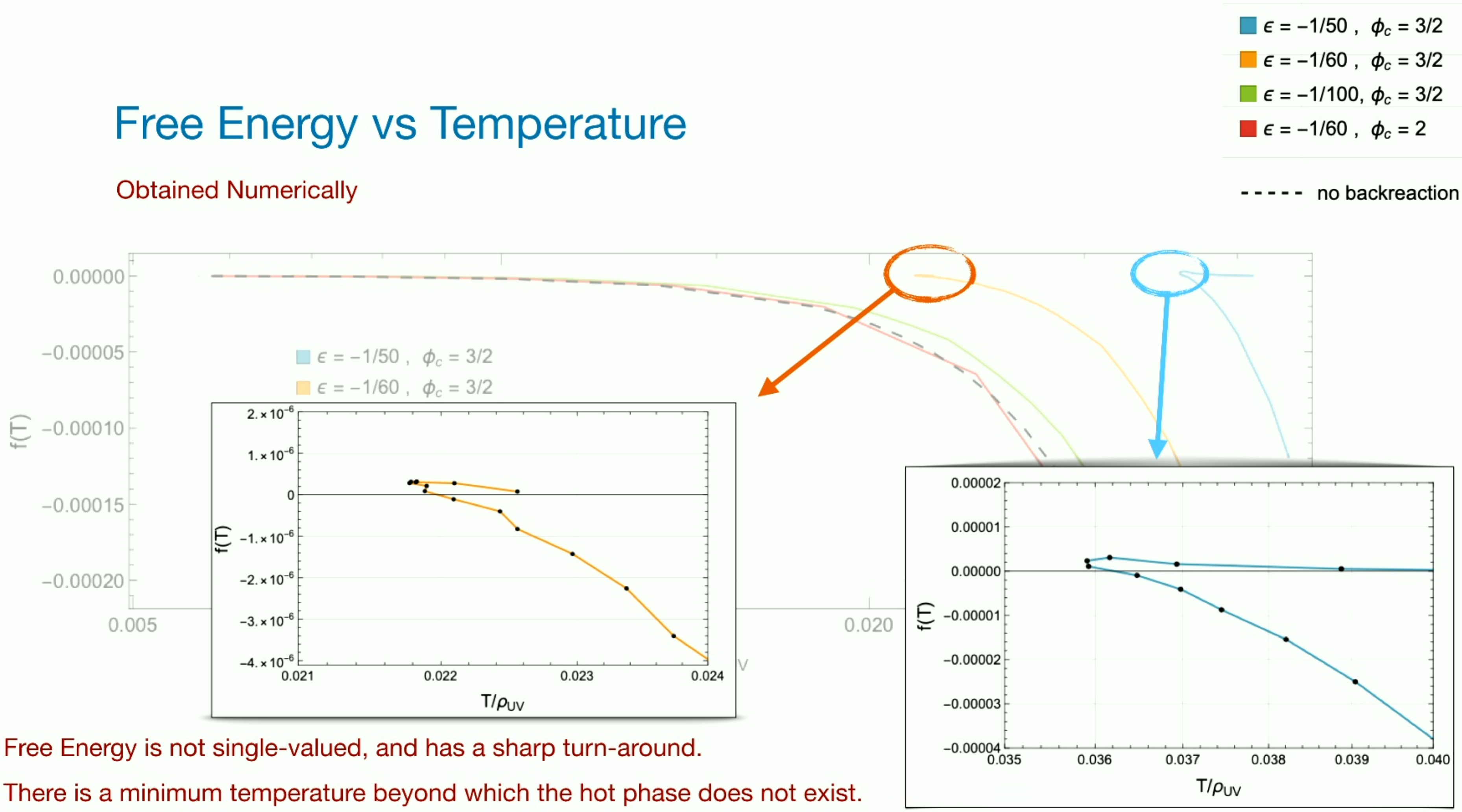

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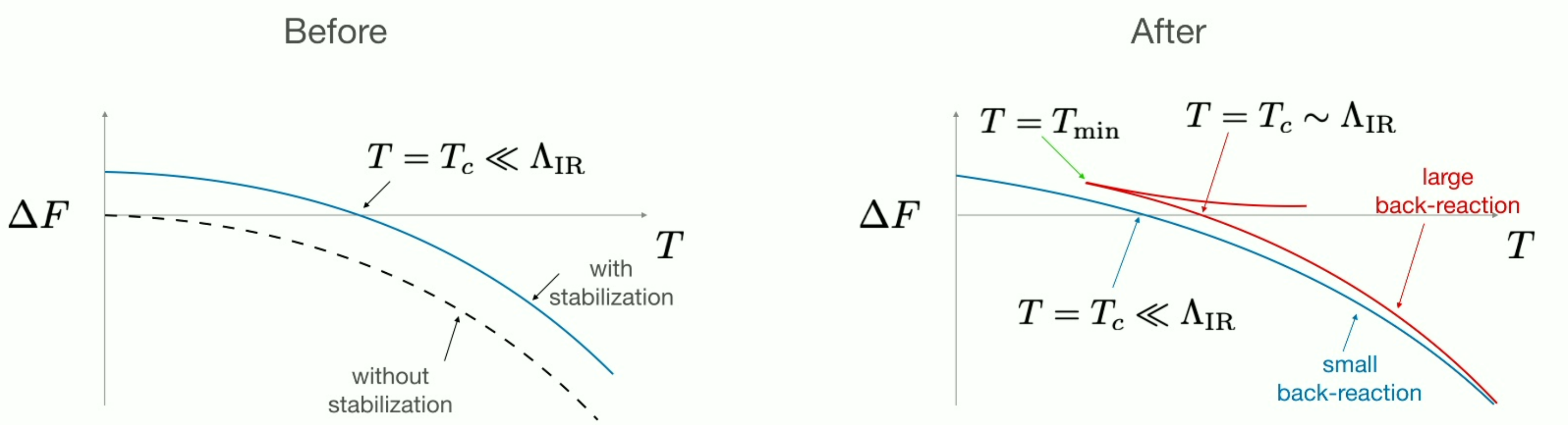
There is a minimum temperature beyond which the hot phase does not exist.

With and Without back reaction

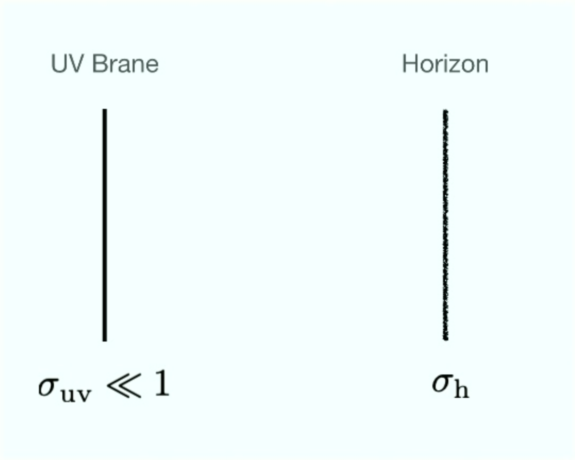


Analytical Estimates

A more useful gauge: use the scalar as the radial coordinate.

$$\mathrm{d}s^2 = e^{2A(\sigma)}\left(-f(\sigma)\,\mathrm{d}t^2 + \mathrm{d}x^2\right) + e^{2B(\sigma)}\frac{\mathrm{d}\sigma^2}{f(\sigma)} \quad , \ \sigma_{\mathrm{uv}} \leq \sigma \leq \sigma_h$$

Horizon: $f(\sigma = \sigma_h) = 0$



The equations of motion are simplified in this gauge.

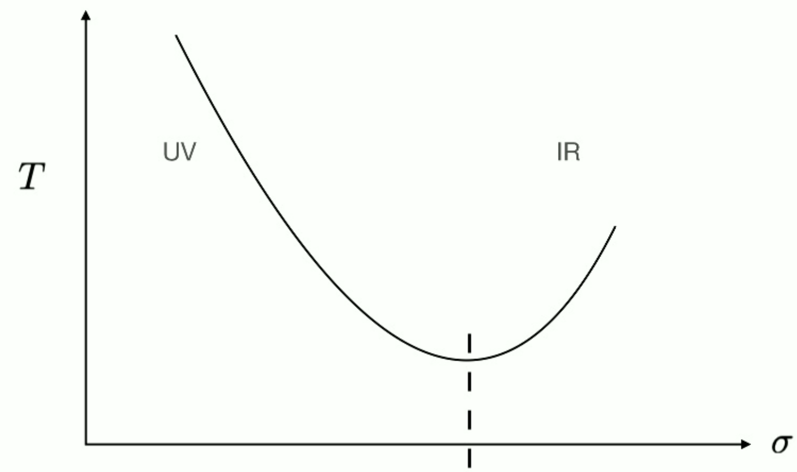
$$\begin{aligned} A'' - A'B' + 1/6 &= 0\,, \\ f'' + (4A' - B')f' &= 0\,, \\ 6A'f' + f(24A'^2 - 1) + 2e^{2B}V &= 0\,, \\ f(4A' - B') + f' - e^{2B}V' &= 0\,. \end{aligned}$$

Analytical Estimate for Temperature

$$\frac{\mathrm{d}\log T}{\mathrm{d}\sigma_h} \propto \frac{1}{3} - \frac{1}{2}\left(\frac{V'(\sigma_h)}{V(\sigma_h)}\right)^2 + \ldots$$

The derivative of temperature changes sign as the potential gets larger in the IR.

The result does not depend on a specific potential.



$\sigma_{\rm uv} \ll 1$ $\quad$ $\sigma_{\rm min} \sim 1$

$V'/V \sim 0$ $\quad$ $V'/V \sim \sqrt{2/3}$

Analytical Estimates for other quantities

$$c_s^2 \equiv \frac{\mathrm{d}\log T}{\mathrm{d}\log s} = \frac{1}{3} - \frac{1}{2}\left(\frac{V'(\sigma_h)}{V(\sigma_h)}\right)^2 + \ldots$$

The speed of sound of the plasma starts real but eventually becomes imaginary, signalling an instability.

$$c_V = 8\pi M_5^3 e^{3A(\sigma_h)} \left(\frac{1}{3} - \frac{1}{2}\left(\frac{V'(\sigma_h)}{V(\sigma_h)}\right)^2\right)^{-1} + \ldots$$

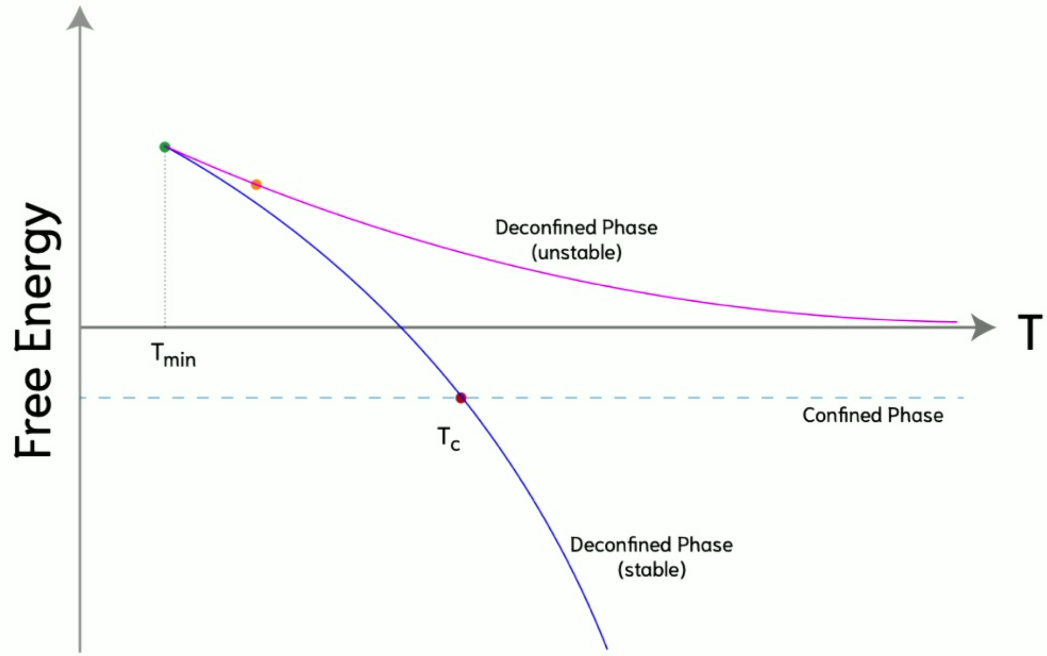
Specific heat starts positive, goes to infinity and then is eventually negative.

$$s/T^3 \propto |V(\sigma_h)|^{-3/2} + \ldots$$

The degrees of freedom reduce from UV to IR.

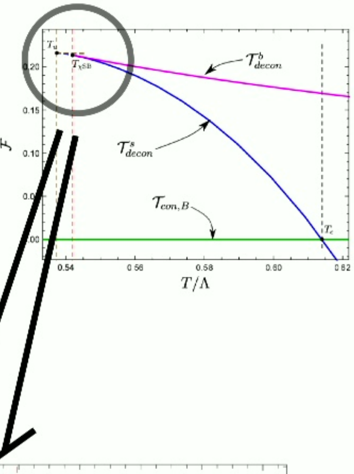
Summary: Qualitative differences from back-reaction

The system shows a minimum Temperature, and leads to unstable branches of solutions that are not present in the limit of small back-reaction.



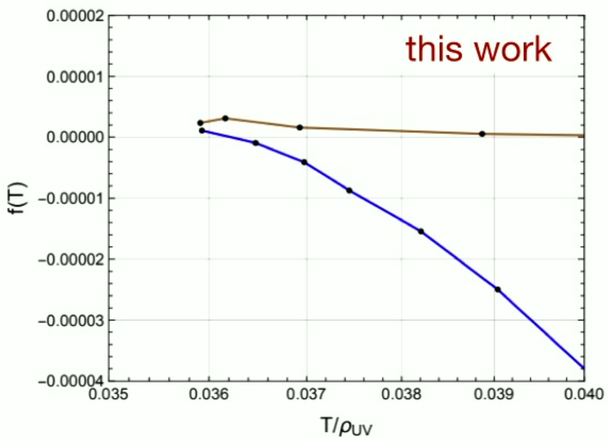
Other examples with similar behavior

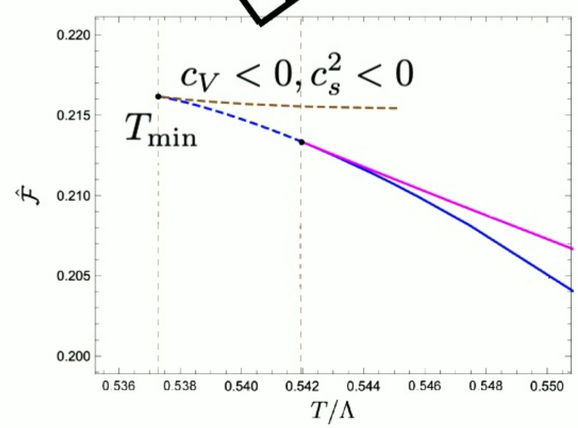
Buchel (2103.15188)



The behaviour is actually generic, and related to the presence of an IR scale.

- Explicit numerical solutions of Klebanov Strassler (KS) geometry show a similar behavior. IR scale is the confinement scale.





Other examples with similar behavior

The behaviour is actually generic, and related to the presence of an IR scale.

- Explicit numerical solutions of Klebanov Strassler (KS) geometry show a similar behavior. IR scale is the confinement scale.

- Black holes in global AdS have a minimum temperature. IR scale is the radius of the sphere.

- Indirect probes of an IR scale (confinement) show similar features for Witten model of confinement.

Supercooling?

Existence of a minimum temperature means there can not be an arbitrary long period of supercooling.

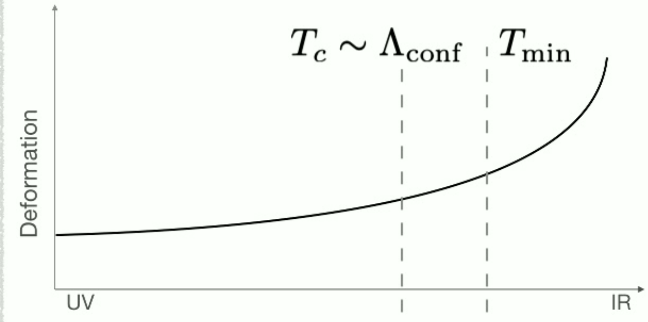
(Closely related to the dynamics which is not addressed at the moment. Likely the unstable branch is not populated because the speed of sound becoming imaginary signals instabilities)

In the explicit examples, T_{min}/T_c is order 1.

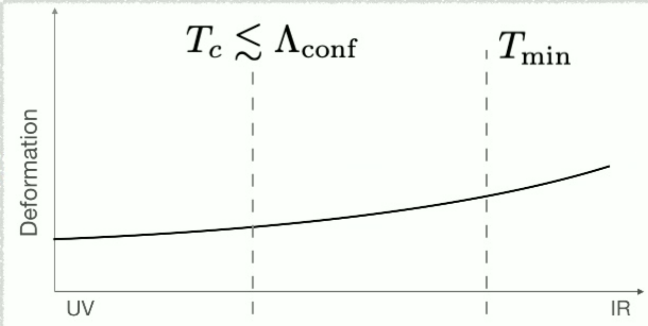
General lesson seems to be that if the CFT breaking is strong in the IR (and back-reaction is important), T_{min} will be close to T_c

Weak breaking of CFT in the IR: $T_{\rm min}/T_c \ll 1$

Strong breaking of CFT in the IR: $T_{\rm min}/T_c \lesssim 1$



Strong breaking of CFT near IR brane



Weak breaking of CFT near IR brane

Conclusion

- Early universe dynamics is qualitatively different when the CFT breaking is large in the IR.
- In our work, we have attempted to include (expected) IR effects in a systematic way.
- Strong IR effects necessitate a 5D analysis.

Open Questions

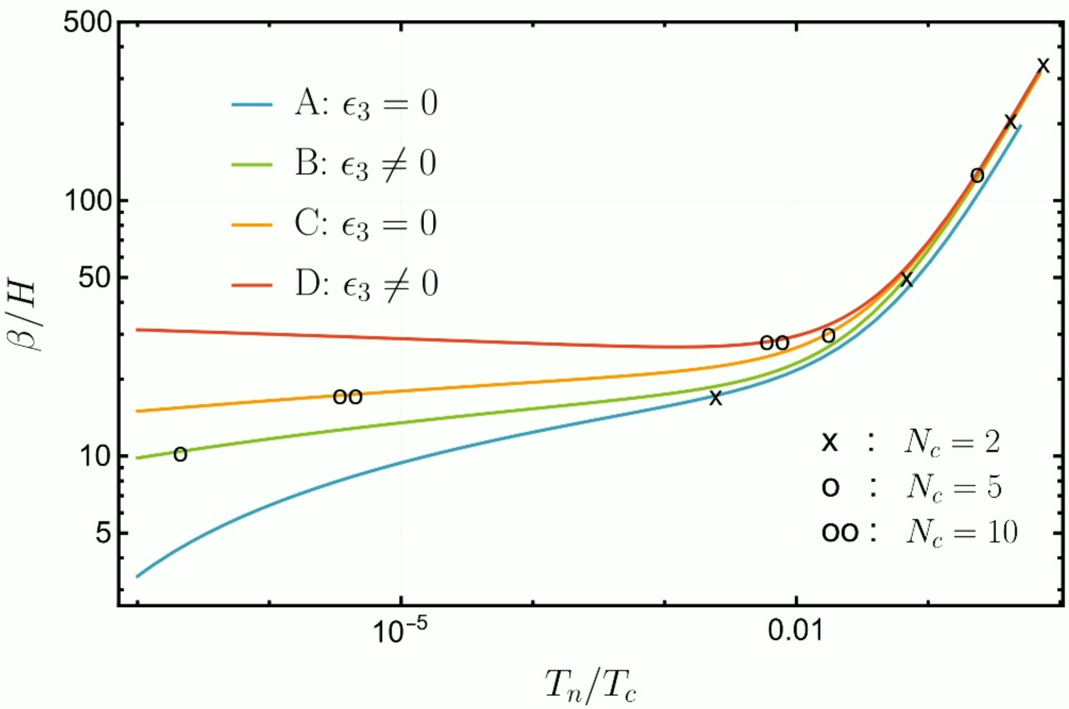
- Dynamics: How does the dynamics lead to a transition to the confined phase eventually?

  Related to the 5D structure of the bounce.
- Signals: GW signals/how do these instabilities play out in cosmology?

Gravitational Wave Signal

Focus on the GW from bubble collision

$$\frac{\beta}{H} = -\frac{\mathrm{d}\log\Gamma}{\mathrm{d}\log T}\bigg|_{T=T_n} \approx -4 + \frac{\mathrm{d}S_b}{\mathrm{d}\log T}\bigg|_{T=T_n}$$



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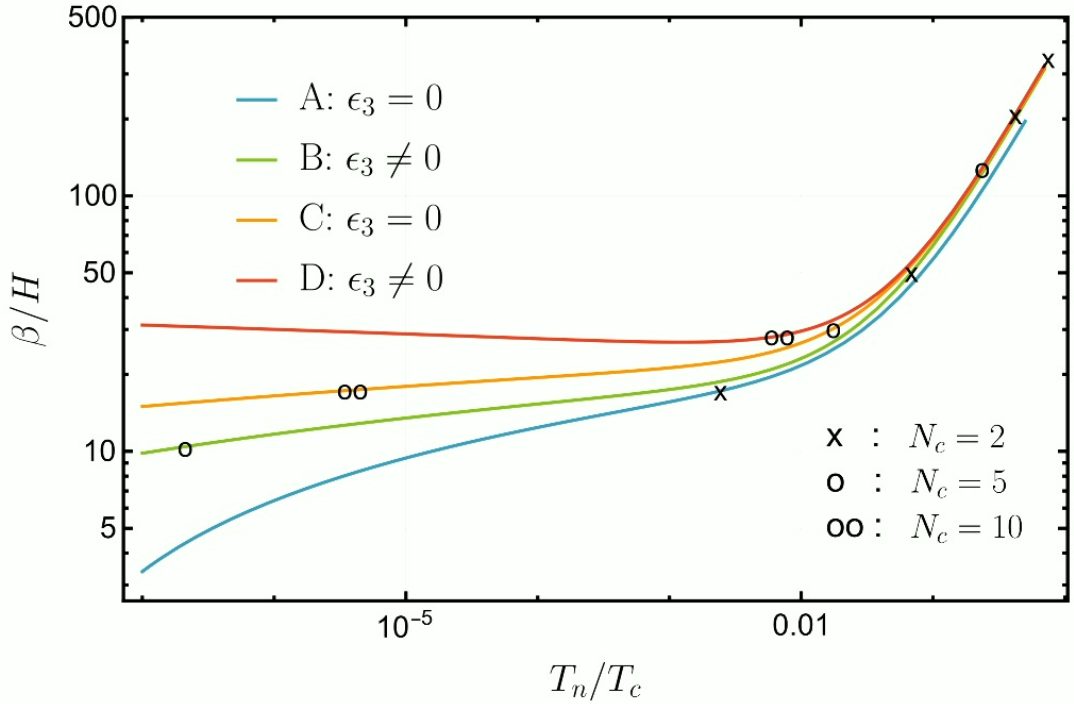

Gravitational Wave Signal

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Focus on the GW from bubble collision

$$f_p = 0.037\ \mathrm{mHz}\ \left(\frac{\beta}{H}\right)\left(\frac{T_*}{\mathrm{TeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

Different values of N_c corresponds to different values of T_n/T_c

$$\Omega_{\mathrm{GW}}\, h^2(f) = 1.3\times 10^{-6}\ \left(\frac{H}{\beta}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\frac{3.8(f/f_p)^{2.8}}{1+2.8(f/f_p)^{3.8}}$$