Title: Energy and speed bound in GPTs - VIRTUAL

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Series: Quantum Foundations

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Abstract: Information-theoretic insights have proven fruitful in many areas of quantum physics. But can the fundamental dynamics of quantum systems be derived from purely information-theoretic principles, without resorting to Hilbert space structures such as unitary evolution and self-adjoint observables? Here we provide a model where the dynamics originates from a condition of informational non-equilibrium, the deviation of the system's state from a reference state associated to a field of identically prepared systems. Combining this idea with three basic information-theoretic principles, we derive a notion of energy that captures the main features of energy in quantum theory: it is observable, bounded from below, invariant under time-evolution, in one-to-one correspondence with the generator of the dynamics, and quantitatively related to the speed of state changes. Our results provide an information-theoretic reconstruction of the Mandelstam-Tamm bound on the speed of quantum evolutions, establishing a bridge between dynamical and information-theoretic notions.

Zoom link

Energy and speed bound in GPTs

L. Giannelli & G. Chiribella

April 2024

Energy in Quantum Theory

- Energy is the expectation value of the Hamiltonian

$$\langle\psi|H|\psi\rangle = E$$

- Hamiltonian $\equiv$ generator of reversible evolutions

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

- This dual role of the Hamiltonian is not an accident!
  Rather it is intrinsecally connected with the quantum formalism [1]

[1] E. Grgin and A. Petersen, Journal of Mathematical Physics **15**, 764 (1974)

Energy in informational terms

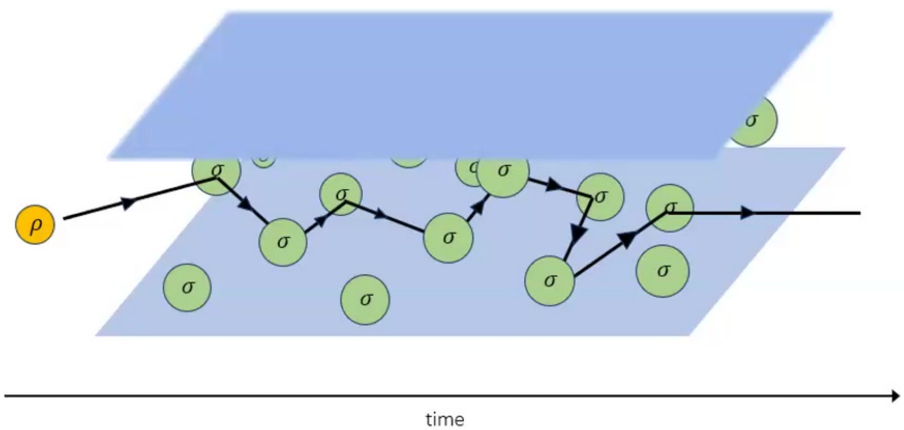
- Despite a central role in quantum mechanics, energy is often neglected in information-theoretic derivation

- Operational theories focus on statistical predictions rather than dynamics

Reconstruct quantum dynamics from informational principles

An informational perspective on the dynamics

Dynamical evolution is triggered
by **information non-equilibrium.**



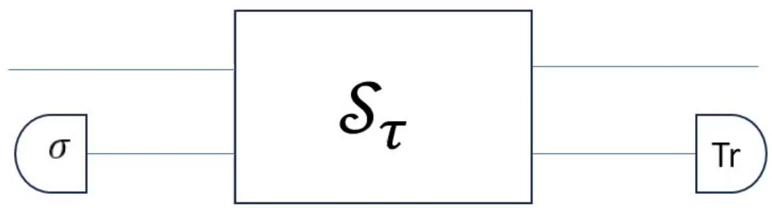
We can picture a field,

composed of identical bosons.

A particle crossing the field evolves if and only if it is in a different state

Quantum collision models

$\mathcal{C}_\tau :=$



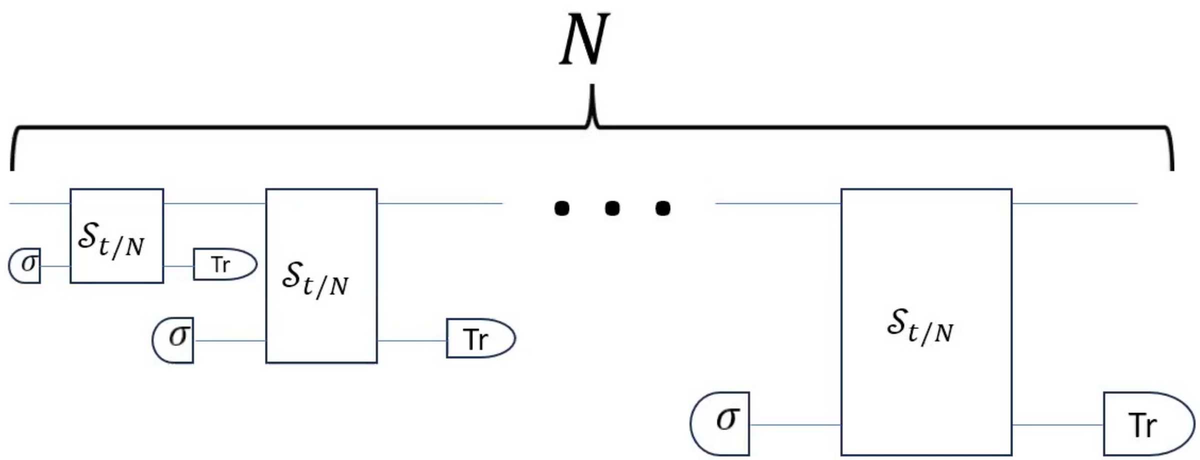
if $\mathcal{S}_{\pi/2} = \text{Swap}$

$\mathcal{C}_\tau \circ \ \ldots \circ \mathcal{C}_\tau$ approximate the unitary evolution $e^{-i\sigma t} \cdot e^{i\sigma t}$ by applying in sequence the operation $\mathcal{C}_\tau$

[1] S. Lloyd, M. Mohseni, and P. Rebentrost, Nature physics **10**, 631 (2014)

Collision models in the continuous-time limit

$$\lim_{N\to\infty} \mathcal{C}_{t/N}^{N} = \lim_{N\to\infty}$$



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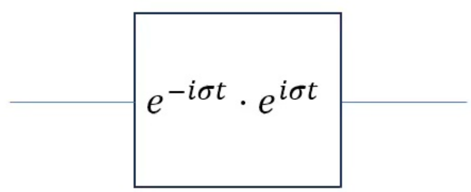

$$= e^{-i\sigma t} \cdot e^{i\sigma t}$$

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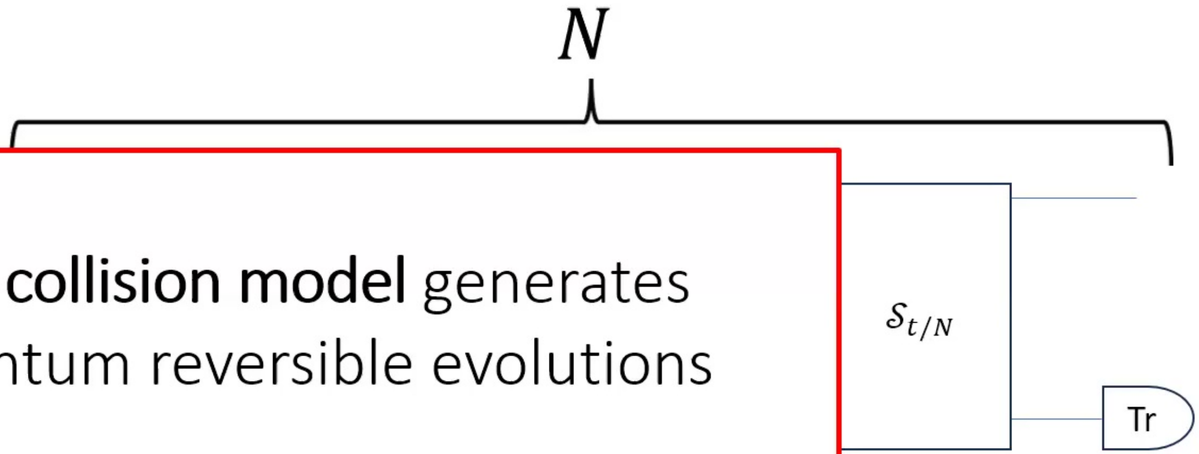

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SWAP **collision model** generates all quantum reversible evolutions

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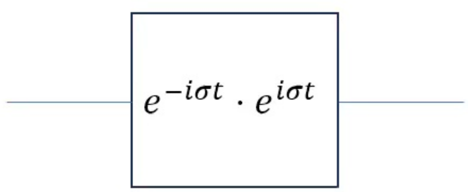

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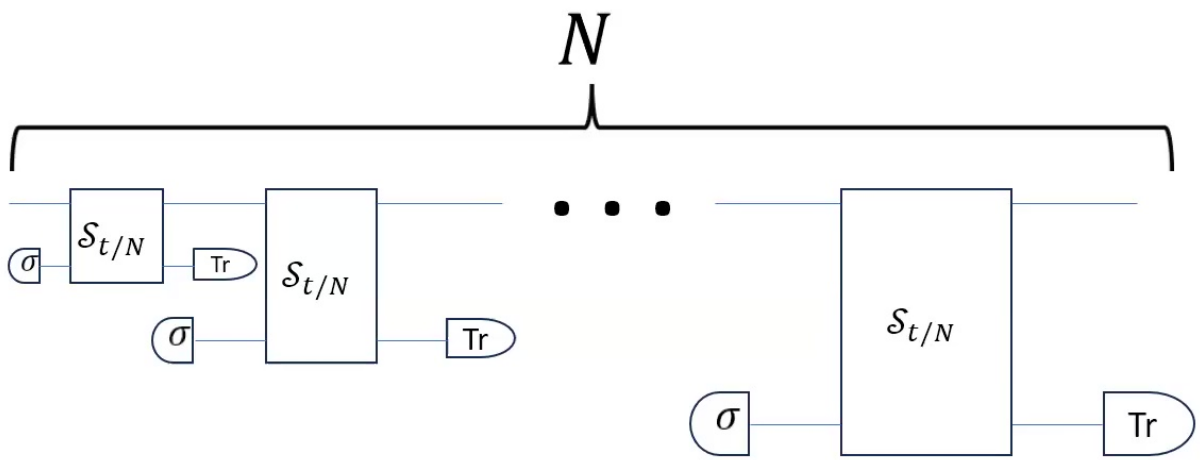

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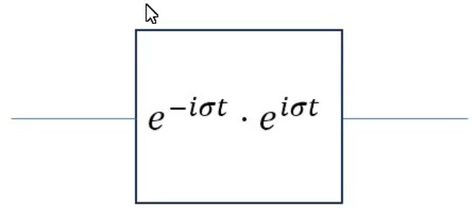

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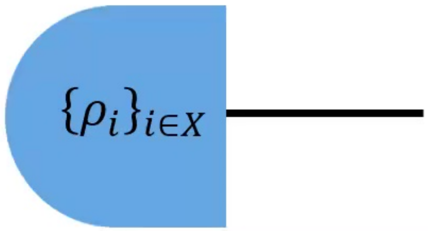
Outline

1. Introduction of the framework:
   a) GPTs
   b) generalized collision models
   c) our assumptions

2. Derivation of a **observable – generator correspondence:**
   a) state – generator
   b) observable

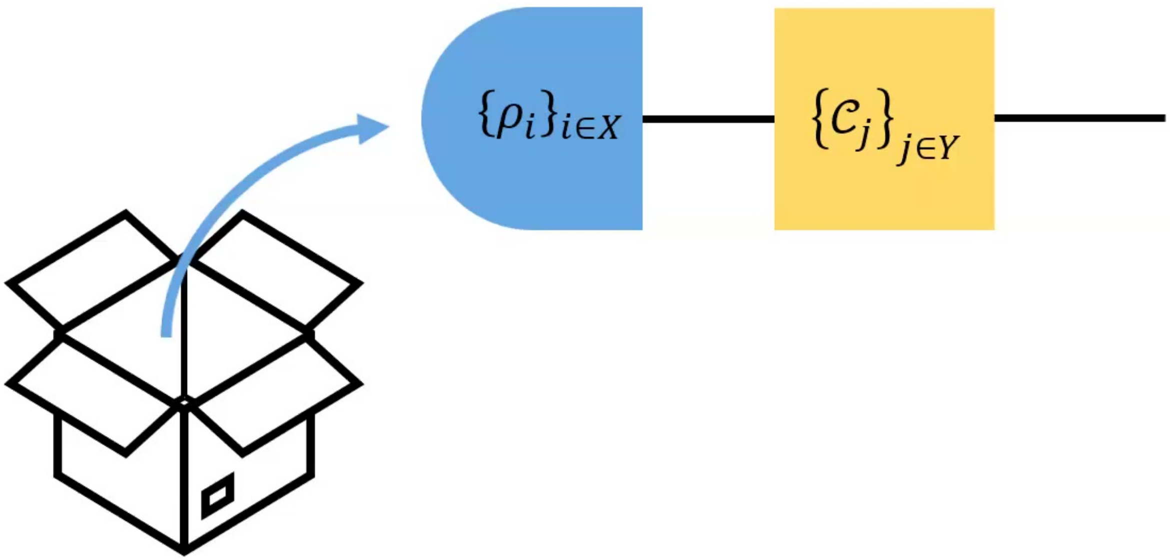
3. Derivation of an operational speed bound

Operational framework…



L. Hardy, *Quantum theory from five reasonable axioms* (2001), arxiv:quant-ph/0101012
J. Barrett, Phys. Rev. A **75**, 032304 (2007)
H. Barnum, J. Barrett, M. Leifer, and A. Wilce, Phys. Rev. Lett. **99**, 240501 (2007)

Operational framework…



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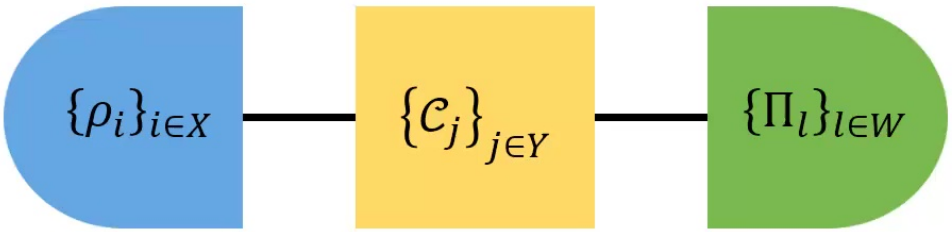
… combined with probability



$$= \quad p(\Pi_l \,|\mathcal{C}_j, \rho_i)$$

$$\begin{cases} p(\Pi_l \,|\mathcal{C}_j \; \rho_i) \geq 0 \\ \sum_{l,j,i} p(\Pi_l|\mathcal{C}_j \rho_i) = 1 \end{cases}$$

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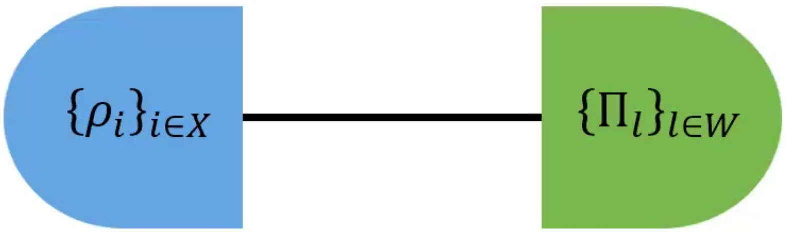

$$= \quad p(\Pi_l \mid \rho_i)$$

$$\rho_i = \begin{pmatrix} p_1^i \\ p_2^i \\ \vdots \\ p_n^i \end{pmatrix}$$

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J. Barrett, Phys. Rev. A **75**, 032304 (2007)
H. Barnum, J. Barrett, M. Leifer, and A. Wilce, Phys. Rev. Lett. **99**, 240501 (2007)

GPT framework

- A **system** $\mathcal{S}$ is a finite dimension real vector space
- A **state** ρ is an element of a convex set $\subset \mathcal{S}$
- A **measurement** is a collection of linear functional $\{e_j\}$ such that $\sum_j e_j(\rho) = 1$;
- An **effect** is an element in a measurement: a linear functionals over the state space with value in [0,1]
- A (**reversible) transformation** is an (invertible) linear map between state spaces

GPT framework

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We assume
- the state and effect spaces,
- the set of transformations,

to be **CONVEX** and **CLOSED**

What is already there about the dynamics?

From the GPT framework ONLY, we can go as far as identifying

- Reversible transformations as a subgroup of SO(N)
- The generators of reversible transformations as skew-symmetric matrices, or a subalgebra thereof

Analogous to unitary and skew-Hermitian matrices in quantum theory

Collision models in GPT

- Same architecture of the quantum version, this time

$$\mathcal{C}_\tau :=$$

$$\lim_{N\to\infty} \mathcal{C}_{t/N}^{N} = e^{G_\sigma t} \quad \text{where } G_\sigma := (I \otimes u)G_{tot}(\cdot \otimes \sigma)$$

Operational assumptions

3. **Diagonalization [4]:** every state ρ can be written as a convex sum of perfectly distinguishable pure states

4. **Purity Preservation [5]:** the parallel or sequential composition of pure states (transformations, effects) is a pure state (transformation, effect)

[4] H. Barnum, M. P. Müller, and C. Ududec, New Journal of Physics 16, 123029 (2014)
[5] G. Chiribella and C. M. Scandolo, *Entanglement as an axiomatic foundation for mechanics* (2016), arXiv:1608.04459

State-generator duality

For every collision model, the correspondence $\sigma \mapsto G_\sigma$ between states and generators is injective:

- if two states generate the same collisional dynamics, then they are the same state
- the maximally mixed state is the only state generating the trivial dynamics

How to define an observable

We take inspiration from [5,6]

- Observables are linear combinations of co-existing pure effects:

$$X = \sum_i c_i e_i$$

where $c_i \in \mathbb{R}$ and

$\{e_i\}_{i=1}^{d}$ is a valid measurement of the theory composed by pure effects

[5] G. Chiribella and C. M. Scandolo, *Entanglement as an axiomatic foundation for mechanics* (2016), arXiv:1608.04459
[6] G. Chiribella, C. M. Scandolo, L. Giannelli, *Sharp theories with purification,* in preparation

How to define an observable

We take inspiration from [5,6]

- Observables are linear combinations of co-existing pure effects:

$$X = \sum_i c_i e_i$$

- Expectation value: $\langle X \rangle_\rho := X(\rho) = \sum_i c_i \, e_i(\rho)$ for every state ρ

[5] G. Chiribella and C. M. Scandolo, *Entanglement as an axiomatic foundation for mechanics* (2016), arXiv:1608.04459
[6] G. Chiribella, C. M. Scandolo, L. Giannelli, *Sharp theories with purification,* in preparation

The generator-observable correspondence

- $G_\sigma \leftrightarrow \sigma$, for every state $\sigma = \sum_i \mathrm{p}_i \, \psi_i$
- $\psi_i \overset{[3,4]}{\leftrightarrow} e_{\psi_i}$ such that $\{e_{\psi_i}\}$ form a measurement

[3] M. P. Müller and C. Ududec, Physical Review Letters 108, 10.1103/physrevlett.108.130401 (2012)
[4] H. Barnum, M. P. Müller, and C. Ududec, New Journal of Physics 16, 123029 (2014)

The generator-observable correspondence

- $G_\sigma \leftrightarrow \sigma$, for ever

- $\psi_i \leftrightarrow e_{\psi_i}$ such t

- ENERGY observable: $H := \sum_i p_i \, e_{\psi_i}$

$\boldsymbol{e^{G_\sigma t}}$ and $\boldsymbol{e^{\alpha G_\sigma t}}$ are different dynamics, for any $\alpha \neq 1$

The generator-observable correspondence

- $G_\sigma \leftrightarrow \sigma$, for every state $\sigma = \sum_i \mathrm{p}_i\, \psi_i$
- $\psi_i \leftrightarrow e_{\psi_i}$ such that $\{ e_{\psi_i} \}$ form a measurement
- ENERGY observable:

$$H := \lambda_{max} \sum_i p_i\, e_{\psi_i}$$

where λ_{max} is the maximum singular value of the generator G_σ

Energy as $\langle H \rangle_\rho := \lambda_{max} \sum_i p_i \, e_{\psi_i}(\rho)$

- $\langle H \rangle_\rho \geq 0$ for every state ρ
- Performing $\{e_{\psi_i}\}$ estimates the expectation value of the energy
- Energy is invariant under time evolution [Informational equilibrium]:

  for $\rho_t = e^{G_\sigma t}\rho$, $\quad \langle H \rangle_{\rho_t} = \langle H \rangle_\rho$ for every t

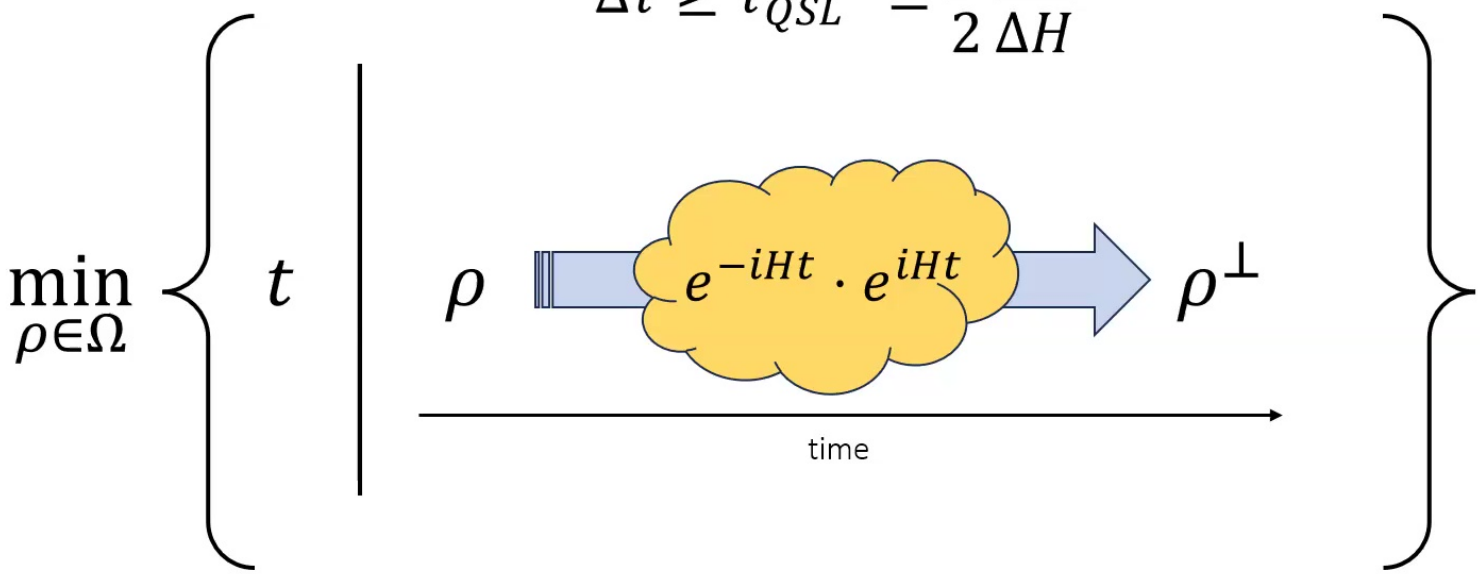
- The whole probability distribution of the ideal energy measurement is invariant:

  for $\rho_t = e^{G_\sigma t}\rho$, $\quad \mathrm{e}_{\psi_\mathrm{i}}(\rho_t) = \mathrm{e}_{\psi_\mathrm{i}}(\rho)$ for every t, i

Quantum speed bound

Lower bound on the time necessary for a system to evolve between every two **orthogonal** states, according to a given Hamiltonian H

$$\Delta\tau \geq \tau_{QSL} \equiv \frac{\pi}{2}\frac{\hbar}{\Delta H}$$



An improper uncertainty relation

- Time is not an observable [7]
- Time has to be interpreted as the internal time of the system [8-10]
- the relation expresses the minimum time necessary to evolve accordingly to a certain dynamics -> speed bound

[7] W. Pauli, General principles of quantum mechanics
[8] L. Mandelstam and I. Tamm, J. Phys. **9** (1945)
[9] Y. Aharonov and D. Bohm, Phys. Rev. **122**, 1649 (1961)
[10] J. Uffink, American journal of physics **61**, 935 (1993)

Let’s prepare the ground in GTPs

Let D_t be a dynamic. For any state ρ we define the quantity

$$v_\rho(t, t_0) \coloneqq \frac{||D_t\rho - D_{t_0}\rho||}{t - t_0} \quad ,$$

as the *evolution speed* of ρ from t_0 to t

Bound #1

Let $U_t = e^{At}$ be a reversible dynamics.

For any state ρ

$$v_\rho(t, t_0) \leq ||A\rho|| = v_\rho(t)$$

the average speed of the evolution is upper bounded by

the instantaneous speed $v_\rho(t) = \lim_{h \to 0} v_\rho(t + h, t)$

Rearranging the terms…

- $\Delta t = t - t_0$,
- $D(\rho_t, \rho_{t_0}) \coloneqq || \rho_t - \rho_{t_0} || / \sqrt{2}$

then the previous equation gives

$$\Delta t \geq \frac{D(\rho_t, \rho_{t_0})}{\Delta H}$$

Quantum speed limit

$$\Delta \tau \geq \frac{\pi}{2} \frac{\hbar}{\Delta H}$$

Take-home points

Characterized the dynamics in informational terms:

- Introduced the **informational equilibrium** assumption
- Derived a **generator-observable duality** in GPTs
- Derived an operational **speed limit**

Future works

- Compare toy theories with our informational-dynamical assumptions
- Derive tighter speed limits
- Can we give up strong symmetry? (sharp theories with purification)

Thank you for the attention!