Title: The Stability of Gapped Quantum Matter and Error-Correction with Adiabatic Noise - VIRTUAL

Speakers: Ali Lavasani

Series: Quantum Matter

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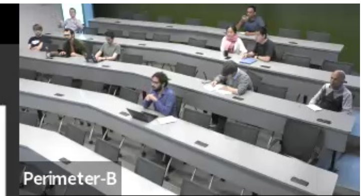
Abstract:

The code space of a quantum error-correcting code can often be identified with the degenerate ground-space within a gapped phase of quantum matter. We argue that the stability of such a phase is directly related to a set of coherent error processes against which this quantum error-correcting code (QECC) is robust: such a quantum code can recover from adiabatic noise channels, corresponding to random adiabatic drift of code states through the phase, with asymptotically perfect fidelity in the thermodynamic limit, as long as this adiabatic evolution keeps states sufficiently "close" to the initial ground-space. We further argue that when specific decoders -- such as minimum-weight perfect matching -- are applied to recover this information, an error-correcting threshold is generically encountered within the gapped phase. In cases where the adiabatic evolution is known, we explicitly show examples in which quantum information can be recovered by using stabilizer measurements and Pauli feedback, even up to a phase boundary, though the resulting decoding transitions are in different universality classes from the optimal decoding transitions in the presence of incoherent Pauli noise. This provides examples where non-local, coherent noise effectively decoheres in the presence of syndrome measurements in a stabilizer QECC.

Zoom link



Stability of Gapped Quantum Matter and Error-Correction with Adiabatic Noise

arXiv:2402.14906

Ali Lavasani (UCSB/KITP), April 2024

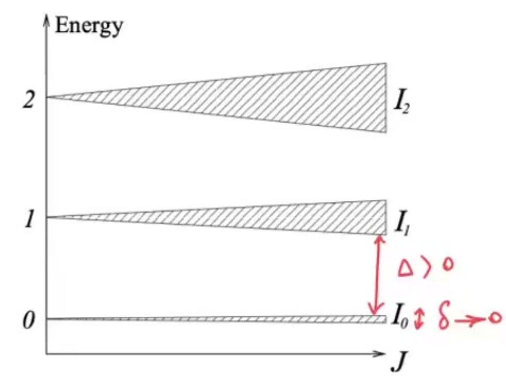
In Collaboration with
Sagar Vijay (UCSB)

Kavli Institute for Theoretical Physics

Two notions of stability

- **Stability of gapped quantum matter**: robust gap and robust ground state degeneracy against local perturbations .

$$H_J = H_0 + J\sum_i V_i$$



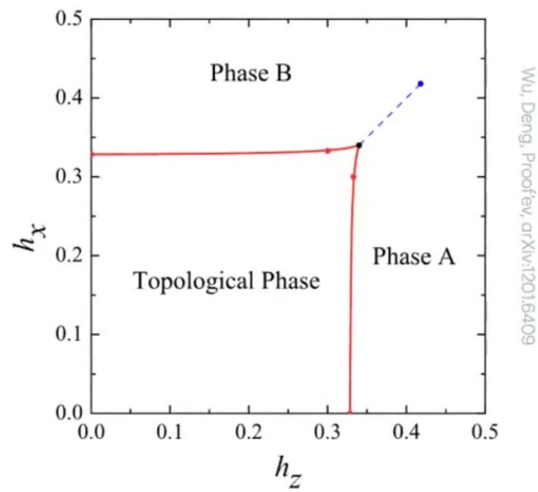
Bravyi, Hastings, Michalakis, arXiv:1001.0344

Two notions of stability

- **Stability of gapped quantum matter**: robust gap and robust ground state degeneracy against local perturbations .

- This notion of stability ensures the existence of an extended quantum phase of matter

$$H = H_{TC} - h_x \sum_i X_i - h_z \sum_i Z_i$$



Two notions of stability

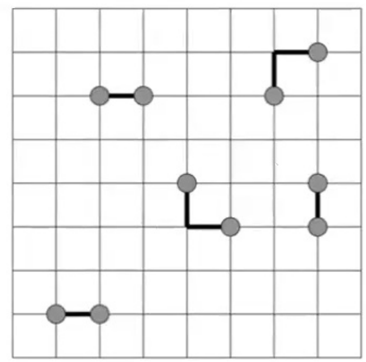
- **Quantum error correction:** quantum information encoded in the ground state subspace can be recovered perfectly after weak decoherence.

$$|\psi\rangle = \alpha_1 |\varphi_1\rangle + \alpha_2 |\varphi_2\rangle$$

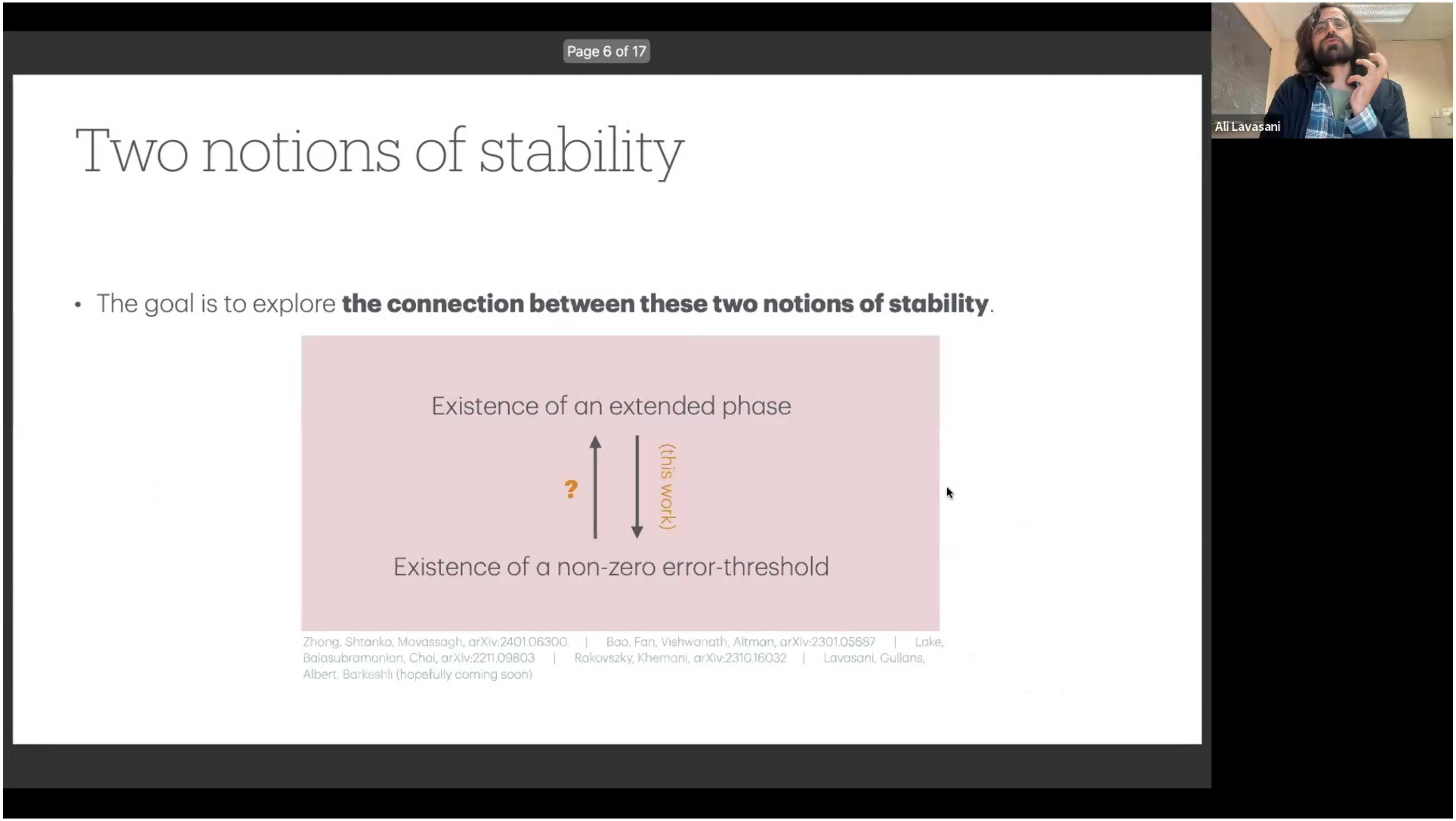
$$\mathcal{N} = \bigotimes_i \mathcal{N}_i$$

$$\mathcal{N}_i(\rho) = (1-p)\rho + pX_i\rho X_i$$

$$\exists \mathcal{R} : \quad \mathcal{R}(\mathcal{N}(|\psi\rangle\langle\psi|)) = |\psi\rangle\langle\psi|$$



Denis, Kitaev, Landahl, Preskill, arXiv:quant-ph/0110143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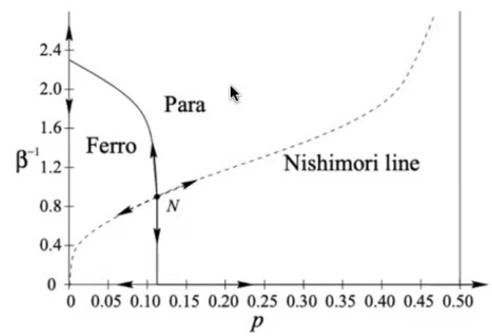
Ali Lavasani
Two notions of stability
• The goal is to explore **the connection between these two notions of stability**.
Existence of an extended phase
?
(this work)
Existence of a non-zero error-threshold
Zhong, Shtanko, Movassagh, arXiv:2401.06300. | Bao, Fan, Vishwanath, Altman, arXiv:2301.05687 | Lake, Balasubramanian, Choi, arXiv:2211.09803 | Rakovszky, Khemani, arXiv:2310.16032 | Lavasani, Gullans, Albert, Barkeshli (hopefully coming soon)

Two notions of stability

- **Quantum error correction:** quantum information encoded in the ground state subspace can be recovered perfectly after weak decoherence.

- This notion of stability ensures the existence of a non-zero error-threshold under which perfect recovery is possible.



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Two notions of stability

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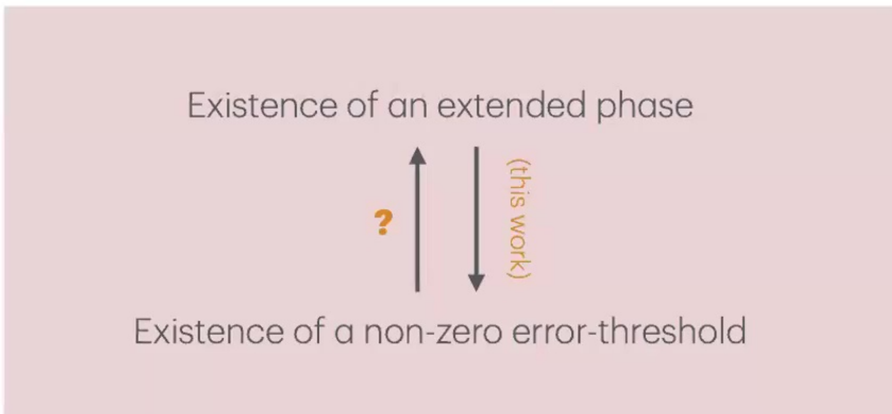

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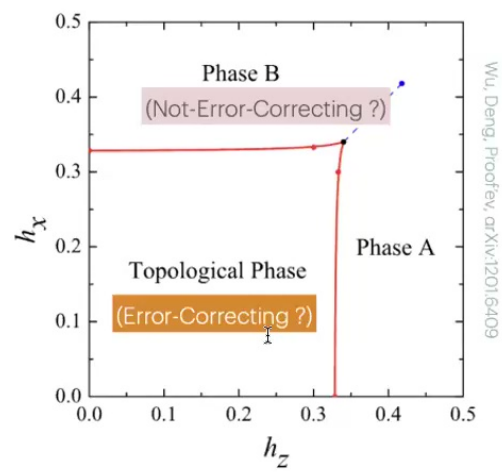
Two notions of stability

- We would like to know **whether the gapped phase can be viewed as an error-correcting phase** with the phase boundary representing the error-threshold for recovery.

- To this end, we need to define a relevant noise channel.



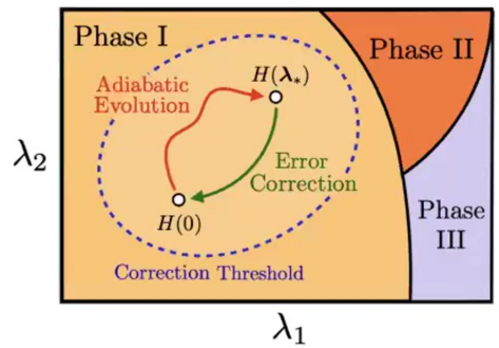
Adiabatic Noise

- Let $H(\lambda)$ denote a specific family of perturbations around some local stabilizer code Hamiltonian $H(0) = -\sum_i S_i$. Let $|\psi\rangle$ be an arbitrary state in the ground state subspace of $H(0)$.

- Let $\mathcal{U}_\gamma$ denote the adiabatic unitary evolution along the path γ

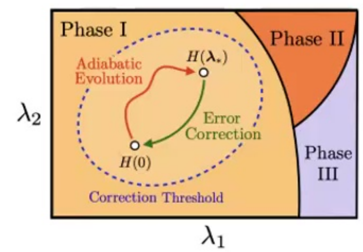
$$|\tilde{\psi}\rangle = \mathcal{U}_\gamma |\psi\rangle$$

Noisy state ($|\tilde{\psi}\rangle$) — Code state ($|\psi\rangle$)



- Given $|\tilde{\psi}\rangle$, can we recover $|\psi\rangle$ through error-correction?

Stability of Gapped Quantum Matter and Error-Correction with Adiabatic Noise



This work investigates the validity of the following statements:

Statement I: If provided with the state $|\tilde{\psi}\rangle = \mathcal{U}_\gamma|\psi\rangle$ and the knowledge of the adiabatic path γ, one can recover $|\psi\rangle$ with perfect fidelity in the thermodynamic limit by measuring stabilizers of $H(0)$ and applying a Pauli feedback.

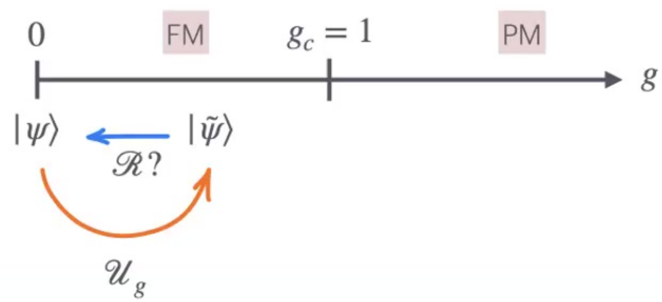
Statement II: If provided with the state $\tilde{\rho} = \sum_\gamma p_\gamma \mathcal{U}_\gamma |\psi\rangle\langle\psi| \mathcal{U}_\gamma^\dagger$, one can recover $|\psi\rangle$ with perfect fidelity in the thermodynamic limit.

Reversing Adiabatic Evolution

Example 1: repetition code

- Consider the repetition code / transverse field Ising chain Hamiltonian

$$H(g) = -\sum_i Z_i Z_{i+1} - g \sum_i X_i$$



$$|\psi\rangle = |\uparrow\uparrow\uparrow\cdots\rangle = \frac{1}{\sqrt{2}}|GHZ_+\rangle + \frac{1}{\sqrt{2}}|GHZ_-\rangle$$

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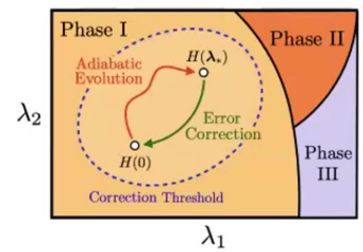

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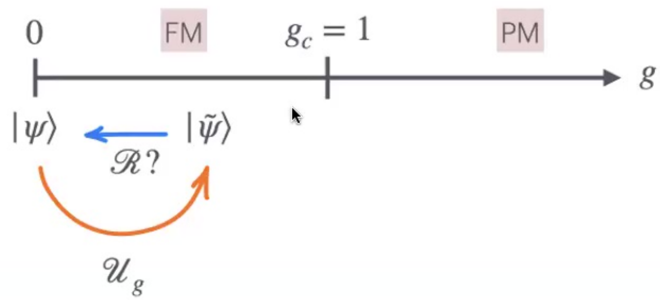
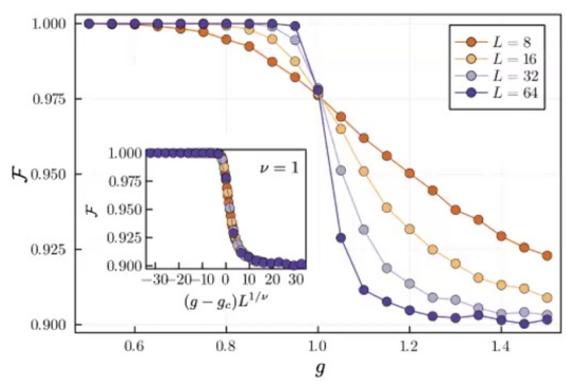


Reversing Adiabatic Evolution

Example 1: repetition code

- letting $\mathscr{R}$ bethe MWPM decoder, perfect recovery is possible throughout the ferromagnetic phase.

- The typical weight of the inferred error is related to the phase order parameter, $k = (1 - \sqrt{m})\frac{N}{2}$, so in the FM phase ($m > 0$), the effective error rate is less than 1/2, which is the threshold of the repetition code.



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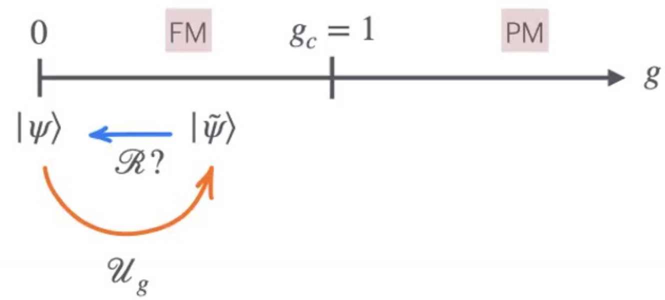

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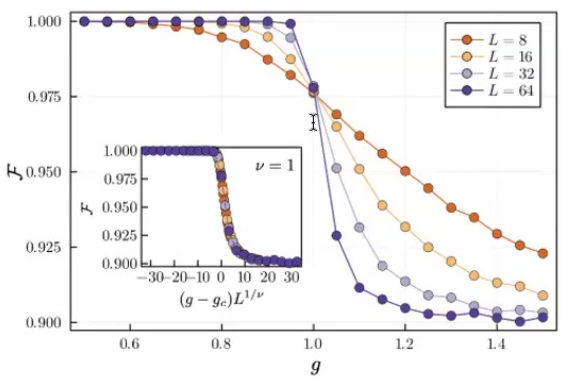
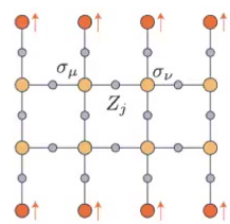
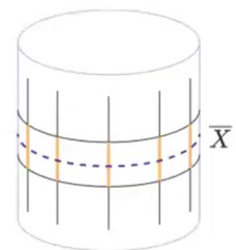


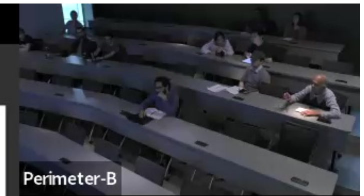
Reversing Adiabatic Evolution

Example 2: Toric code

- Playing the same game with the perturbed toric code Hamiltonian,
  $$H(\beta) = -\sum_s A_s - \sum_p B_p + \sum_s \exp(-\beta \sum_{j\in s} Z_j)$$
- The average infidelity of the probabilistic likelihood decoder is prop. to, $1 - \mathcal{F} \propto \sqrt{\frac{\mathcal{Z}_0}{\mathcal{Z}_1}} - 1$, where $\mathcal{Z}_0$ and $\mathcal{Z}_1$ are the partition function of classical 2d Ising with/without a domain wall
- Perfect recovery is possible throughout the topological phase.







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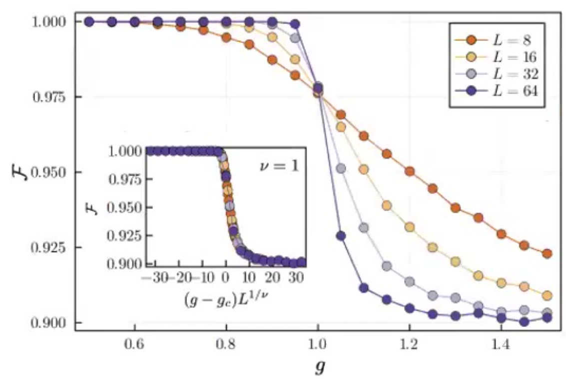

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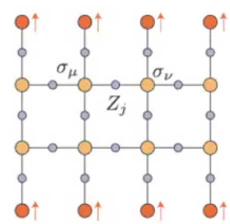
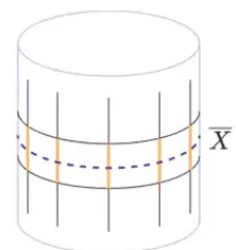




Reversing Adiabatic Evolution

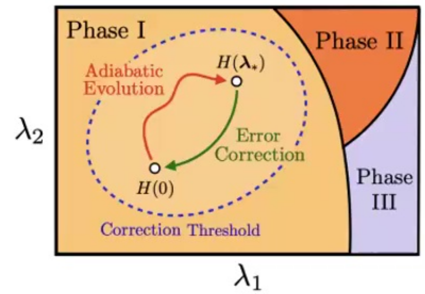
- Whether these observations are more generally true for arbitrary perturbations of gapped quantum phases is left as an open question...

Reversing Adiabatic Noise

- When the adiabatic evolution is unknown, by providing a counter example, we show that the error correction threshold under an adiabatic noise channel need not coincide with the phase boundary:

$$H(\theta) = \prod e^{-i\frac{\theta}{2}X_i} \; H_0 \; \prod e^{+i\frac{\theta}{2}X_i}$$

with H_0 denoting the repetition code Hamiltonian.



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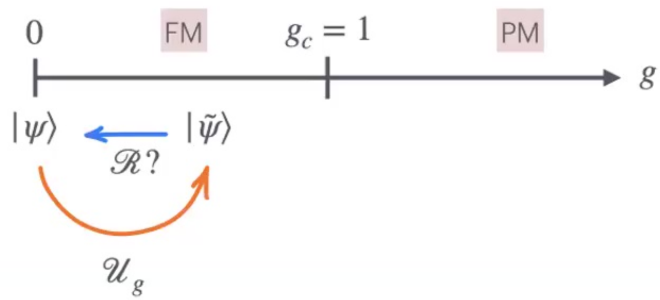

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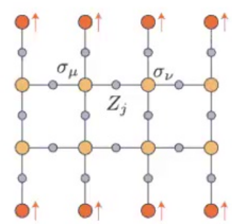

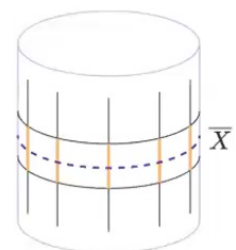

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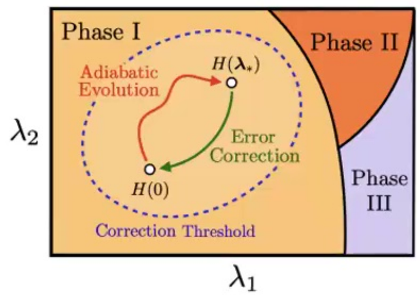
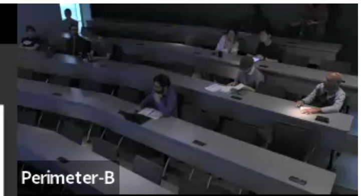


Summary

- We relate the stability of gapped phase to recovery from adiabatic noise channels
- When the adiabatic evolution is known, we study examples where recovery is possible throughout the phase.
- When the adiabatic evolution is not known, we show that the error correction might not be possible throughout the phase.
- We drive conditions similar to KL conditions which can be used to establish a finite neighborhood for which recovery from an unwon adiabatic noise is possible.



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