Title: Curvature correlators in nonperturbative 2D Lorentzian quantum gravity

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Series: Quantum Gravity

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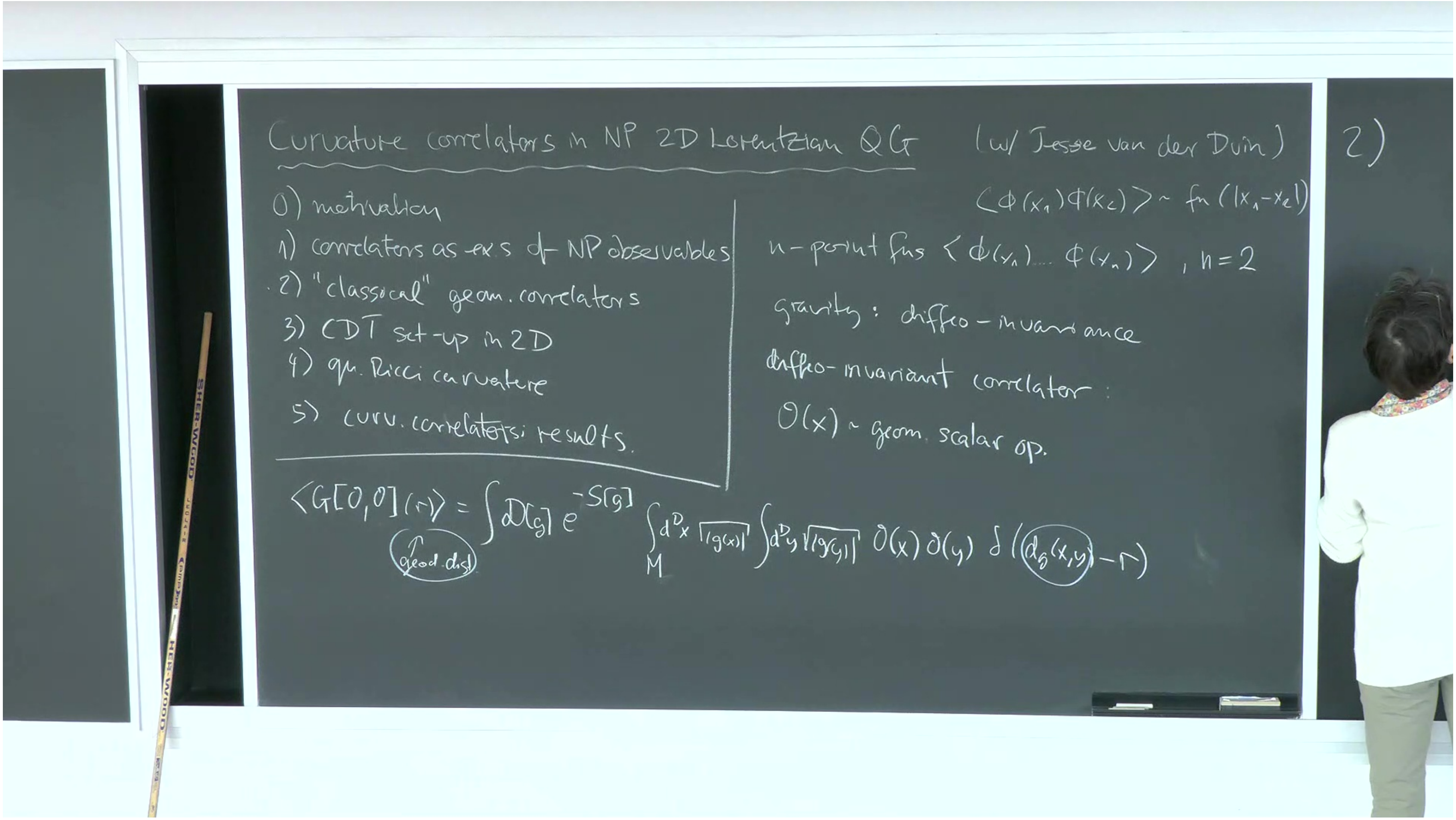
Abstract: Correlation functions are ubiquitous in quantum field theory, but their use in quantum gravity beyond perturbative and asymptotically flat regimes has been limited, due to difficulties with diffeomorphism invariance and the dynamical nature of geometry. We address these challenges in the context of nonperturbative quantum gravity, motivated by the goal of computing geometric correlators from first principles, as a possible window to early-universe cosmology. I will report on recent work in 2D Lorentzian quantum gravity, formulated in terms of causal dynamical triangulations, which demonstrates the feasibility of constructing and measuring manifestly diffeomorphism-invariant curvature correlators. I will discuss some unusual features of the connected correlators and new results on their behaviour in two dimensions, obtained with the help of Monte Carlo simulations.

Zoom link


Curvature correlators in NP 2D Lorentzian QG (w/ Jesse van der Duin)
2)
0) motivation
1) correlators as ex.s of NP observables
2) "classical" geom. correlators
3) CDT set-up in 2D
4) qu. Ricci curvature
5) curv. correlators: results.
$\langle \phi(x_1)\phi(x_2)\rangle \sim fn(|x_1-x_2|)$
n-point fns $\langle \phi(x_1)\dots\phi(x_n)\rangle$, n=2
gravity: diffeo-invariance
diffeo-invariant correlator:
$\mathcal{O}(x)$ ~ geom. scalar op.
$\langle G[\mathcal{O},\mathcal{O}](r)\rangle = \int \mathcal{D}[g]\, e^{-S[g]} \int_M d^Dx \sqrt{|g(x)|} \int d^Dy \sqrt{|g(y)|}\, \mathcal{O}(x)\,\mathcal{O}(y)\, \delta(d_g(x,y)-r)$
geod. dist

$(M, g_{\mu\nu})$, compact, Riem., $D=2$, $vol(M) = \int d^2x \sqrt{g}$, $\bar{O}|_g = \frac{1}{vol(M)} \int d^2x \sqrt{g}\, O(x)$

$$G_g[O_1,O_2](r) := \int d^2x \sqrt{g(x)} \int d^2y \sqrt{g(y)}\, O_1(x) O_2(y)\, \delta(d_g(x,y) - r) \qquad (*_1)$$

$O_1 = O_2 = 1$: $G_g[1,1](r) = \int d^2x \sqrt{g} \int d^2y \sqrt{g}\, \delta(d_g(x,y) - r) = \int d^2x \sqrt{g}\, vol(S^r_x) = vol(M)\, \overline{vol(S^r)}|_g$

$O_2 = 1$: $G_g[O,1](r) = \int d^2x \sqrt{g}\, O(x)\, vol(S^r_x)$

connected correl. $G^c_g[O_1,O_2](r) = \int d^2x \sqrt{g} \int d^2y \sqrt{g}\, (O_1(x) - \bar{O}_1|_g)(O_2(y) - \bar{O}_2|_g) =$

$= G_g[O_1,O_2](r) - \bar{O}_1|_g\, G_g[O_2,1](r) - \bar{O}_2|_g\, G_g[O_1,1](r) + \bar{O}_1|_g \bar{O}_2|_g\, G[1,1](r)$

Suppose g maxim'ly symmetric

$G_g[O,1](r) = vol(S^r) \int d^2x \sqrt{g}\, O(x) = vol(S^r)\, vol(M)\, \bar{O}|_g$

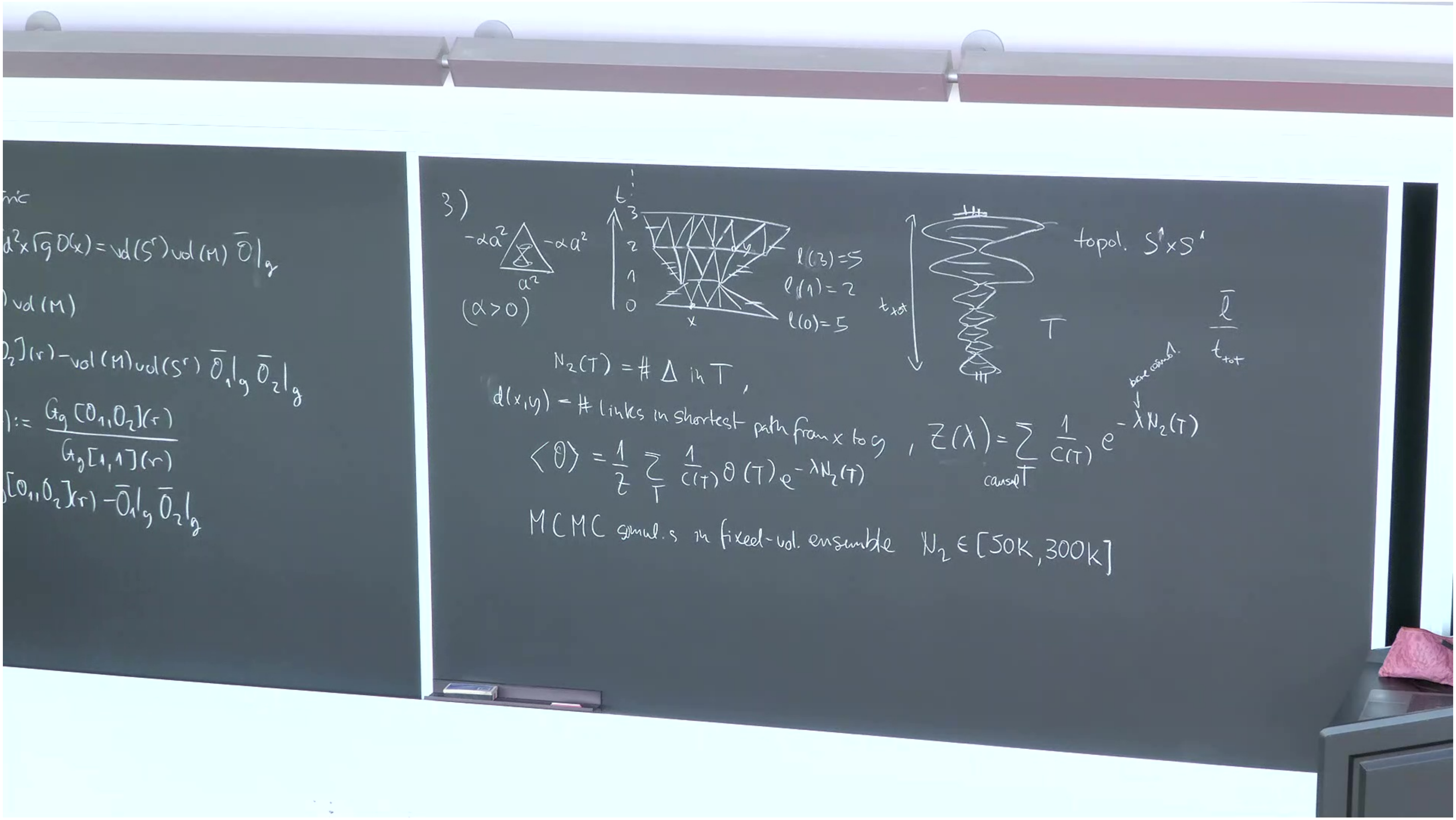
$G_g[1,1](r) = vol(S^r)\, vol(M)$

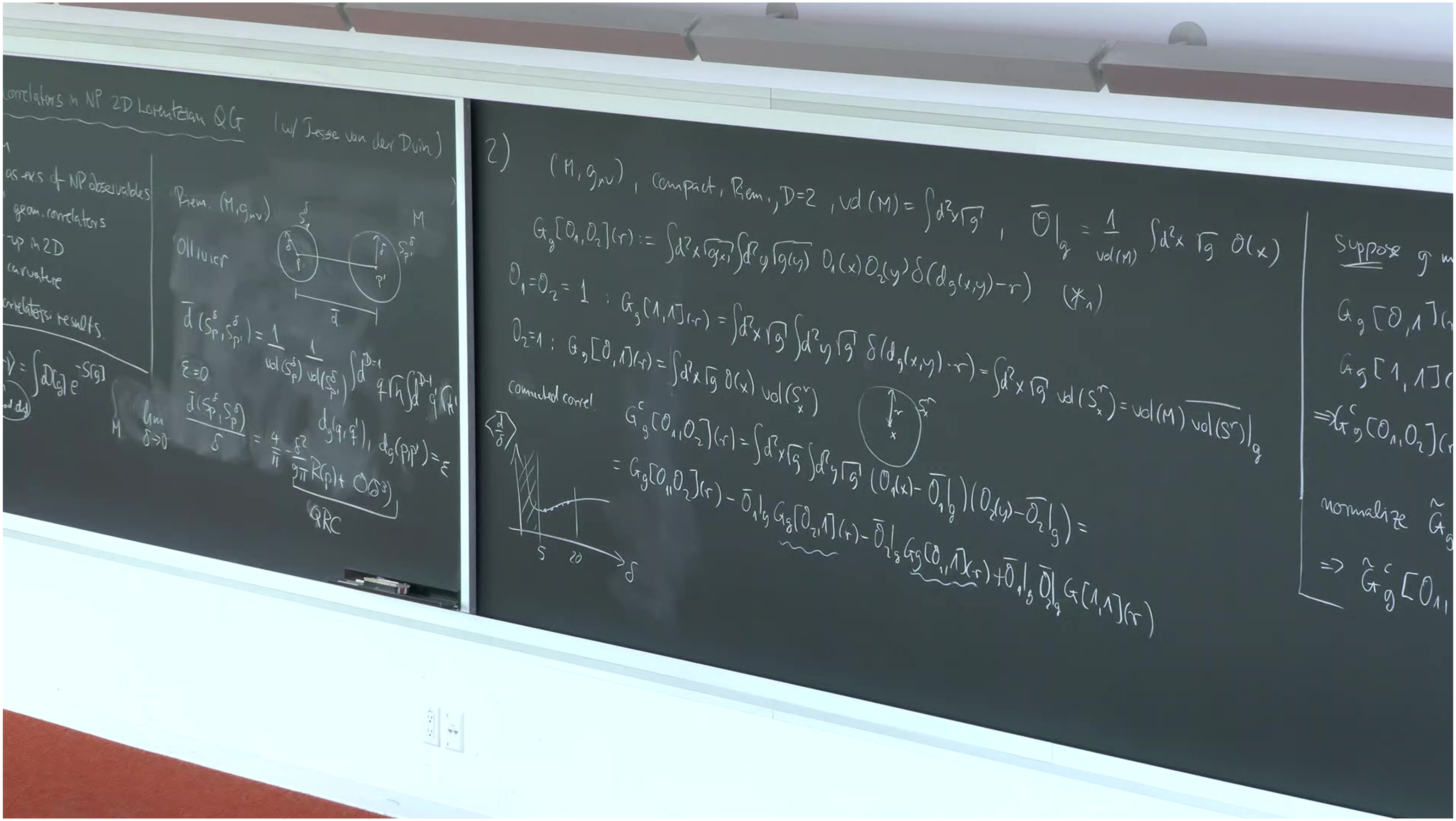
$\Rightarrow G^c_g[O_1,O_2](r) = G_g[O_1,O_2](r) - vol(M)\, vol(S^r)\, \bar{O}_1|_g \bar{O}_2|_g$

normalize $\tilde{G}_g[O_1,O_2](r) := \dfrac{G_g[O_1,O_2](r)}{G_g[1,1](r)}$

$\Rightarrow \tilde{G}^c_g[O_1,O_2](r) = \tilde{G}_g[O_1,O_2](r) - \bar{O}_1|_g \bar{O}_2|_g$

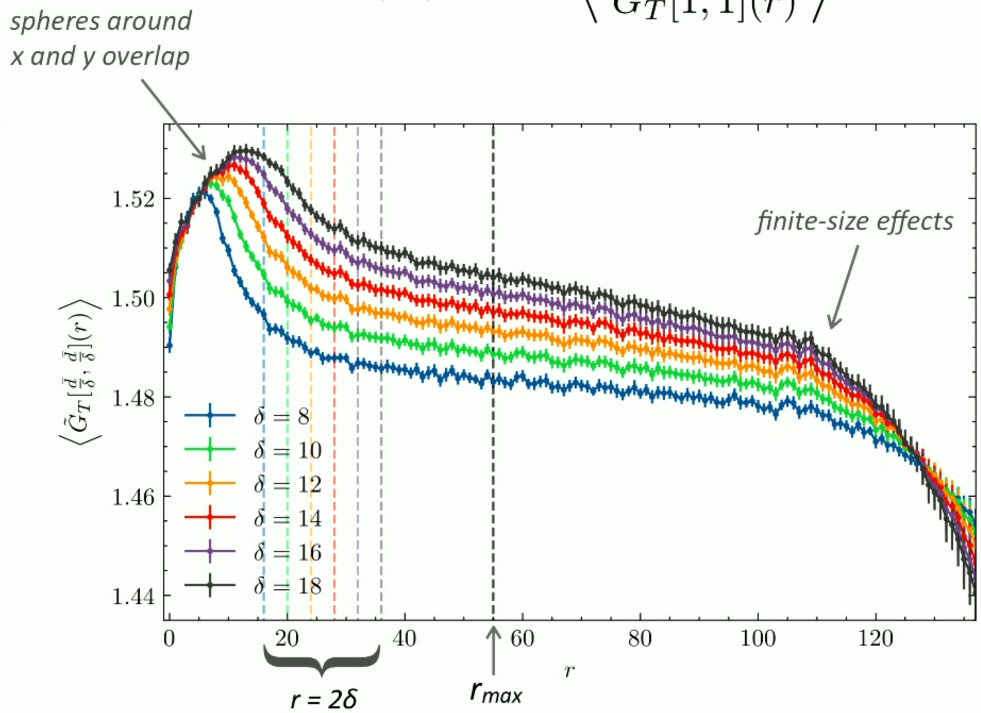
3)


3)
$-\alpha a^2$ $-\alpha a^2$ a^2
$(\alpha > 0)$
t
3 2 1 0
x
$\ell(3)=5$
$\ell(1)=2$
$\ell(0)=5$
t_{tot}
T
topol. $S^1 \times S^1$
$\frac{\bar{\ell}}{t_{tot}}$
$N_2(T) = \# \Delta$ in T,
d(x,y) = # links in shortest path from x to y
$Z(\lambda) = \sum_{\text{causal } T} \frac{1}{C(T)} e^{-\lambda N_2(T)}$
$\langle \mathcal{O} \rangle = \frac{1}{Z} \sum_T \frac{1}{C(T)} \mathcal{O}(T) e^{-\lambda N_2(T)}$
MCMC simul.s in fixed-vol. ensemble $N_2 \in [50K, 300K]$
$d^2x \sqrt{g}\, O(x) = vol(S^r) vol(M) \bar{O}|_g$
vol(M)
$vol(M) vol(S^r) \bar{O}_1|_g \bar{O}_2|_g$
$:= \frac{G_g[O_1,O_2](r)}{G_g[1,1](r)}$
$[O_1,O_2](r) - \bar{O}_1|_g \bar{O}_2|_g$


correlators in NP 2D Lorentzian QG
(w/ Jesse van der Duin)
geom. correlators
curvature
correlators results
Riem. (M, g_μν)
Ollivier
QRC
2)
(M, g_μν), compact, Riem., D=2, vol(M) = ∫d²x √g
O_1 = O_2 = 1
O_2 = 1
connected correl.
Suppose g
normalize

$$\langle \tilde{G}_T[\tfrac{\bar{d}}{\delta}, \tfrac{\bar{d}}{\delta}](r) \rangle = \left\langle \frac{G_T[\frac{\bar{d}}{\delta}, \frac{\bar{d}}{\delta}](r)}{G_T[1,1](r)} \right\rangle$$

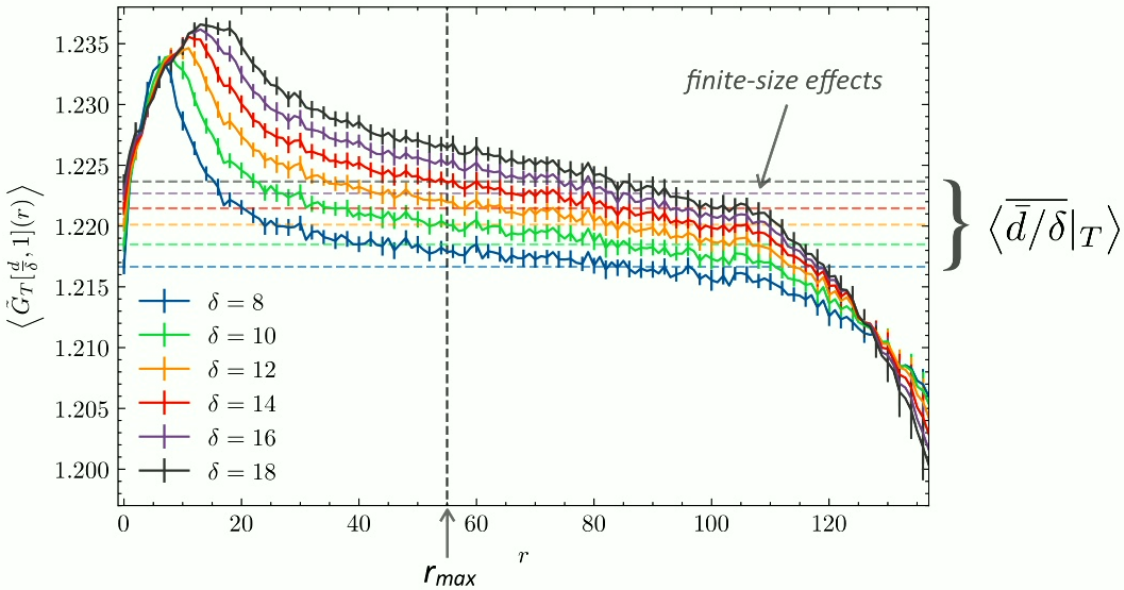
Fig. 1



normalized curvature-curvature correlator as a function of the link distance *r*,
at $N_2 = 300k$ and for sphere radii $\delta \in [8,18]$

$$\langle \tilde{G}_T[\tfrac{\bar{d}}{\delta},1](r)\rangle = \left\langle \frac{G_T[\frac{\bar{d}}{\delta},1](r)}{G_T[1,1](r)} \right\rangle \neq \langle \overline{\bar{d}/\delta}|_T \rangle$$

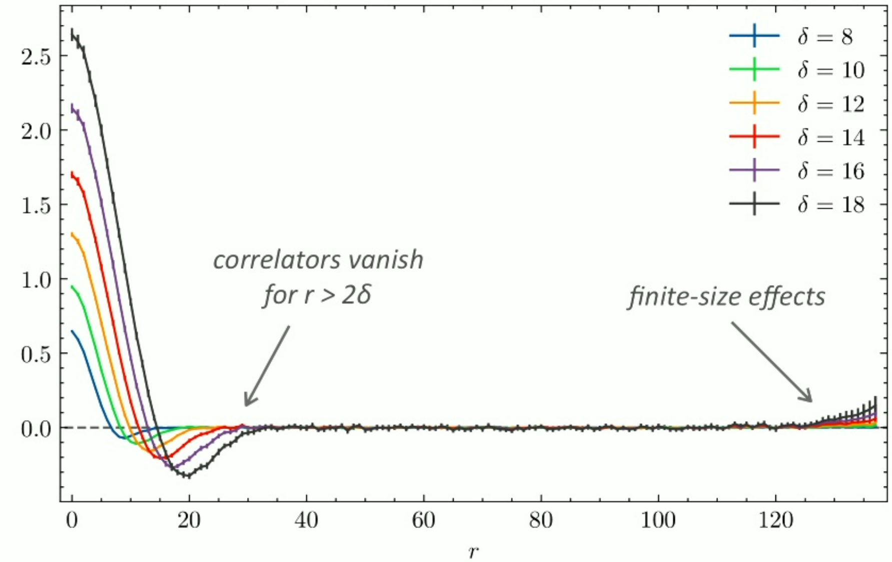
Fig. 2



normalized curvature-"1" correlator as a function of the link distance r, at $N_2 = 300k$ and for sphere radii $\delta \in [8,18]$

$$\langle \tilde{G}^c_T[\tfrac{\bar{d}}{\delta},\tfrac{\bar{d}}{\delta}](r)\rangle = \langle \tilde{G}_T[\tfrac{\bar{d}}{\delta},\tfrac{\bar{d}}{\delta}](r)\rangle - 2\big\langle\overline{\bar{d}/\delta}|_T\big\rangle\langle \tilde{G}_T[\tfrac{\bar{d}}{\delta},1](r)\rangle + \big\langle\overline{\bar{d}/\delta}|_T\big\rangle^2$$

Fig. 3



normalized connected curvature-curvature correlator as a function of the link distance r, at $N_2 = 300k$ and for sphere radii $\delta \in [8,18]$