

Title: Turning Back Time on Space

Speakers: Farnik Nikakhtar

Series: Cosmology & Gravitation

Date: April 23, 2024 - 11:00 AM

URL: <https://pirsa.org/24040109>

Abstract: One big question in the physics of large-scale structures is how we can trace back the evolution of cosmic structures and reconstruct the initial density field. The universe we observe today is dotted with galaxy clusters separated by vast voids, in sharp contrast to its initial state, which was nearly uniform with only minor density fluctuations. The evolution from this early uniformity to today's complex structure of galaxies is a profound transformation, with many intermediate processes still unexplained. This talk focuses on this transformation, aiming to reconstruct both the initial density and the displacement fields of galaxies observed in spectroscopic surveys. I will discuss new reconstruction algorithms that depend weakly, if at all, on a cosmological model. These algorithms have paved the way for new astrophysical achievements in several directions, such as: Baryon Acoustic Oscillations (BAO) analysis, morphological analysis of proto-halos, and kinematic Sunyaev-Zel'dovich (kSZ) effect.

Zoom link

Turning Back Time on Space

Farnik Nikakhtar
YCAA Fellow, Yale University

In collaboration with:

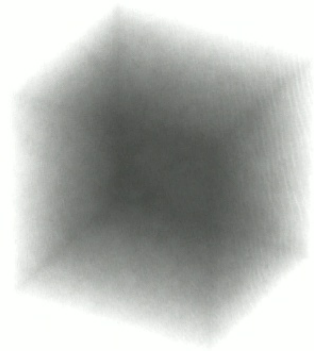
Nikhil Padmanabhan (Yale)	Ravi K. Sheth (Penn)
Daisuke Nagai (Yale)	Bruno Levy (Inria)
Roya Mohayaee (IAP)	Navya Uberoi (Yale)
Sasha Gaines (Yale)	

Perimeter Institute, Cosmology Seminar
April 23, 2024

Interstellar, Christopher Nolan (2014)

Big questions I'm interested in

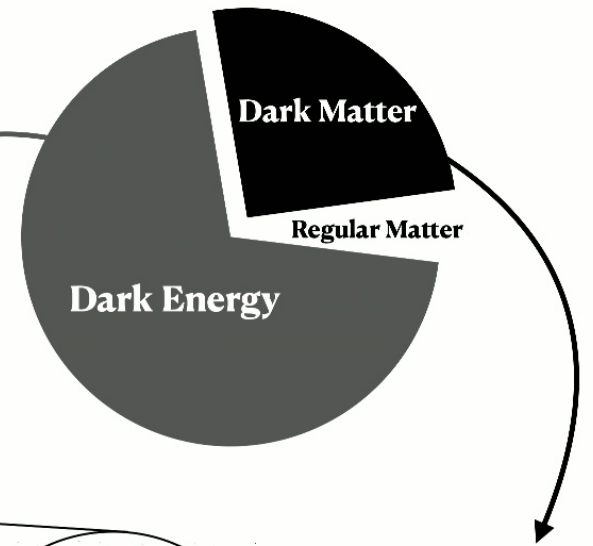
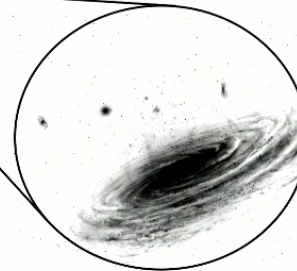
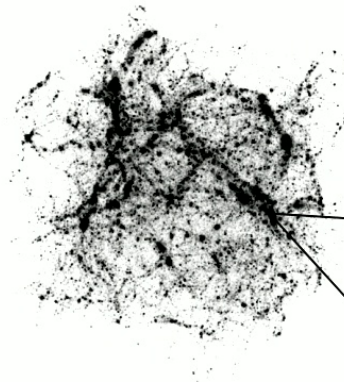
- How can we move back the evolution of cosmic structures?



Cosmic Evolution
→

← ?

Reconstruction



- How can we quantify out-of-equilibrium characteristics of galactic systems?

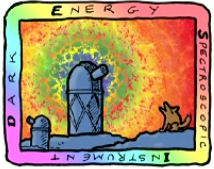


DARK ENERGY SPECTROSCOPIC INSTRUMENT

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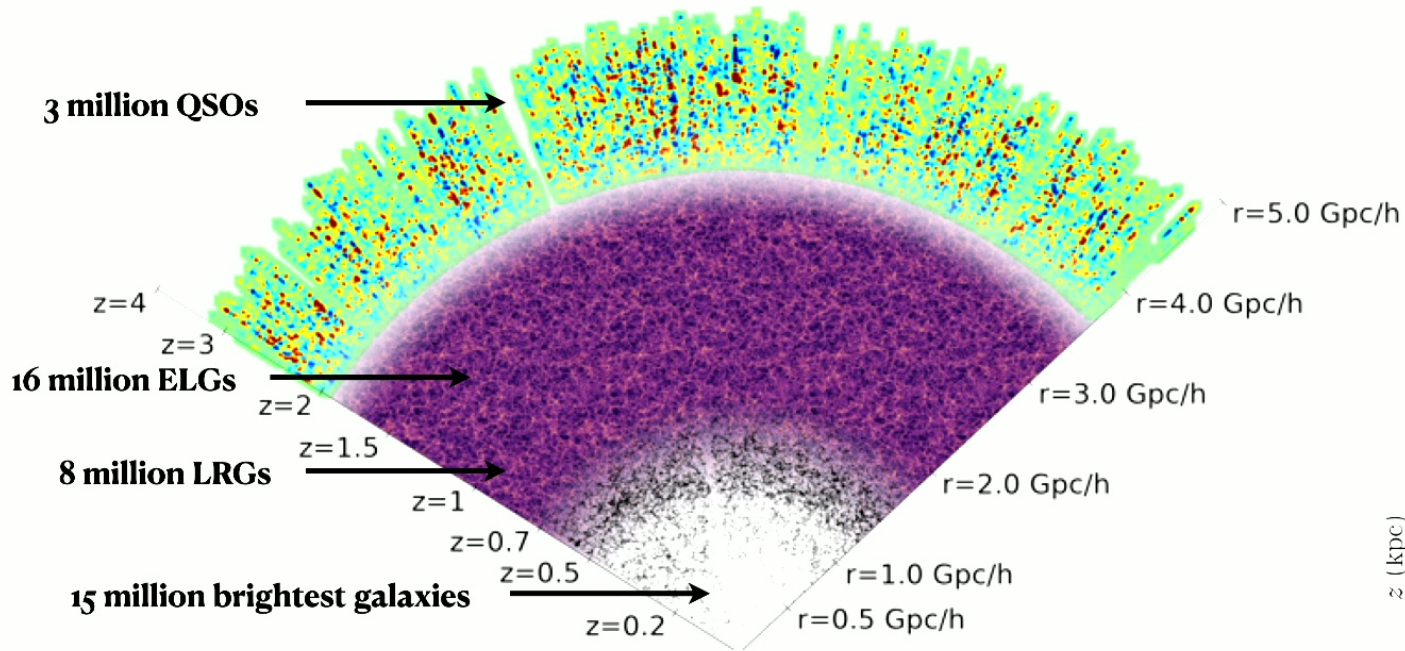
Credit: Credit: DESI Collaboration/NOIRLab/NSF/AURA/R. Proctor



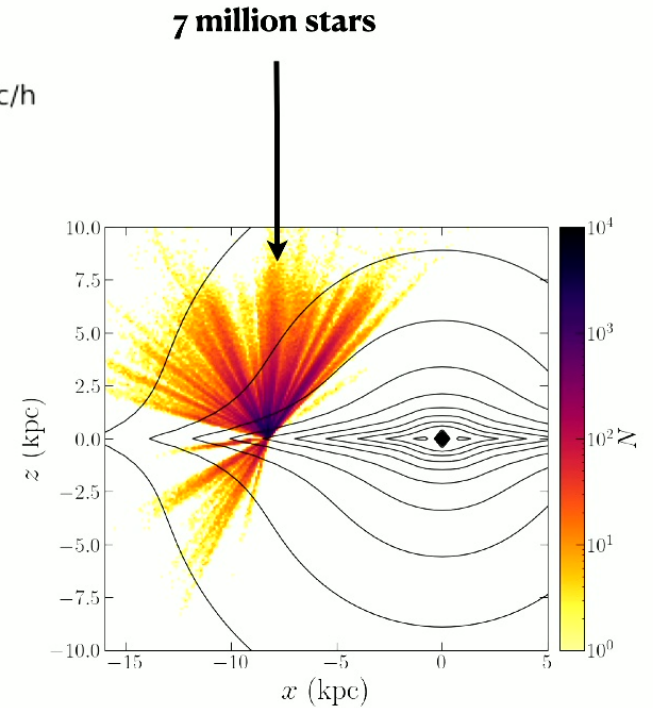
DARK ENERGY
SPECTROSCOPIC
INSTRUMENT

U.S. Department of Energy Office of Science

DESI will be the largest 3D map of the universe this decade!



Credit: D. Schlegel (LBNL)



Turning Back Time on Space or how can we move back the evolution of cosmic structures?

A simple “biased” view of reconstruction

Initial position \vec{q} Displacement \vec{S}

Evolved position $\vec{X} = \vec{q} + \vec{S}$

$$\vec{X}_2 - \vec{X}_1 = \vec{q}_2 - \vec{q}_1 + \vec{S}_2 - \vec{S}_1$$

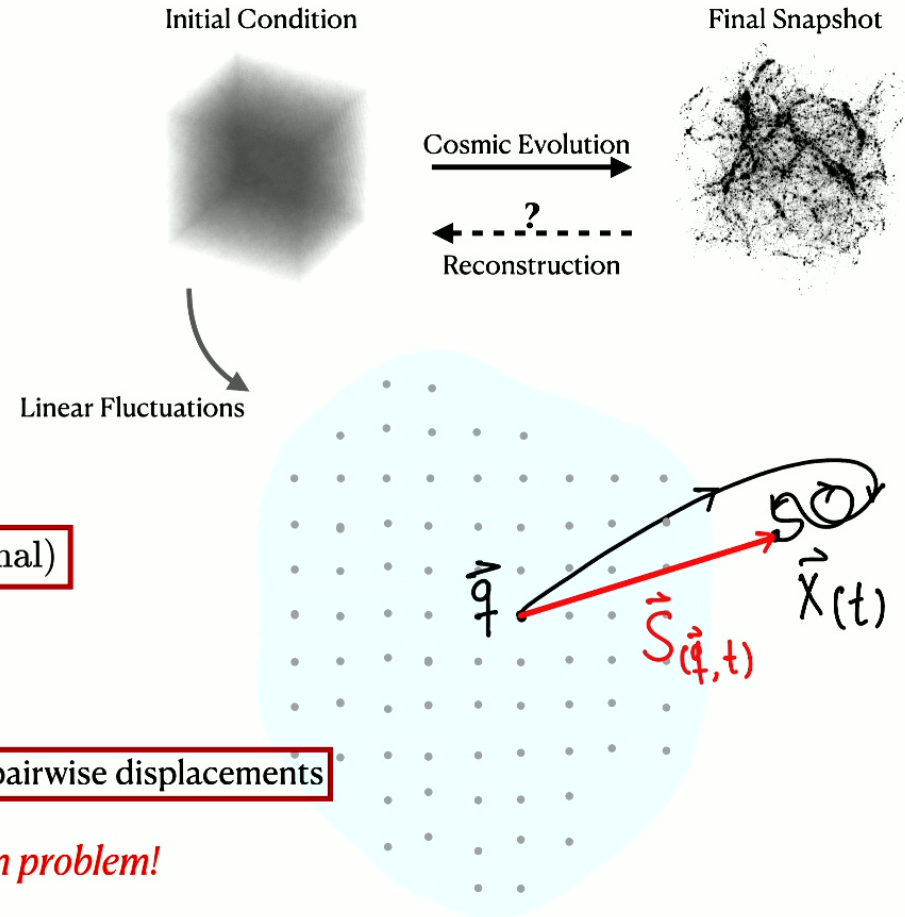
$$\boxed{\vec{X}_{12} \text{ data}} = \boxed{\vec{q}_{12} \text{ signal}} + \boxed{\vec{S}_{12} \text{ noise}}$$

$$\boxed{\mathcal{P}(\text{data})} = \int d(\text{signal}) \boxed{\mathcal{P}(\text{signal})} \boxed{\mathcal{P}(\text{noise} = \text{data} - \text{signal} | \text{signal})}$$

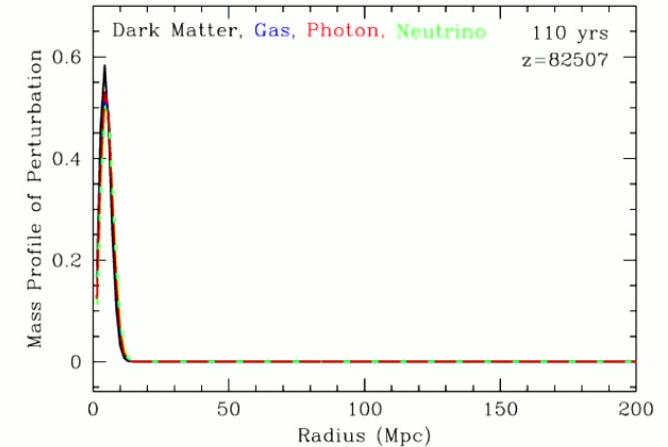
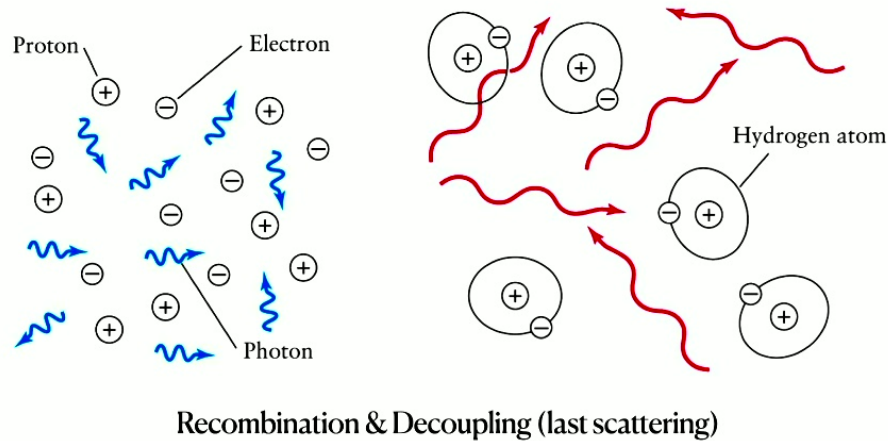
Histogram of pair separations today =

Convolution of **initial pair separation histogram** with **distribution of pairwise displacements**

Reconstruction of ~~pair separations~~ is a deconvolution problem!
2-point correlation function



Baryon Acoustic Oscillations from Soup to Nuts



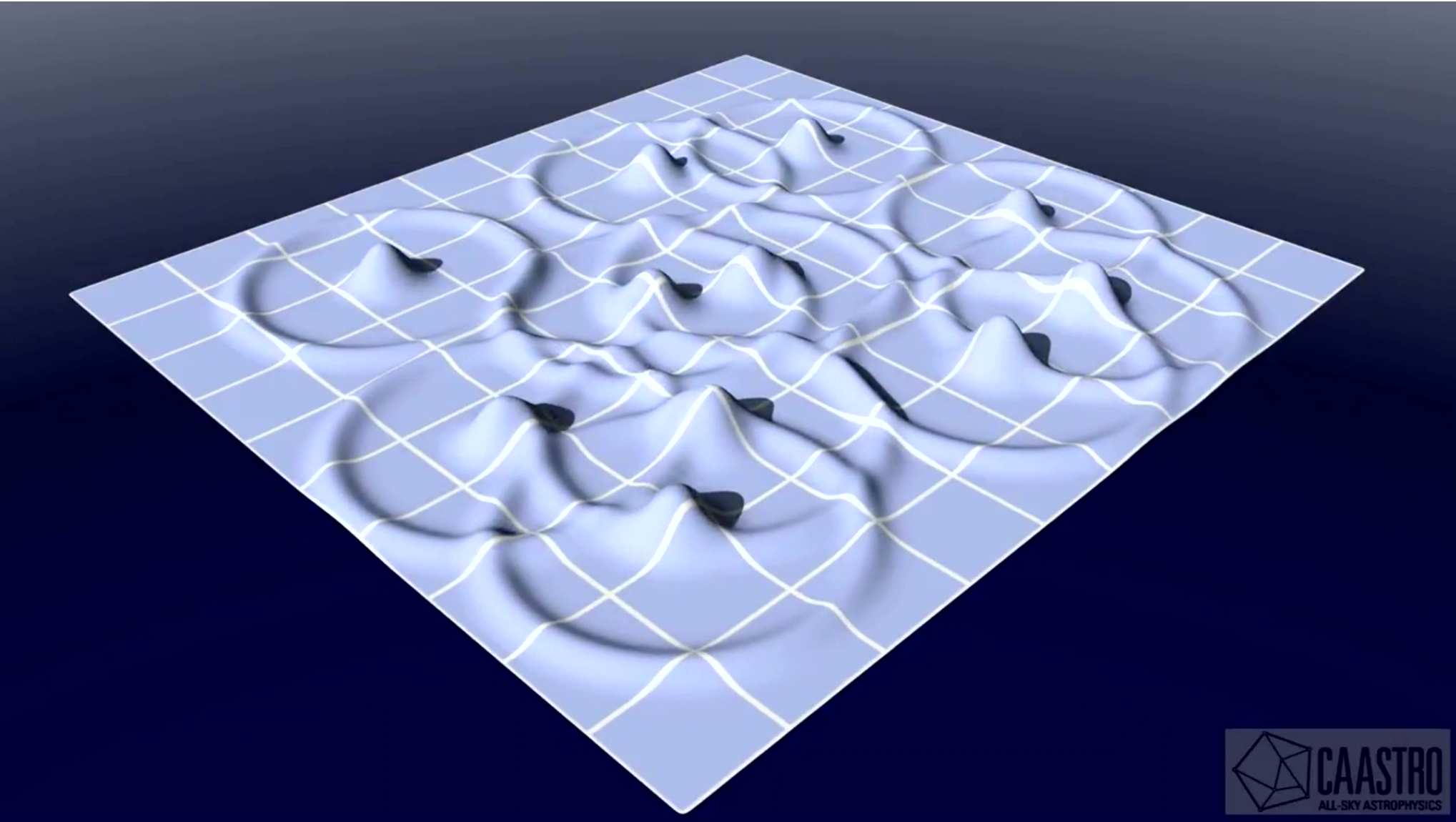
Credit: D. Eisenstein (Harvard)

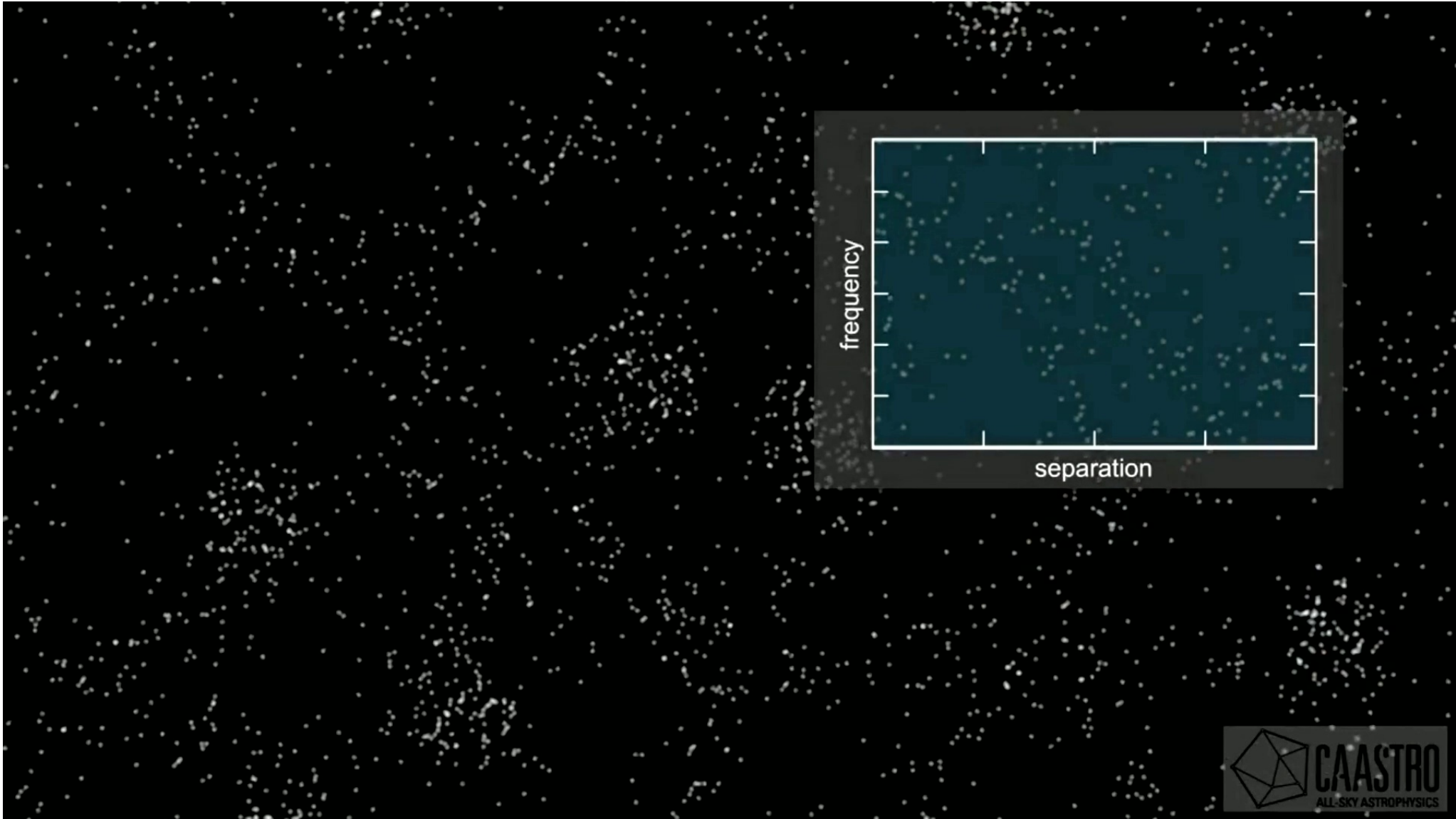
Photons “drag” baryons for ~400,000 years

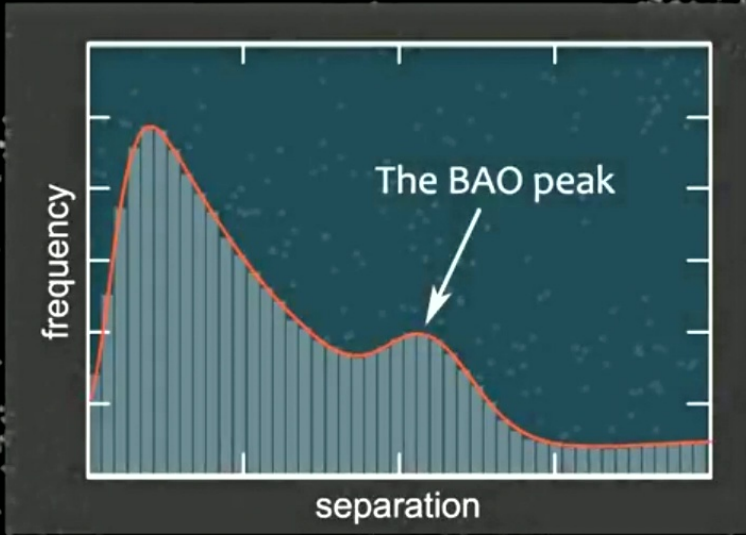
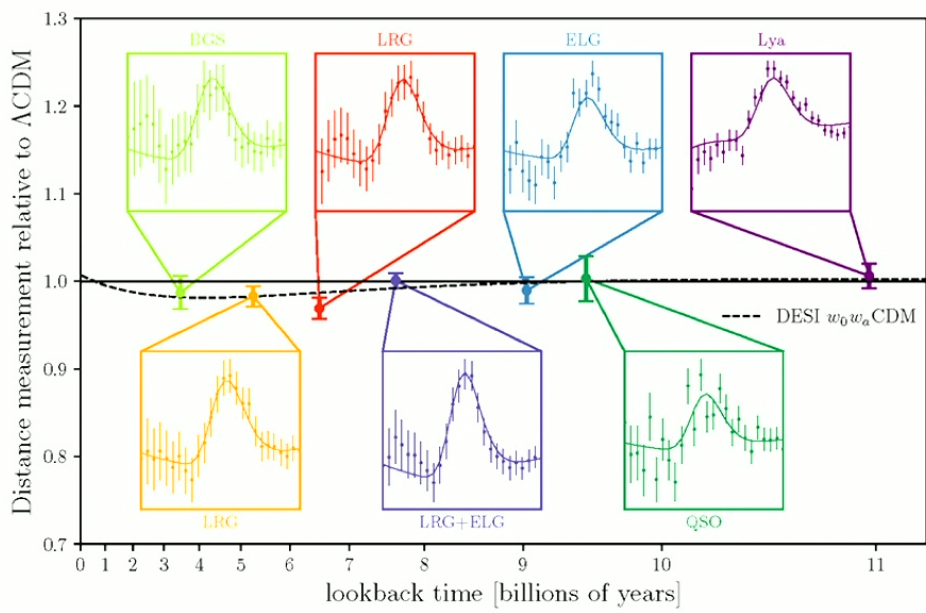
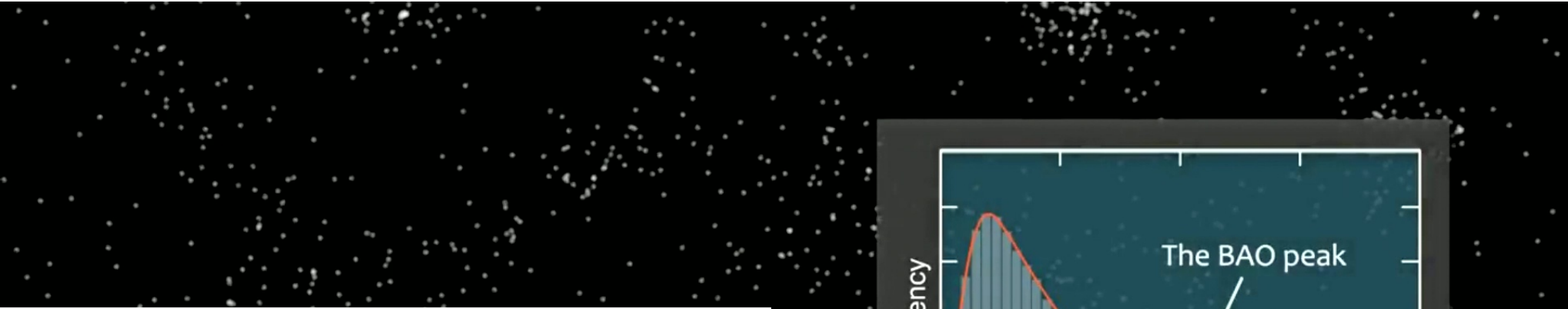
$$\text{at speed } \sim \frac{c}{\sqrt{3}} \left[\frac{3}{4} \frac{\bar{\rho}_b(z)}{\bar{\rho}_\gamma(z)} + 1 \right]^{-1/2}$$

$$v \Delta t \sim 300,000 \text{ light years} \sim 100,000 \text{ pc} \sim 100 \text{ kpc} \text{ (set by } \Omega_m h^2, \Omega_b h^2 \text{)}$$

Expansion of Universe since then stretches this to $(3000/2.725) \times 100 \text{ kpc} \sim 100 \text{ Mpc}$







Reconstruction of 2-point correlation function

2PCF of today = Convolution of initial 2PCF with a displacement kernel

$$\xi_{\text{NL}}(s) = \int_0^\infty \frac{dr}{\Sigma} \chi_3\left(\frac{r}{\Sigma} \middle| \frac{s}{\Sigma}\right) \xi_{\text{L}}(r)$$

smoothing scale/growth of structures

Laguerre Reconstruction: Nikakhtar et al. (2021a, 2021b, 2022)

If we assume: $\xi_{\text{L}}(r) = \sum_{k=0}^n a_k r^k$

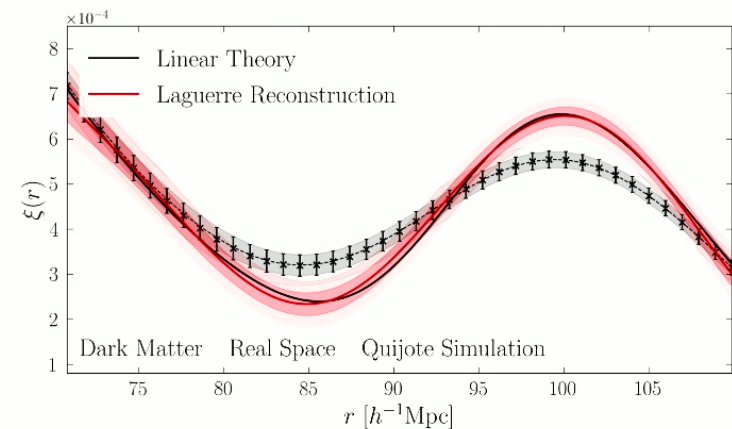
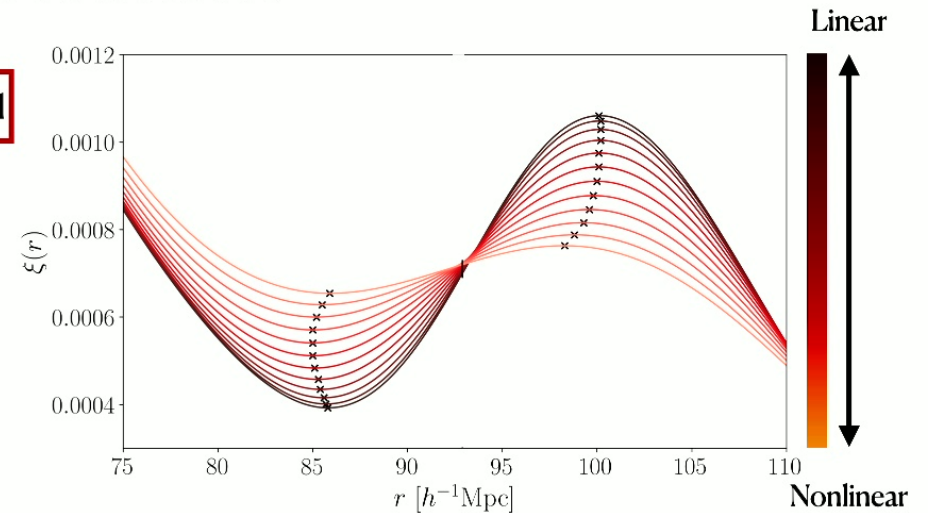
$$\xi_{\text{NL}}(s) = \sum_{k=0}^n c_k \mu_k(x) \quad \text{moments of the kernel}$$

(For specialists: Easy to include mode-coupling)



Navya Uberoi (Yale)

Uberoi, Nikakhtar, Padmanabhan et al. (2024 in prep.)



How to reconstruct the full field ?

(not only its 2-point statistics)

Turning Back Time on Space or how can we move back the evolution of cosmic structures?

Optimal Transport Theory



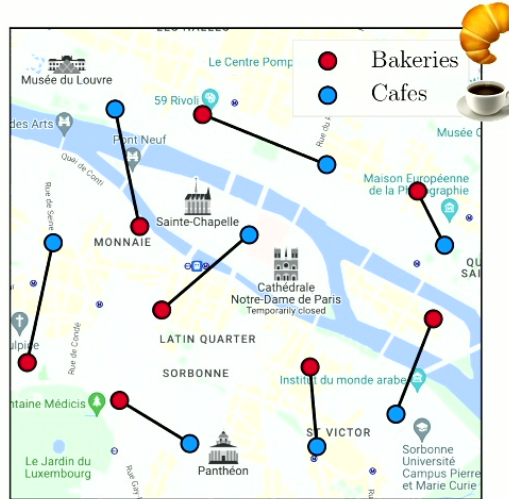
Gaspard Monge
(1746 – 1818)



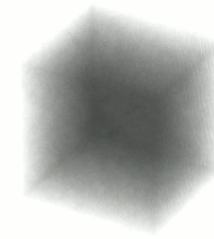
Leonid Kantorovich
(1912 – 1986)



Yann Brenier
Polar factorization theorem
1991



Initial Condition



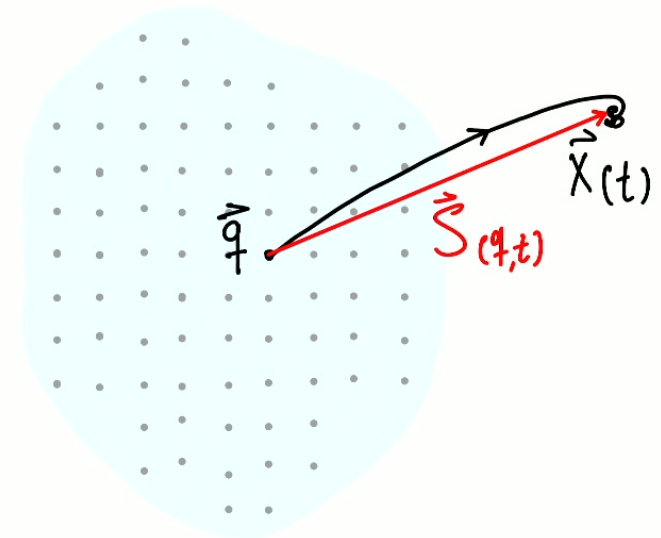
Final Snapshot



Cosmic Evolution



Reconstruction



OT: How to map a distribution into another one while preserving weights and minimizing the effort?

Conservation law

Minimum action principle

Minimum action principle subject to conservation law!

Turning Back Time on Space or how can we move back the evolution of cosmic structures?

Optimal Transport Theory (Assignment Problem)

Mass Conservation

$$\rho_{\text{fin}}(\mathbf{x})d^3\mathbf{x} = \rho_{\text{ini}}(\mathbf{q})d^3\mathbf{q}$$

$$\mathbf{q} = \mathbf{x} + \nabla\Theta(\mathbf{x})$$

$$\frac{\rho_{\text{fin}}(\mathbf{x})}{\bar{\rho}} = \left| \frac{d^3\mathbf{q}}{d^3\mathbf{x}} \right| = \det\left(\frac{\partial q^i}{\partial x^j}\right) = \det[1 + \partial_i\partial_j\Theta(\mathbf{x})]$$

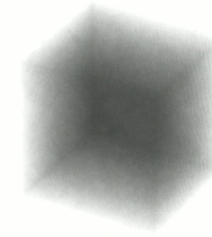
Monge–Ampère equation



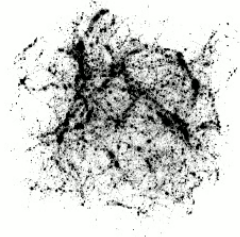
Yann Brenier

Brenier 1991: *Solving MA is equivalent to the solution of OT with quadratic cost (existence and uniqueness)*

Initial Condition



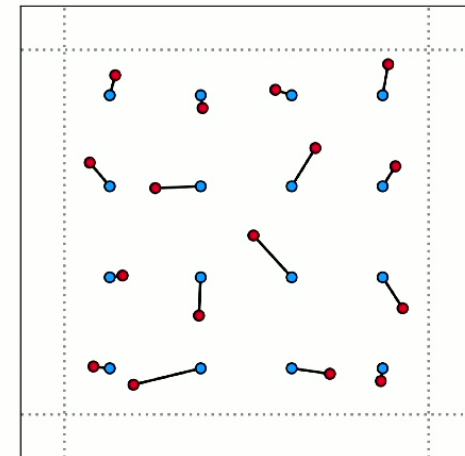
Final Snapshot



Cosmic Evolution



Reconstruction

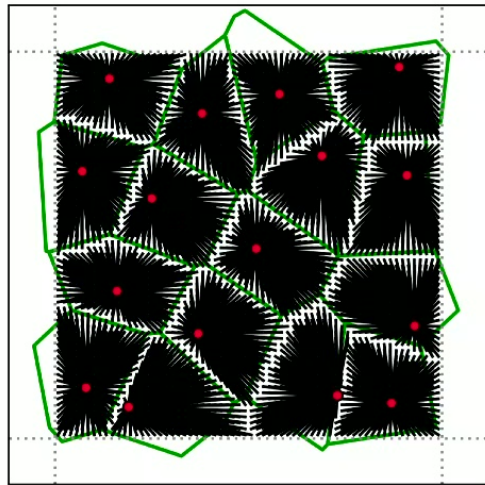


Frisch et al. (2002)

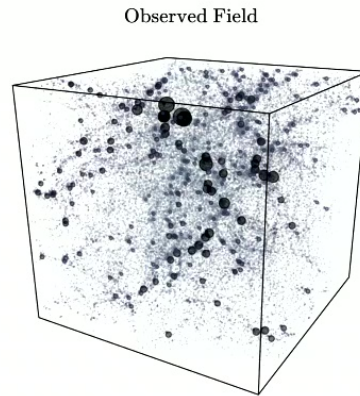
Brenier et al. (2003)

Semi-Discrete Optimal Transport

Mostly everything gets worse before it gets better!



Partition space into Laguerre cells
(power diagram / modified Voronoi diagram)

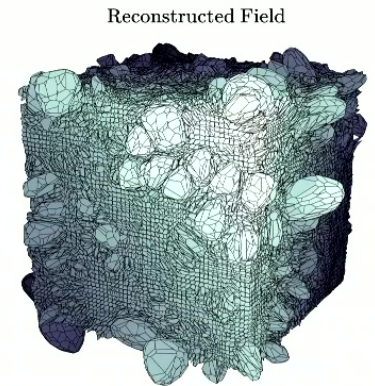


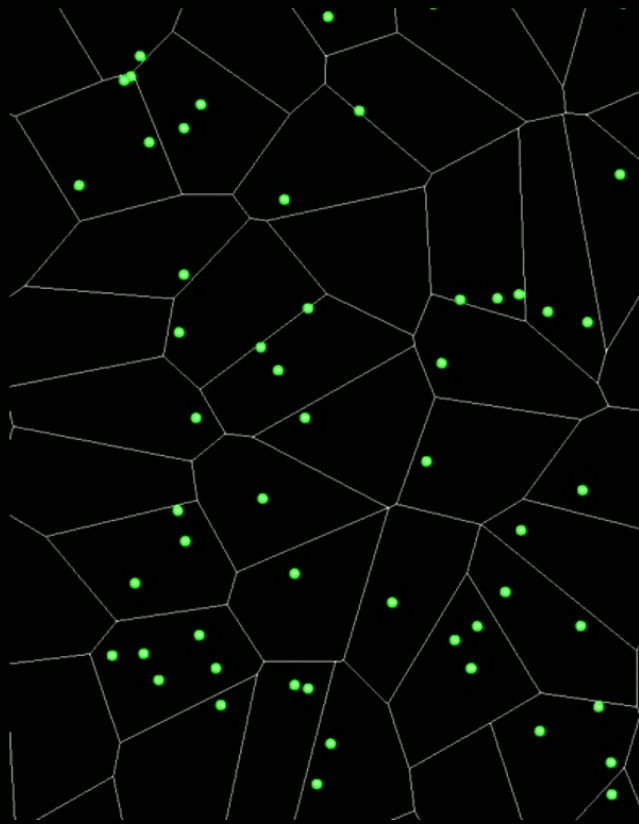
Nikakhtar et al. PRL 129, 251101 (2022)

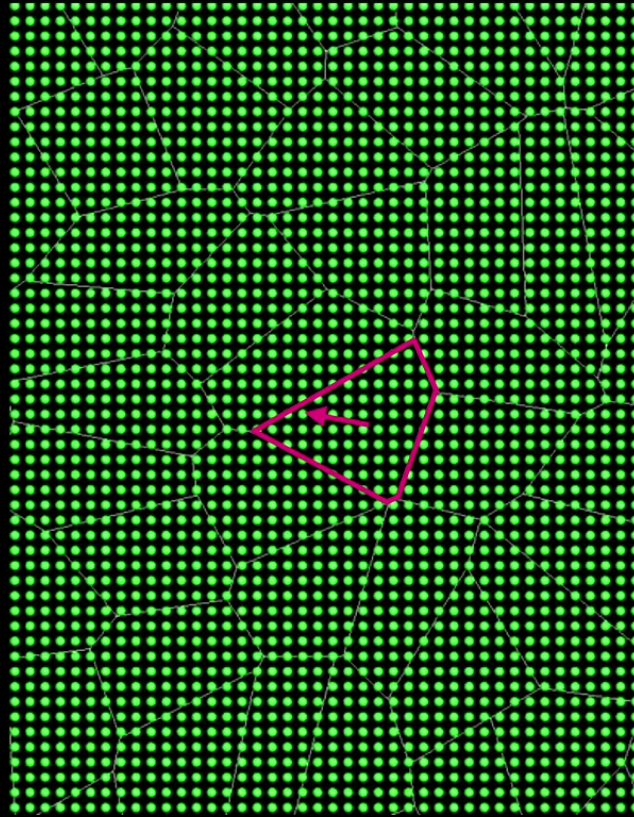
Optimal Transport Reconstruction

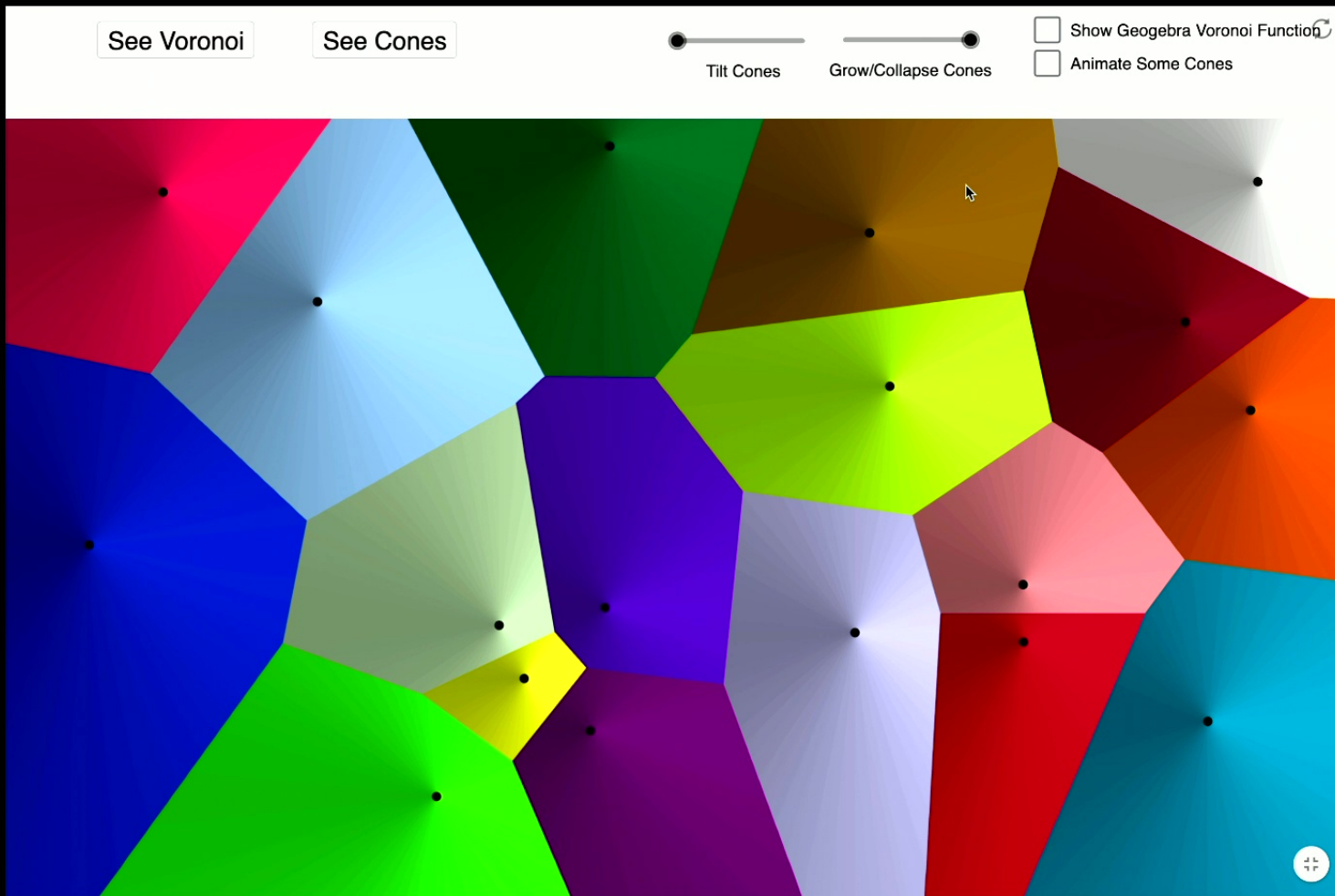
$$V_i^\psi = \left\{ \mathbf{q} \left| \underbrace{\frac{1}{2}|\mathbf{x}_i - \mathbf{q}|^2}_{\text{Kinetic}} - \underbrace{\psi_i}_{\text{Potential}} < \frac{1}{2}|\mathbf{x}_j - \mathbf{q}|^2 - \psi_j, \forall j \neq i \right. \right\}$$

Geometrical Optimization in Mathematics
≡
Action Minimization in Physics

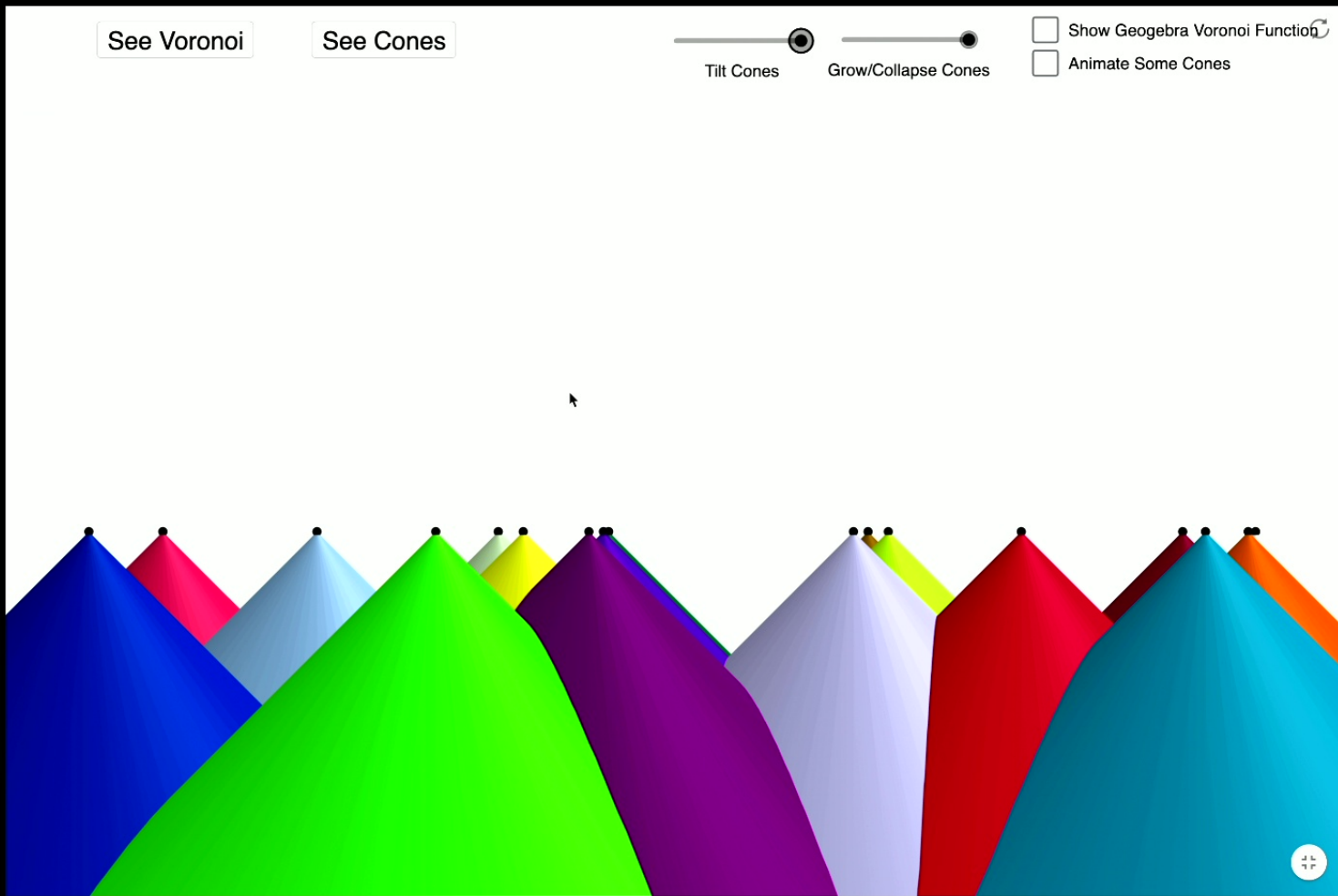








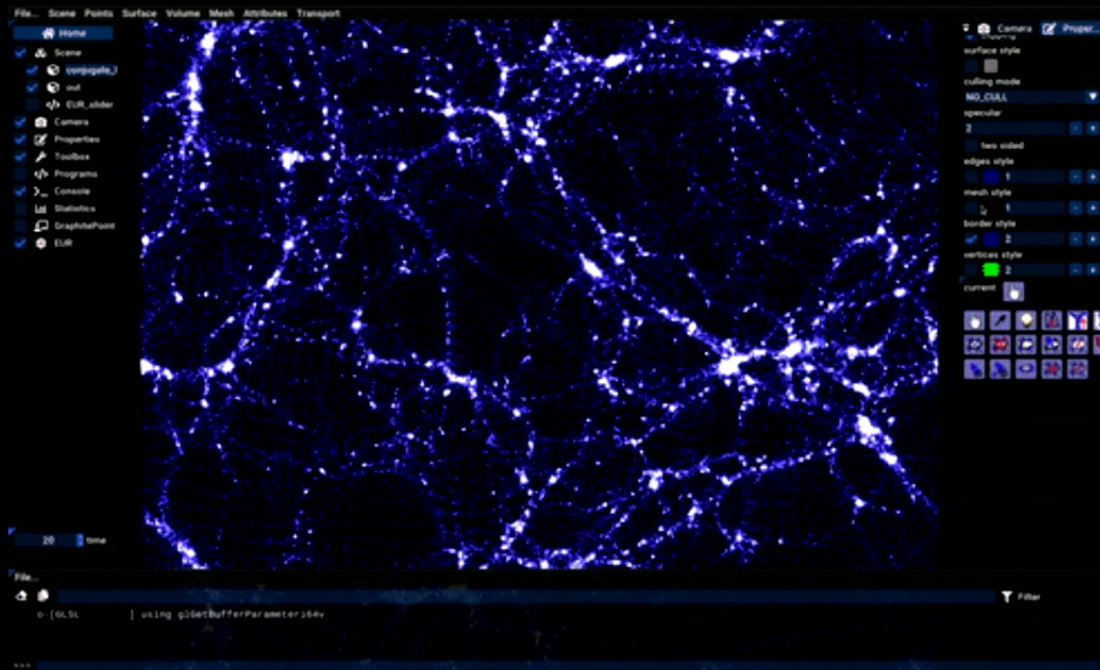
Credit: Tom Walsh (physics.com)



Credit: Tom Walsh (physics.com)

Geogram: programming library with geometric algorithms

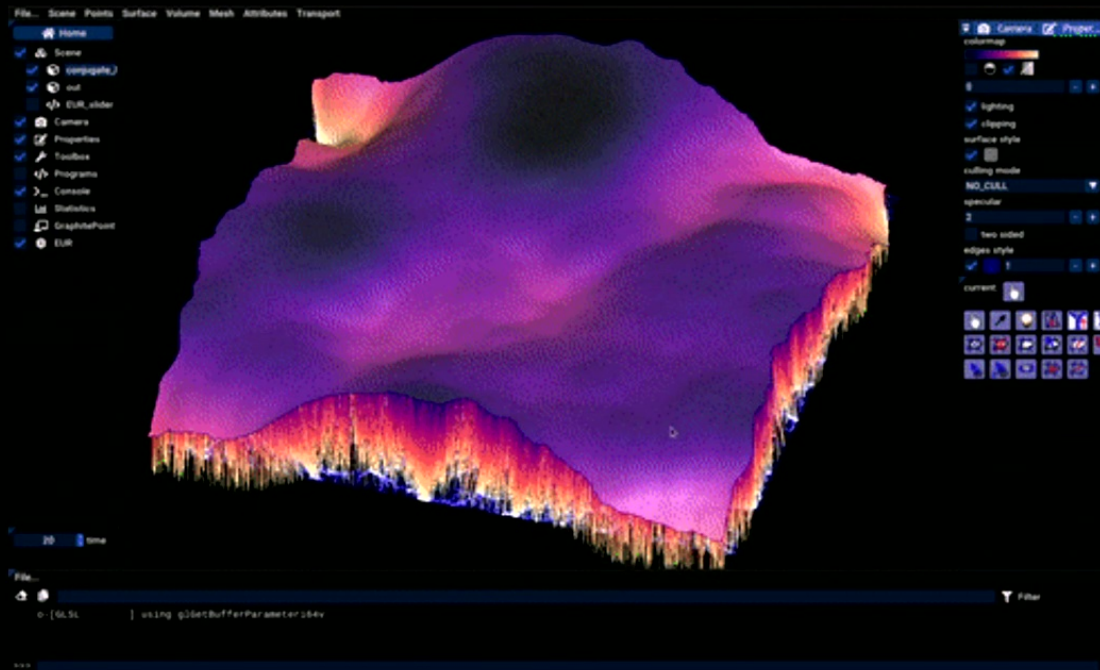
<https://github.com/BrunoLevy/geogram>



Bruno Levy (Inria-Paris)

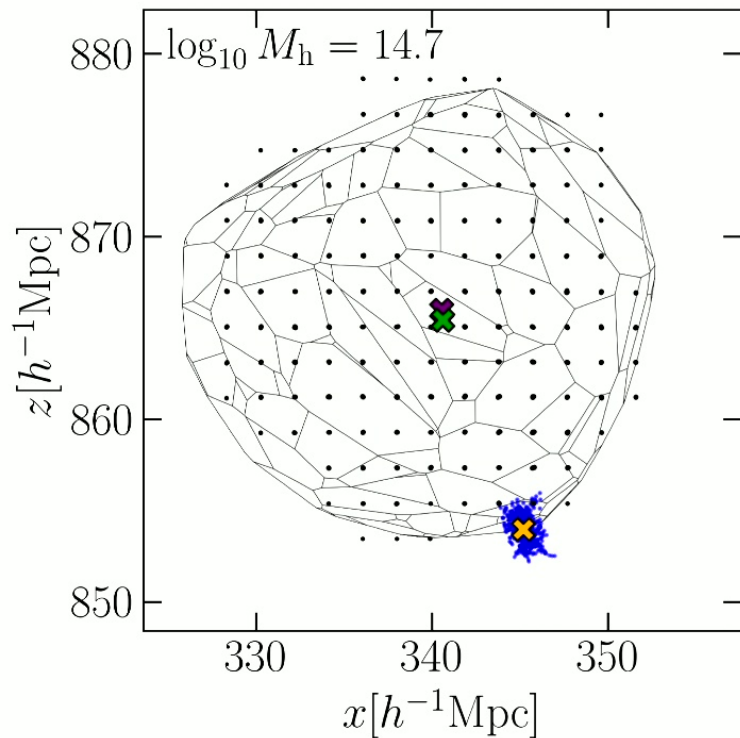
Geogram: programming library with geometric algorithms

<https://github.com/BrunoLevy/geogram>

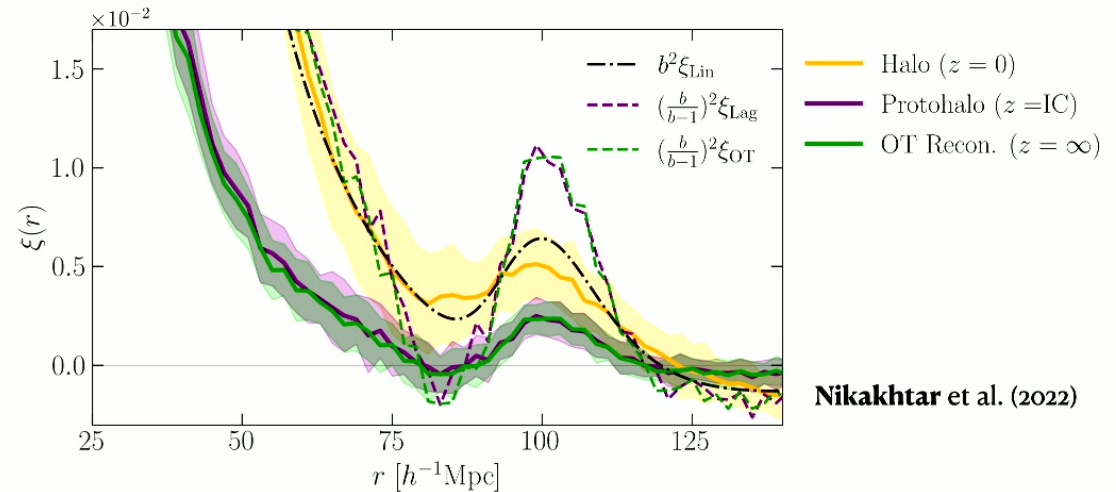


Bruno Levy (Inria-Paris)

Reconstructing Protohalo Positions + Shapes



- ✕ Halo ($z = 0$)
- ✕ Protohalo ($z = \text{IC}$)
- ✕ OT Recon. ($z = \infty$)

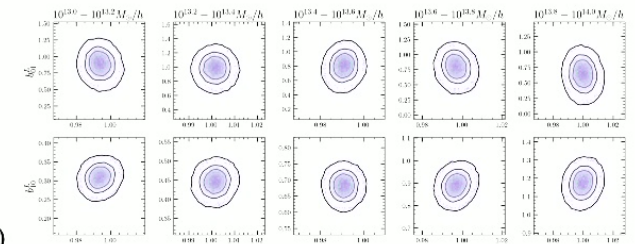


Nikakhtar et al. (2022)

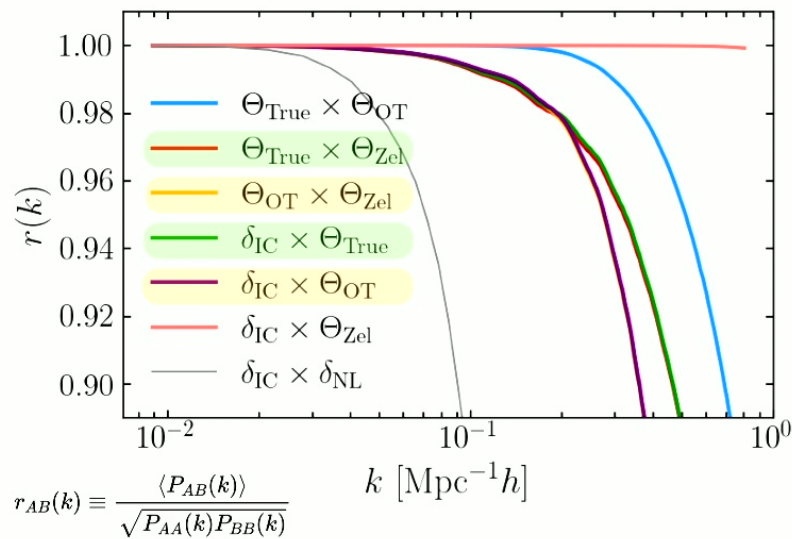


Sasha Gaines (Yale)

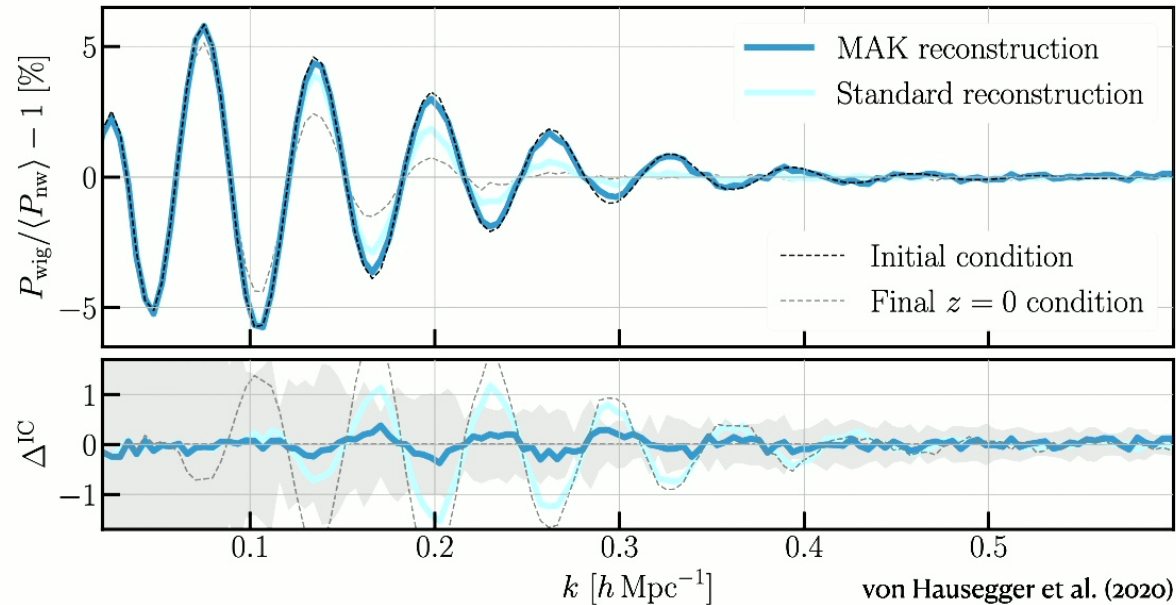
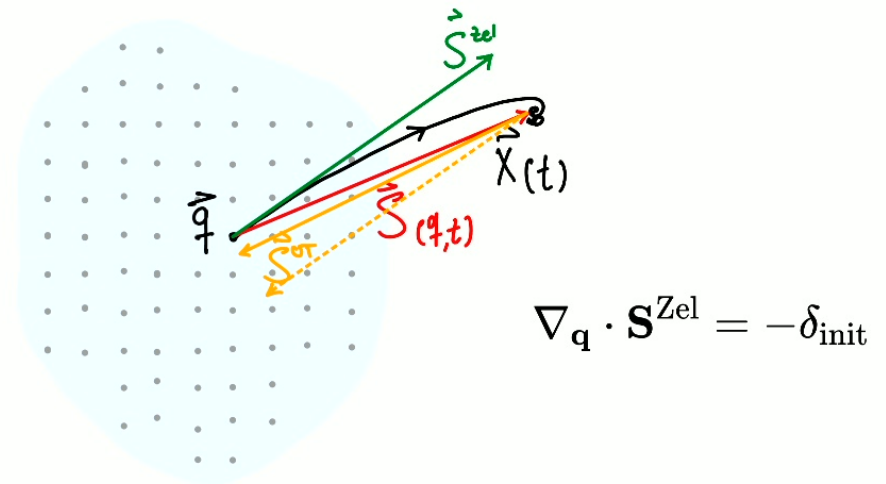
Gaines, Nikakhtar, Padmanabhan, Sheth (2024 in prep.)



Displacement Field Analysis

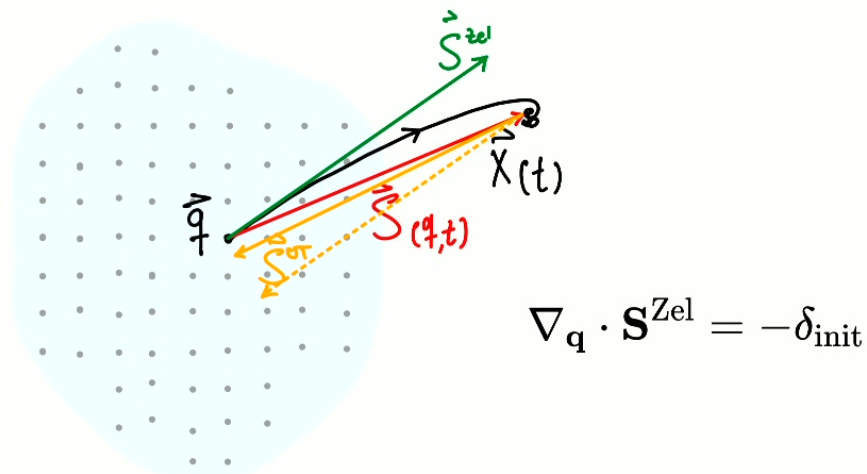
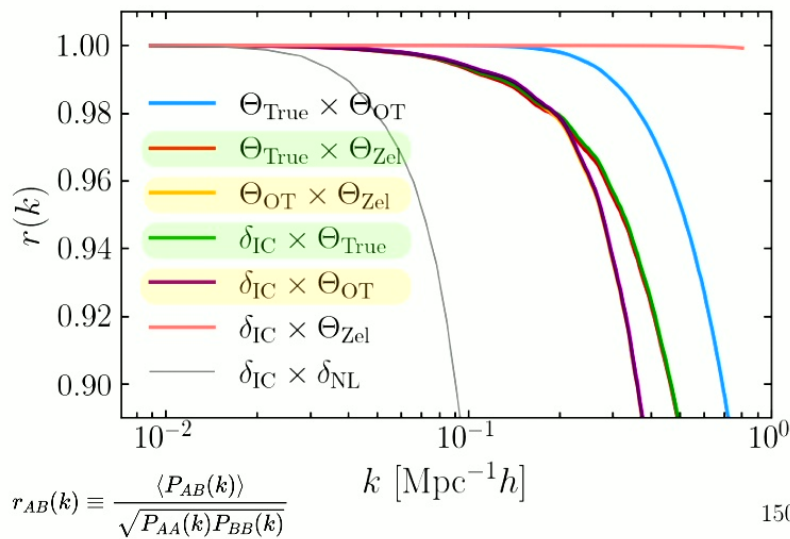


Nikakhtar et al. (2024)



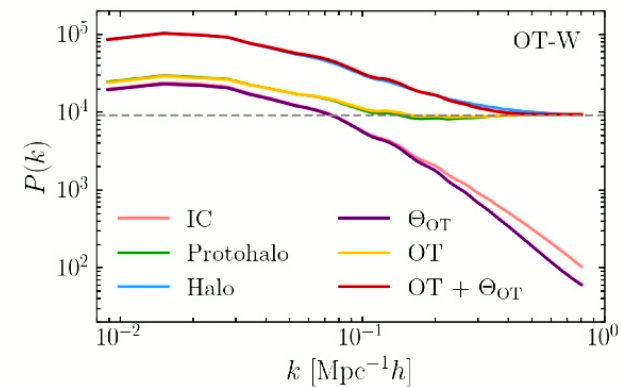
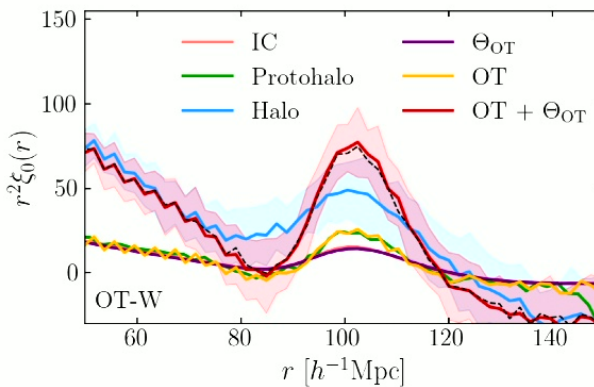
von Hausegger et al. (2020)

Displacement Field Analysis



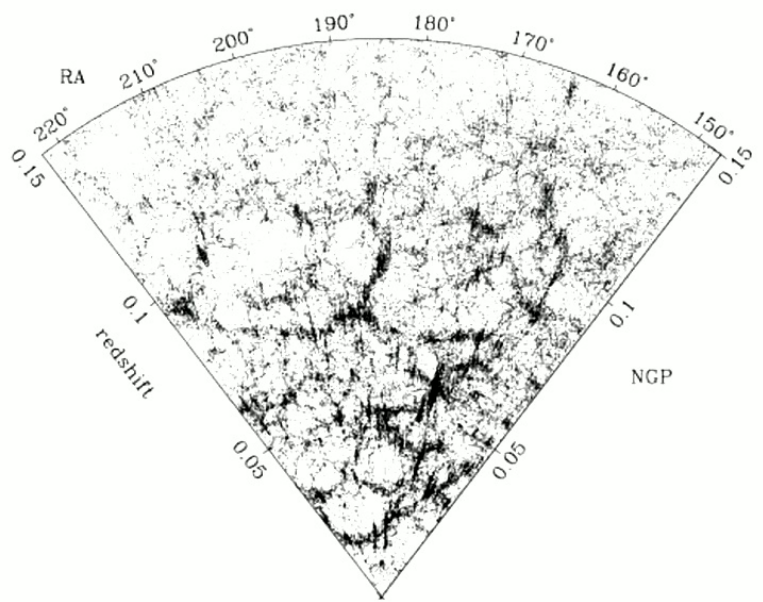
Reconstruction of biased tracers:

$$\delta_{\text{OT(protohalo)}} + \Theta_{\text{OT}}$$



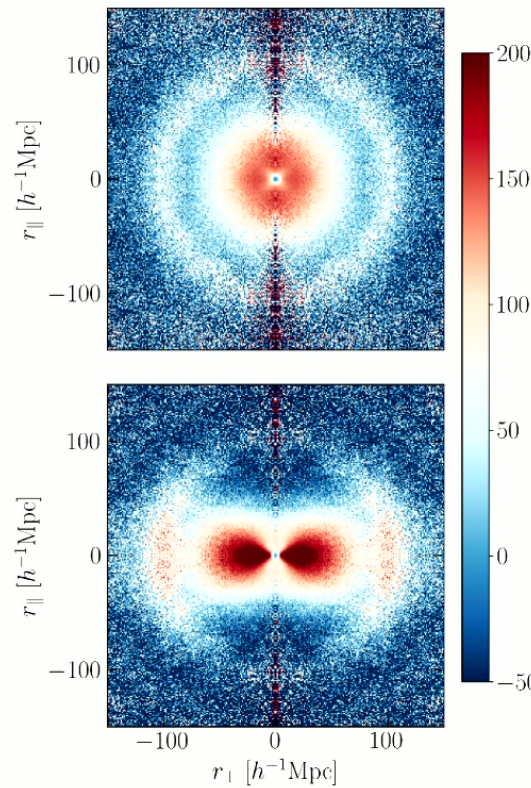
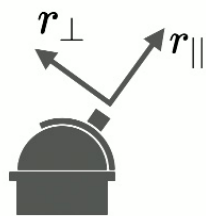
Nikakhtar et al. (2024)

Redshift Space Distortions & Anisotropic OT

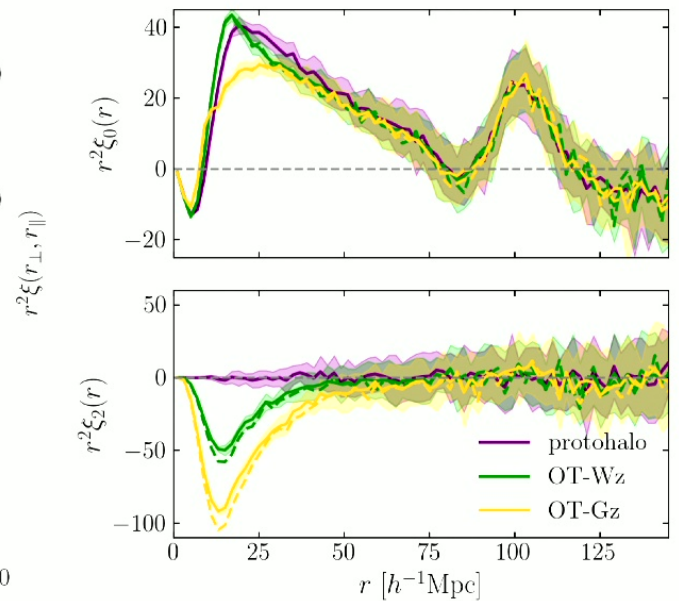


Eke et al. (2003)

$$z_{\text{obs}} = z_{\text{true}} + \frac{v_{\text{pec}}}{c}$$



$$\xi(r_{\parallel}, r_{\perp}) = \xi_0(s)\mathcal{L}_0(\mu) + \xi_2(s)\mathcal{L}_2(\mu) + \xi_4(s)\mathcal{L}_4(\mu)$$



Nikakhtar et al. (2023)

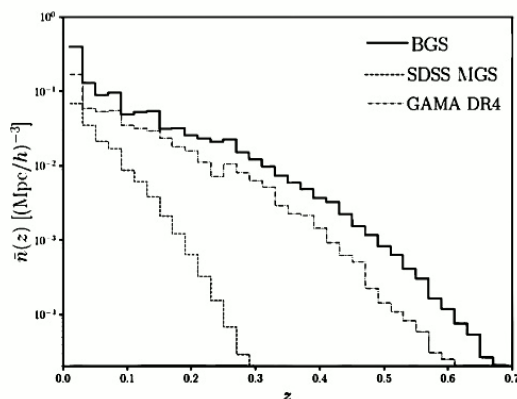
OT Reconstruction on DESI Bright Galaxy Survey

15 million galaxies at $z < 0.6$ in the dark energy dominated epoch

magnitude limited sample to $r < 19.5$, fainter sample to $r < 20.175$ over 14,000 deg² footprint

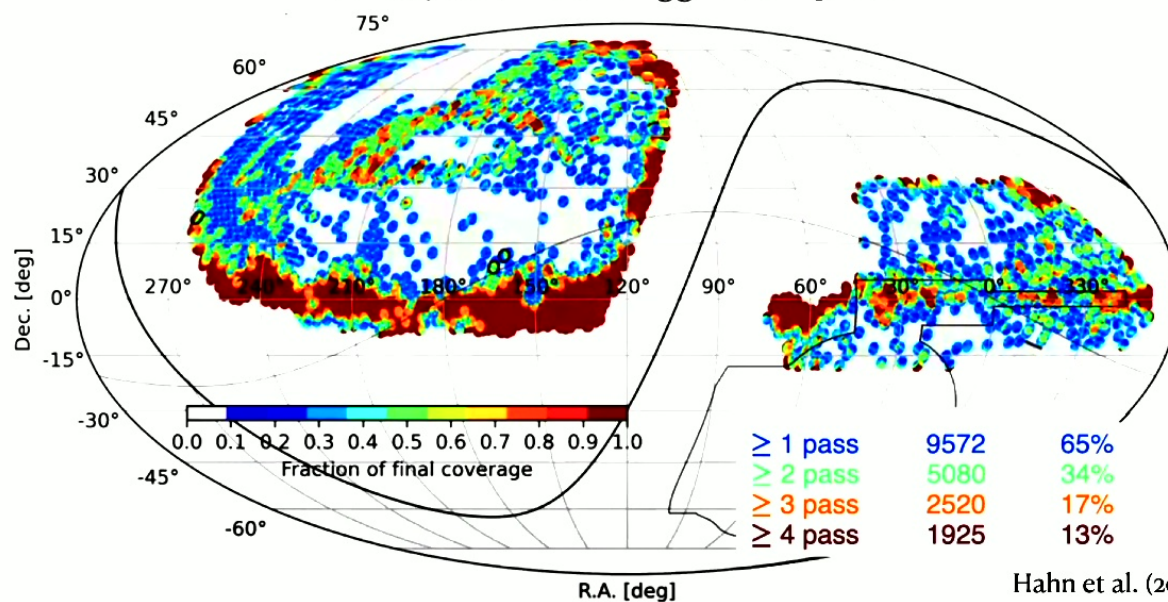
2 magnitude deeper than SDSS main survey

in its 1st year, BGS is >33% complete



high density sample

more than **an order of magnitude** larger than previous surveys



Hahn et al. (2022)

What else can we learn from reconstructed density & displacement?

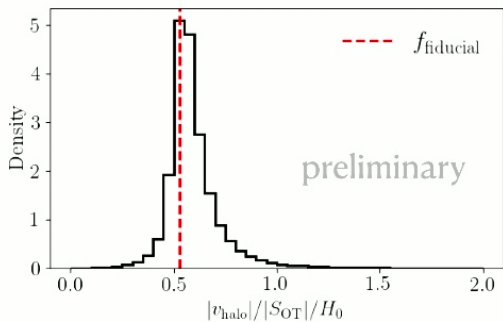
Future Research Avenues

Peculiar Velocity Surveys

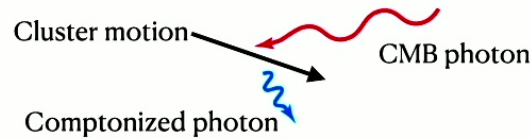
Tully-Fisher / Fundamental Plane relations

$$\mathbf{v}^{\text{rec}} = a f H \mathbf{S}^{\text{rec}}$$

Growth rate of structures



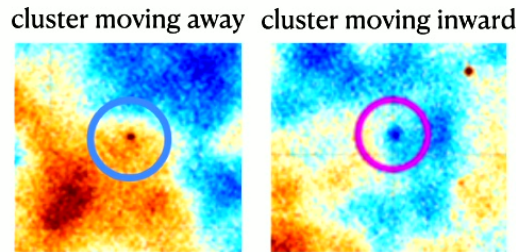
Baryonic feedback with the kSZ effect



$$\frac{\delta T_{\text{kSZ}}(\hat{n})}{T_{\text{CMB}}} = -\tau_{\text{gal}} \left(\frac{v_{e,r}}{c} \right)$$

Optical depth Electron/halo velocity

integral of gas density along LOS



Madhavacheril et al. (2020)

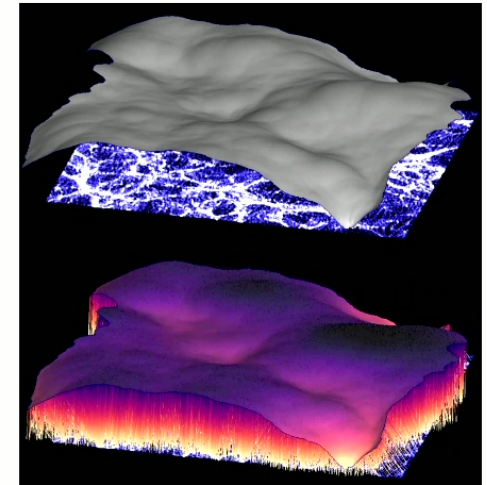
DESI x ACT

Accurate straight-line-evolved sims

$$\mathbf{q} = \mathbf{x} + \nabla \Theta_{\text{OT}}(\mathbf{x}) \quad \mathbf{x} = \mathbf{q} + \nabla \Phi_{\text{OT}}(\mathbf{q})$$

$$\Theta_{\text{OT}}(\mathbf{x}) = \max_{\mathbf{q}} \mathbf{x} \cdot \mathbf{q} - \Phi_{\text{OT}}(\mathbf{q})$$

$$\Phi_{\text{OT}}(\mathbf{q}) = \max_{\mathbf{x}} \mathbf{x} \cdot \mathbf{q} - \Theta_{\text{OT}}(\mathbf{x})$$



Summary

DESI: Exciting Data Ahead for both *Galaxy Clustering & Milky Way Studies*

Reconstruction is interesting: if we reconstruct the *initial density & displacement* fields, we can understand several other things: BAO scale, growth rate, baryon distribution, ...

Optimal Transport theory is not merely a tool — in our language, we can consider it as: *Minimum action principle subject to conservation law!*



DARK ENERGY
SPECTROSCOPIC
INSTRUMENT

U.S. Department of Energy Office of Science

Thank you!

For more details:

- Uberoi, FN, Padmanabhan et al. (2024 in prep.)
- Gaines, FN, Padmanabhan, Sheth (2024 in prep.)
- FN, N Padmanabhan, R Sheth, B Lévy, R Mohayaee; PRD submitted, 2024
- FN, N Padmanabhan, R Sheth, B Lévy, R Mohayaee; PRD, 108 (8), 083534
- FN, RK Sheth, B Lévy, R Mohayaee; PRL, 129 (25), 251101
- FN, RK Sheth, I Zehavi; PRD 105 (4), 043536
- FN, RK Sheth, I Zehavi; PRD 104 (6), 063504
- FN, RK Sheth, I Zehavi; PRD 104, 043530