Title: Realization and Characterization of Topological States on Quantum Processors

Speakers: Frank Pollmann

Series: Colloquium

Date: April 17, 2024 - 2:00 PM

URL: https://pirsa.org/24040106

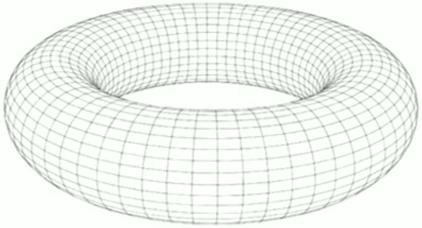
Abstract: The interplay of quantum fluctuations and interactions can yield novel quantum phases of matter with fascinating properties. Understanding the physics of such systems is a very challenging problem as it requires to solve quantum many body problems--which are generically exponentially hard to solve on classical computers. In this context, universal quantum computers are potentially an ideal setting for simulating the emergent quantum many-body physics. In this talk, I will discuss two different classes of quantum phases: First, we consider symmetry protected topological (SPT) phases and show that a topological phase transitions can be simulated using shallow circuits. We then utilize quantum convolutional neural networks (QCNNs) as classifiers and introduce an efficient framework to train them. Second, we focus on the realization of topological ordered phases and simulate the braiding of anyons. Taking into account additional symmetries, we then investigate phase transitions between different symmetry enriched topological (SET) phases.

Zoom link

Realization and Characterization of Topological States on Quantum Processors

Frank Pollmann

Technische Universität München



Bernhard Jobst, Yu-Jie Liu, Adam Smith, Michael Knap, Andrew Green, Kirill Shtengel, Kevin Satzinger, Pedram Roushan

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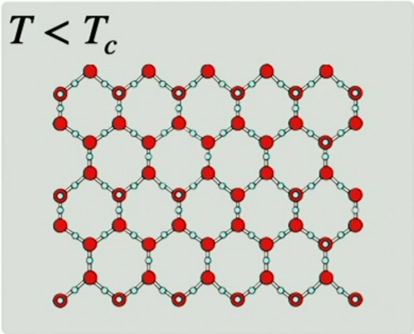
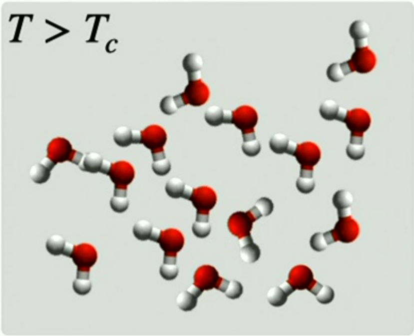
EQUAHUMO Efficient Quantum Algorithms for the Hubbard Model

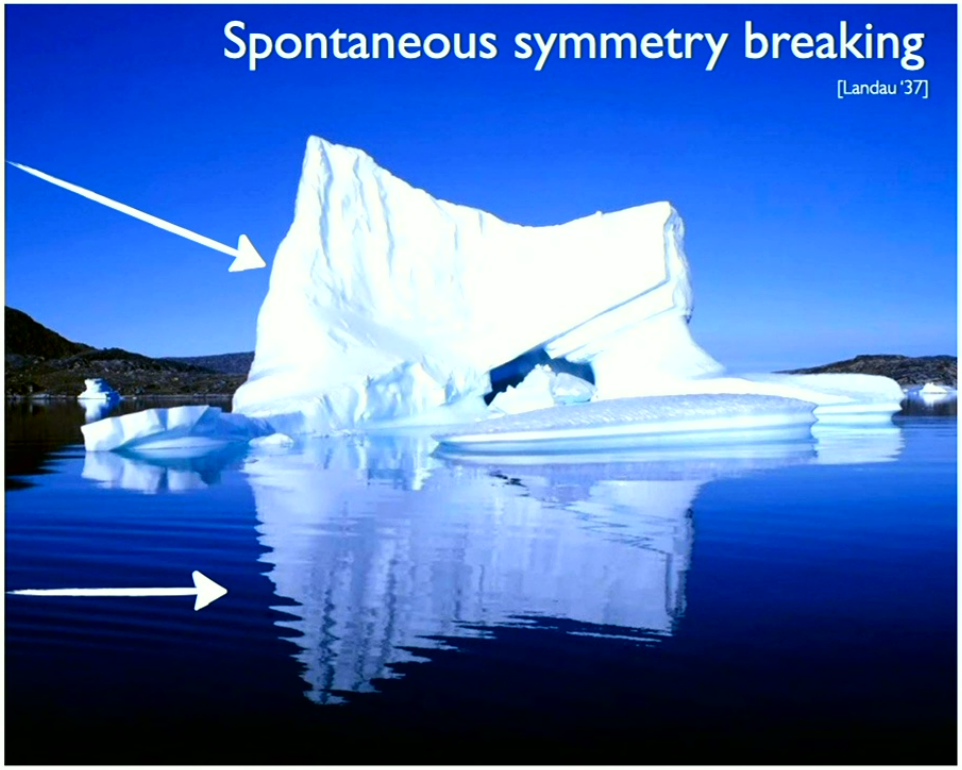
Munich Quantum Valley

Perimeter,
17. Apr. 2024

Matter occurs in different phases



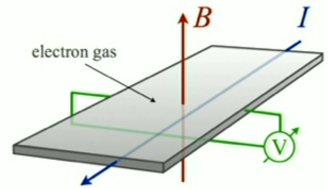


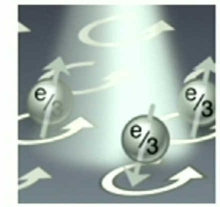


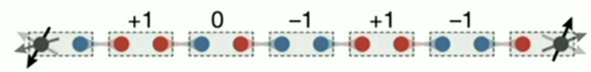
Topological Quantum Phases

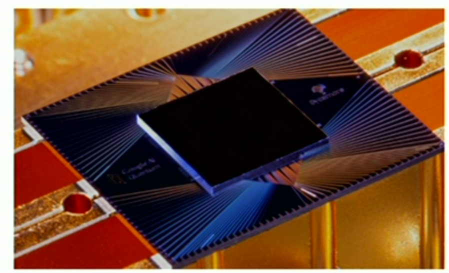
Long range entangled states of matter

- Fractional Quantum Hall effect [Tsui '82, Laughlin '83]
- Quantum Spin-liquids [Anderson '73]
- Symmetry Protected Topological phases [Haldane '83]







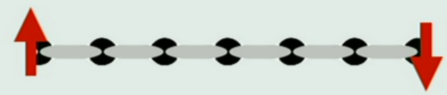


Quantum Computers as a tool to realize exotic states of matter

Realization and Characterization of Topological States on Quantum Processors

(I) Symmetry Protected Topological (SPT) phases

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]
[Liu, Smith, Knap, FP, PRL **130**, 220603 (2023)]



(II) Topologically Ordered phases

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 6572 (2021)]
[Liu, Smith, Shtengel, FP, PRX Quantum **3**, 040315 (2022)]
[Haller, Xu, Liu, FP, PRR **5**, 043078 (2023)]
[Liu, Shtengel, FP, arXiv:2312.05079]

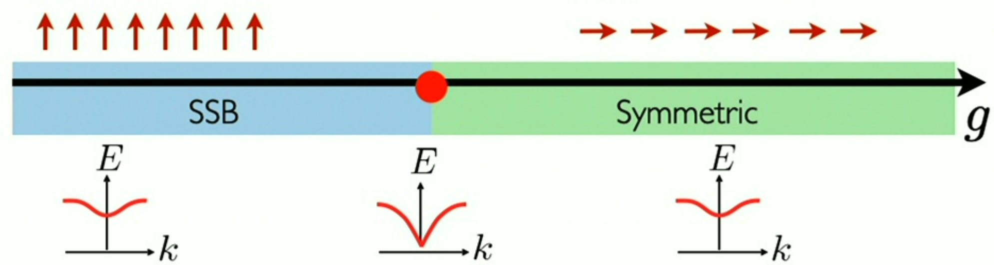
Spontaneous symmetry breaking (SSB)

Quantum phase transition in transverse field Ising model: $\mathbb{Z}_2$ symmetry

$$H = -\sum_j \sigma_j^z \sigma_{j+1}^z - g \sum_j \sigma_j^x$$

↑↑↑↓↓↓↑↑

$$\lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle \neq 0 \qquad \lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle = 0$$

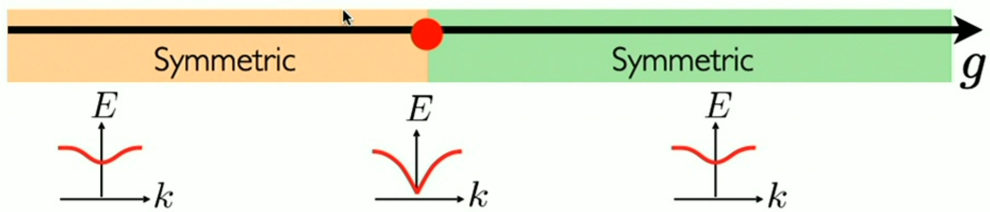


Local order parameter in SSB Phase:
Landau theory of phase transitions

Symmetry Protected Topological (SPT) phases

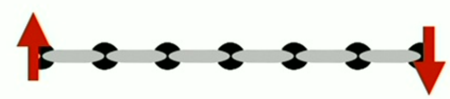
Quantum phase transition in cluster state model: $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

$$H = \sum_j \sigma_j^z \sigma_{j+1}^x \sigma_{j+2}^z - g \sum_j \sigma_j^x$$



Classified by "Symmetry fractionalization"

[FP, Turner, Berg, Oshikawa '10, Chen et al. '11; Schuch et al '11]



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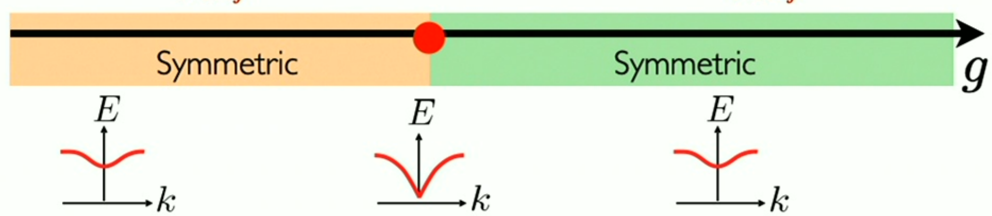
$$H = \sum_j \sigma_j^z \sigma_{j+1}^x \sigma_{j+2}^z - g \sum_j \sigma_j^x$$

$$\lim_{|i-j|\to\infty} \langle \prod_{i<k<j} \sigma_k^x \rangle = 0 \qquad \lim_{|i-j|\to\infty} \langle \prod_{i<k<j} \sigma_k^x \rangle \neq 0$$



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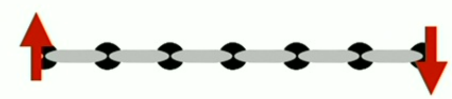
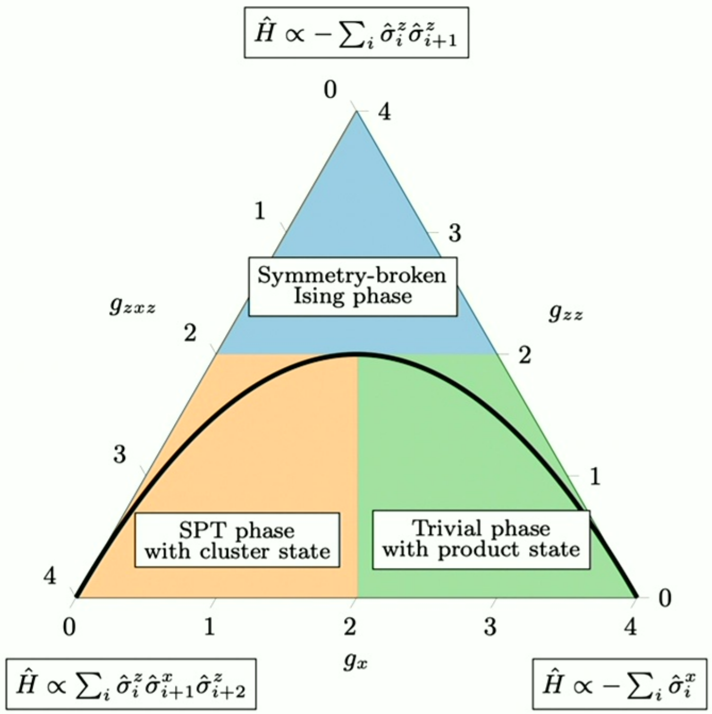


Non-local "string" order parameter

[den Nijs and Rommelse '89, FP and Turner '12]

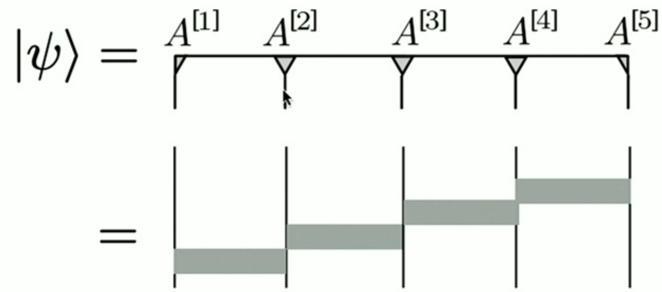
Exact quantum circuit crossing a phase transition

$$\hat{H} = \sum_i \left[-g_{zz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - g_x\, \hat{\sigma}_i^x + g_{zxz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^x \hat{\sigma}_{i+2}^z\right]$$



Exact matrix-product state (MPS) along a path → Exact circuit!

[Wolf et al. '05, Jones et al. PRR **3**, 033265 (2021)]

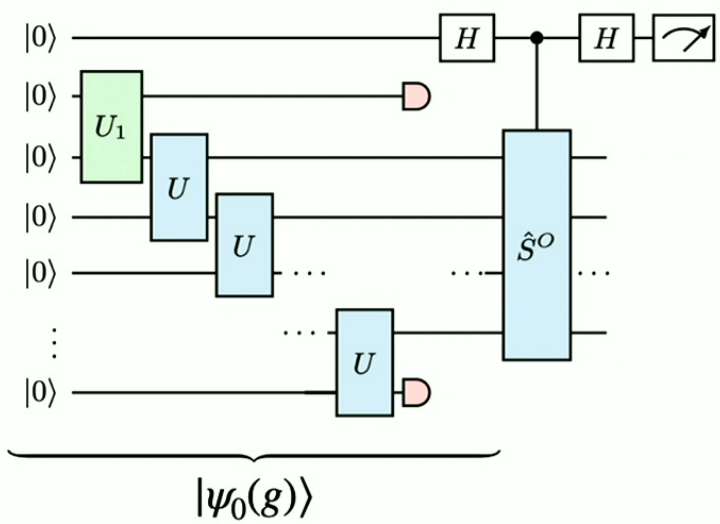


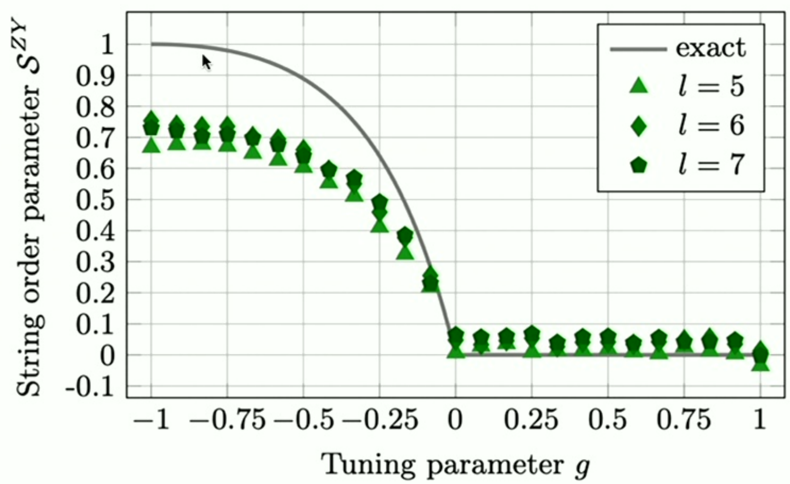
$\chi = 2$

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Measuring string order

Results on a **20 qubit IBM-Q device:** $\mathcal{S}^{O}(g) = \langle\psi|\hat{O}_i\left(\prod_{j=i+2}^{k-2}\hat{\sigma}_j^x\right)\hat{O}_k'|\psi\rangle$



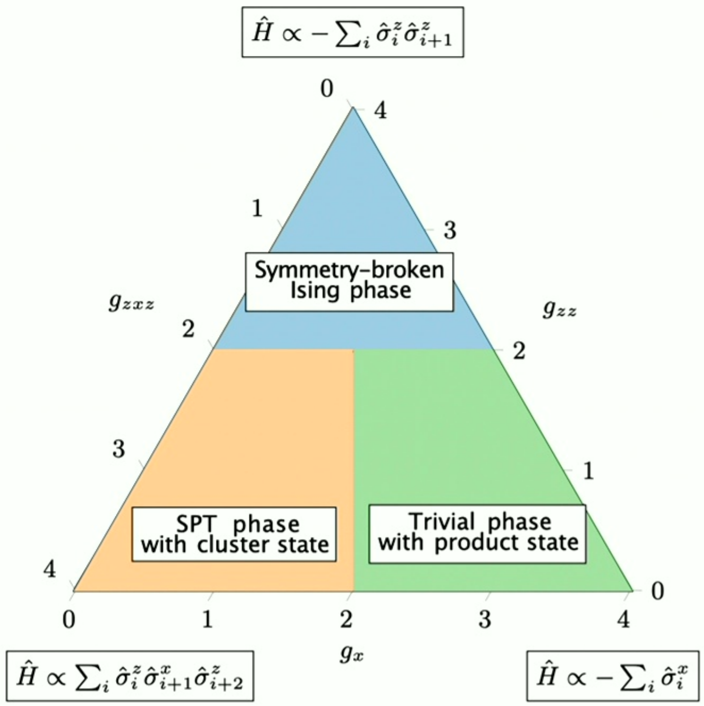


- Accurate representation on quantum processor

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Determining the phase diagram of model systems

$$\hat{H} = \sum_i \left[-g_{zz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - g_x\, \hat{\sigma}_i^x + g_{zxz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^x \hat{\sigma}_{i+2}^z \right]$$



How to automate the discovery of order parameters?

- Detect phases using Quantum Convolutional Networks

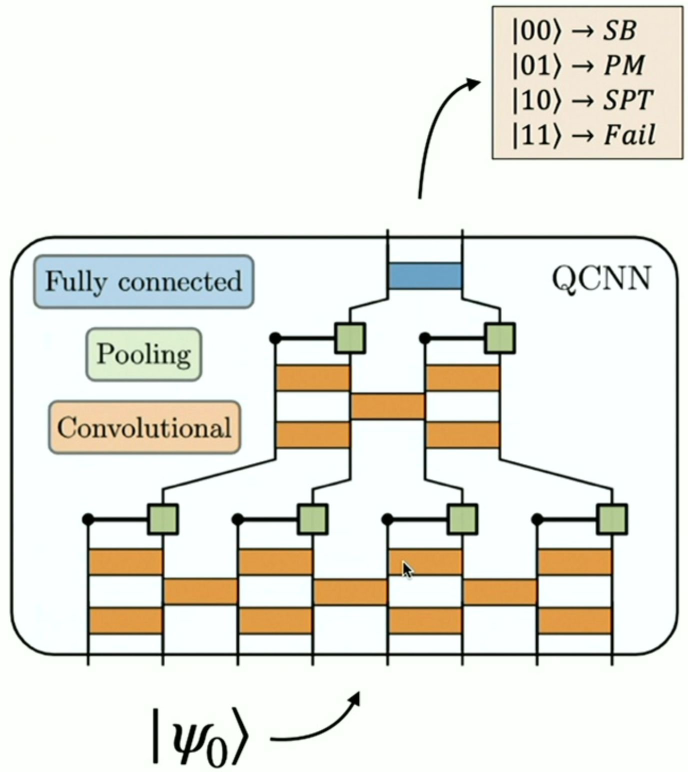
Quantum Convolutional Networks (QCNN)

QCNNs:

- Introduced as phase classifier [Cong, Choi, Lukin, 2019]
- Barren-plateau free [Pesah et al. PRX 2021]

Challenges:

- How to efficiently train a QCNN?
- Hard to generate large data!
- Overfitted QCNN when training on certain models



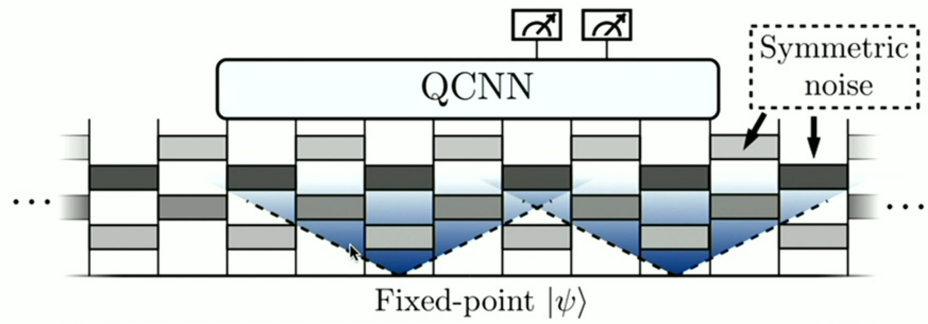
[Yu-Jie Liu, Adam Smith, Knap, FP, PRL **130**, 220603 (2023)]

Model independent learning

Protocol to generate large training data set:

1. Prepare a simple fixed-point state for the phase
2. Apply local symmetric unitary gates
3. Supervised training of QCNN

$|00\rangle \to SB$
$|01\rangle \to PM$
$|10\rangle \to SPT$
$|11\rangle \to Fail$



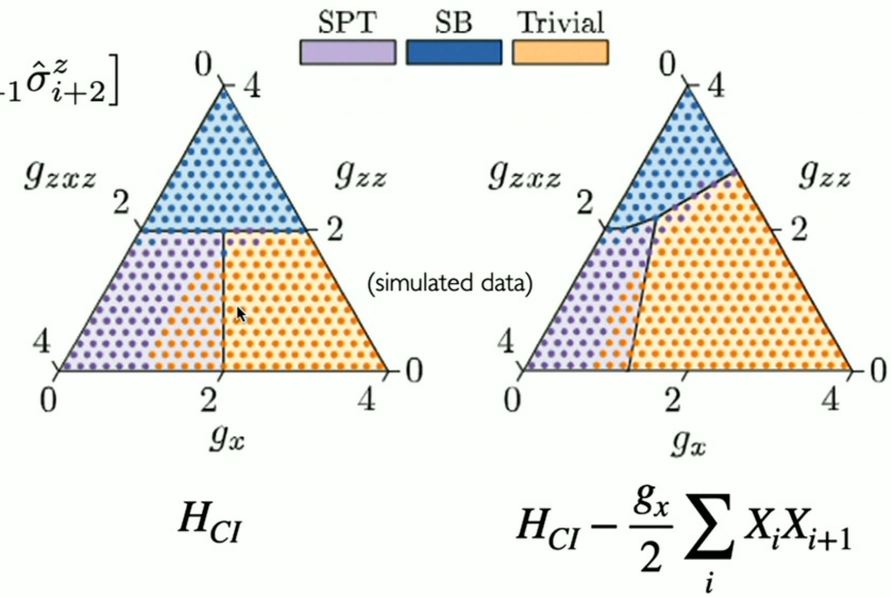
[Yu-Jie Liu, Adam Smith, Knap, FP, PRL **130**, 220603 (2023)]

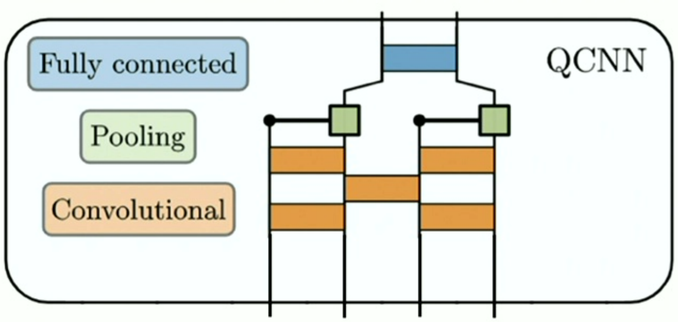
Benchmark: Cluster Ising Model (4 qubit QCNN)

$$\hat{H} = \sum_i \left[-g_{zz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - g_x\, \hat{\sigma}_i^x + g_{zxz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^x \hat{\sigma}_{i+2}^z \right]$$

Time-Reversal Symmetry

⟶ 3 Quantum Phases





[Yu-Jie Liu, Adam Smith, Knap, FP, PRL **130**, 220603 (2023)]

Benchmark: Cluster Ising Model (8 qubit QCNN)

$$\hat{H} = \sum_i \left[-g_{zz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - g_x\, \hat{\sigma}_i^x + g_{zxz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^x \hat{\sigma}_{i+2}^z \right]$$

Time-Reversal Symmetry

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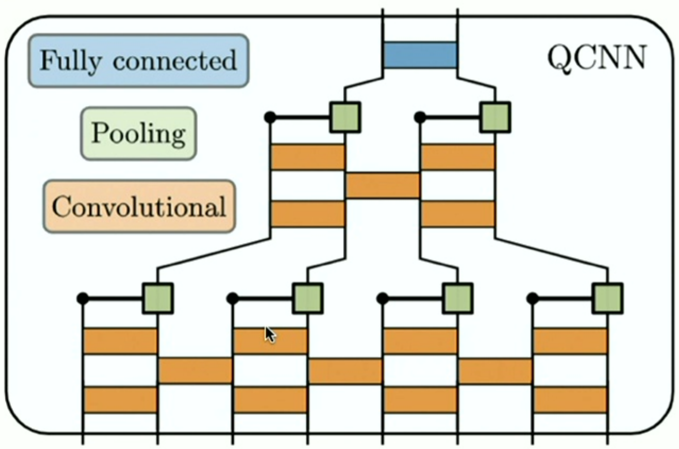
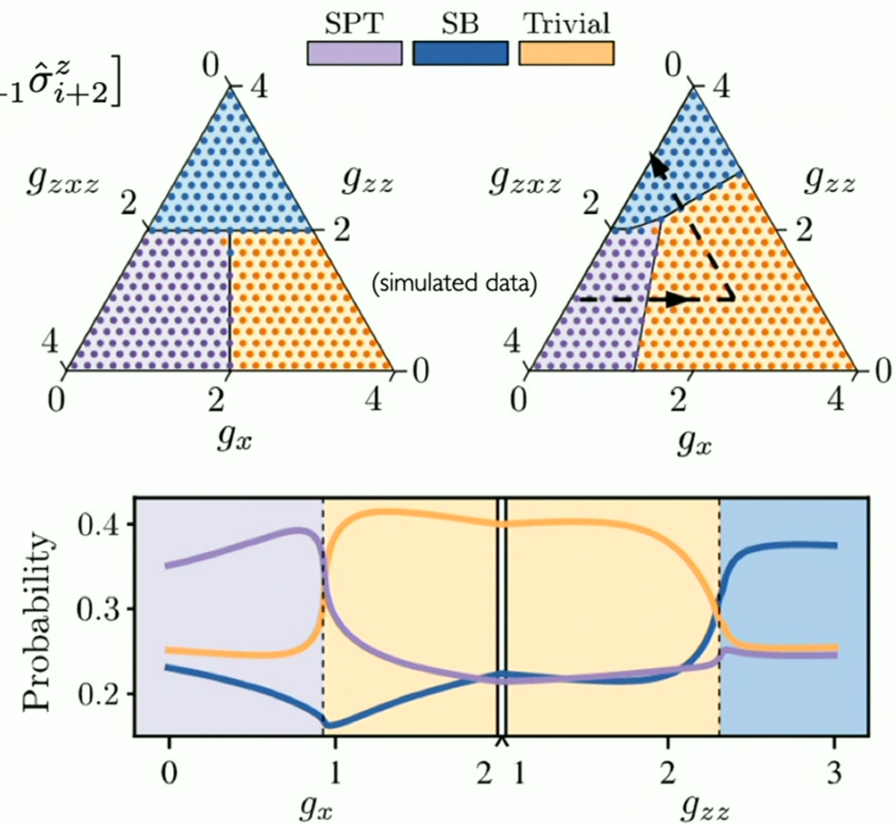




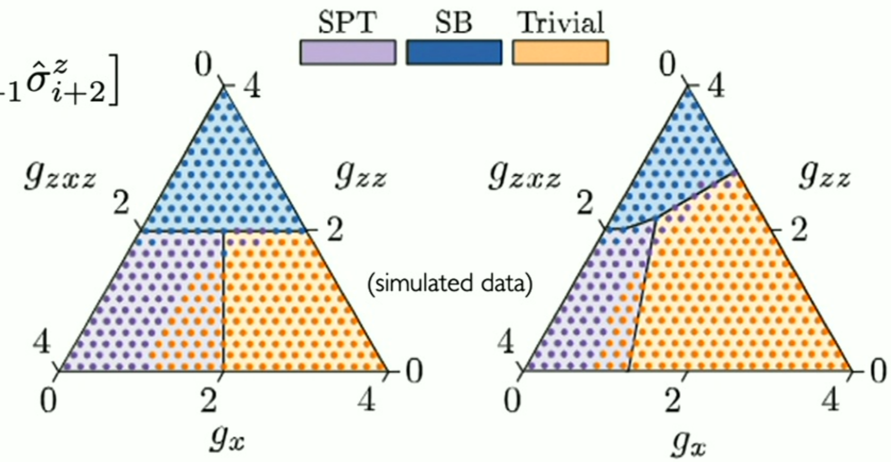
[Yu-Jie Liu, Adam Smith, Knap, FP, PRL **130**, 220603 (2023)]

Benchmark: Cluster Ising Model (4 qubit QCNN)

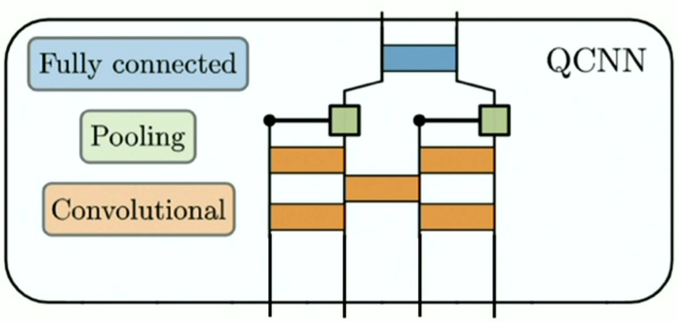
$$\hat{H} = \sum_i \left[-g_{zz}\ \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - g_x\ \hat{\sigma}_i^x + g_{zxz}\ \hat{\sigma}_i^z \hat{\sigma}_{i+1}^x \hat{\sigma}_{i+2}^z\right]$$

Time-Reversal Symmetry

⟶ 3 Quantum Phases



H_{CI} $\qquad$ $H_{CI} - \frac{g_x}{2}\sum_i X_i X_{i+1}$



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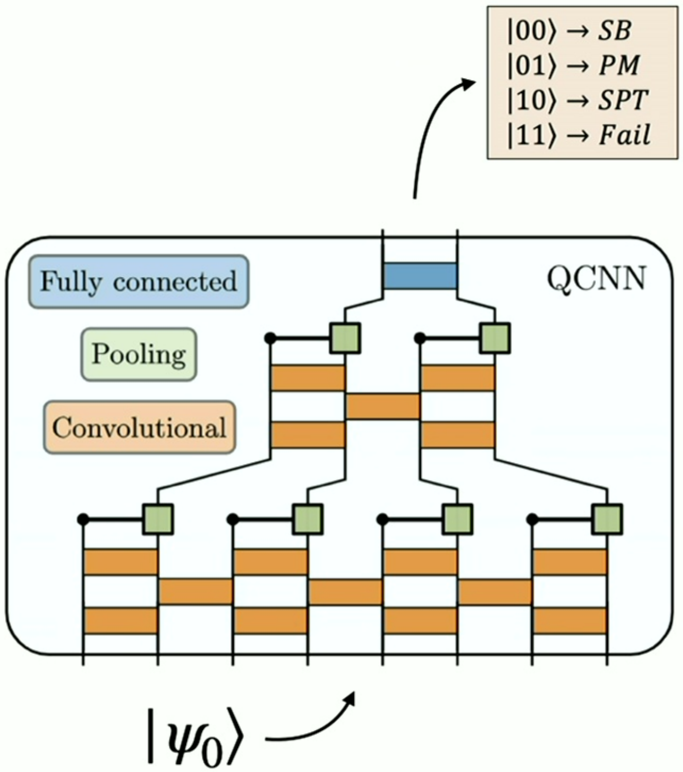

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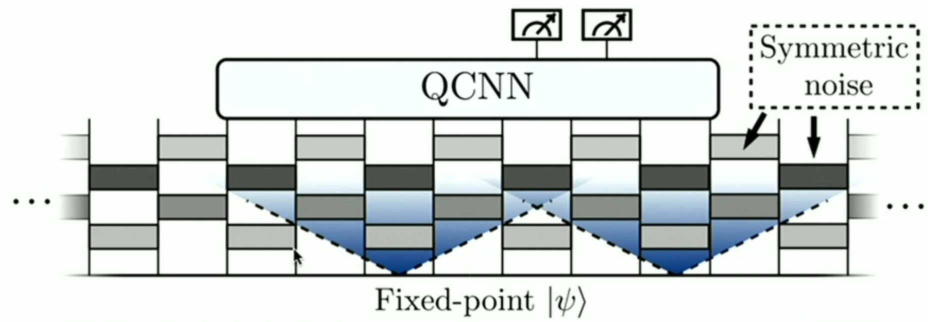

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Realization and Characterization of Topological States on Quantum Processors

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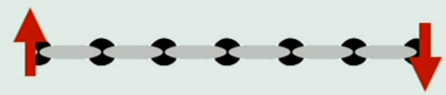

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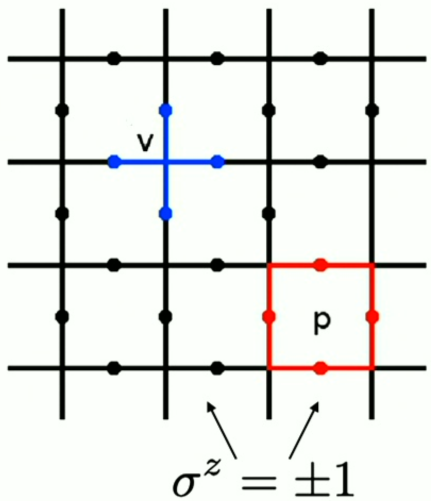
[Haller, Xu, Liu, FP, PRR **5**, 043078 (2023)]
[Liu, Shtengel, FP, arXiv:2312.05079]

Topological order and fractionalized excitations

Toric code model $H_{TC} = -J\sum_v A_v - J\sum_p B_p, \;\; J > 0$

[Kitaev '03]



$$A_v = \prod_{i\in v} \sigma_i^z, \; B_p = \prod_{i\in p} \sigma_i^x$$

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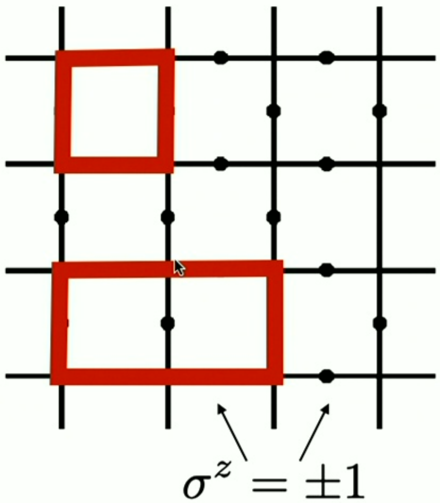
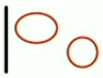


$$A_v = \prod_{i\in v} \sigma_i^z, \; B_p = \prod_{i\in p} \sigma_i^x$$

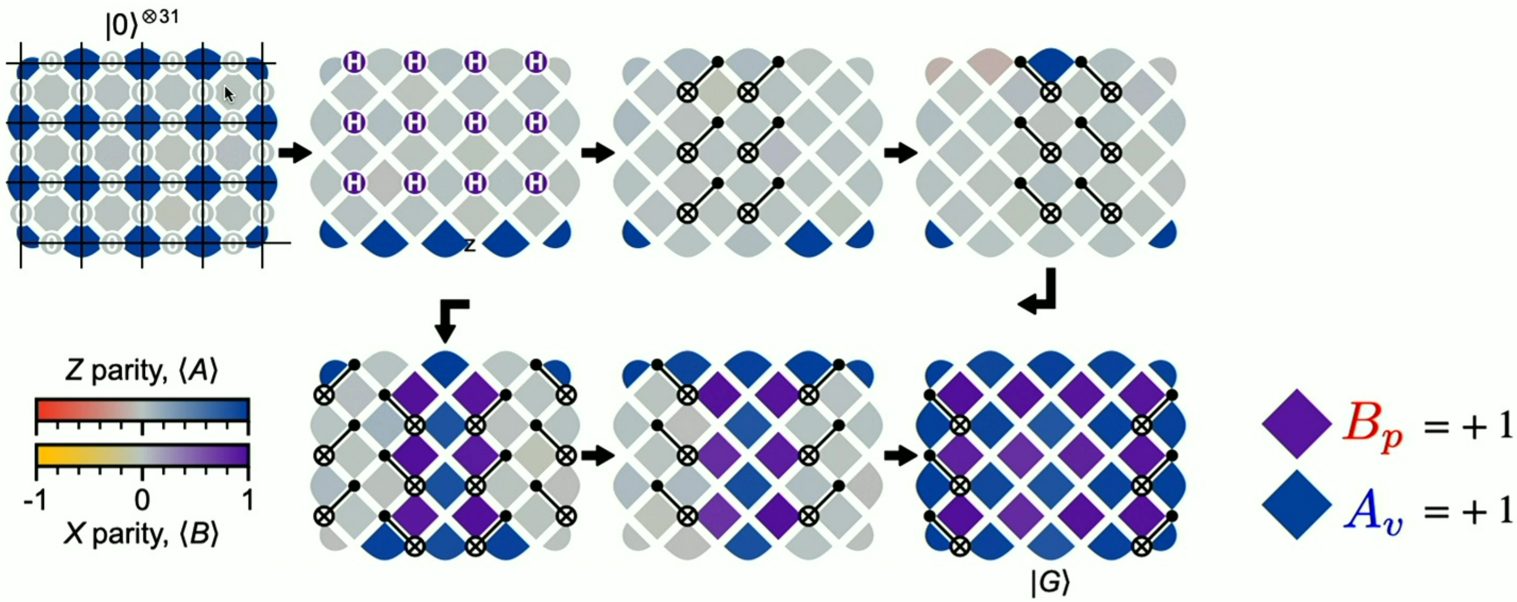
$[A_v, B_p] = 0$ ➡ Exactly solvable

$|\psi_0\rangle = |$  $\rangle + |$  $\rangle + \ldots$

▸ $\mathbb{Z}_2$ topological order

Realizing the toric code on a quantum processor

Toric code ground state $|G\rangle \propto \prod_p (1 + B_p)|0\rangle$



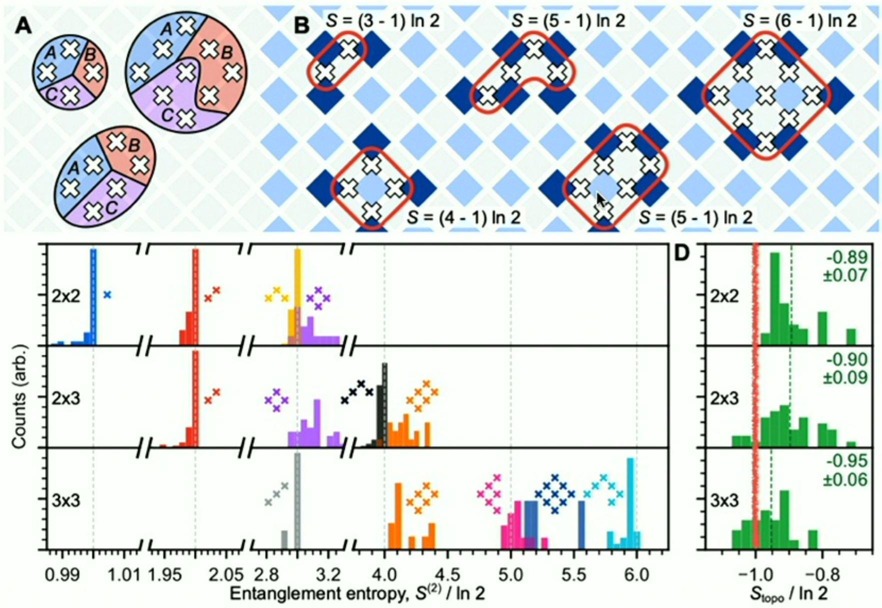
▶ Linear depth in system width

[Satzinger, Liu, Smith, et al., FP, Roushan, Science **374**, 1237 (2021)]
[See also Semeghini et al., Science **374**, 1242 (2021)]

Probing topological entanglement

Topological entanglement entropy $S = \alpha L - \gamma$ $\gamma = \ln 2$

[Kitaev and Preskill '06, Levin and Wen '06]



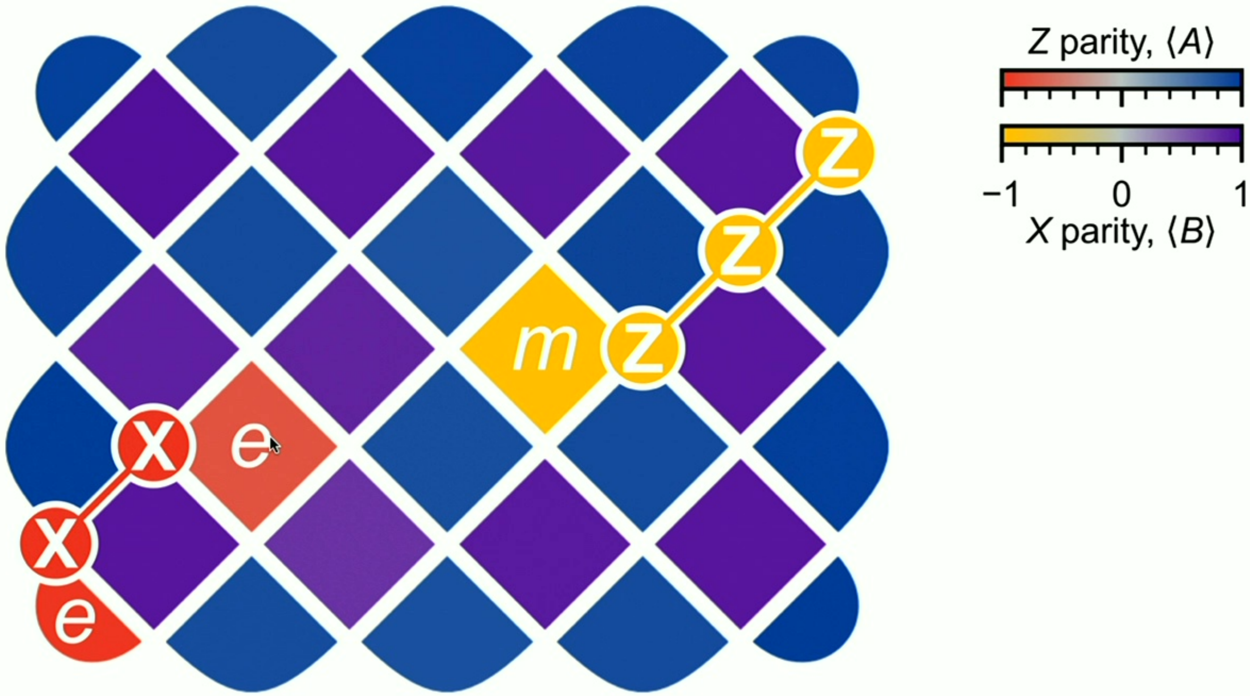
Full state tomography for 4 and 6 qubits

Randomized measurements for 9 qubits

Average over location and orientation

[Satzinger, Liu, Smith, et al., FP, Roushan, Science **374**, 1237 (2021)]

Simulating anyonic statistics



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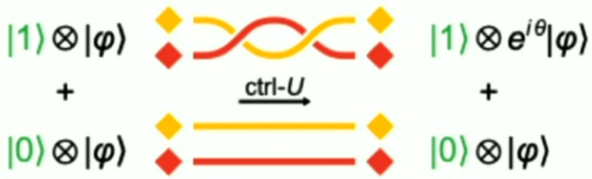
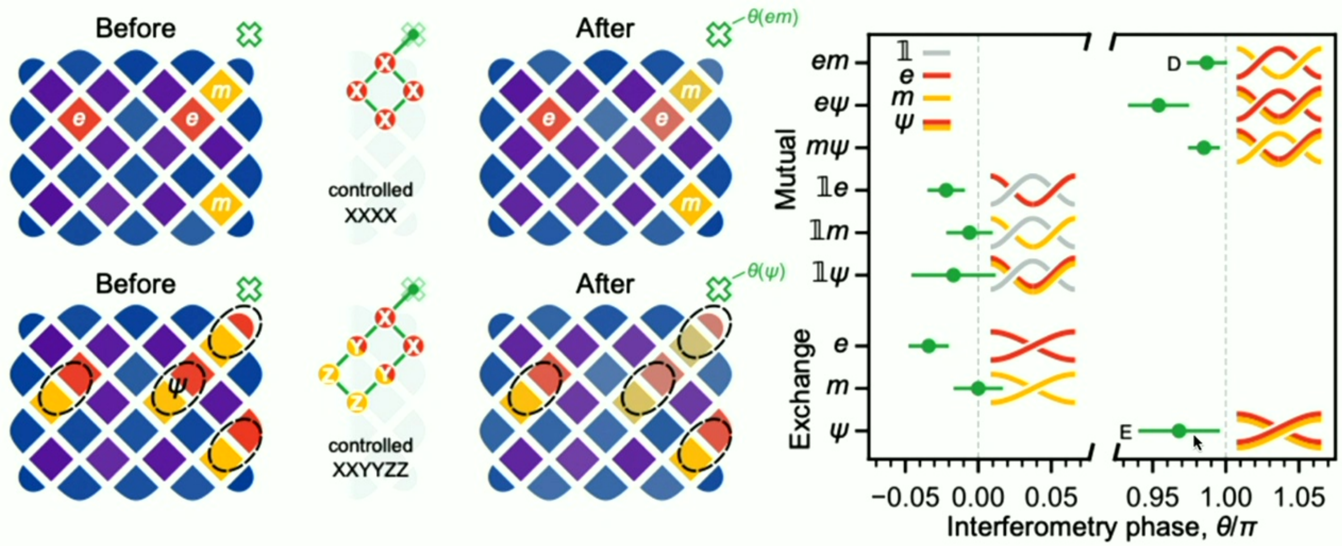


[Jiang et al. '08]



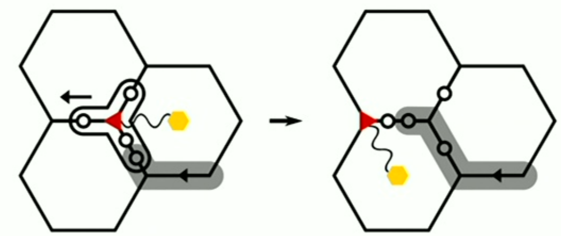
[Satzinger, Liu, Smith, et al., FP, Roushan, Science **374**, 1237 (2021)]

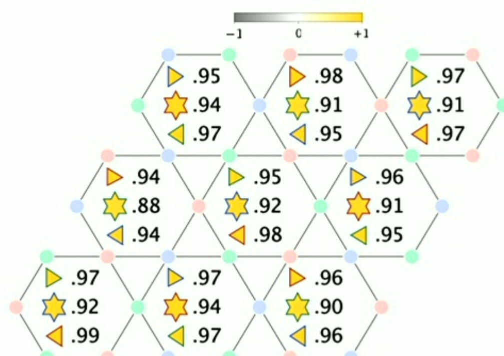
Non-Abelian Topological Order

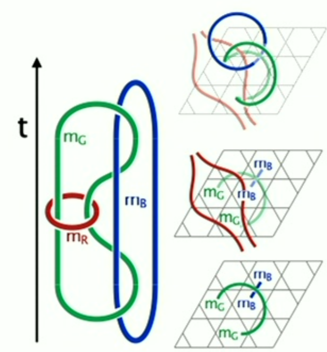
Linear depth circuits for realizing general string-net models and braiding **non-abelian anyons**
[Liu, Smith, Shtengel, FP, PRX Quantum **3**, 040315 (2022)]

Shortcut: non-abelian topological order using measurements [Iqbal et al. '23]

Non-abelian

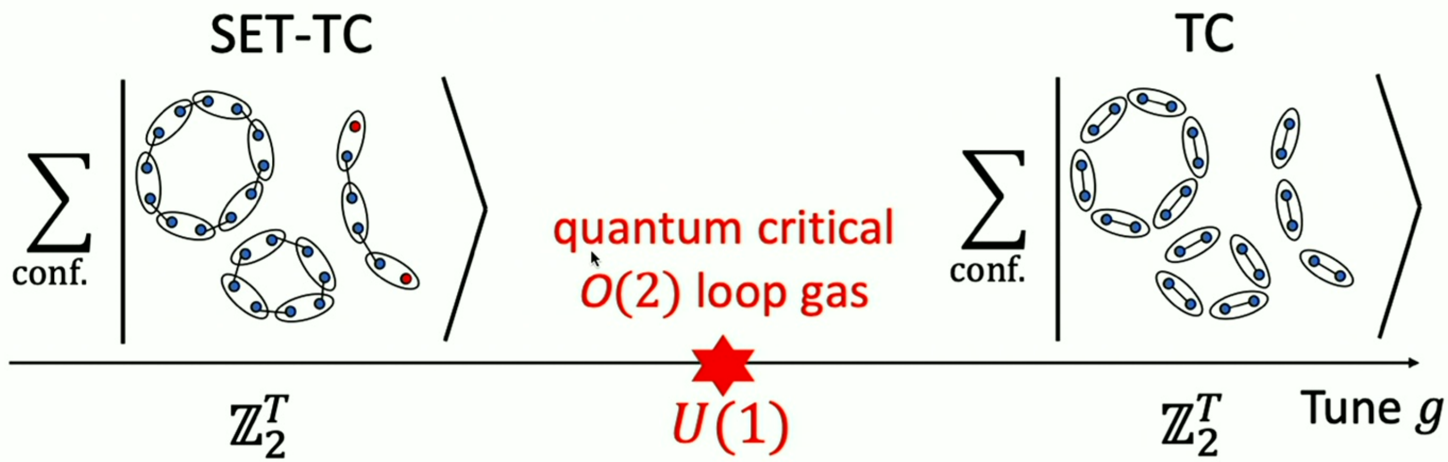






Exact tensor network crossing a phase transition in 2D

Tensor-network solvable: Quantum phase transition between **symmetry enriched topological (SET) phases** [Essin and Hermele '13, …]



- Decorate 2D loops (in toric code) by 1D SPT

[Haller, Xu, Liu, FP, arXiv:2305.02432]

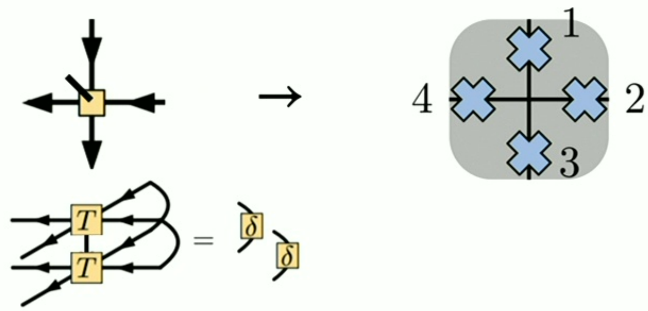
Isometric tensor-network states: Sequential generation

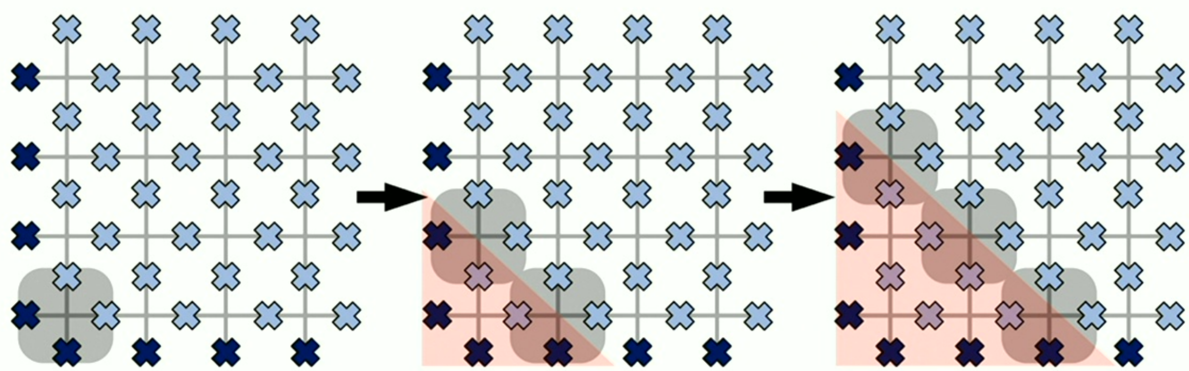
Problem: Not every 2D TN can be simply prepared by simple circuit!

Sequential generation of states

isoTNS → Linear depth circuit

[Zaletel and FP '20, Slattery and Clark '21
Wei, Malz, and Cirac '22]



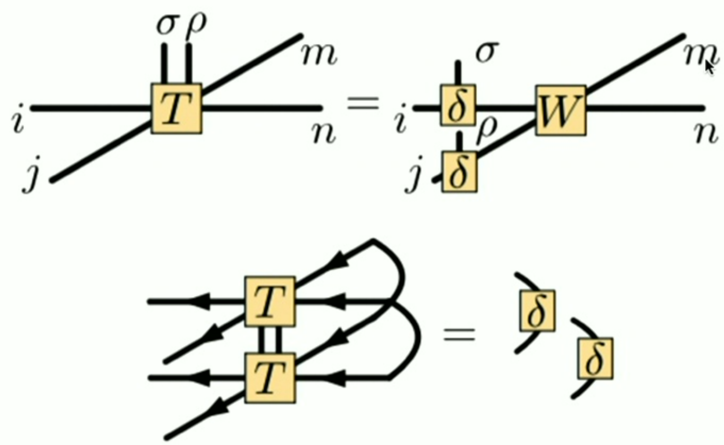


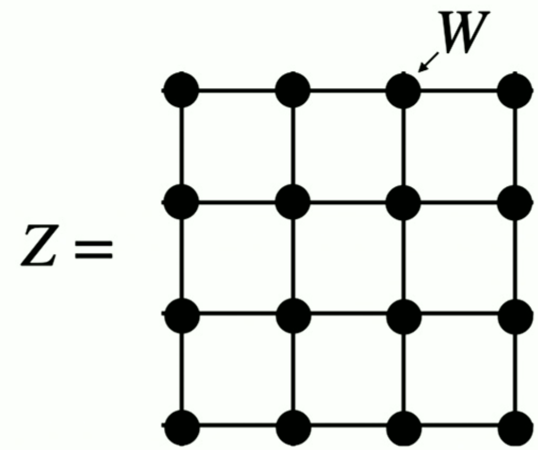
[Liu, Shtengel, FP, arXiv:2312.05079]

Exact isoTNS crossing a phase transition in 2D

Family of states that can be efficiently created on quantum computers!

isoTNS from classical partition functions $\sum_{m,n} |W_{ijmn}|^2 = 1$

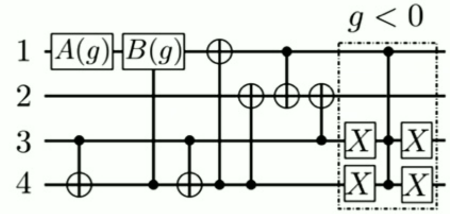


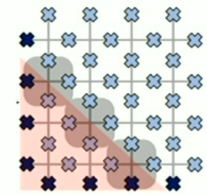


[Liu, Shtengel, FP, arXiv:2312.05079]

Exact isoTNS crossing a phase transition in 2D

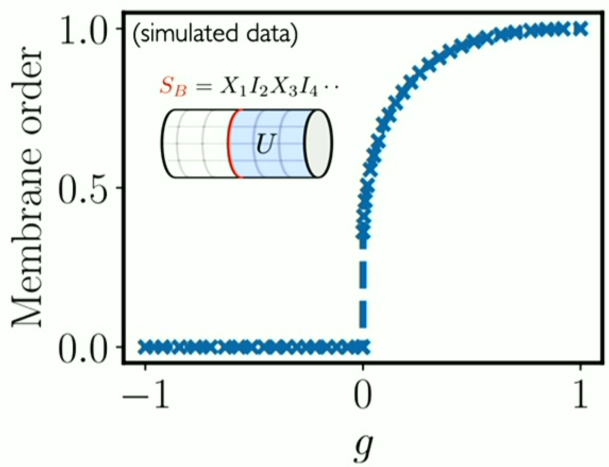
Linear depth circuit between SET and trivial TC phase!

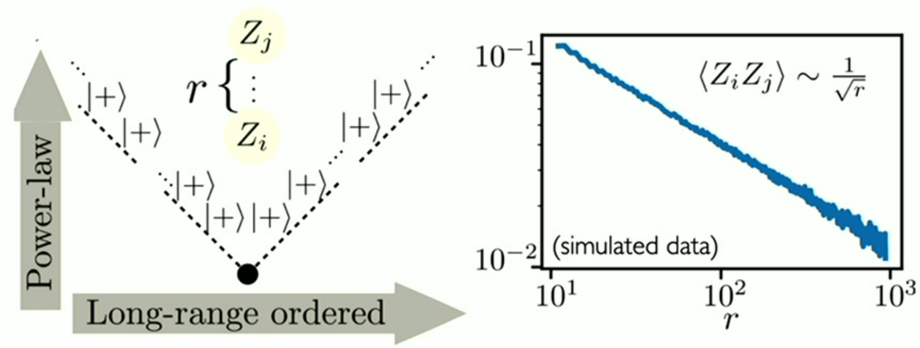




Membrane order parameter

Quantum Critical Point with algebraic correlation





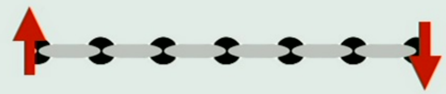
[Liu, Shtengel, FP, arXiv:2312.05079]

Realization and Characterization of Topological States on Quantum Processors

Thank you!

(I) Symmetry Protected Topological (SPT) phases

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]
[Liu, Smith, Knap, FP, PRL **130**, 220603 (2023)]



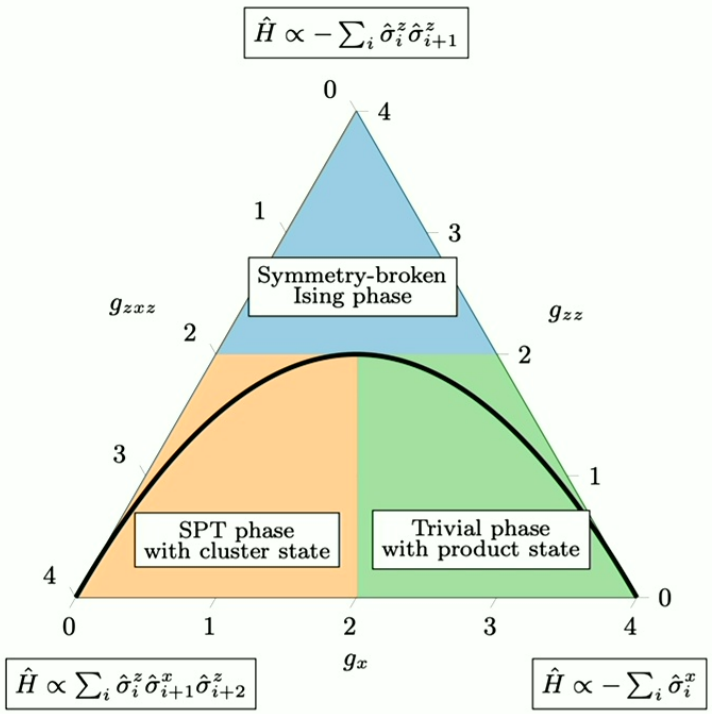
(II) Topologically Ordered phases

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 6572 (2021)]
[Liu, Smith, Shtengel, FP, PRX Quantum **3**, 040315 (2022)]
[Haller, Xu, Liu, FP, PRR **5**, 043078 (2023)]
[Liu, Shtengel, FP, arXiv:2312.05079]

Bernhard Jobst **Yu-Jie Liu** Adam Smith Michael Knap Andrew Green Kirill Shtengel Kevin Satzinger Pedram Roushan

Exact quantum circuit crossing a phase transition

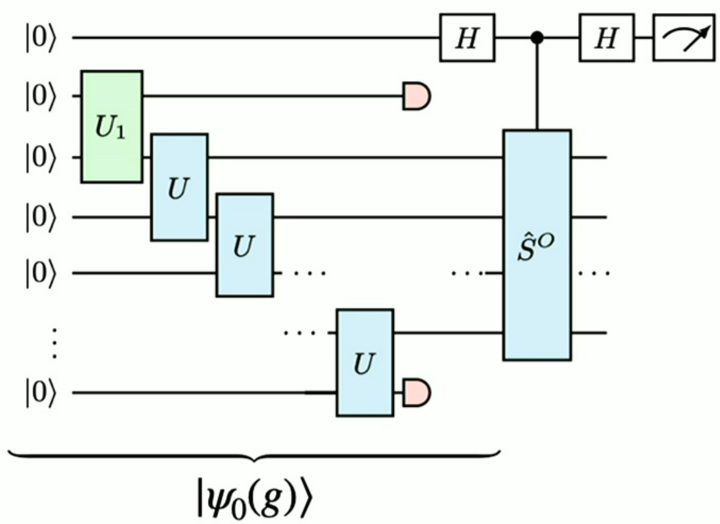
$$\hat{H} = \sum_i \left[-g_{zz}\, \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} - g_x\, \hat{\sigma}^x_i + g_{zxz}\, \hat{\sigma}^z_i \hat{\sigma}^x_{i+1} \hat{\sigma}^z_{i+2} \right]$$

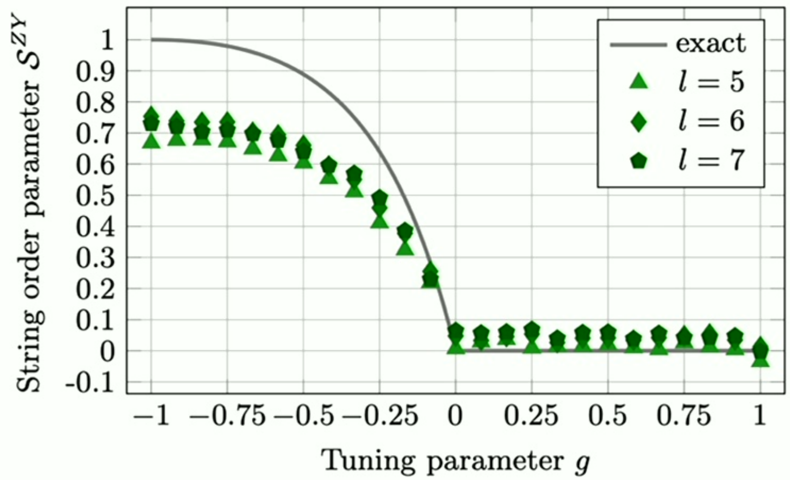


[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Measuring string order

Results on a **20 qubit IBM-Q device:** $\mathcal{S}^O(g) = \langle\psi|\hat{O}_i\left(\prod_{j=i+2}^{k-2}\hat{\sigma}_j^x\right)\hat{O}'_k|\psi\rangle$



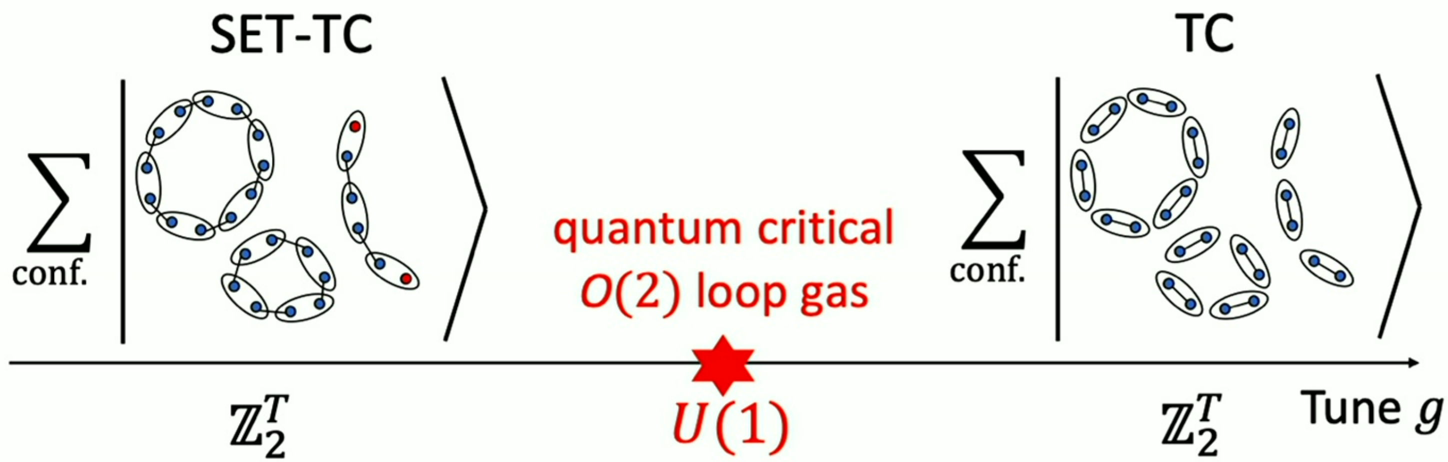


- **Accurate representation on quantum processor**

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Exact tensor network crossing a phase transition in 2D

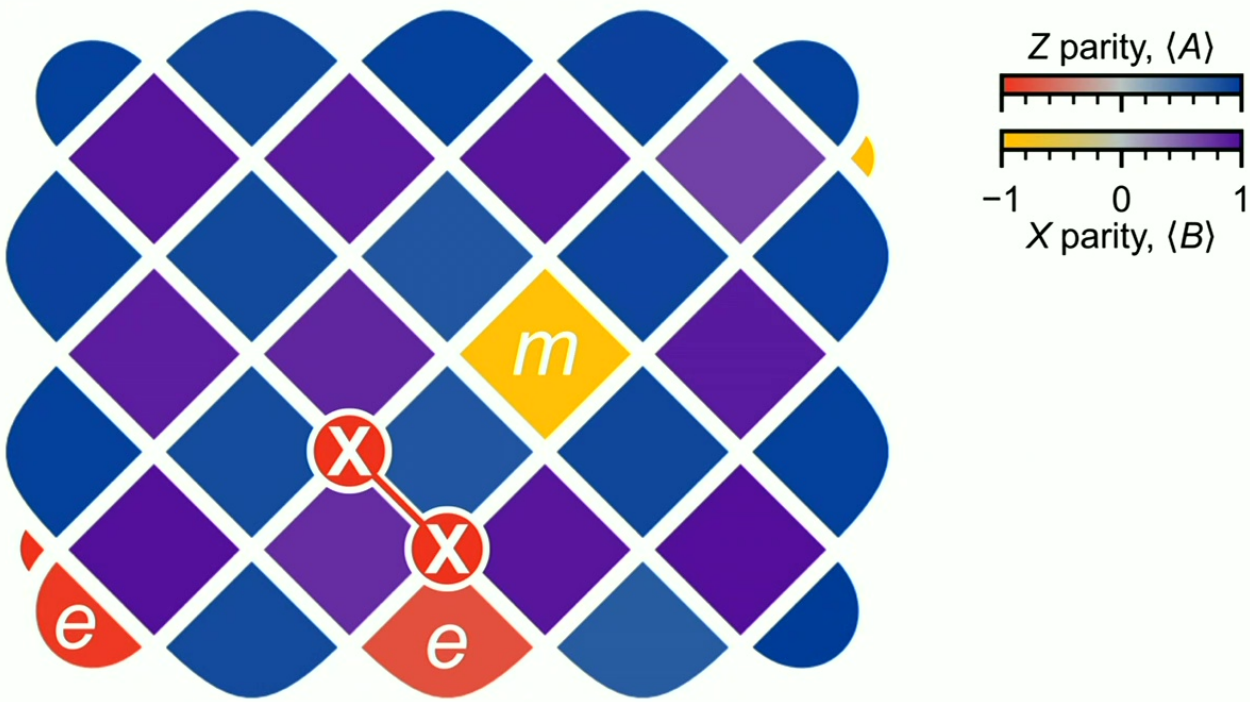
Tensor-network solvable: Quantum phase transition between **symmetry enriched topological (SET) phases** [Essin and Hermele '13, …]



- Decorate 2D loops (in toric code) by 1D SPT

[Haller, Xu, Liu, FP, arXiv:2305.02432]

Simulating anyonic statistics



[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]