Title: Hardware-efficient quantum computing using qudits

Speakers: Christine Muschik

Series: Perimeter Institute Quantum Discussions

Date: April 17, 2024 - 11:00 AM

URL: https://pirsa.org/24040105

Abstract: Particle physics underpins our understanding of the world at a fundamental level by describing the interplay of matter and forces through gauge theories. Yet, despite their unmatched success, the intrinsic quantum mechanical nature of gauge theories makes important problem classes notoriously difficult to address with classical computational techniques. A promising way to overcome these roadblocks is offered by quantum computers, which are based on the same laws that make the classical computations so difficult. Here, we present a quantum computation of the properties of the basic building block of two-dimensional lattice quantum electrodynamics, involving both gauge fields and matter. This computation is made possible by the use of a trapped-ion qudit quantum processor, where quantum information is encoded in different states per ion, rather than in two states as in qubits. Qudits are ideally suited for describing gauge fields, which are naturally high-dimensional, leading to a dramatic reduction in the quantum register size and circuit complexity. Using a variational quantum eigensolver, we find the ground state of the model and observe the interplay between virtual pair creation and quantized magnetic field effects. The qudit approach further allows us to seamlessly observe the effect of different gauge field truncations by controlling the qudit dimension. Our results open the door for hardware-efficient quantum simulations with qudits in near-term quantum devices.

Zoom link

Qudit workshop

9-11 July 2024, Waterloo

Simulating 2D lattice gauge theories on a qudit quantum computer

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[6] *Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, Technikerstraße 21a, Innsbruck, Austria*
[7] *Alpine Quantum Technologies GmbH, Innsbruck, Austria*
[8] *Universität Innsbruck, Institut für Theoretische Physik, Technikerstraße 21a, Innsbruck, Austria*
[9] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada*

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https://arxiv.org/abs/2310.12110

Overview

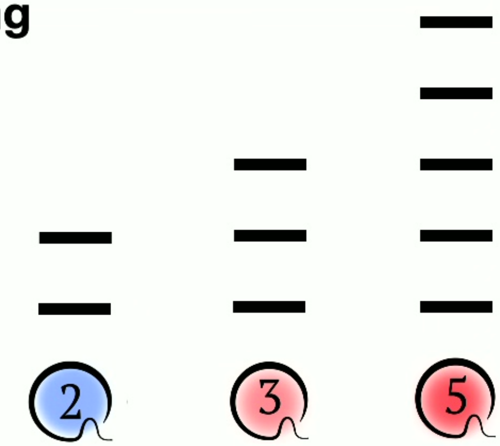
1. Introduction
2. Using qudits for problems in particle physics
3. Simulating both: gauge fields and matter
4. Increasing d
5. Conclusions

Classical computing today:
Almost exclusively based on binary encoding.

Beyond binary

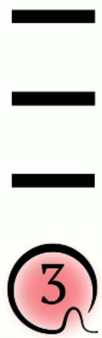
Today's quantum hardware: **capable of qudit encoding**

- Trapped ions
- Superconducting architectures
- Rydberg atoms in optical tweezers
- Ultracold atoms in optical lattices
- Nuclear spins
- Photonic systems

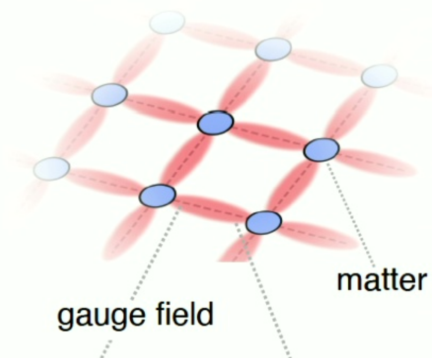


Beyond binary

Short-depth qu**d**it circuits



Lattice gauge theories



QuDit review article:

Y. Wang, Z. Hu, B. C. Sanders, and S. Kais, Front. Phys. 8 (2020).

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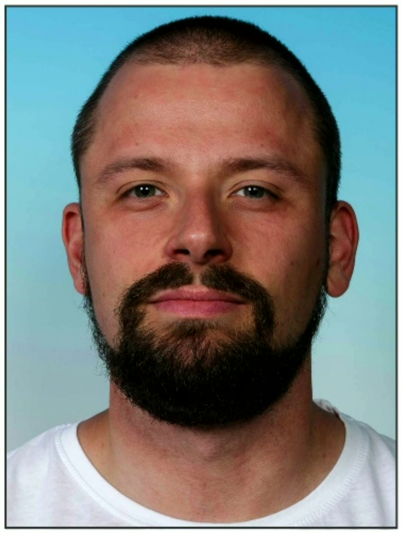
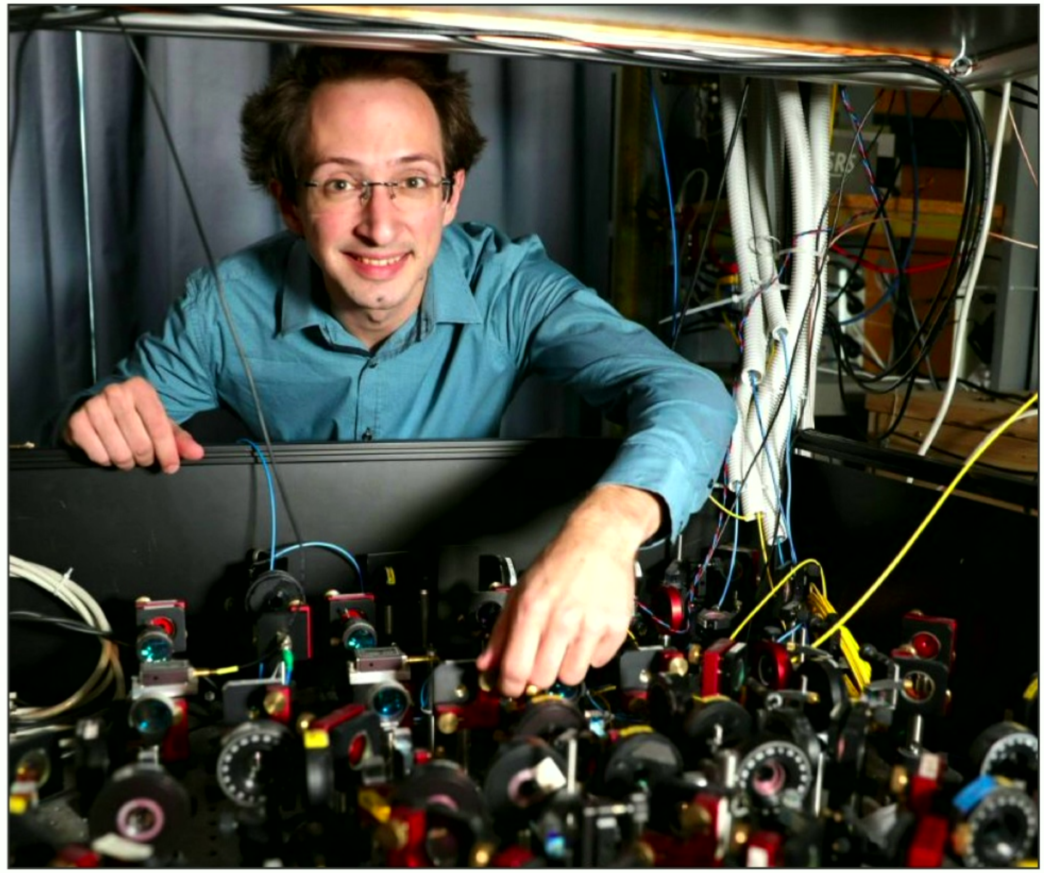


Mike Meth



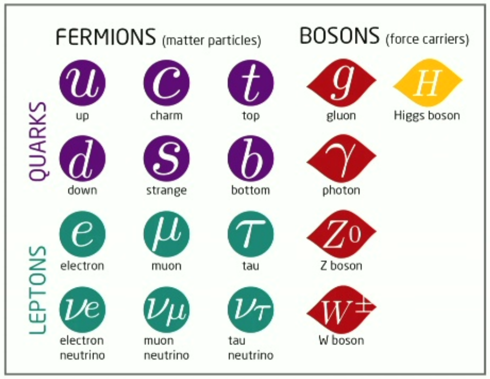
Martin Ringbauer

Gauge theories

Condensed matter systems
(Frustration, topological order)

Gauge theories in particle physics
(Quantum Electrodynamics, Quantum Chromodynamics,…)

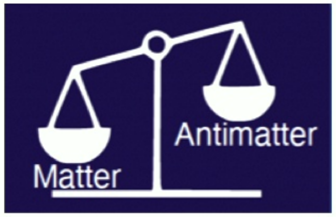
Standard model of particle physics

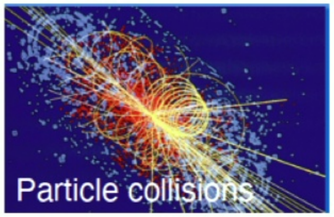


Inaccessible to traditional lattice gauge theory

Afflicted by sign problems

Topological terms	Real time evolutions	Chemical potentials
Fundamental symmetries	Dynamical processes	High matter density





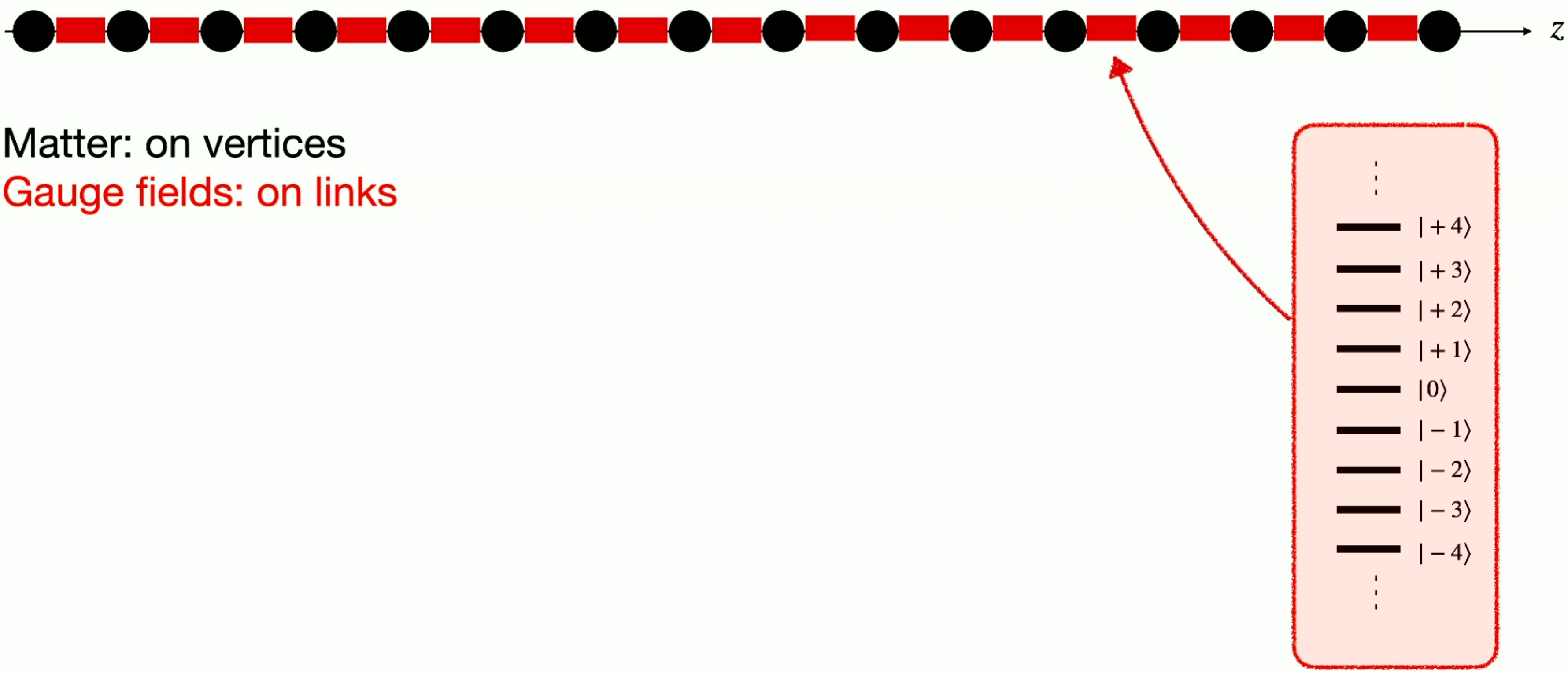
Using qudits
for problems in particle physics

Two major hurdles in quantum LGT calculations

#1: Representing gauge fields

#2: Quantum simulations beyond 1D

Representing gauge fields



Matter: on vertices

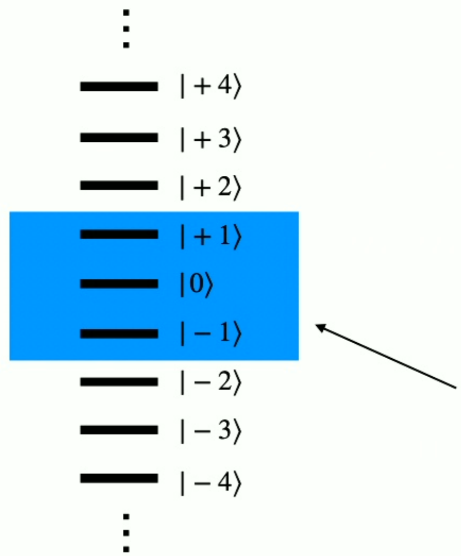
Gauge fields: on links

Gauge fields

Example: lattice QED

$\hat{E}\,|E\rangle = E\,|E\rangle,$

$E = 0, \pm 1, \pm 2, \ldots$



Classical simulations and finite dimensional quantum hardware require a **truncation**:

Minimal truncation: quTrit
- field in positive direction
- no flux
- field in negative direction

Truncations for bosonic systems:

- Schwinger boson representation
- Holstein-Primakoff-representation
- Dysen-Maleev transformation
- Highly occupied boson model

Truncations for qubit systems:

$$\hat{E} \longmapsto \hat{S}^z, \qquad \hat{U} \longmapsto \hat{V}^-,$$

$$\hat{V}^- \equiv \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ 1 & \cdots & \cdots & 0 \\ 0 & \ddots & \vdots & 0 \\ 0 & \cdots & 1 & 0 \end{bmatrix}$$

Alternative:

$\hat{V}^- \equiv \hat{S}^- / |l|$

$\hat{S}^- = \hat{S}^x - i\hat{S}^y$

Gauge fields

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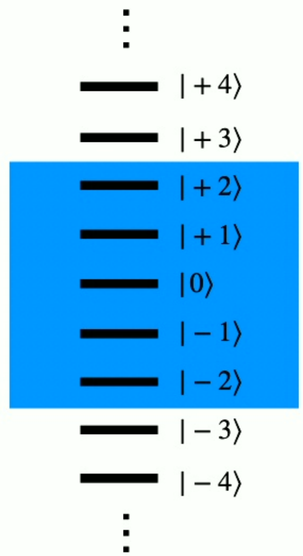

Classical simulations and finite dimensional quantum hardware require a **truncation**:

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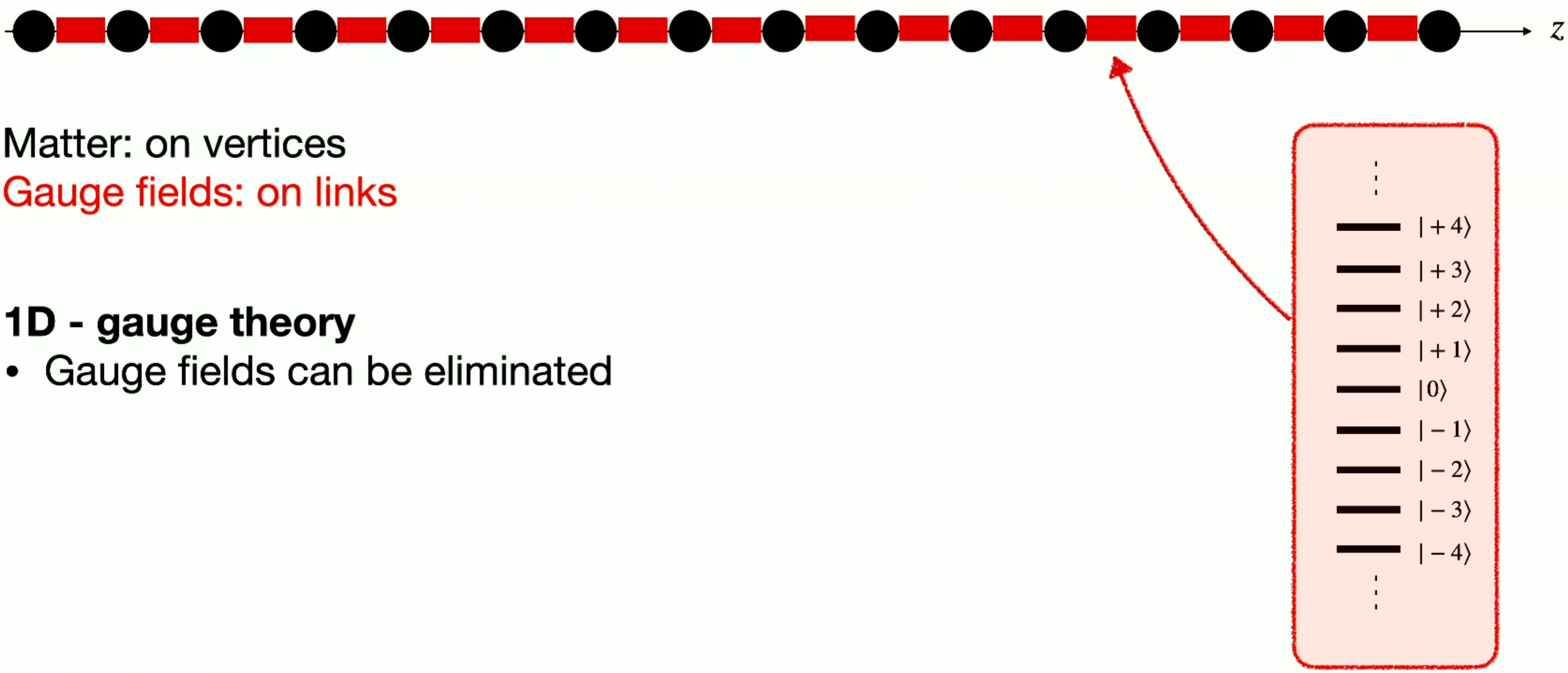
Representing gauge fields



Matter: on vertices
Gauge fields: on links

1D - gauge theory

- Gauge fields can be eliminated

Open boundary conditions

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

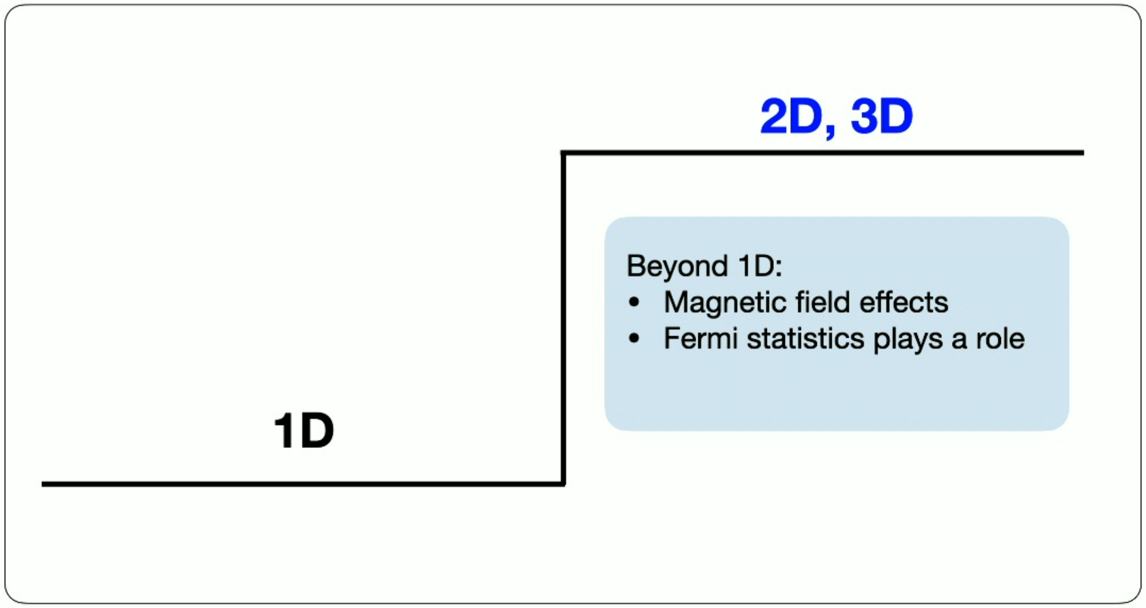
Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

Nature **534**, 516–519 (2016)

Theory + Experiment

- Trapped ion quantum computer
- Quantum simulation of 1D-QED
- Real-time dynamics of electron-positron pair creation.

Gauge theories for particle physics beyond 1D

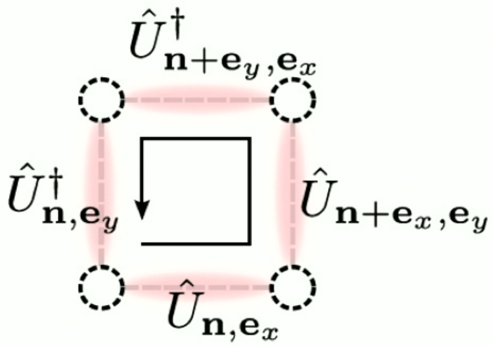


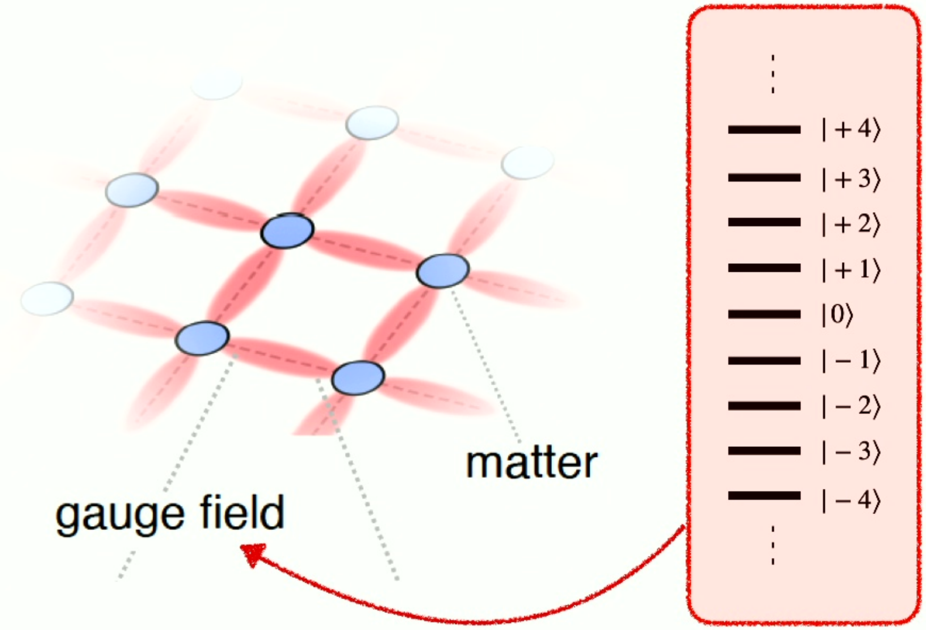
Gauge theories beyond 1D

2D gauge theory

- Not all gauge fields can be eliminated
- i.e. "dynamical" gauge fields

→ Magnetic field effects





Experimental demonstrations

Gauge theories for particle physics beyond 1D

- N. Klco, M. J. Savage, and J. R. Stryker, Phys. Rev. D **101**, 074512 (2020).
- A. Ciavarella, N. Klco, and M. J. Savage, Phys. Rev. D **103**, 094501 (2021).
- S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D **104**, 034501 (2021).
- A. N. Ciavarella and I. A. Chernyshev, Phys. Rev. D **105**, 074504 (2022).
- S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D **106**, 074502 (2022).
- S. A. Rahman, R. Lewis, E. Mendicelli, and S. Powell, "Real time evolution and a traveling excitation in SU(2) pure gauge theory on a quantum computer," (2022), arXiv:2210.11606 [hep-lat].
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Impressive Advances!
(but gauge fields or matter fields are trivial)

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Continuous gauge groups, e.g. U(1), SU(3),…

As opposed to e.g. $\mathbb{Z}_2$

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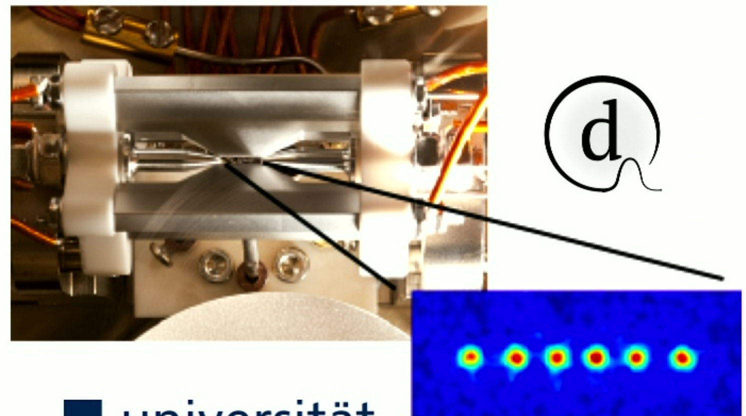
Experimental qudit system

Linear ion-trap quantum processor
with all-to-all connectivity

Extending qubit entangling gates
to mixed-dimensional qudit systems



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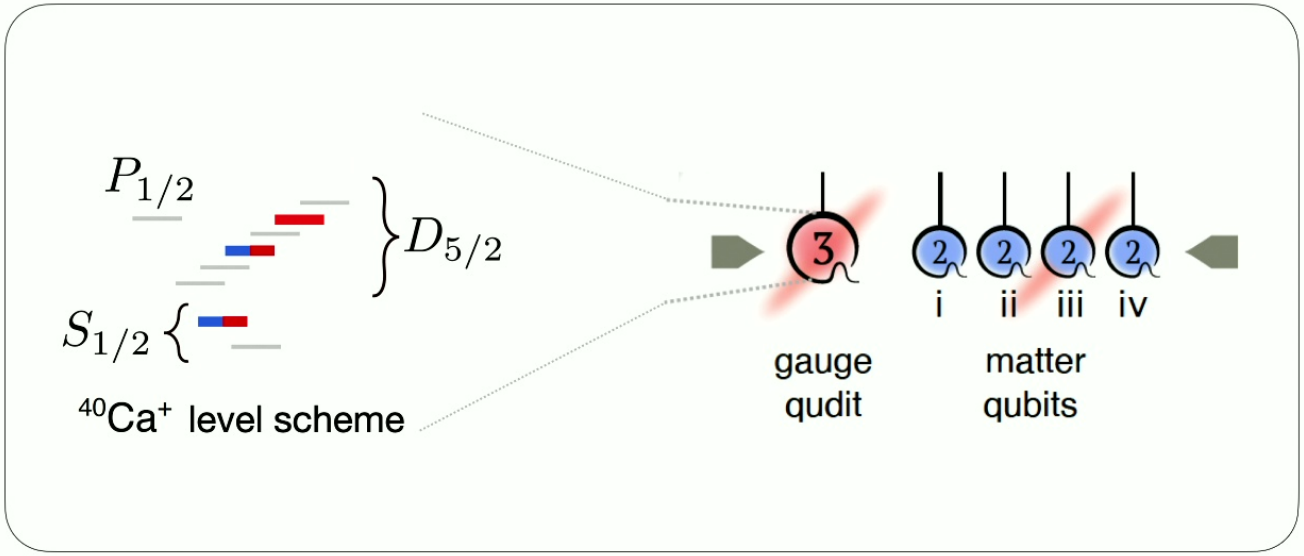
Martin Ringbauer

M. Ringbauer, M. Meth, L. Postler, R. Stricker, R. Blatt, P. Schindler, T. Monz, Nature Physics 18, 1053 (2022)

Encoding qudits in trapped ions

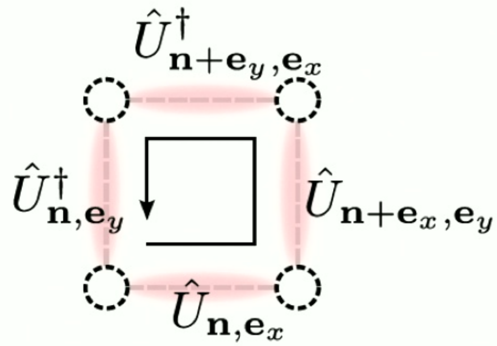


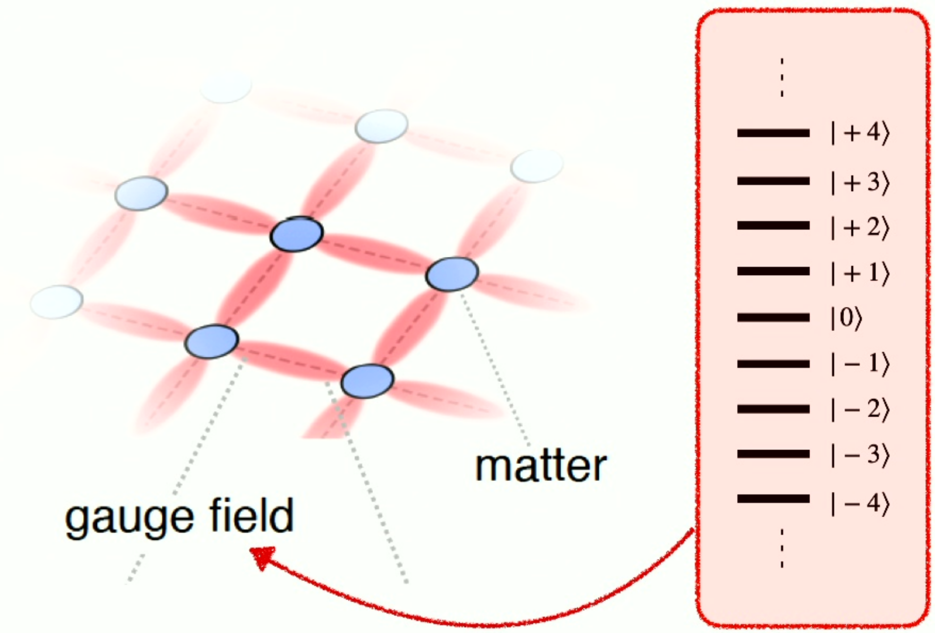
Gauge theories <u>beyond 1D</u>

2D gauge theory

- Not all gauge fields can be eliminated
- i.e. <u>“dynamical” gauge fields</u>

→ Magnetic field effects





Gauge fields

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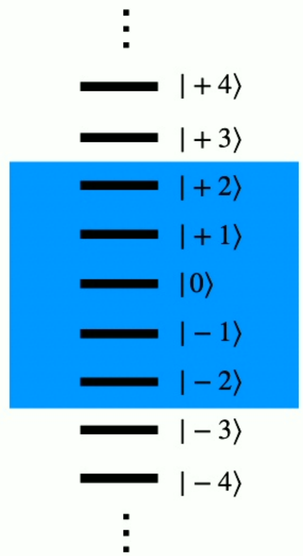

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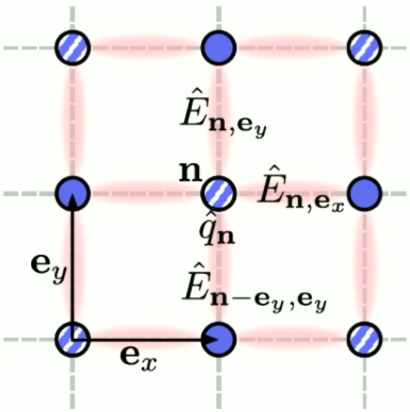
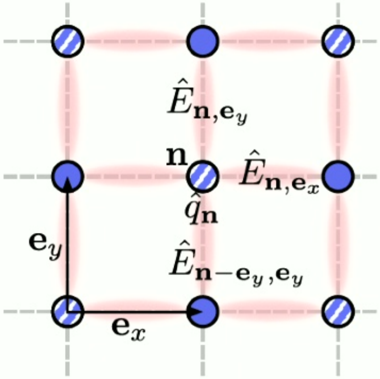
2D lattice QED



Even $n_x + n_y$ lattice sites	Odd $n_x + n_y$ lattice sites
◌ $\cong |\downarrow\rangle \cong |e\rangle\,, \quad \hat{q}_{\mathbf{n}} = -1$	◌ $\cong |\downarrow\rangle \cong |v\rangle\,, \quad \hat{q}_{\mathbf{n}} = 0$
● $\cong |\uparrow\rangle \cong |v\rangle\,, \quad \hat{q}_{\mathbf{n}} = 0$	● $\cong |\uparrow\rangle \cong |p\rangle\,, \quad \hat{q}_{\mathbf{n}} = +1$



2D lattice QED

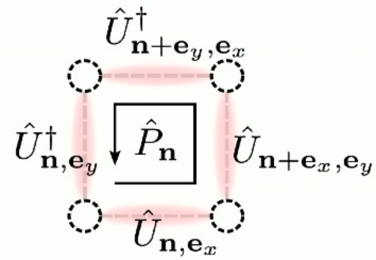
$$\boxed{\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2}\hat{H}_B + m\hat{H}_m + \Omega \hat{H}_k}$$

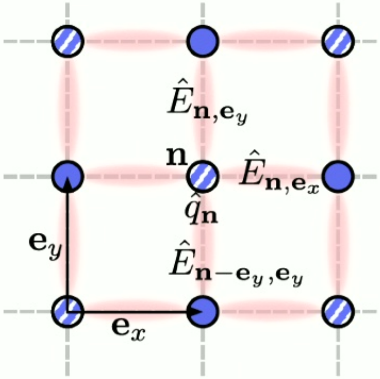
Electric field energy $\longrightarrow$ $\hat{H}_E = \frac{1}{2}\sum_{\mathbf{n}}\left(\hat{E}^2_{\mathbf{n},\mathbf{e}_x} + \hat{E}^2_{\mathbf{n},\mathbf{e}_y}\right),$

Magnetic field energy $\longrightarrow$ $\hat{H}_B = -\frac{1}{2}\sum_{\mathbf{n}}\left(\hat{P}_{\mathbf{n}} + \hat{P}^\dagger_{\mathbf{n}}\right),$ $\quad \hat{P}_{\mathbf{n}} = \hat{U}_{\mathbf{n},\mathbf{e}_x}\hat{U}_{\mathbf{n}+\mathbf{e}_x,\mathbf{e}_y}\hat{U}^\dagger_{\mathbf{n}+\mathbf{e}_y,\mathbf{e}_x}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_y}.$

Particle rest mass $\longrightarrow$ $\hat{H}_m = \sum_{\mathbf{n}}(-1)^{n_x+n_y}\hat{\phi}^\dagger_{\mathbf{n}}\hat{\phi}_{\mathbf{n}},$

Kinetic energy term (pair creation) $\longrightarrow$ $\hat{H}_k = \sum_{\mathbf{n}}\sum_{\mu=x,y}\left(\hat{\phi}_{\mathbf{n}}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_\mu}\hat{\phi}^\dagger_{\mathbf{n}+\mathbf{e}_\mu} + \mathrm{H.c.}\right).$





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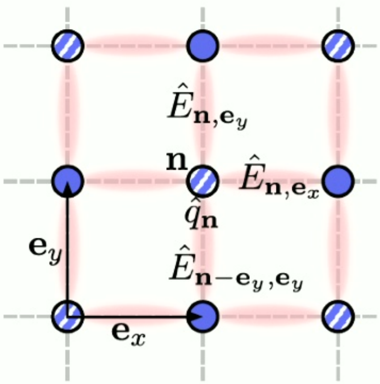
Classical Gauss law: $\nabla \mathbf{E}(\mathbf{r}) - \rho(\mathbf{r}) = 0$

QED Gauss law: $\hat{G}_{\mathbf{n}}|\Psi_{\text{phys}}\rangle = 0,$

$$\hat{G}_{\mathbf{n}} = \sum_{\mu}(\hat{E}_{\mathbf{n},\mathbf{e}_\mu} - \hat{E}_{\mathbf{n}-\mathbf{e}_\mu,\mathbf{e}_\mu}) - \hat{q}_{\mathbf{n}}$$



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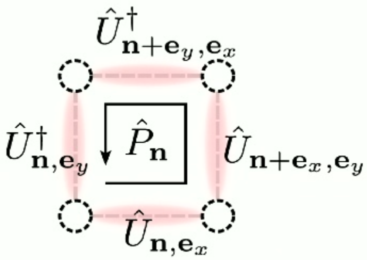

Gauge theories beyond 1D

2D gauge theory

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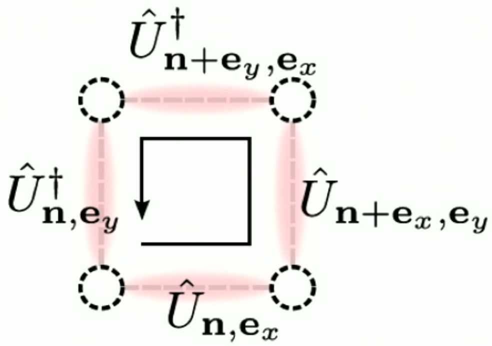
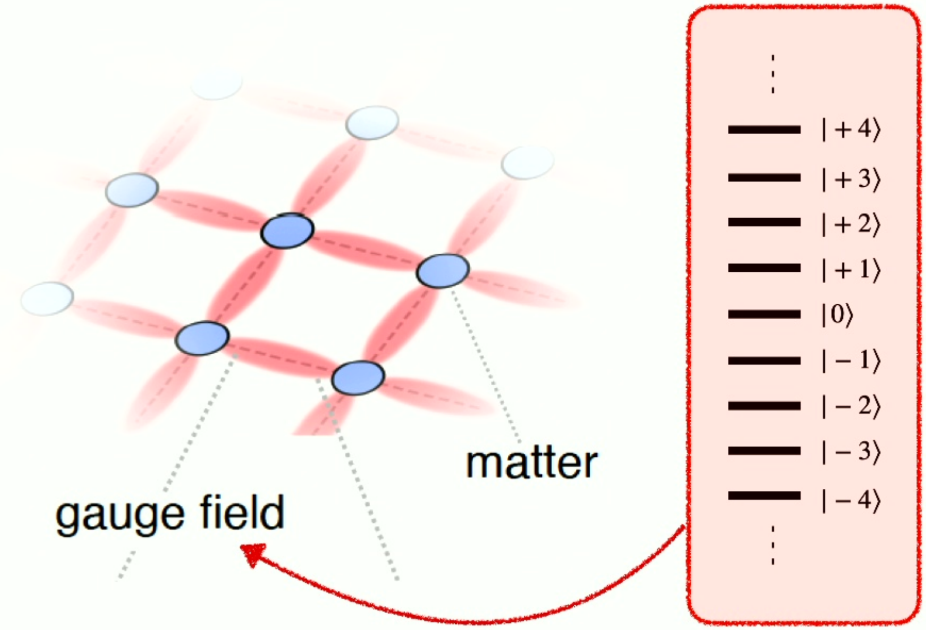
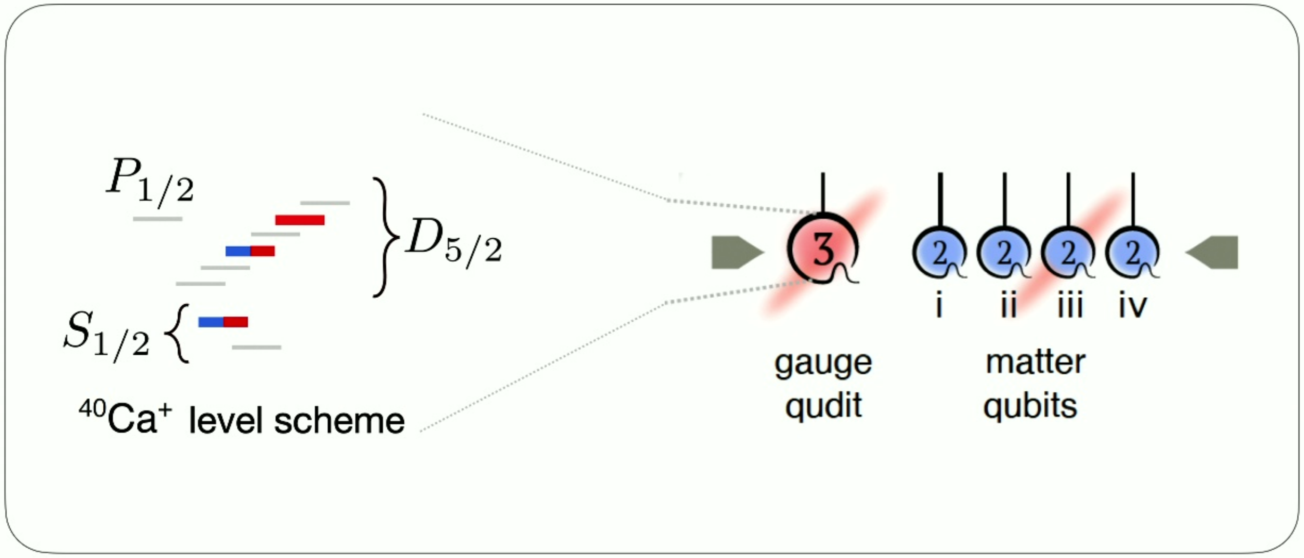
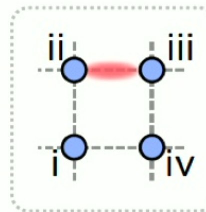




Experiment



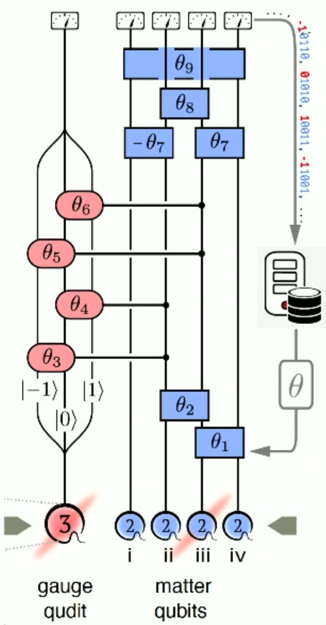


Basic building block: Plaquette

Experiment

Variational Quantum Eigensolver

Hybrid qubit-quTrit variational circuit



➡ Ground state preparation ➡ Measure plaquette expectation value

$\langle \hat{\square} \rangle \propto \langle \hat{H}_B \rangle$

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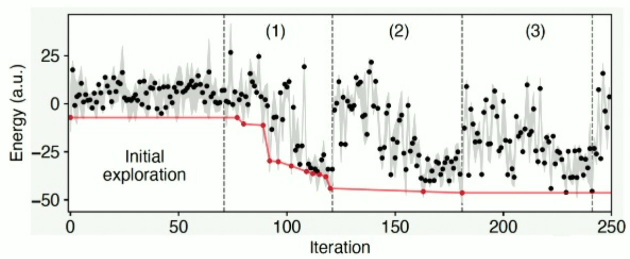

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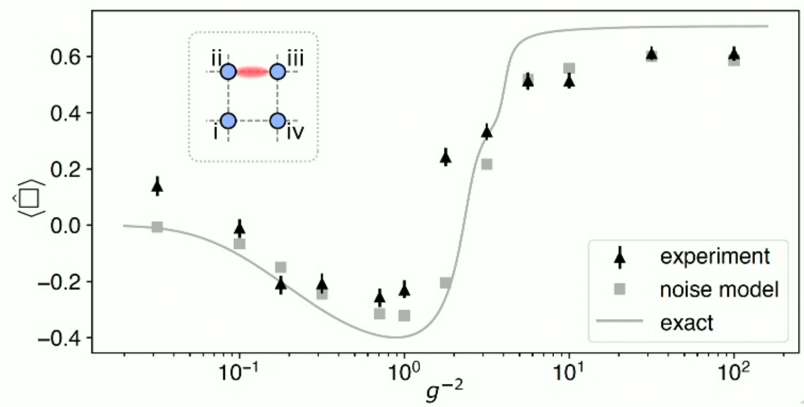
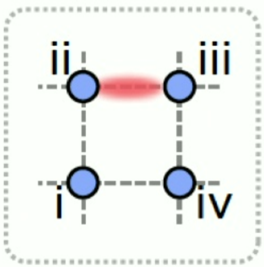
Measure plaquette expectation value

$\langle \hat{\square} \rangle \propto \langle \hat{H}_B \rangle$



Qudit encoding: short circuits

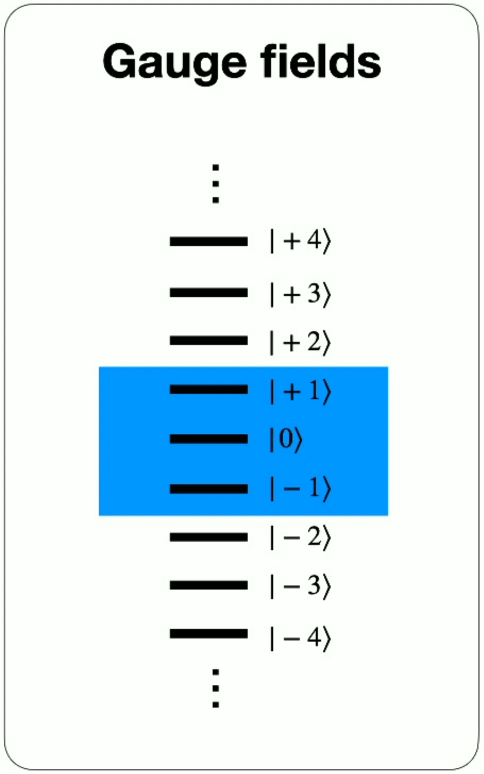
Basic building block: Single plaquette:



	Qudit enc			Qubit enc.		
dimension d	3	5	7	3	5	7
register size	5	5	5	7	9	11
CNOT count	$\boxed{26}$	34	42	$\boxed{96}$	174	252
CNOT fidelity	99%					
approx. circ. fid.	77%	71%	66%	38%	17%	8%
CNOT fidelity	99.5%					
approx. circ. fid.	88%	84%	81%	62%	42%	28%

So far:

- Relevant physics could be simulated.
- Using the minimal truncation $d = 3$.

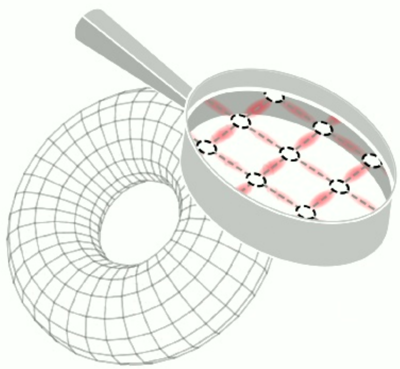


Increasing d

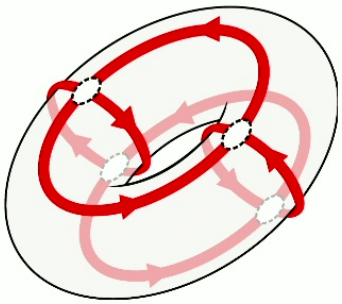
Improving the gauge-field truncation

Concrete example:

- Study the dependence of $\langle \hat{\square} \rangle$ on the bare coupling at different discretizations.
- Consider QED on a 2D lattice with periodic boundary conditions.
- No matter fields.



2D-QED with periodic boundary conditions



One plaquette with periodic boundary conditions

Increasing d

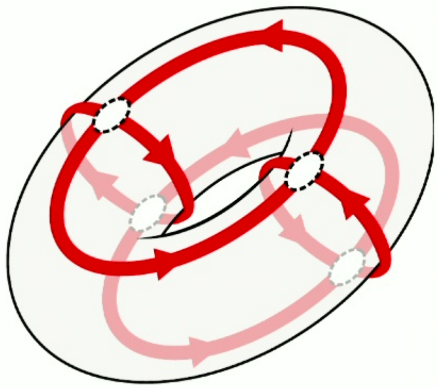
Improving the gauge-field truncation

Pure gauge theory:

$$\hat{H} = g^2 \hat{H}_E + (1/g^2)\hat{H}_B$$

Single plaquette:

- 4 vertices.
- 8 links = 8 gauge fields.
- Gauss law: 5 independent gauge fields.
- For ground state properties **3 gauge fields**.



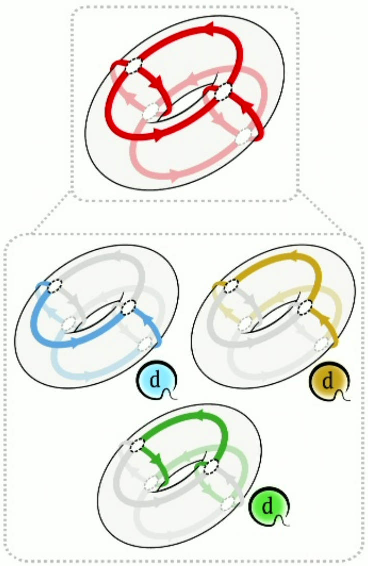
J. F. Haase, L. Dellantonio, A. Celi, D. Paulson, A. Kan, K. Jansen, and C. A. Muschik, Quantum **5**, 393 (2021).

Increasing d

Improving the gauge-field truncation

Gauge fields:
3 relevant gauge fields: convenient description as “rotators” (operators that can be visualised as loops).

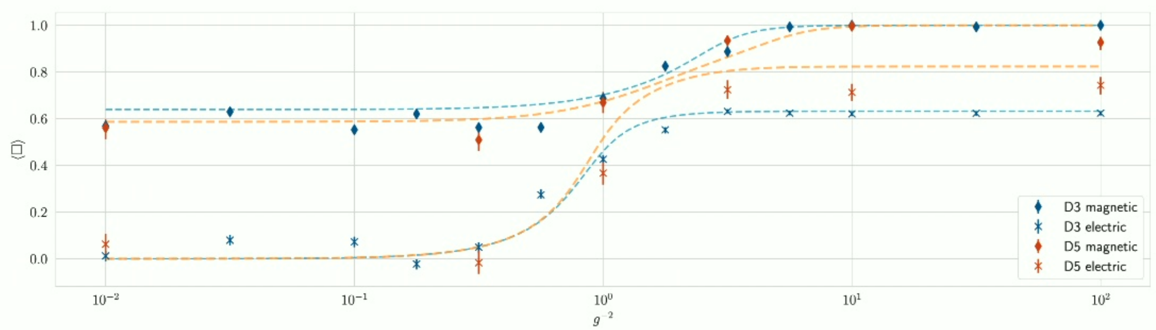
Qudits representation:
Each gauge field is represented d levels.
We investigate $d = 3$ and $d = 5$.

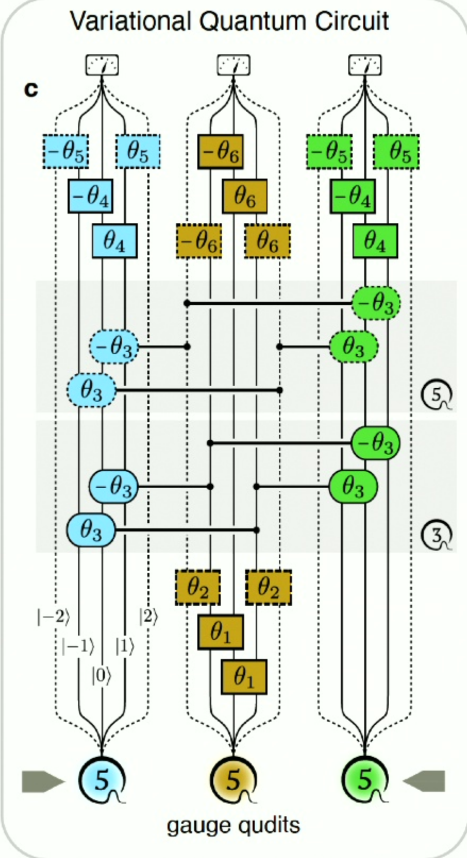


J. F. Haase, L. Dellantonio, A. Celi, D. Paulson, A. Kan, K. Jansen, and C. A. Muschik, Quantum **5**, 393 (2021).

Experiment:

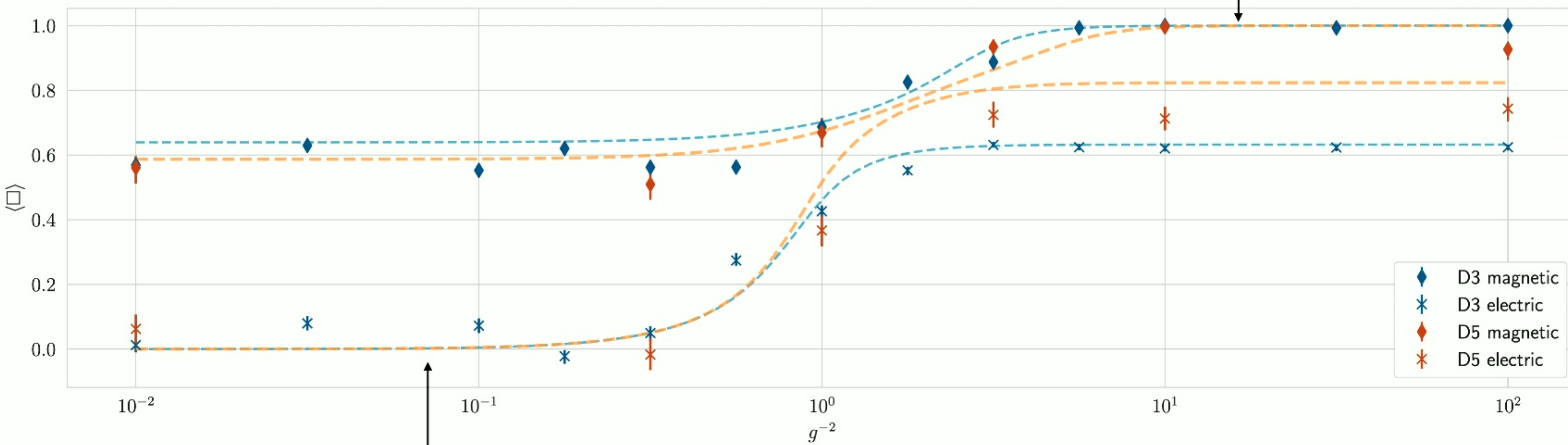
- VQE protocol to prepare the ground state.
- Measurement of $\langle \hat{\square} \rangle$ in the ground state
- for different values of the bare coupling g.





$$\boxed{\hat{H} = g^2 \hat{H}_E + (1/g^2)\hat{H}_B}$$

Large values of g^{-2} : Magnetic representation is efficient



Small values of g^{-2} : Electric representation is efficient

J. F. Haase, L. Dellantonio, A. Celi, D. Paulson, A. Kan, K. Jansen, and C. A. Muschik, Quantum **5**, 393 (2021).

Take home messages:

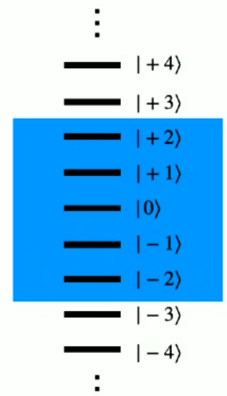
- Full VQE ground state search for 3-level qutrits and 5-level ququints.

- Qudits allows to seamlessly observe the effect of different gauge field truncations by controlling the qudit dimension.

Experimental demonstration

Gauge theories for particle physics beyond 1D

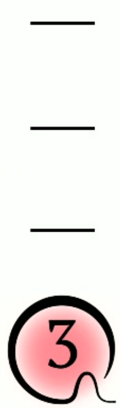
Including both - dynamical gauge and matter fields

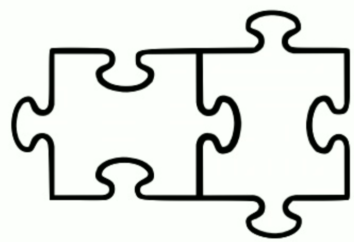


Adjustable discretisation of the gauge fields

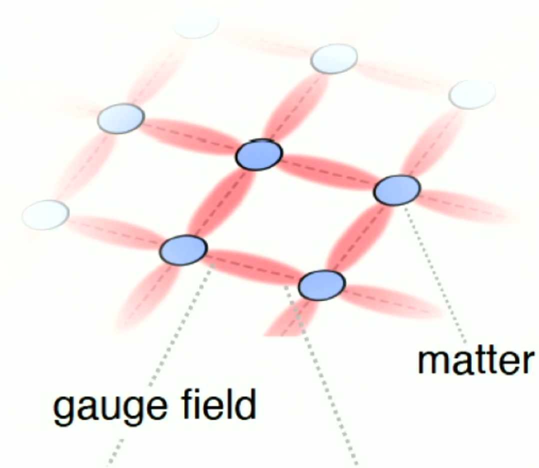
Beyond binary

short-depth qu**d**it circuits

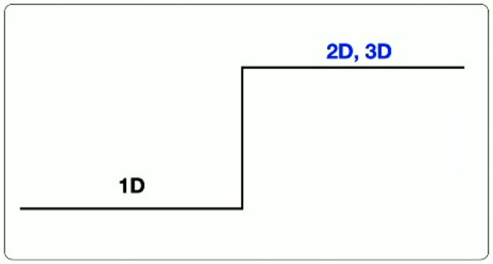




Lattice Gauge theories for particle physics



Outlook



Long-term vision:
- Simulate nature in 3 spatial dimensions
- Scaling up the system sizes
- Both make an efficient gauge field representation even more important

Qudit techniques:
- Can be implemented on many quantum computing platforms
- Are directly adaptable to digital quantum simulations of real-time dynamics
  Exciting perspective: quantum LGT calculations in regimes that are classically intractable due to sign problems
- Applications in chemistry and materials science
  e.g. in the form of exotic large-spin models: see e.g. work at Harvard (N. Maskara et. al.)

Qudit workshop

9-11 July 2024, Waterloo