Title: Embedding generalised LTB models in polymerised spherically symmetric models: formalism and applications

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Abstract: At the classical level, one can restrict a spherically symmetric model to the corresponding LTB sector by requiring that a so-called LTB condition is satisfied. In this talk we will discuss how this can be generalised in the context of effective models containing quantum gravity inspired modifications of the classical theory in the form of polymerisations. The formalism presented allows us to consider more general polymerisations than in previous work in the literature. Applications of this framework are then considered, focusing on a particular class of models that have the property that the effective dynamics is completely decoupled along the radial direction. Examples of effective models that fall into this class are discussed and their physical properties are compared with existing models in the literature.

Zoom link

Embedding generalised LTB models in polymerised spherically symmetric models: formalism and applications

Friedrich-Alexander-Universität Erlangen-Nürnberg

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

joint work with Hongguang Liu, Eric Rullit, Stefan Weigl and Parampreet Singh
submitted to Phys.Rev.D.
arXiv:2308.10949, arXiv:2308.10953

Quantum Gravity Seminar
Perimeter Institute
19.04.24

Funded by DFG Deutsche Forschungsgemeinschaft German Research Foundation

Kristina Giesel, Institute for Quantum Gravity, FAU Erlangen-Nürnberg

Motivation

Investigate quantum black hole models interesting for quantum gravity

Many people contribute:

[Ashtekar, Bojowald, Modesto, Cartin, Khanna, Boehmer, Vandersloot, Chiou, Campiglia, Gambini, Pullin, Sabharwal, Brannlund, Kloster, De Benedictis, Olmedo, Dadhich, Joe, Singh, Haggard, Rovelli, Vidotto, Corichi, Saini, Cortez, Cuervo, Morales-Técotl, Ruelas, Pawlowski, Bianchi,, Christodoulo, D'Ambrosio, Alesci, Bahrami, Pranzetti, Husain, Kelly, Santacruz, Wilson-Ewing, Lewandowski, Zhang, Ma, Song, Bodendorfer, Mele, Münch, Navascués, Mena Marugán, García-Quismondo, Perez, Speziale, Viollet, Han, K.G.,Liu, Li, Weigl,Alonso-Bardaji, Brizuela, Vera,...]

Recent reviews: [Gambini,Olmedo,Pullin '22], [Ashtekar, Olmedo, Singh '23]

Dynamical formulation of gravitational collapse: consider spherically spherically symmetric models with dust
LTB models, Oppenheimer-Snyder collapse

Here we will consider effective models to formulate such models which involve (LQG inspired) quantum corrections

Aim: Develop formalism that allows to investigate a broad class of effective models [seminal work by Bojowald, Harada, Reyes, Tibrewala '08 '09]

Plan of the talk

I. Brief summary on classical LTB models
II. Effective models: embedding LTB models into spherically symmetric models
III. Applications
IV. Comparison with existing models
V. Summary and conclusions

I. Classical LTB models

LTB: Spherically symmetric solution with dust

We consider Ashtekar-Barbero variables for spherical symmetry $\left(A_a^j, E_j^a\right)$

After implementing the Gauss constraint: [Bojowald Kastrup '00], [Bojowald, Swiderski'03]

$$(x, \theta, \phi)$$

$$\begin{aligned} A_a^j \tau_j \, \mathrm{d}X^a =& 2\beta K_x(x)\tau_1 \, \mathrm{d}x + \left(\beta K_\phi(x)\tau_2 + \frac{\partial_x E^x(x)}{2E^\phi(x)}\tau_3\right)\mathrm{d}\theta \\ &+ \left(\beta K_\phi(x)\tau_3 - \frac{\partial_x E^x(x)}{2E^\phi(x)}\tau_2\right)\sin(\theta)\mathrm{d}\phi + \cos(\theta)\tau_1 \, \mathrm{d}\phi \\ E_j^a \tau^j \frac{\partial}{\partial X^a} =& E^x(x)\sin(\theta)\tau_1\partial_x + \left(E^\phi(x)\tau_2\right)\sin(\theta)\partial_\theta + \left(E^\phi(x)\tau_3\right)\partial_\phi, \end{aligned}$$

Reduced phase space including dust:

$$\{K_x(x), E^x(y)\} = G\delta(x,y) \quad \{K_\phi(x), E^\phi(y)\} = G\delta(x,y) \quad \{T(x), P_T(y)\} = \delta(x,y)$$

I. Classical LTB models

General spherically symmetric metric

$$\mathrm{d}s^2 = -N(x,t)^2 dt^2 + \frac{\left(E^\phi\right)^2}{|E^x|}\left(dx + N^x dt\right)^2 + |E^x|\, d\Omega^2$$

Consider the form of the LTB metric [Lemaître '33], [Tolman '34], [Bondi '47]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{\left((E^x)'\right)^2}{4\,|E^x|\,(1+\mathcal{E}(x))}\mathrm{d}x^2 + |E^x|\,\mathrm{d}\Omega^2 \qquad \partial_t E^x = \pm 2\sqrt{E^x}\sqrt{\mathcal{E}(x) + \frac{\mathcal{F}(x)}{(E^x)^2}}$$

shells decouple classically

To match both metric we need

$$N = 1 \quad N^x = 0 \quad G_x(x) = \frac{E^{x\prime}}{2E^\phi}(x) - \sqrt{1+\mathcal{E}(x)} = 0$$

Dust time gauge + LTB condition

$$C \longrightarrow G_T = T(x) - t, \quad C_x \longrightarrow G_x = \frac{E^{x\prime}}{2E^\phi}(x) - \sqrt{1+\mathcal{E}(x)}$$

II. Effective models

II. Effective models: assumptions

Work in the framework of effective theories: Polymerisations

$K_\phi \to \frac{\sin(\alpha K_\phi)}{\alpha}$ more general $K_\phi \to f(E^x, E^\phi, K_x, K_\phi)$ $\lim_{\alpha\to 0} f = K_\phi$

Inverse triad corrections

$$\frac{1}{\sqrt{E^x}} \to \frac{h_1(E^x)}{\sqrt{E^x}}, \quad \frac{E^x}{\sqrt{E^x}} = \sqrt{E^x} \to \sqrt{E^x} h_2(E^x)$$

Main assumptions of the framework:

Function preserving polymerisation

No polymerisation of the spatial diffeomorphism constraint

Temporal gauge fixed model —> partial gauge fixing

Allow polymerisations of the LTB condition

Large class of existing models can be embedded into this framework

II. Effective models

We consider the following class of models in dust time gauge

$$H_P^\Delta[N^x] = \int \mathrm{d}x \left(C^\Delta + N^x C_x\right)(x), \quad C_x = \frac{1}{G}\left(E^\phi K_\phi' - K_x \left(E^x\right)'\right)$$

Polymerised gravitational contribution to Hamiltonian constraint

$$C^\Delta(x) = \frac{E^\phi}{2G\sqrt{E^x}}\left[-(1+f)E^x\left(\frac{4K_x K_\phi}{E^\phi} + \frac{K_\phi^2}{E^x}\right) + h_1\left(\left(\frac{E^{x\prime}}{2E^\phi}\right)^2 - 1\right) + 2\frac{E^x}{E^\phi}h_2\left(\frac{E^{x\prime}}{2E^\phi}\right)'\right]$$

Polymerisation functions with following classical limit

$$h_1\left(E^x\right) \to 1 \quad h_2\left(E^x\right) \to 1 \quad f\left(K_x/E^\phi, K_\phi, E^x\right) \to 0$$

$\frac{K_x}{E^\phi}$ more convenient combination

We have

$$\left\{C^\Delta[N], C_x\left[N^x\right]\right\} = C^\Delta\left[N^x\left(\partial_x N\right)\right]$$

For Poisson bracket involving two C^Δ we will get additional restrictions on polymerisation

II. Conservation of C^Δ

Condition for conservation of C^Δ

[K.G, Liu, Rullit, Singh, Weigl '23] [Tibrewala '12], [Alonso-Bardaji, Brizuela '21]

One has

$$\left\{H_P^\Delta\left[N^x=0\right], C^\Delta(y)\right\}\Big|_{C_x=0}=0$$

if K_x is not polymerised and the condition holds

$$\frac{h_1-2E^x\partial_{E^x}h_2}{h_2}=\frac{-4E^x\partial_{E^x}f^{(2)}+\partial_{K_\phi}f^{(1)}}{2f^{(2)}}.$$

This restricts the form of C^Δ to

$$C^\Delta(x)=-\frac{E^\phi}{2G\sqrt{E^x}}\left[\frac{4K_xf^{(2)}\left(K_\phi,E^x\right)}{E^\phi}+\frac{f^{(1)}\left(K_\phi,E^x\right)}{E^x}-h_1\ldots\right](x)$$

then we have

$$\left\{C^\Delta\left[N_1\right], C^\Delta\left[N_2\right]\right\}=\left(\left(\partial_{K_\phi}f^{(2)}\right)\frac{E^x}{\left(E^\phi\right)^2}C_x\right)\left[N_1N_2'-N_2N_1'\right]$$

since partial gauge fixed could also relax condition of conservation here

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then we have

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since partial gauge fixed could also relax condition of conservation here

II. Examples

Example I: conserved C^{Δ}

[Tibrewala '12]

$$f^{(1)} = \frac{\sin\left(\beta\sqrt{\Delta}K_\phi\right)^2}{\beta^2\Delta}, \quad f^{(2)} = \frac{\sin\left(2\beta\sqrt{\Delta}K_\phi\right)}{2\beta\sqrt{\Delta}} \quad h_1 = 1, \quad h_2 = 1$$

factor 2 important here, cos-deformation in algebra

Example II: C^{Δ} is not conserved [Bojowald, Harada, Tribewala '06]

$$f^{(1)} = h_1\left(f^{(2)}\right)^2, \quad f^{(2)} = \frac{\sin\left(\beta\sqrt{\Delta}K_\phi\right)}{\beta\sqrt{\Delta}}, \quad h_2 = l\left(E^x\right) \quad h_1\left(E^x\right) = \sqrt{E^x}\frac{\sqrt{E^x + \beta l_p^2/2} - \sqrt{E^x - \beta l_p^2/2}}{\beta l_p^2/2}$$

condition not satisfied since RHS not only function of E^x

$$\frac{-4E^x\partial_{E^x}f^{(2)} + \partial_{K_\phi}f^{(1)}}{2f^{(2)}} = \cos\left(\beta\sqrt{\Delta}K_\phi\right)h_1$$

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Example II: C^{Δ} is not conserved [Bojowald, Harada, Tibrewala '06]

$$f^{(1)} = h_1\left(f^{(2)}\right)^2, \quad f^{(2)} = \frac{\sin\left(\beta\sqrt{\Delta}K_\phi\right)}{\beta\sqrt{\Delta}}, \quad h_2 = l\left(E^x\right) \quad h_1\left(E^x\right) = \sqrt{E^x}\frac{\sqrt{E^x+\beta l_p^2/2}-\sqrt{E^x-\beta l_p^2/2}}{\beta l_p^2/2}$$

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$$\frac{-4E^x\partial_{E^x}f^{(2)} + \partial_{K_\phi}f^{(1)}}{2f^{(2)}} = \cos\left(\beta\sqrt{\Delta}K_\phi\right)h_1$$

II. Compatible LTB condition

First consider general case where C^{Δ} does not need to be conserved

Asumme $f\left(K_x/E^{\phi}, K_{\phi}, E^x\right)$

Ansatz for polymerised LTB condition $\quad G_x^{\Delta}(x) := \dfrac{E^{x\prime}}{2E^{\phi}}(x) - g_{\Delta}$

with $\quad g_{\Delta}\left(K_{\phi}, E^x, \partial_x K_{\phi}, \partial_x E^x, \cdots, \partial_x^n K_{\phi}, \partial_x^n E^x, \Xi\right)(t,x)$

Classical limit $\quad G_x^{\Delta}(x) \to G_x = \dfrac{E^{x\prime}}{2E^{\phi}} - \sqrt{1+\mathcal{E}(x)}$

Require stability under effective dynamics, then compatible

Technically simpler to work at the level of the equations of motion

For which G_x^{Δ} do the four EOM of $\dot{K}_x, \dot{K}_{\phi}, \dot{E}^x, \dot{E}^{\phi}$ reduce to only two in the sector

$$N^x = 0, \quad C_x = 0, \quad G_x^{\Delta} = 0 \quad ?$$

Compatible LTB conditions can be written in the form

$$g_{\Delta} = g_{\Delta}^{(1)}\left(K_{\phi}, E^x, \mathcal{E}\right) + g_{\Delta}^{(2)}\left(\widetilde{K}_x = \frac{\partial_x K_{\phi}}{\partial_x E^x}, K_{\phi}, E^x\right)$$

$g_{\Delta}^{(1)}$ non-marginally

$g_{\Delta}^{(2)}$ marginally

II. Compatible LTB condition

Interplay between polymerisation and compatible LTB condition

Also restriction on K_x polymerisation

non-marginally bound: no polymerisation for K_x allowed
marginally bound: if $g_\Delta^{(2)} = g_\Delta^{(2)}(K_\phi, E^x)$ then no K_x polymerisation allows

In case the LTB function can be factored out $g_\Delta^{(1)} = \tilde{g}_\Delta(K_\phi, E^x)\sqrt{1+\mathcal{E}}$

Then $g_\Delta^{(1)} = \tilde{g}_\Delta(E^x)\sqrt{1+\mathcal{E}}$ $\quad 1 - \frac{2E^x \partial_{E^x}\tilde{g}_\Delta}{\tilde{g}_\Delta} = \frac{-4E^x\partial_{E^x} f^{(2)} + \partial_{K_\phi} f^{(1)}}{2f^{(2)}}$ ⟵

Conservation of C^Δ if in addition holds

$$2E^x\partial_{E^x}\tilde{g}_\Delta = \left(1 - \frac{h_1 - 2E^x\partial_{E^x}h_2}{h_2}\right)\tilde{g}_\Delta$$

Special case classical LTB condition $\tilde{g}_\Delta = 1$

$$\partial_{K_\phi} f^{(1)} = 2f^{(2)} + 4E^x\partial_{E^x} f^{(2)} \qquad h_1 = h_2 + 2E^x\partial_{E^x}h_2$$

compatibility — conservation

II. Compatible LTB condition

Again the two examples we considered earlier

Example I: [Tibrewala '12]

$$f^{(1)} = \frac{\sin\left(\beta\sqrt{\Delta}K_\phi\right)^2}{\beta^2\Delta}, \quad f^{(2)} = \frac{\sin\left(2\beta\sqrt{\Delta}K_\phi\right)}{2\beta\sqrt{\Delta}} \quad h_1 = 1, \quad h_2 = 1$$

satisfies $\partial_{K_\phi} f^{(1)} = 2f^{(2)} + 4E^x \partial_{E^x} f^{(2)} \quad h_1 = h_2 + 2E^x \partial_{E^x} h_2$

classical LTB condition is compatible: $\tilde{g}_\Delta = 1 \qquad G_x^\Delta = G_x = \frac{E^{x\prime}}{2E^\phi}(x) - \sqrt{1+\mathcal{E}(x)}$

Example II: [Bojowald, Harada, Tibrewala '06]

$$f^{(1)} = h_1\left(f^{(2)}\right)^2, \quad f^{(2)} = \frac{\sin\left(\beta\sqrt{\Delta}K_\phi\right)}{\beta\sqrt{\Delta}}, \quad h_2 = l\left(E^x\right) \quad h_1\left(E^x\right) = \sqrt{E^x}\frac{\sqrt{E^x + \beta l_p^2/2} - \sqrt{E^x - \beta l_p^2/2}}{\beta l_p^2/2}$$

Complicated equation to solve to obtain compatible LTB condition non-marginally bound case $g_\Delta = g_\Delta^{(1)}\left(K_\phi, E^x\right)$

$$0 = \frac{\sin\left(\beta\sqrt{\Delta}K_\phi\right)}{\beta\sqrt{\Delta}}\left(g_\Delta^{(1)}\left(-1 + h_1\cos\left(\beta\sqrt{\Delta}K_\phi\right)\right) + 2E^x\partial_{E^x}g_\Delta^{(1)}\right)$$

$$-\left(\left(h_1 + \left(g_\Delta^{(1)}\right)^2\left(h_1 - 2h_2 - 4E^x\partial_{E^x}h_2\right)\right) + h_1\frac{\sin^2\left(\beta\sqrt{\Delta}K_\phi\right)}{\beta^2\Delta}\right)\partial_{K_\phi}g_\Delta^{(1)}$$

II. Specific class of effective models

We consider effective models where a compatible LTB condition exist and C^{Δ} is conserved

Equations of motion in both marginally and non-marginally bound case

$$\partial_t E^x = 2\sqrt{E^x} f^{(2)}$$

$$\partial_t K_\phi = -\frac{1}{2\sqrt{E^x}} \left(f^{(1)} - \tilde{g}_\Delta^2 (1+\mathcal{E}) \left(2h_2 + 4E^x \partial_{E^x} h_2 - h_1\right) + h_1 \right)$$

decouple along the radial coordinate x

for each x solution involves energy $\mathcal{E}(x)$ and conserved mass $M(x)$

Some models have assumed shell decoupling by hand here shown when given

[Kiefer, Schmitz '19], [K.G,Li,Singh '21]

Next we want to discuss an example that belongs to this class.

III. General Strategy in this class of models

We consider effective models where a compatible LTB condition exist and C^{Δ} is conserved (Corollary 4 in [K.G, Liu, Rullit, Singh, Weigl '23])

As the EOM completely decouple along x we can choose an LQC model as a starting point

Then results of corollary 4 allow us to determine the underlying spherically symmetric model and corresponding compatible LTB condition

Advantages:

we obtain spherically symmetric model where no areal gauge has been applied

model involves dynamically stable reduction to its LTB sector at the effective level

Makes the comparison with other existing models more straight forward

for a subclass of these models we can relate the polymerised model to a covariant Lagrangian of extended mimetic gravity (modified gravity)

III. Specific model

Choose LQC model to start with [Ashtekar,Pawloswki, Singh '06]

Basic variables: $v = (E^x)^{3/2}\,, b = \frac{K_\phi}{\sqrt{E^x}}, \alpha = \beta\sqrt{\Delta}.$

Dynamics: $\partial_t v = 3v\frac{\sin(2\alpha b)}{2\alpha}, \quad \partial_t b = -\frac{1}{2}\left(\frac{\mathcal{E}(x)}{v^{\frac{2}{3}}} + \frac{3\sin^2(\alpha b)}{\alpha^2}\right)$

Now we can use corollary 4 to determine $f^{(1)}, f^{(2)}, h_1, h_2$ and compatible LTB condition by construction C^Δ is conserved

[Tibrewala '12]

$$C^\Delta = -\frac{E^\phi\sqrt{E^x}}{2G}\left[\frac{3}{\alpha^2}\sin^2\left(\frac{\alpha K_\phi}{\sqrt{E^x}}\right) + \frac{(2E^xK_x - E^\phi K_\phi)}{\alpha\sqrt{E^x}E^\phi}\sin\left(\frac{2\alpha K_\phi}{\sqrt{E^x}}\right) + \frac{1-\left(\frac{E^{x\prime}}{2E^\phi}\right)^2}{E^x} - \frac{2}{E^\phi}\left(\frac{E^{x\prime}}{2E^\phi}\right)'\right]$$

no inverse triad corrections: $h_1 = h_2 = 1$

Classical LTB condition is compatible: $G_x^\Delta = G_x = \frac{E^{x\prime}}{2E^\phi} - \sqrt{1+\mathcal{E}}$

III. Specific model

Effective LTB sector in LTB coordinates

LTB metric: $\mathrm{d}s^2 = -\mathrm{d}t^2 + (\partial_x R)^2 \ \mathrm{d}x^2 + R^2 \ \mathrm{d}\Omega^2 \qquad R = \sqrt{E^x}$

Dynamics modified Friedmann eqn $\frac{\dot{R}^2}{R^2}(x) = \left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\left(1 - \alpha^2\left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\right)(x)$

Specialise to marginally bound case $\mathcal{E} = 0$

Solution of modified Friedmann eqn

$$R(x,t) = \sqrt{E^x} = \left(\mathcal{F}(x)\left(\frac{9}{4}(\tilde{\beta}(x) - t)^2 + \alpha^2\right)\right)^{\frac{1}{3}}$$

Classical LTB solution for $\alpha = 0$

For homogeneous dust solution [K.G., Han, Li, Liu, Singh '22], [Fazzini, Rovelli, Soltani '23]

III. Concrete model

We can specialise to polymerised vacuum case for

LTB metric: $\mathrm{d}s^2 = -\mathrm{d}t^2 + (\partial_x R)^2 \ \mathrm{d}x^2 + R^2 \ \mathrm{d}\Omega^2 \qquad R = \sqrt{E^x}$

Dynamics modified Friedmann eqn $\frac{\dot{R}^2}{R^2}(x) = \left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\left(1 - \alpha^2\left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\right)(x)$

Specialise to marginally bound case $\mathcal{E} = 0$

Solution $R(x,t) = \sqrt{E^x} = \left(\mathcal{F}(x)\left(\frac{9}{4}(\tilde{\beta}(x) - t)^2 + \alpha^2\right)\right)^{\frac{1}{3}}$

Classical LTB solution for $\alpha = 0$

For homogeneous dust solution [K.G., Han, Li, Liu, Singh '22], [Fazzini, Rovelli, Soltani '23]

No shell crossing singularity for OS collapse, different for inhom. dust profiles

[Fazzini, Husain, Wilson-Ewing '23]

Horizons form if $M(x) = \frac{\mathcal{F}}{2G} > M_c = \frac{8\alpha}{3\sqrt{3}G}$

[Kelly, Santacruz, Wilson-Ewing '20] [K.G., Li, Singh '21] [K.G., Han, Li, Liu, Singh '21][Lewandowski, Ma, Zhang '22]

III. Polymerised vacuum solutions

We can specialise to polymerised vacuum case for $\mathcal{F} = R_s = \mathrm{const}$

Solution $R(x,t) = \left(R_s\left(\frac{9}{4}z^2 + \alpha^2\right)\right)^{\frac{1}{3}}$ here $\tilde{\beta}(x) = x \quad z := x - t$

Schwarzschild solution in LTB coordinates for $\alpha = 0$

Since solution depends on z only

Insensitive to sign of z since it depends quadratically on z.

Curvature invariants

$$\mathcal{R} = -\frac{96\alpha^2}{(4\alpha^2 + 9z^2)^2}, \quad \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} = \frac{576\left(160\alpha^4 - 96\alpha^2 z^2 + 27z^4\right)}{(4\alpha^2 + 9z^2)^4}$$

At z=0 different from classical theory: $\mathcal{R} \neq 0$ and bounded at z=0.

III. Underlying covariant Lagrangian

Extended mimetic gravity

$$S[g_{\mu\nu},\phi,\lambda] = \frac{1}{8\pi G}\int_{M_4} d^4x\sqrt{-g}\left[\frac{1}{2}R^{(4)} + L_\phi(\phi,\chi_1,\cdots,\chi_p) + \frac{1}{2}\lambda(\phi_\mu\phi^\mu+1)\right]$$

mimetic field ϕ, λ is a Lagrange multiplier for the mimetic condition, 2+1 dof

$$\chi_n \equiv \sum \phi^{\mu_2}_{\mu_1}\phi^{\mu_3}_{\mu_2}\cdots\phi^{\mu_n}_{\mu_{n-1}}\phi^{\mu_1}_{\mu_n}, \quad \phi_\mu = \nabla_\mu\phi, \quad \phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi.$$

Spherically symmetric model: Sufficient to have $L_\phi(\chi_1,\chi_2), \psi = ln(E^x)$ 2D action

[[Achour, Lamy, Liu, Noui '18], [Han, Liu '22]

$$S_2 = \frac{1}{4G}\int_{\mathcal{M}_2} \mathrm{d}^2x\,\det(e)e^{2\psi}\left\{\mathcal{R} + L_\phi(X,Y) + \frac{\lambda}{2}\left(\phi_{,j}\phi^{,j}+1\right)\right\}$$

(Smooth) mimetic field defines foliation into spacelike hyper surfaces $\phi = \mathrm{const}$

Generalised Einstein's equation

$$G^\Delta_{\mu\nu} := G_{\mu\nu} - T^\phi_{\mu\nu} = -\lambda\partial_\mu\phi\partial_\nu\phi, \quad \partial_\mu\phi\partial^\mu\phi = -1$$

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Spherically symmetric model: Sufficient to have $L_\phi\left(\chi_1,\chi_2\right)$, $\psi=ln(E^x)$ 2D action

[[Achour, Lamy, Liu, Noui '18], [Han, Liu '22]

$$S_2=\frac{1}{4G}\int_{\mathcal{M}_2}\mathrm{d}^2x\,\det(e)e^{2\psi}\left\{\mathcal{R}+L_\phi(X,Y)+\frac{\lambda}{2}\left(\phi_{,j}\phi^{,j}+1\right)\right\}$$

(Smooth) mimetic field defines foliation into spacelike hyper surfaces $\phi=\mathrm{const}$

Generalised Einstein's equation

$$G^{\Delta}_{\mu\nu}:=G_{\mu\nu}-T^{\phi}_{\mu\nu}=-\lambda\partial_\mu\phi\partial_\nu\phi,\quad \partial_\mu\phi\partial^\mu\phi=-1$$

III. Underlying covariant Lagrangian

Now for models with have no inverse triad corrections + compatible with $\overline{\mu}$-scheme we can relate the choice of the mimetic potential to specific choices of polymerisation function [Achour, Lamy, Liu, Noui '18], [Han, Liu '22]

$$S_2 = \frac{1}{4G} \int_{\mathcal{M}_2} \mathrm{d}^2 x \det(e) e^{2\psi} \left\{ \mathcal{R} + L_\phi(X,Y) + \frac{\lambda}{2} \left(\phi_{,j} \phi^{,j} + 1 \right) \right\}$$

Higher derivative couplings can be expressed in terms of X,Y and relate to extrinsic curvature

$$X = -\Box_h \phi + Y = \frac{\partial_t E^\phi}{E^\phi}, \quad Y = -h^{ij} \partial_i \psi \partial_j \phi = \frac{\partial_t E^x}{2E^x} = \frac{\sin(2\alpha b)}{2\alpha}$$

Underlying covariant model allows to gauge-unfix temporal gauge with respect to mimetic field and consider coordinate taros in (t,x)

Interpretation: effective model has different clock than classical model

IV. Comparison with other models

Polymerised vacuum solution in LTB coordinates

$$R(x,t) = \left(R_s\left(\frac{9}{4}z^2+\alpha^2\right)\right)^{\frac{1}{3}} \qquad z := x - t$$

Transform to Schwarzschild coordinates: $(t,x) \to (\tau, r)$

$$\mathrm{d}s^2 = -A(r)\mathrm{d}\tau^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2 \quad A(r) = 1 - \frac{2Gm_s}{r}\left(1 - \frac{\alpha^2}{r^2}\frac{2Gm_s}{r}\right)$$

Agrees with models in [Kelly,Santacruz,Wilson-Ewing '20],[Parvizi,Pawlowski,Tavakoli,Lewandowski '21], [Lewandowski,Ma,Yang,Zhang '22]

Metric given for all $r > 0$ except when horizons occur, coord. singularities

However, coordinate trafo to Schwarzschild coordinates only valid for monotonic R(z)

comparison: model presented here is an initial value problem

III. Quantum OS-collapse

Question: can we recover the model in [Husain, Kelly, Santacruz, Wilson-Ewing '22] ?

For this purpose need to apply areal gauge (Gullstrand-Painleve coord)

$G^{\rm ar} = E^x - r^2$ gauge fixing for C_x yields $N^r = -\frac{r}{2\alpha}\sin\left(\frac{2\alpha}{r}K_\phi\right)$

Exactly reproduces model with dynamics

$$\partial_t E^\phi = -r^2 \partial_r \left(\frac{E^\phi}{r}\right) \frac{\sin\left(\frac{2\alpha K_\phi}{r}\right)}{2\alpha}$$

$$\partial_t K_\phi = -\frac{1}{2r\alpha^2}\partial_r\left(r^3 \sin^2\left(\frac{\alpha K_\phi}{r}\right)\right) - \frac{1}{2r} + \frac{r}{2E^2}.$$

Strong evidence that model here is gauge-unfixed version of

[Husain, Kelly, Santacruz, Wilson-Ewing '22]

III. Quantum OS-collapse

Different properties shock solutions present?

We can transform our solution from LTB to Gullstrand-Painleve coordinates

$$r = R(x,t) \qquad N^r = -\partial_t R(t,x) = -\,\mathrm{sign}\,(\partial_t R(t,x))\sqrt{\frac{2GM(x)}{r}(\ldots)}$$

Obtain smooth change of sign of shift in the model presented here

Note observed when working directly in areal gauge and GP coordinates

Have discontinuity/shocks in the quantum OS collapse model

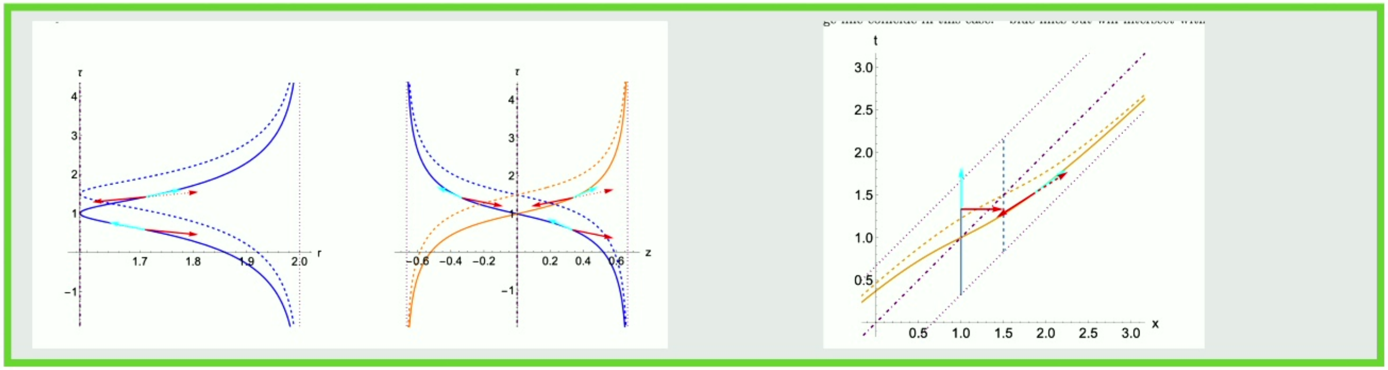
Rather discontinuity in the clock field than in the metric, coordinate artefact [Fazzini, Rovelli, Soltani '23]

Can we carry this over to the mimetic field which is the clock in the effective model?

III. Quantum OS-collapse

Different globale structure can be seen by looking at x=const geodesics

Here since mimetic field is the clock these are also its world lines



world lines intersect in (τ, r) after the bounce but blue lines do not in $(\tau, z), (t, x)$ coordinates

OS collapse: glueing with effective junction condition does not allow shocks without violating smoothness of mimetic clocks field.

IV. Summary & Conclusions

Formalism allows to investigate a broad class of effective models

The formalism can be used in two ways

1.) start with a spherically symmetric effective models and then consistent reduction to its LTB sector

2.) Consider LTB model and obtain underlying spherically symmetric model from it

A certain class of effective LTB models decouples along the radial direction here direct link to effective LQC models for OS collapse models

Could link those models in LTB coordinates to existing models

Saw that gauge fixing and/or coordinate choice is more subtle in effective models in general underlying covariant mimetic model helpful.

Next steps:

More detailed investigation on inhomogeneous dust profiles

Consider more general polymerisation not necessarily LQG inspired.

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