Title: Information dynamics or dynamics from information

Speakers: Matteo Scandi

Series: Quantum Foundations

Date: April 18, 2024 - 11:00 AM

URL: https://pirsa.org/24040099

Abstract: In this talk the role of information theory in the description of physical evolutions will be discussed. After defining information quantifiers, their contractivity with respect to physical dynamics will be explained, a requirement which simply encodes the intuition that noisy transformations should lose information. The interplay between the two concepts will be exemplified for Markovian evolutions, showing how Markovianity can be defined in purely information theoretic terms. Extending on this result, we prove our main theorem: that all physical maps can be defined solely in terms of a particular metric on the space of density matrices, the Fisher information. This result should be understood in the context of reconstruction of quantum mechanics, proving once again the key role of information in shaping our description of the world.

Zoom link

INFORMATION DYNAMICS

DYNAMICS FROM INFORMATION

Matteo Scandi

DYNAMICS

$$\pi_t = \mathrm{Tr}_A \left[U \left(\pi_0 \otimes \omega_A \right) U^\dagger \right]$$

π ω

CPTP Maps

$$\mathbb{I}_A \otimes \Phi[\rho_{AB}] \geq 0$$

$$\mathrm{Tr}\left[\Phi(\rho)\right] = 1$$

A map is CP if it maps positive states into positive states (together with ancillas)

Unitaries

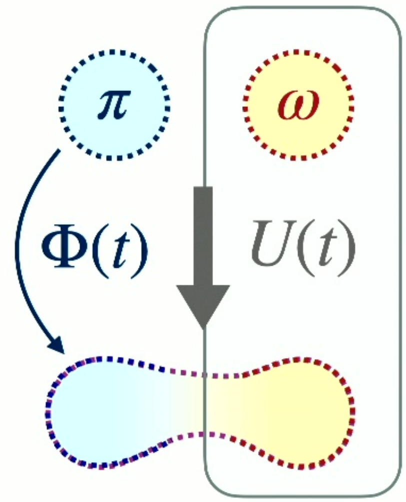
$$\mathcal{U}[\rho] = U \rho U^\dagger$$

$$U U^\dagger = \mathbb{I}$$

The evolution is physically invertible

DYNAMICS

$$\pi_t = \mathrm{Tr}_A \left[U (\pi_0 \otimes \omega_A) U^\dagger \right]$$



CPTP Maps

$$\mathbb{I}_A \otimes \Phi[\rho_{AB}] \geq 0$$

$$\mathrm{Tr}\left[\Phi(\rho)\right] = 1$$

The evolution is generated by the marginal of a unitary on a bigger system

Unitaries

$$\mathscr{U}[\rho] = U \rho U^\dagger$$

$$U U^\dagger = \mathbb{I}$$

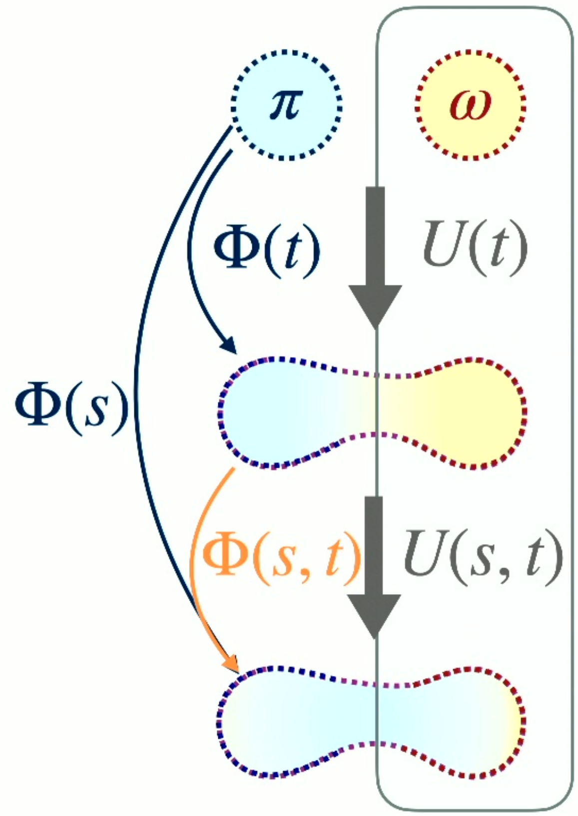
The evolution is physically invertible

DYNAMICS

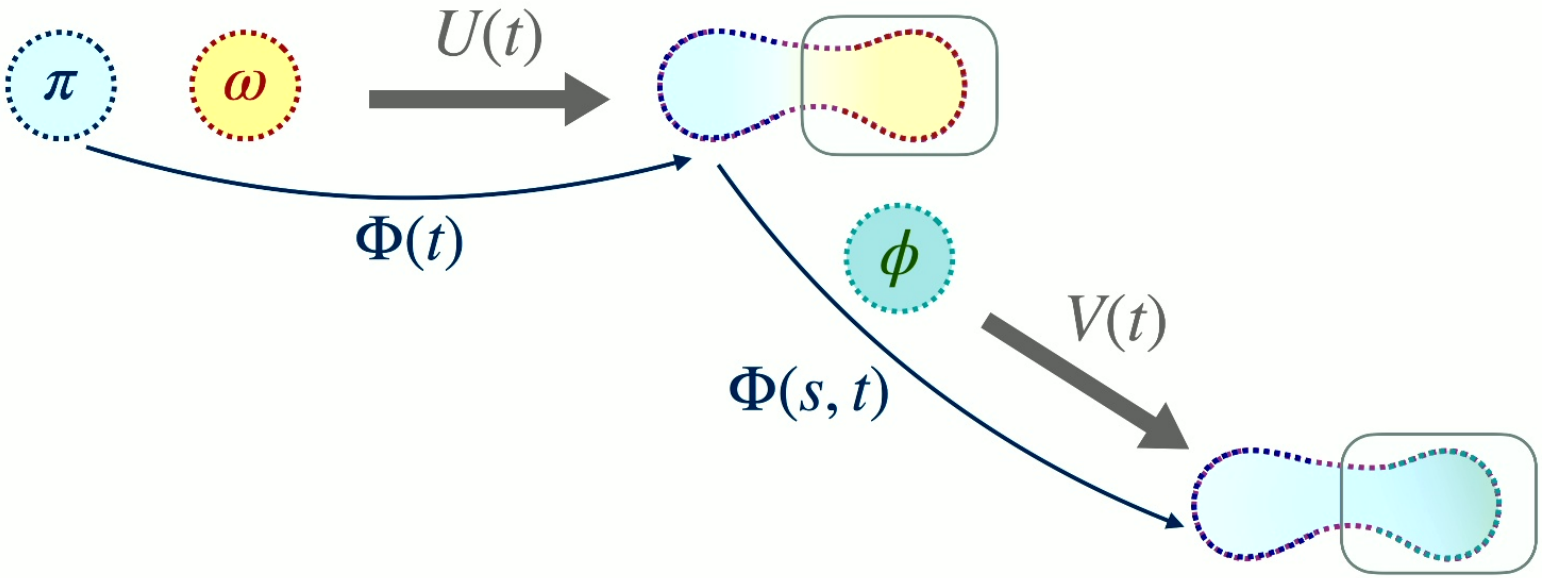
$$\Phi(s) = \Phi(s,t)\Phi(t)$$

The evolution is divisible if there exists an intermediate map $\Phi(s,t)$ for any $s \geq t$

The evolution is Markovian if all the maps $\Phi(s,t)$ are CP



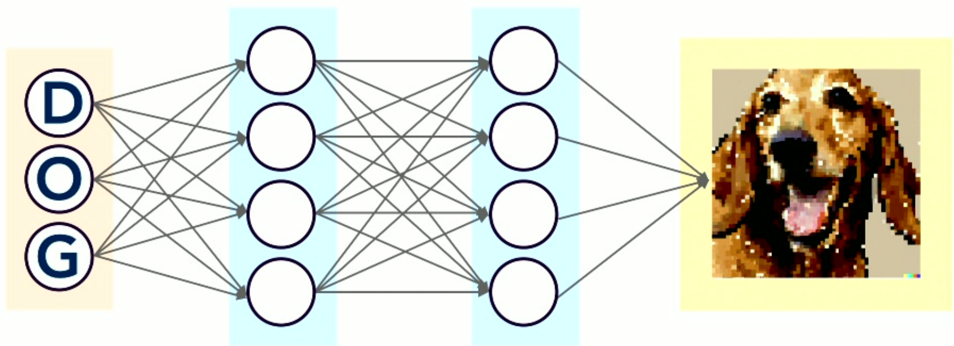
DYNAMICS



The evolution is Markovian if all the maps $\Phi(s, t)$ are CP

The evolution does not need access to the environmental states at earlier times to continue

INFORMATION



INFORMATION

INFORMATION

$p \rightarrow \{x_1, x_2, \ldots, x_N\}$

$q \rightarrow \{\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N\}$

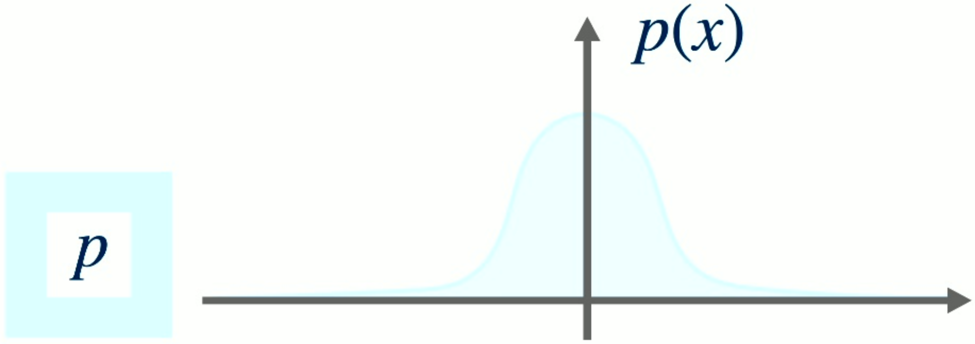
$$\frac{q(\{x_1, x_2, \ldots, x_N\})}{p(\{x_1, x_2, \ldots, x_N\})} = \prod_{i=1}^{N} \frac{q(x_i)}{p(x_i)} =$$

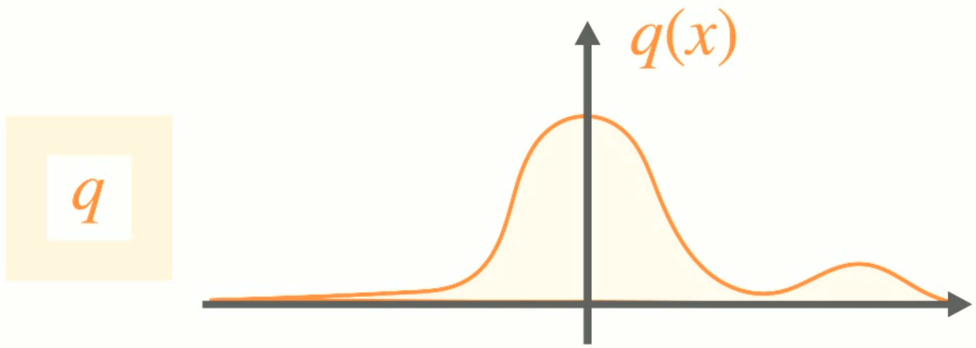
$$= \exp\left(-N \cdot H_L(p\,||\,q)\right)$$

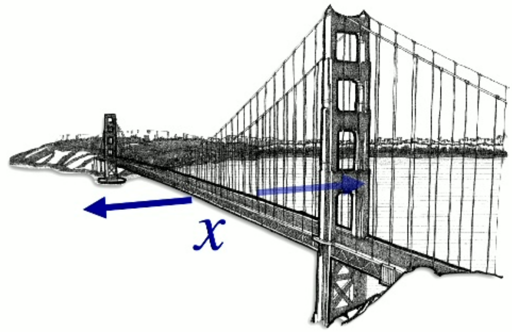
$N \gg 1$

$$-H_L(p\,||\,q) = \sum_x p(x) \log \frac{q(x)}{p(x)}$$

INFORMATION







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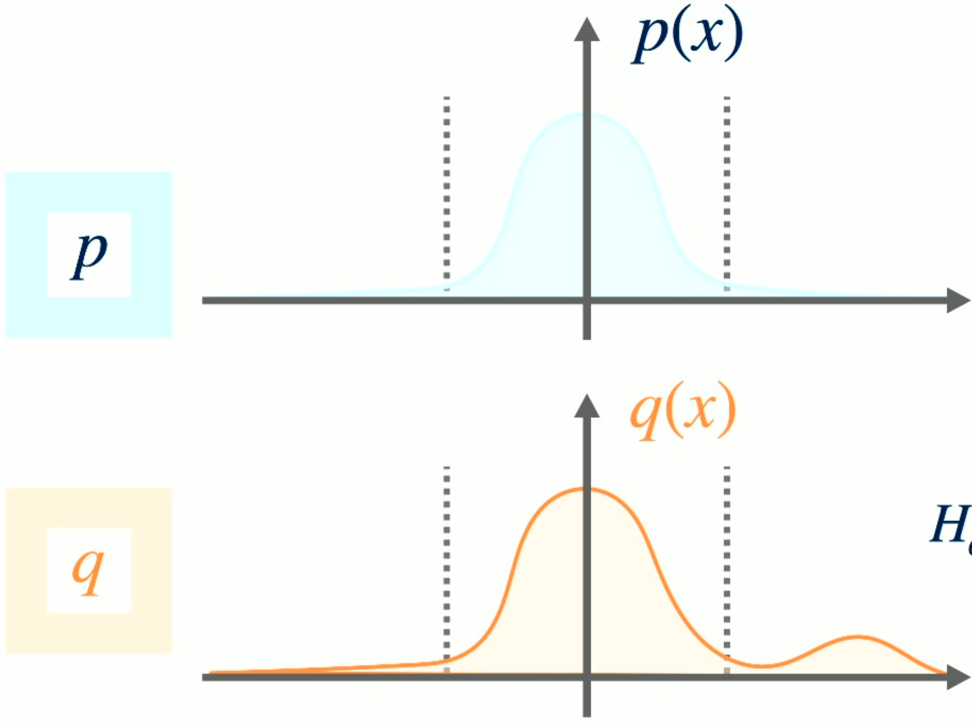

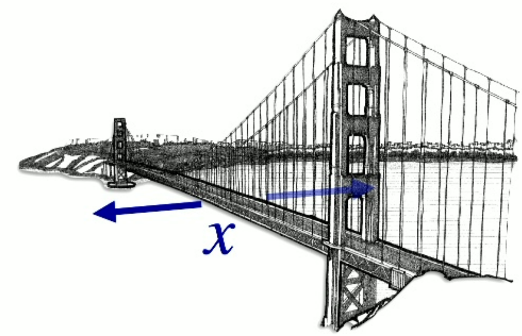

$$H_\alpha(p||q) := $$

$$= \frac{1}{\alpha(\alpha-1)} \sum_x p(x) \left(\left(\frac{q(x)}{p(x)} \right)^\alpha - 1 \right)$$

INFORMATION

$$H_\alpha(p\,||\,q) := \frac{1}{\alpha(\alpha-1)} \sum_x p(x) \left(\left(\frac{q(x)}{p(x)} \right)^\alpha - 1 \right)$$

p

q

INFORMATION

$$H_g(p\,||\,q) := \sum_x p(x)\, g\left(\frac{p(x) + \mathrm{d}p(x)}{p(x)}\right)$$

p

q

- Positivity
- Homogeneity
- Joint convexity
- Local smoothness

INFORMATION

$$H_g(p\,||\,q) := \frac{g''(0)}{2} \sum_x \frac{(\mathrm{d}p(x))^2}{p(x)} = \frac{g''(0)}{2} \sum_{x,y} \mathrm{d}p(x)\, \eta_{x,y}\Big|_p \mathrm{d}p(y)$$

$$||\mathrm{d}p|| \ll 1$$

$\eta_{x,y}\Big|_p$ is:

- Symmetric
- Positive
- Smooth in p

All contrast functions are locally described by the same quantity:

INFORMATION DYNAMICS

$H_g(p||q)$

$$p \xrightarrow{A} p' \xrightarrow{B} p'' \xrightarrow{C} p'''$$

$$q \xrightarrow{A} q' \xrightarrow{B} q'' \xrightarrow{C} q'''$$

Monotonicity

INFORMATION DYNAMICS

$H_g(p||q)$

$$p \xrightarrow{A} p' \xrightarrow{B} p'' \xrightarrow{C} p'''$$

$$H_g(p||q) \geq H_g(p'||q') \geq H_g(p''||q'') \geq H_g(p'''||q''')$$ Monotonicity

$$q \xrightarrow{A} q' \xrightarrow{B} q'' \xrightarrow{C} q'''$$

Information decreases under noisy transformations

INFORMATION DYNAMICS

$$H_g(p\,||\,q) = \frac{g''(0)}{2} \sum_{x,y} \mathrm{d}p(x)\, \eta_{x,y}\Big|_p \mathrm{d}p(y)$$

Information decreases under noisy transformations

Chentsov theorem: **the only metric that is contractive under all noisy transformations is the Fisher information metric**

Contractivity constrains the local behaviour of all contrast functions

$$H_g(p\,||\,q) := \sum_x p(x)\, g\left(\frac{q(x)}{p(x)}\right) \longrightarrow H_g(\pi\,||\,\sigma) := \mathrm{Tr}\left[g(\mathbb{L}_\sigma \mathbb{R}_\pi^{-1})[\pi]\right]$$

$$\sum_{x,y} \mathrm{d}p(x)\, \eta_{x,y}\Big|_p \mathrm{d}p(y) \longrightarrow \mathrm{Tr}\left[\delta\pi\, \mathbb{J}_{f(g)}^{-1}\Big|_\pi [\delta\pi]\right]$$

$$\mathbb{J}_B^{-1}[A] = \int_0^\infty \mathrm{d}t\, e^{-t\pi/2} A\, e^{-t\pi/2} \qquad \mathbb{J}_L^{-1}[A] = \int_0^\infty \mathrm{d}t\, (\pi + t)^{-1} A\, (\pi + t)^{-1}$$

$$\mathbb{J}_{SQ}^{-1}[A] = \sqrt{\pi^{-1}}\, A \sqrt{\pi^{-1}} \qquad \mathbb{J}_H^{-1}[A] = \frac{1}{2}\{\pi^{-1}, A\}$$

$$H_g(p||q) := \sum_x p(x)\, g\left(\frac{q(x)}{p(x)}\right) \longrightarrow H_g(\pi||\sigma) := \mathrm{Tr}\left[g(\mathbb{L}_\sigma \mathbb{R}_\pi^{-1})[\pi]\right]$$

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$g(x)$	$H_g(\rho\|\|\sigma)$	$f(x)$	$\mathbb{J}_{f(g)}[A]$	$\mathbb{J}_{f(g)}^{-1}[A]$
$\frac{(x-1)^2}{x+1}$	$\mathrm{Tr}\left[(\rho-\sigma)(\mathbb{L}_\sigma+\mathbb{R}_\rho)^{-1}(\rho-\sigma)\right]$	$\frac{x+1}{2}$	$\frac{1}{2}\{\pi, A\}$	$\int_0^\infty \mathrm{d}t\, e^{-t\pi/2} A\, e^{-t\pi/2}$
$\frac{(x-1)^2}{x^\gamma+x^{1-\gamma}}$	$\mathrm{Tr}\left[(\rho-\sigma)(\mathbb{L}_\sigma^\gamma\mathbb{R}_\rho^{1-\gamma}+\mathbb{L}_\sigma^{1-\gamma}\mathbb{R}_\rho^\gamma)^{-1}[(\rho-\sigma)]\right]$	$\frac{x^\gamma+x^{1-\gamma}}{2}$	$\frac{1}{2}\{\pi^{1-2\gamma}, \pi^\gamma A\, \pi^\gamma\}$	$\int_0^\infty \mathrm{dt}\, \mathrm{e}^{-(\mathrm{t}\pi^{1-2\gamma})/2}\, \pi^{-\gamma} \mathrm{A}\, \pi^{-\gamma}\, \mathrm{e}^{-(\mathrm{t}\pi^{1-2\gamma})/2}$
$\frac{x^\alpha-1}{\alpha(\alpha-1)}$	$\frac{1}{\alpha(\alpha-1)}(\mathrm{Tr}\left[\sigma^\alpha\rho^{1-\alpha}\right]-1)$	$\frac{\alpha(\alpha-1)(x-1)^2}{(x-x^\alpha)(x^{-\alpha}-1)}$	???	$\int_0^\alpha \frac{\mathrm{d}\beta}{\alpha(1-\alpha)}\int_\beta^{1-\beta} \mathrm{d}\gamma\, \mathbb{J}_L^{-2}[\pi^{1-\gamma} A\, \pi^\gamma]$
$4(1-\sqrt{x})$	$4(1-\mathrm{Tr}[\sqrt{\rho}\sqrt{\sigma}])$	$\frac{1}{4}\left(1+\sqrt{x}\right)^2$	$\frac{1}{4}\{\sqrt{\pi}, \{\sqrt{\pi}, A\}\}$	$\int_0^\infty \mathrm{d}t \int_0^\infty \mathrm{d}s\, e^{-(t+s)\sqrt{\pi}/2} A\, e^{-(t+s)\sqrt{\pi}/2}$
$-\log x$	$\mathrm{Tr}\left[(\rho\,(\log\rho-\log\sigma)\right]$	$\frac{x-1}{\log x}$	$\mathbb{J}_L = \int_0^1 \mathrm{d}t\, \pi^{1-t} A\, \pi^t$	$\mathbb{J}_L^{-1} = \int_0^\infty \mathrm{d}t\, (\pi+t)^{-1} A\, (\pi+t)^{-1}$
$\frac{1}{2}(\log x)^2$	$\mathrm{Tr}\left[(\rho\,(\log\rho-\log\sigma)^2\right]$	$\frac{2\,(x-1)^2}{(x+1)(\log x)^2}$	$\int_0^\infty \mathrm{d}t\, e^{-t\pi/2}\, \mathbb{J}_L^2[A]\, e^{-t\pi/2}$	$\frac{1}{2}\mathbb{J}_L^{-2}\left[\{\pi, A\}\right]$
$\sqrt{x^{-1}}-\sqrt{x}$	$\mathrm{Tr}\left[\sqrt{\rho}(\rho-\sigma)\sqrt{\sigma^{-1}}\right]$	$\sqrt{x}$	$\sqrt{\pi}\, A \sqrt{\pi}$	$\sqrt{\pi^{-1}}\, A \sqrt{\pi^{-1}}$
$\frac{(x-1)^2}{4x(x^\gamma+x^{1-\gamma})^{-1}}$	$\mathrm{Tr}\left[(\rho-\sigma)(\mathbb{L}_\sigma^{\gamma-1}\mathbb{R}_\rho^{-\gamma}+\mathbb{L}_\sigma^{-\gamma}\mathbb{R}_\rho^{\gamma-1})[(\rho-\sigma)]\right]$	$\frac{2x}{x^\gamma+x^{1-\gamma}}$	$\int_0^\infty \mathrm{dt}\mathrm{e}^{-(\mathrm{t}\pi^{2\gamma-1})/2}\, \pi^\gamma \mathrm{A}\, \pi^\gamma\, \mathrm{e}^{-(\mathrm{t}\pi^{2\gamma-1})/2}$	$\frac{1}{2}\{\pi^{2\gamma-1}, \pi^{-\gamma} A\, \pi^{-\gamma}\}$
$\frac{(x-1)^2}{2}$	$\frac{1}{2}(\mathrm{Tr}\left[\sigma^2\rho^{-1}\right]-1)$	$\frac{2x}{x+1}$	$\int_0^\infty \mathrm{d}t\, e^{-t\pi^{-1}/2} A\, e^{-t\pi^{-1}/2}$	$\frac{1}{2}\{\pi^{-1}, A\}$

$$H_g(p\,||\,q) := \sum_x p(x)\, g\left(\frac{q(x)}{p(x)}\right) \longrightarrow H_g(\pi\,||\,\sigma) := \mathrm{Tr}\left[g(\mathbb{L}_\sigma \mathbb{R}_\pi^{-1})[\pi]\right]$$

$$\sum_{x,y} \mathrm{d}p(x)\, \eta_{x,y}\Big|_p \mathrm{d}p(y) \longrightarrow \mathrm{Tr}\left[\delta\pi\, \mathbb{J}_{f(g)}^{-1}\Big|_\pi [\delta\pi]\right]$$

All contrast functions are locally described by the same family

The Quantum Fisher Information metrics

These are the only metrics contracting under physical maps

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$\frac{(x-1)^2}{4x(x^\gamma+x^{1-\gamma})^{-1}}$	$\mathrm{Tr}\left[(\rho-\sigma)(\mathbb{L}_\sigma^{\gamma-1}\mathbb{R}_\rho^{-\gamma}+\mathbb{L}_\sigma^{-\gamma}\mathbb{R}_\rho^{\gamma-1})[(\rho-\sigma)]\right]$	$\frac{2x}{x^\gamma+x^{1-\gamma}}$	$\int_0^\infty \mathrm{dt}\mathrm{e}^{-(\mathrm{t}\pi^{2\gamma-1})/2}\, \pi^\gamma \mathrm{A}\, \pi^\gamma\, \mathrm{e}^{-(\mathrm{t}\pi^{2\gamma-1})/2}$	$\frac{1}{2}\{\pi^{2\gamma-1}, \pi^{-\gamma} A\, \pi^{-\gamma}\}$
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All contrast functions are locally described by the same family

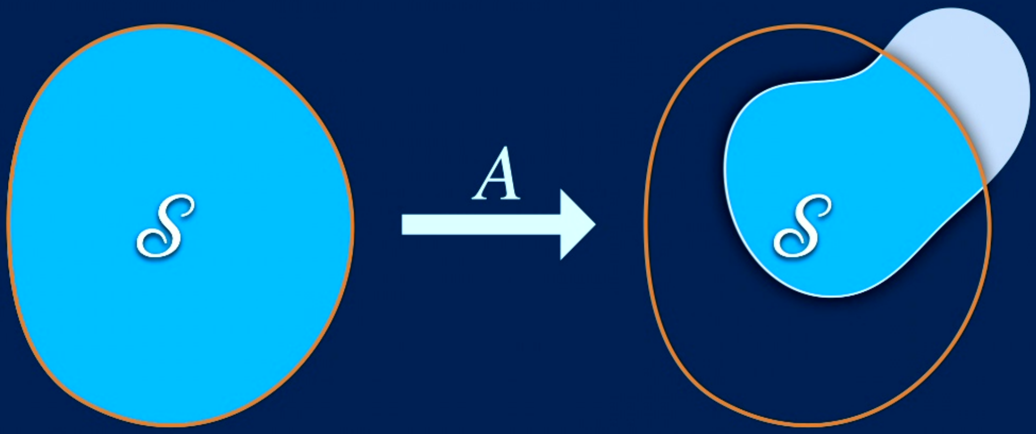
The Quantum Fisher Information metrics

These are the only metrics contracting under physical maps

DYNAMICS FROM INFORMATION

THEOREM 1: *Consider a linear map A that:*

- *maps at least one positive vector into a positive vector*
- *is Fisher contractive on positive vectors*



DYNAMICS FROM INFORMATION

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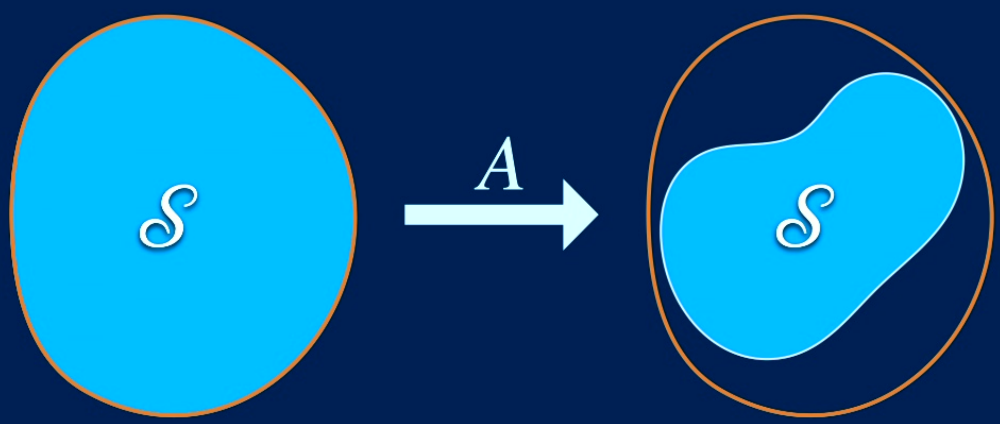


Then $A(\mathscr{S}) \subseteq \mathscr{S}$

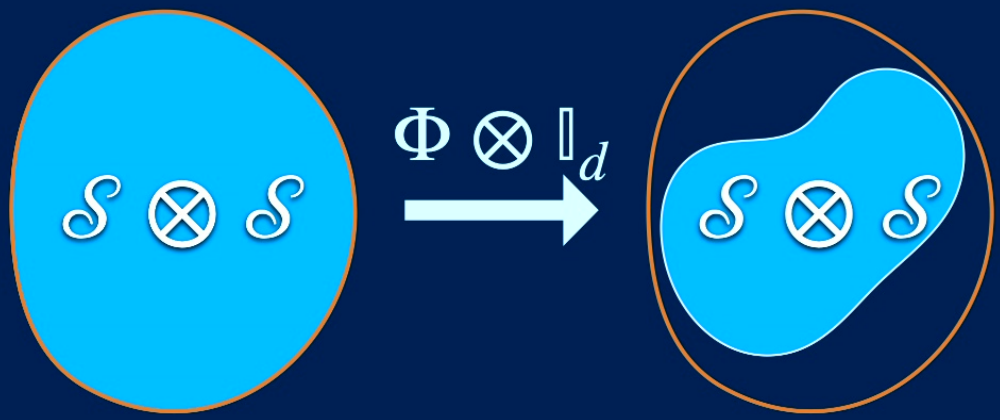
$\Updownarrow$

A *is physical*

DYNAMICS FROM INFORMATION

COROLLARY: *Consider a quantum channel Φ that:*

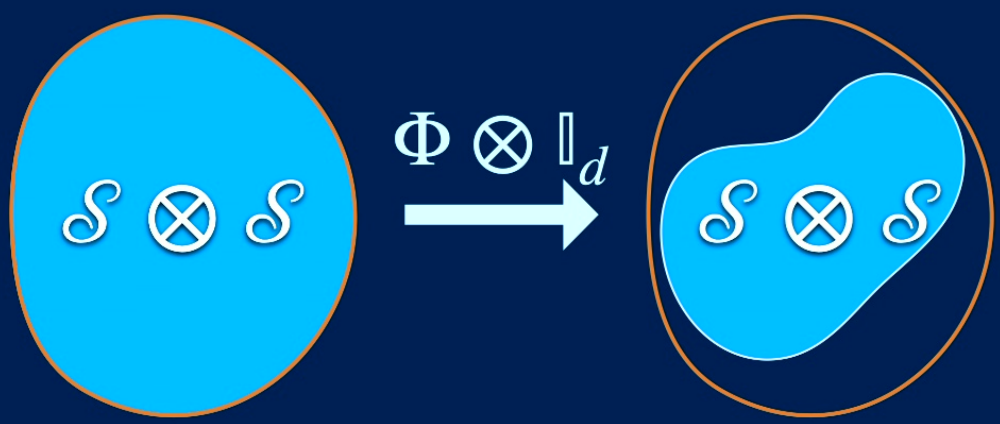
- *maps at least one positive vector into a positive vector*



DYNAMICS FROM INFORMATION

COROLLARY: *Consider a quantum channel Φ that:*

- *maps at least one positive vector into a positive vector*
- $\Phi \otimes \mathbb{I}_d$ *is Fisher contractive*



Then

$$\Phi \otimes \mathbb{I}_d(\mathcal{S} \otimes \mathcal{S}) \subseteq \mathcal{S} \otimes \mathcal{S}$$

$\Updownarrow$

Φ *is physical*

DYNAMICS FROM INFORMATION

PHYSICAL EVOLUTIONS

STATISTICAL QUANTIFIERS

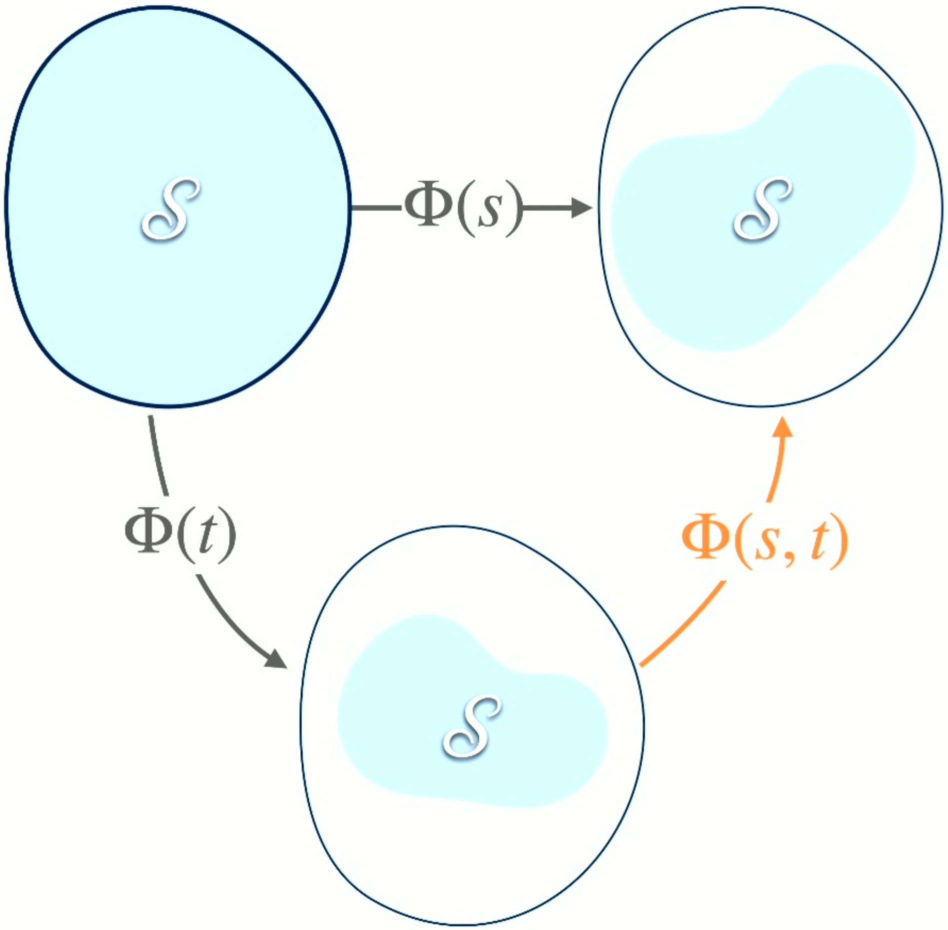
DYNAMICS FROM INFORMATION

PHYSICAL EVOLUTIONS

CHENTSOV THEOREM ↓ ↑ THEOREM 1

FISHER INFORMATION

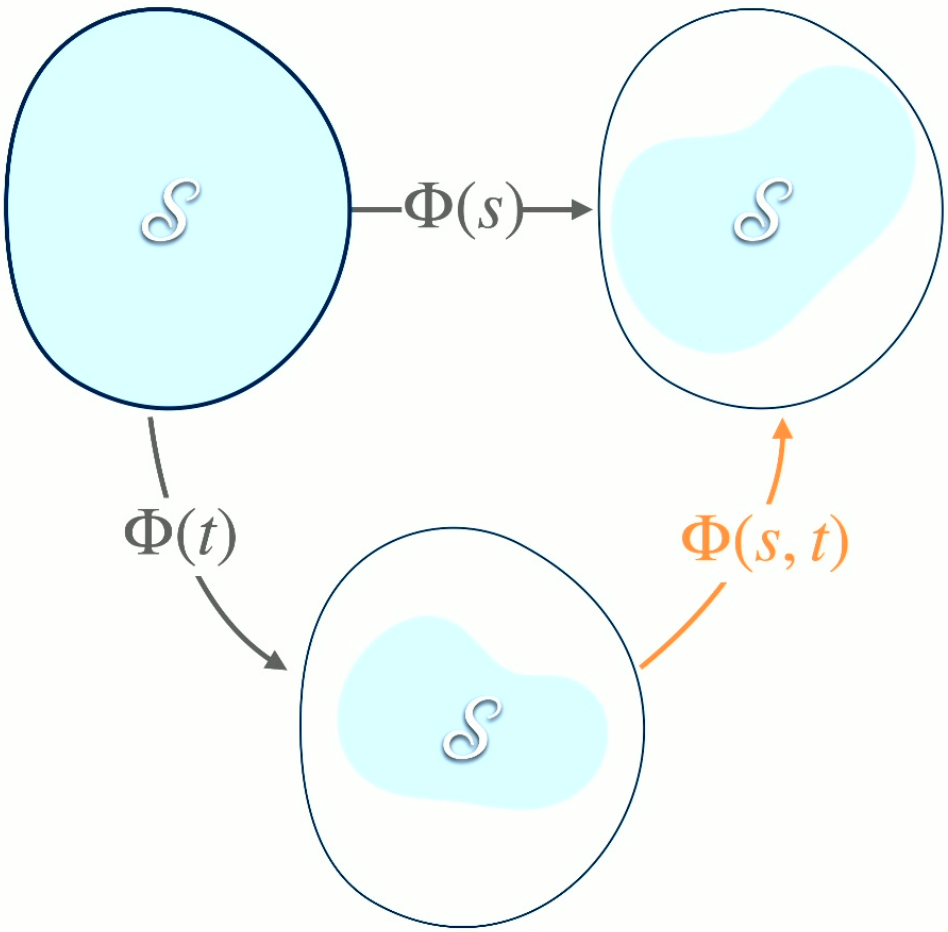
MARKOVIANITY



$$H_g(\rho\,||\,\sigma) \geq H_g\big(\Phi(t)[\rho]\,||\,\Phi(t)[\sigma]\big)$$

$$H_g(\rho\,||\,\sigma) \geq H_g\big(\Phi(s)[\rho]\,||\,\Phi(s)[\sigma]\big)$$

MARKOVIANITY



$$H_g(\rho\,||\,\sigma) \geq H_g\big(\Phi(t)[\rho]\,||\,\Phi(t)[\sigma]\big)$$

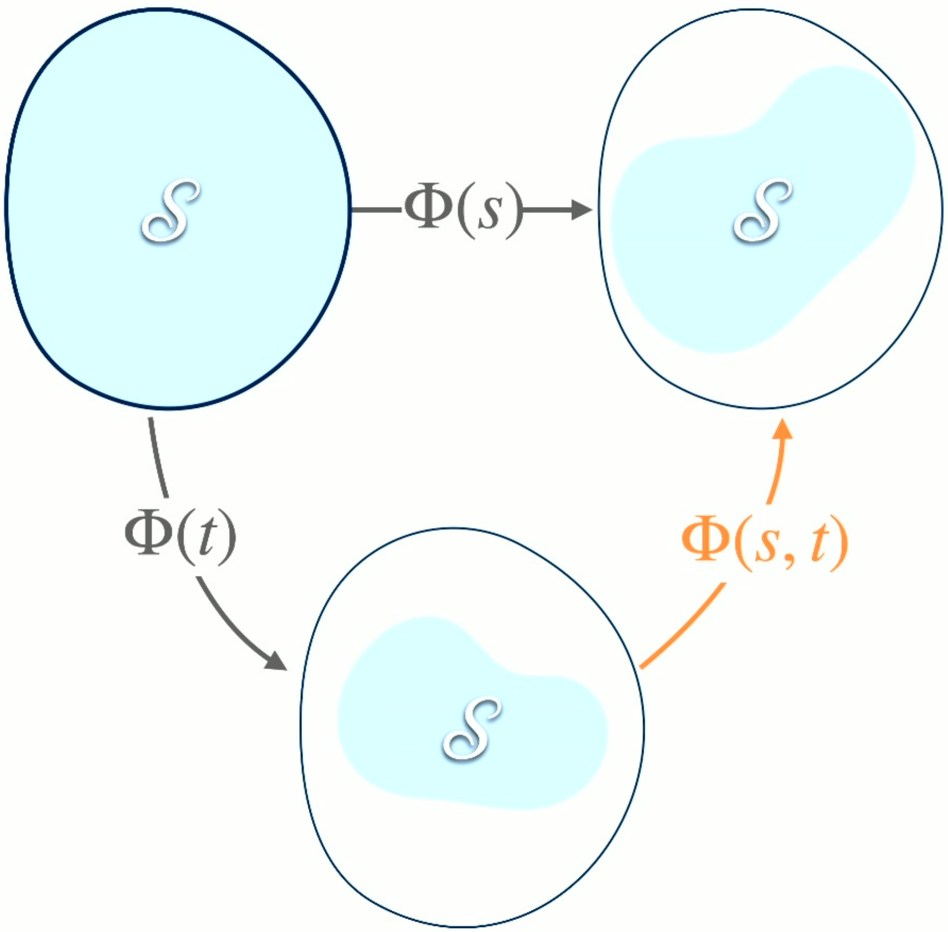
Markovian $\quad\quad \geq$

$$H_g\big(\Phi(s,t)\Phi(t)[\rho]\,||\,\Phi(s,t)\Phi(t)[\sigma]\big)$$

$$=$$

$$H_g(\rho\,||\,\sigma) \geq H_g\big(\Phi(s)[\rho]\,||\,\Phi(s)[\sigma]\big)$$

MARKOVIANITY



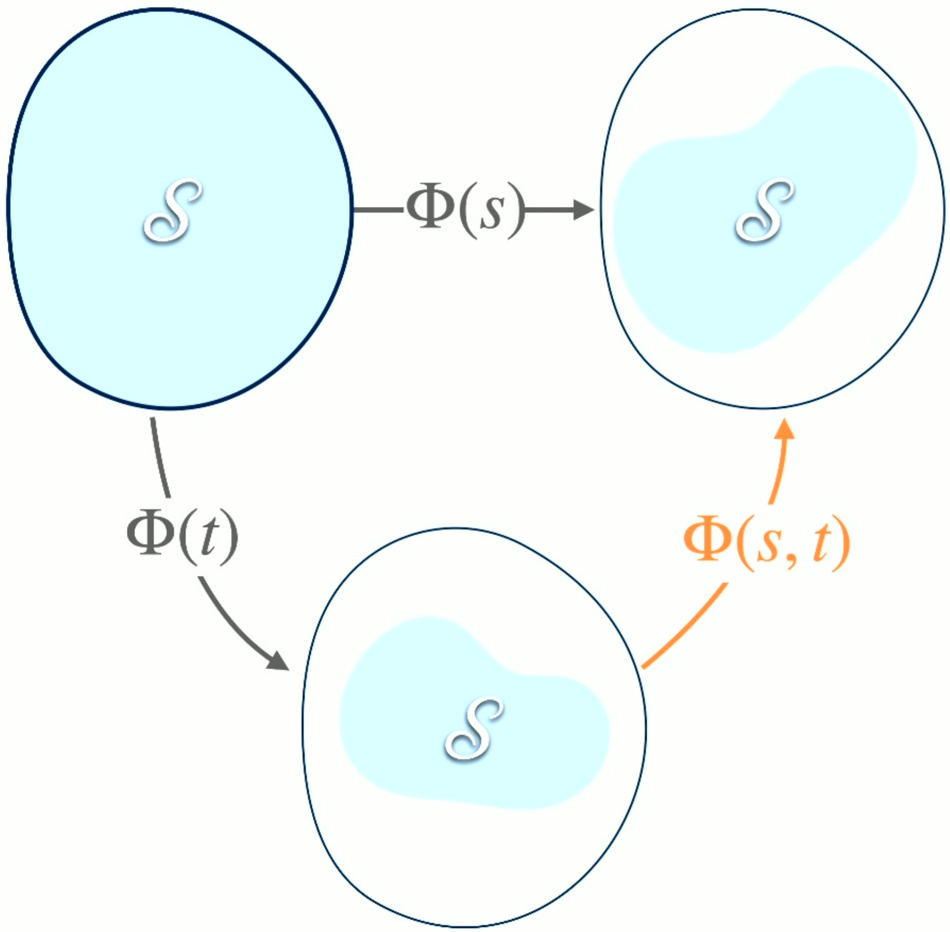
Markovianity implies a monotonic contractivity of information quantities

$$\frac{d}{dt}\mathcal{I}_f(\Phi(t)[\rho] \,\|\, \Phi(t)[\sigma]) \leq 0$$

THEOREM 1:

Monotonic contractivity of information quantities implies Markovianity

MARKOVIANITY



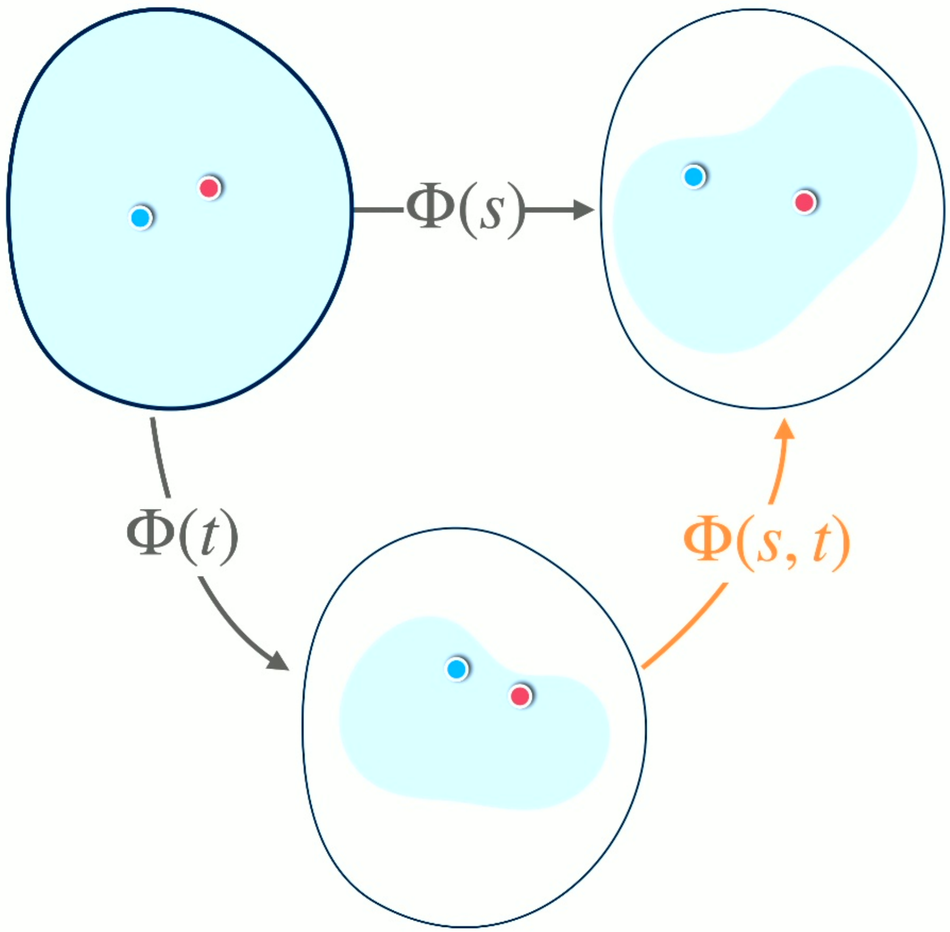
Markovianity implies a monotonic contractivity of information quantifiers

$$\frac{\mathrm{d}}{\mathrm{d}t} H_g\big(\Phi(t)[p] \,||\, \Phi(t)[q]\big) \leq 0$$

THEOREM 1:

Monotonic contractivity of information quantifiers implies Markovianity

MARKOVIANITY

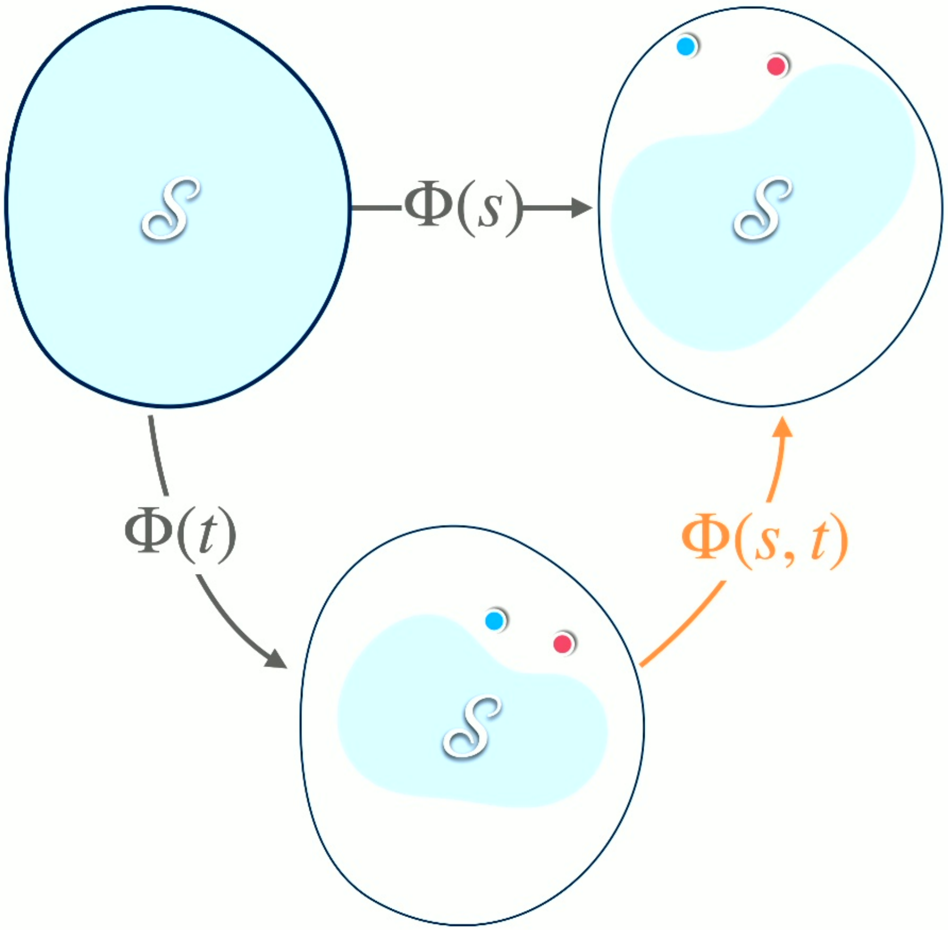


NON MARKOVIANITY

⇕

EXPANSION OF THE FISHER INFORMATION ON THE IMAGE

MARKOVIANITY



NON MARKOVIANITY

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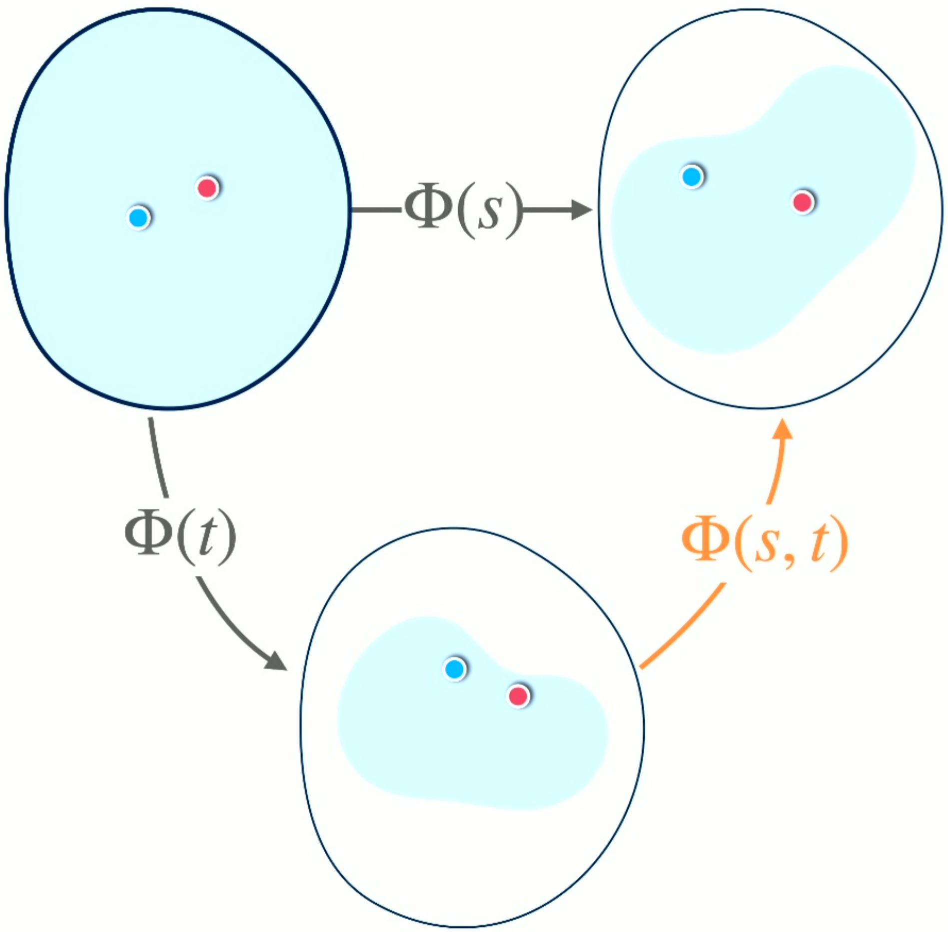
EXPANSION OF THE FISHER INFORMATION

MARKOVIANITY



NON MARKOVIANITY

↕

EXPANSION OF THE
FISHER INFORMATION
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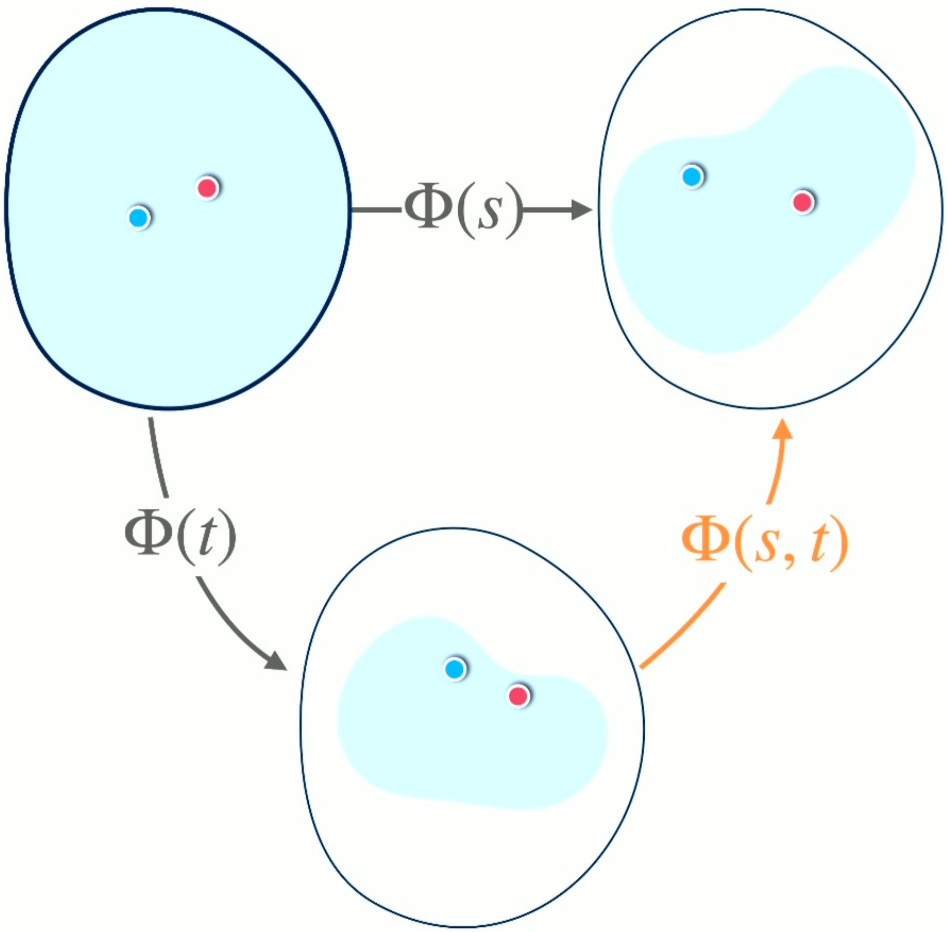
MARKOVIANITY



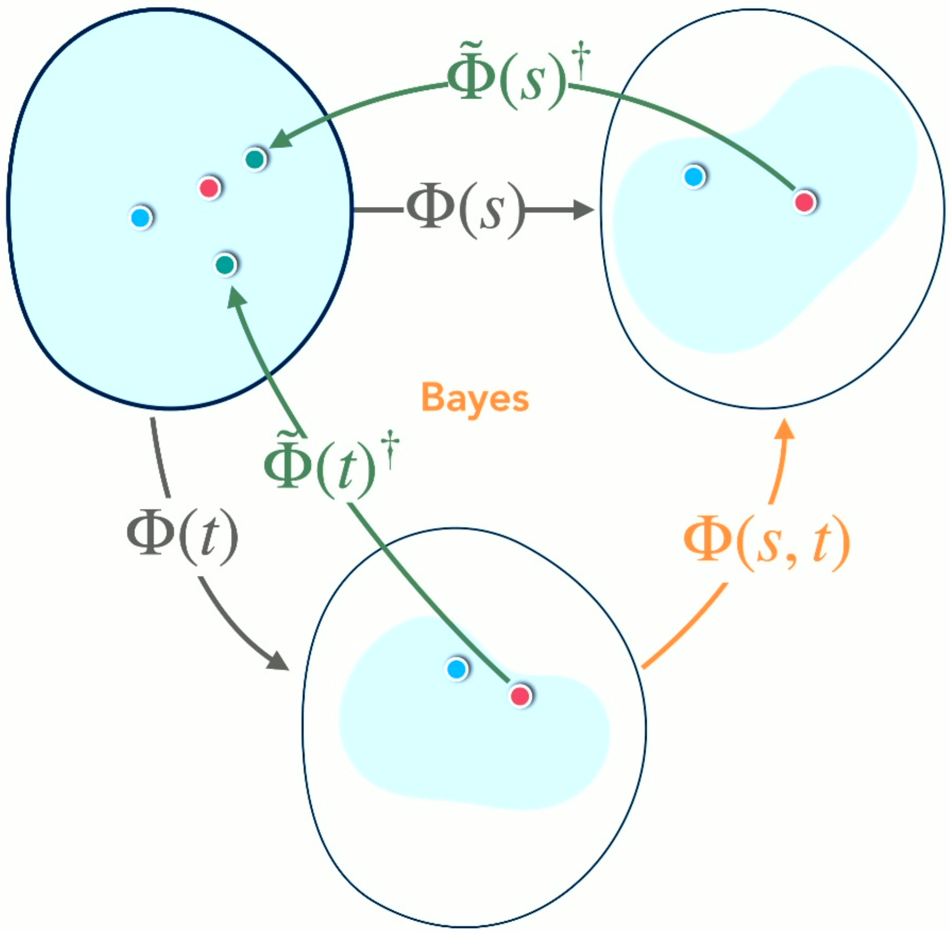
NON MARKOVIANITY

⇕

EXPANSION OF THE FISHER INFORMATION ON THE IMAGE

$$\langle X \,|\, \Phi Y \rangle_{\Phi(\rho)} = \langle \tilde{\Phi}^{\dagger} X \,|\, Y \rangle_{\rho}$$

MARKOVIANITY



NON MARKOVIANITY

EXPANSION OF THE FISHER INFORMATION ON THE IMAGE

IMPROVED RETRODICTION

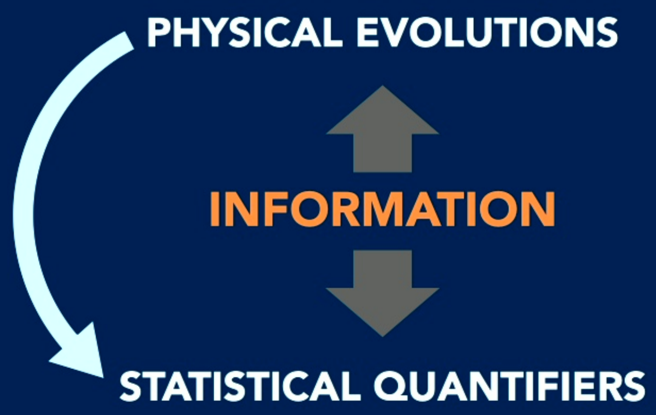
$$\langle X \,|\, \Phi Y \rangle_{\Phi(\rho)} = \langle \tilde{\Phi}^\dagger X \,|\, Y \rangle_\rho$$

CONCLUSIONS

PHYSICAL EVOLUTIONS

STATISTICAL QUANTIFIERS

CONCLUSIONS



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I. Introduction

II. From quantum contrast functions to quantum Fisher information
Contrast functions are introduced, and their general properties are discussed. Thm. 1 gives a characterisation of their local behaviour in terms of a family of functionals. The same objects also appear in the characterisation of monotone metrics on the space of states (Thm. 2). These are the two ways of defining the family of quantum Fisher information metrics. The informative boxes of this sections are dedicated to give more details about the relation between contrast functions and their g function (Box 1), provide a first characterisation of standard monotones (Box 2) and describe other possible quantifiers of statistical difference (Box 3).

III. Dynamical properties of Fisher information
The equivalence between the positivity of linear maps and the contraction of Fisher information is proved (Thm. 3). This implies that one can define the physicality of an evolution by looking at whether it contracts everywhere the Fisher information on the space of states tensor an ancilla of the same dimension (Corollary 3.1). In Box 4 the same results are derived by considering contrast functions.

A. Markovianity as monotonous contraction of Fisher information
A novel expression of the Fisher information currents is presented (Thm. 4). The relation between Fisher information and Markovianity is completely characterised: it is shown that the monotonous contraction of Fisher information on the space of states (adjoined with an ancilla of the same dimension) is equivalent to the Markovianity of the corresponding evolution (Thm. 5). Moreover, it is discussed how, despite this deep connection between the two notions, it is impossible to operationally witness non-Markovianity through the expansion of Fisher information for all evolutions only by the use of extra ancillas and copies of the channel (Thm. 6). Still, if one allows for a post-processing at the end of the dynamics, it is also shown that one can actually provide an operational witness (Thm. 7).

B. Retrodiction and Fisher information
A generalised version of the Petz recovery map is introduced, and it is shown how this allows to canonically map evolved states (close to a prior) back into their initial condition. Markovianity is then equivalent to a monotonous increase of the error committed during this procedure (Thm. 9). In Box 9 we discuss the topic of universal recovery, and we give some preliminary results using χ^2_f*-divergences (Thm. 10 and corollaries thereof). Moreover, we find that the traditional Petz recovery map can be characterised in two different ways: either as the unique universal recovery map that is a quantum channel (Corollary 10.1 and 10.2); or as the one whose spectrum dominates all other maps in the family (Thm. 11).*

C. Fisher information and detailed balance
Detailed balance corresponds to a stronger form of equilibration in which the dynamics appears to be at equilibrium with its reverse process close to the steady state. In Thm. 12 we show how this condition can naturally be formulated for classical systems in terms of the self-adjointness of the generator with respect to the Fisher scalar product. A similar result is also proven for quantum dynamics (Thm. 13) where we find a slightly more general form of the Lindbladian compared to the most used choice in the literature.

IV. Mathematical properties of quantum Fisher information

A. The set of standard monotone functions
The set of standard monotone functions is characterised, focusing on an important symmetry of the set and its convex structure, generated by a continuous family of vertices.

B. Properties of quantum Fisher operators
The way in which the properties of the defining functions are mirrored on their corresponding Fisher operators is presented, giving a first general characterisation.

C. Complete positivity of the Fisher information operators
A full characterisation of the complete positive Fisher information operator is provided. In particular, Thm. 14 gives the most general expression for such maps.

V. A garden of quantum Fisher information
We present the most notable examples of quantum Fisher operators, summarised in table 4, and in Fig. 5.

VI. Conclusions and open questions

Thank you for the attention

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arXiv:2204.04072

arXiv:2304.14984

arXiv:2310.16107

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