Title: Simulating 2D lattice gauge theories on a qudit quantum computer

Speakers:

Collection: Foundations of Quantum Computational Advantage

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Abstract: Particle physics underpins our understanding of the world at a fundamental level by describing the interplay of matter and forces through gauge theories. Yet, despite their unmatched success, the intrinsic quantum mechanical nature of gauge theories makes important problem classes notoriously difficult to address with classical computational techniques. A promising way to overcome these roadblocks is offered by quantum computers, which are based on the same laws that make the classical computations so difficult. Here, we present a quantum computation of the properties of the basic building block of two-dimensional lattice quantum electrodynamics, involving both gauge fields and matter. This computation is made possible by the use of a trapped-ion qudit quantum processor, where quantum information is encoded in d different states per ion, rather than in two states as in qubits. Qudits are ideally suited for describing gauge fields, which are naturally high-dimensional, leading to a dramatic reduction in the quantum register size and circuit complexity. Using a variational quantum eigensolver we find the ground state of the model and observe the interplay between virtual pair creation and quantized magnetic field effects. The qudit approach further allows us to seamlessly observe the effect of different gauge field truncations by controlling the qudit dimension. Our results open the door for hardware-efficient quantum simulations with qudits in near-term quantum devices.

Simulating 2D lattice gauge theories on a qudit quantum computer

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Foundations of Quantum Computational Advantage
30 April 2024, Perimeter Institute

UNIVERSITY OF WATERLOO | IQC Institute for Quantum Computing

Outline

M Meth, JF Haase, J Zhang, C Edmunds, L Postler, AJ Jena, A Steiner,
L Dellantonio, R Blatt, P Zoller, T Monz, P Schindler, C Muschik, M Ringbauer
arXiv:2310.12110

- Motivation and background
- The model: QED in (2+1)D
- Using qudits to simulate matter and gauge fields
- Using qudits to refine truncation effects
- Summary & Outlook

Why simulations?

Fundamental laws are known:

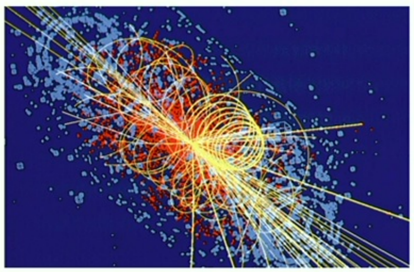
- Quantum chemistry
- Condensed matter systems
- Standard model of high energy physics
- …

… but analytical results are not feasible
-> need numerical simulations!

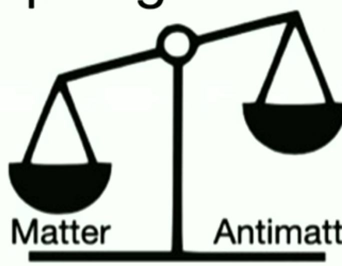
Classical lattice QCD methods

Sign problem
afflicted settings:

Real-time evolution



Topological terms



High baryon density

Quantum computing: a possible solution?

Hamiltonian formulation:
sign-problem free

J. Kogut and L. Susskind, Hamiltonian Formulation of Wilson's Lattice Gauge Theories, Phys. Rev. D 11, 395 (1975).

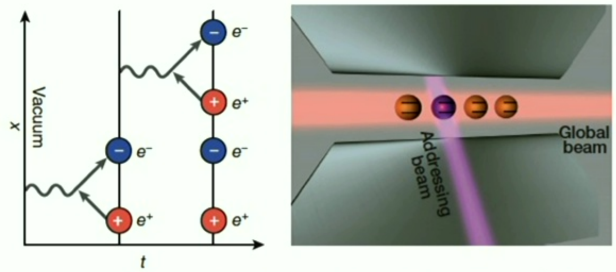
Quantum computing: a possible solution?

How do we simulate Nature with quantum computers?

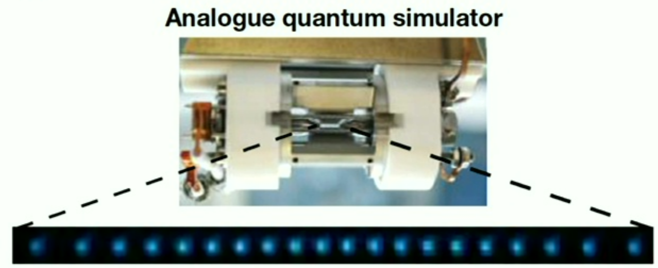
- How do we describe (1, 2, 3)D systems?
- How do we study colour and flavour degrees of freedom?
- How to best utilise quantum resources?
- Which are the most suitable algorithms?

- **First theoretical proposal**: S. P. Jordan, K. S. M. Lee, and J. Preskill, Quantum Algorithms for Quantum Field Theories, Science 336, 1130 (2012).
- **First experiment**: E. A. Martinez et al., Real-Time Dynamics of Lattice Gauge Theories with a Few-Qubit Quantum Computer, Nature 534, 516 (2016).
- **White paper**: C. W. Bauer et al., Quantum Simulation for High Energy Physics, arXiv:2204.03381.
- Di Meglio et al., Quantum Computing for High-Energy Physics: State of the Art and Challenges, arXiv:2307.03236

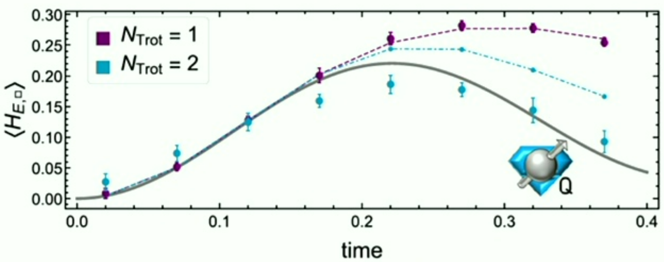
Quantum computing: a possible solution?



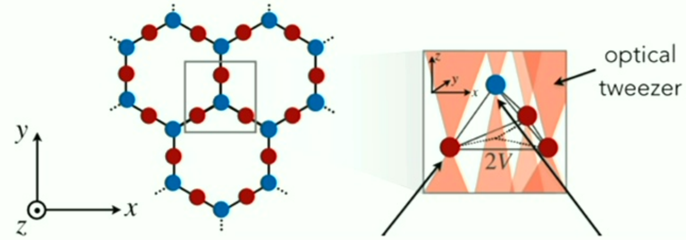
E. A. Martinez et al., Nature 534, 516 (2016).



C. Kokail et al., Nature 569, 355 (2019).



N. Klco et al., Phys. Rev. D **101**, 074512 (2020).



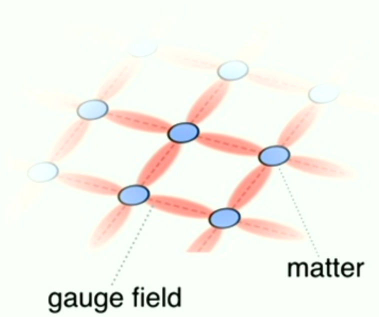
L. Homeier et al., Commun Phys **6**, 1 (2023).

Many more...!

Why qudits?

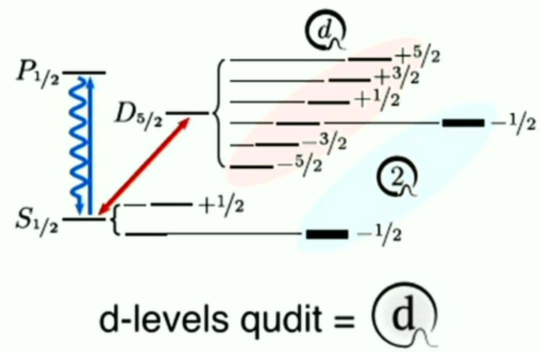
Natural have multilevel systems:

- gauge fields
- locally gauge invariant dressed basis [see G. Calajò et al., arXiv:2402.07987]

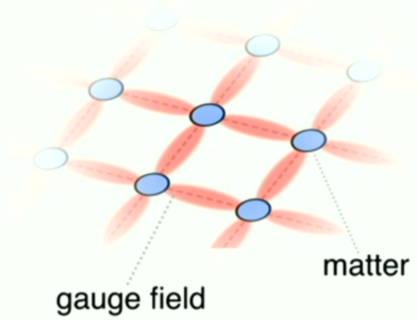


Hardware-efficient quantum simulation:

- smaller registers
- low-depth circuits
- high-fidelity entangling gates



The model: 2D-QED



The model: 2D-QED

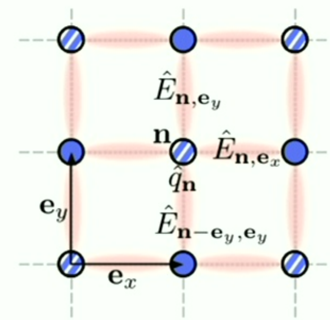
$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2}\hat{H}_B + m\hat{H}_m + \Omega\hat{H}_k$$

$$\hat{H}_E = \frac{1}{2}\sum_{\mathbf{n}}\left(\hat{E}^2_{\mathbf{n},\mathbf{e}_x} + \hat{E}^2_{\mathbf{n},\mathbf{e}_y}\right),$$

$$\hat{H}_B = -\frac{1}{2}\sum_{\mathbf{n}}\left(\hat{P}_{\mathbf{n}} + \hat{P}^\dagger_{\mathbf{n}}\right),$$

$$\hat{H}_m = \sum_{\mathbf{n}}(-1)^{n_x+n_y}\hat{\phi}^\dagger_{\mathbf{n}}\hat{\phi}_{\mathbf{n}},$$

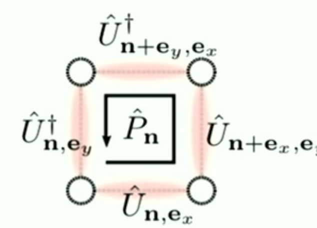
$$\hat{H}_k = \sum_{\mathbf{n}}\Big(i\hat{\phi}^\dagger_{\mathbf{n}}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_x}\hat{\phi}_{\mathbf{n}+\mathbf{e}_x} + (-1)^{n_x+n_y+1}\hat{\phi}^\dagger_{\mathbf{n}}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_y}\hat{\phi}_{\mathbf{n}+\mathbf{e}_y} + \text{H.c.}\Big).$$



Gauss' law

$$\hat{G}_{\mathbf{n}}|\Psi_{\text{phys}}\rangle = 0,$$

$$\hat{G}_{\mathbf{n}} = \sum_{\mu}(\hat{E}_{\mathbf{n},\mathbf{e}_\mu} - \hat{E}_{\mathbf{n}-\mathbf{e}_\mu,\mathbf{e}_\mu}) - \hat{q}_{\mathbf{n}}$$



How do we map the fields on a quantum processor?

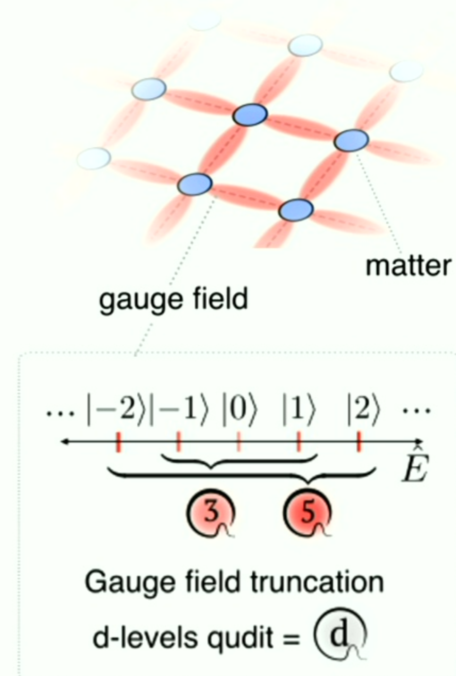
The model: 2D-QED

Matter field:

- Use Jordan-Wigner transformation to map to two-level qubit 

Gauge field:

- Approximate U(1) with $\mathbb{Z}_{2L+1}$
- Truncate to $d = 2l + 1$ relevant states to be included in the simulation, we choose $L = l + 1$
- Map the gauge field onto a d-level qudit 



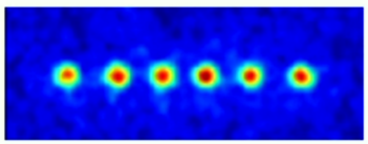
D. Paulson et al., PRX Quantum 2, 030334 (2021)

Ground state calculation: VQE

Cost function

$C(\theta) = \langle \Psi(\theta) | H | \Psi(\theta) \rangle$

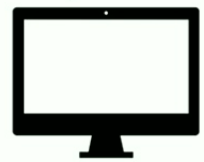
Trapped ion quantum processor



Parametrized ansatz state

$|\Psi(\theta)\rangle = U(\theta)|\Psi_0\rangle$

Classical optimization algorithm

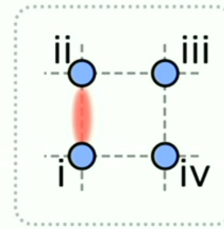


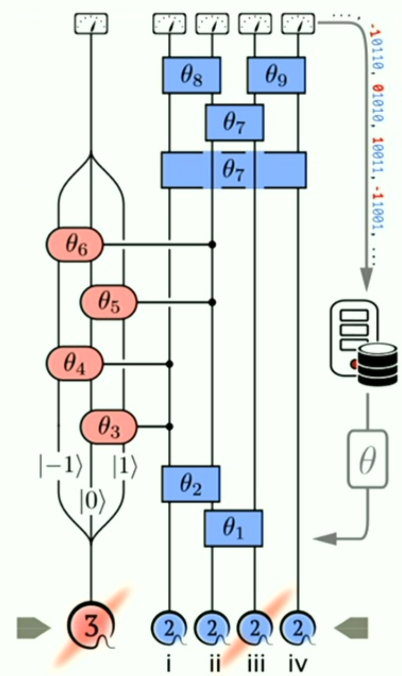
Variational parameters θ

Hybrid qubit-qudit circuit

- Consider a single plaquette with open boundary condition
- Gauss Law can be used to eliminate three gauge field degrees of freedom

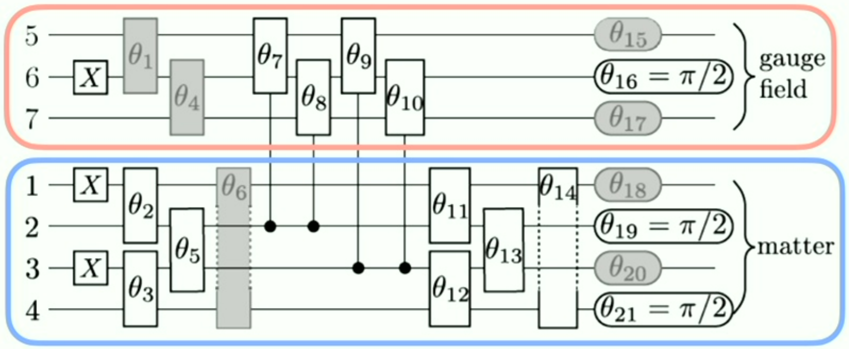
- Variational circuit inspired by the form of the Hamiltonian
- Study expectation value of $\hat{\square} = -\frac{1}{V}\hat{H}_B$



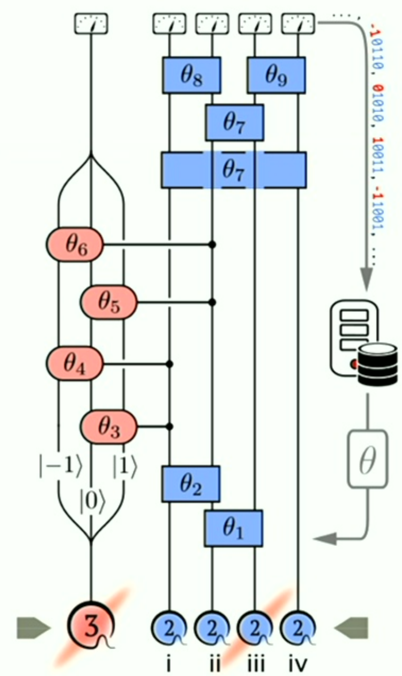


D. Paulson et al., PRX Quantum 2, 030334 (2021)

Hybrid qubit-qudit circuit

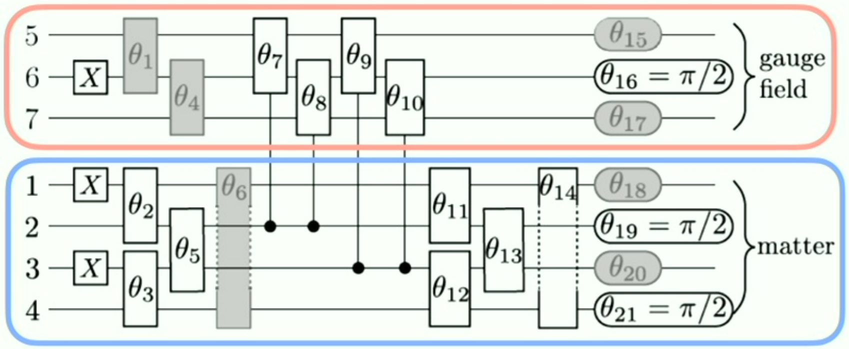


- One-hot encoding that uses d qubits for a d-level system $|0,\ldots,010,\ldots,0\rangle$

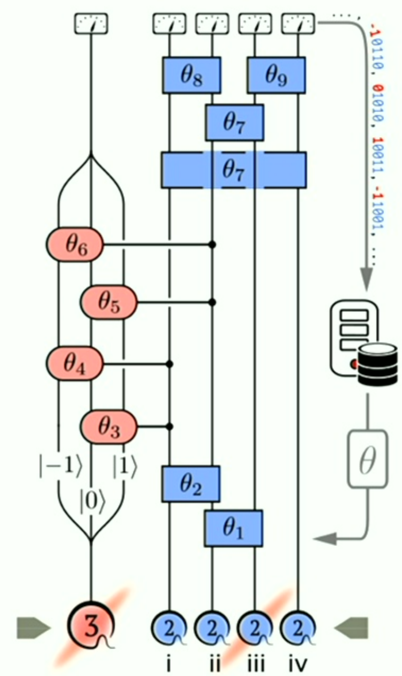


D. Paulson et al., PRX Quantum 2, 030334 (2021)

Hybrid qubit-qudit circuit

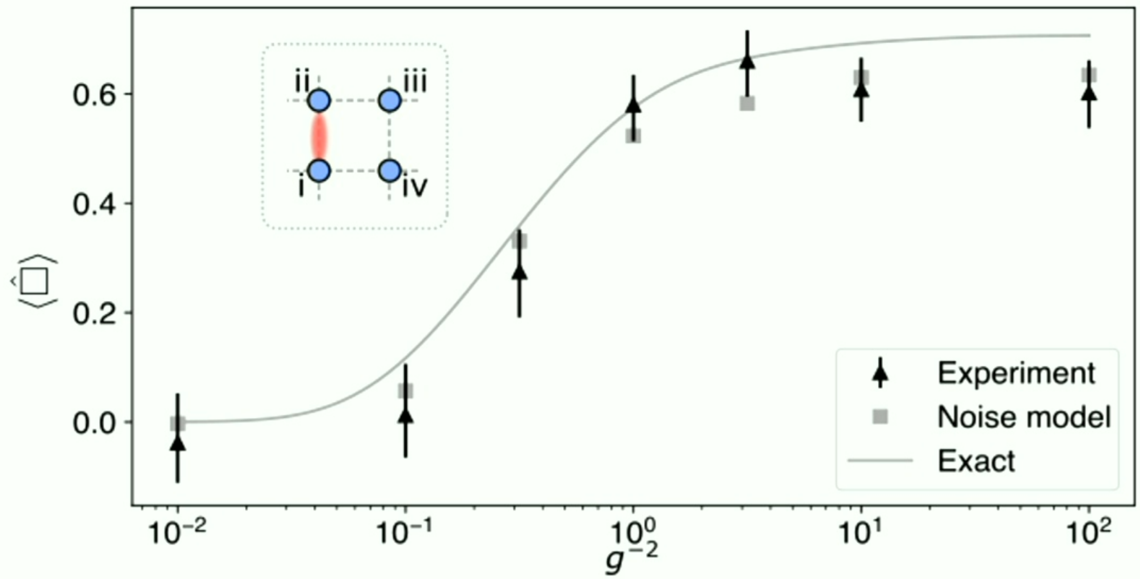


	Qudit encoding			Qubit encoding		
dimension d	3	5	7	3	5	7
register size	5	5	5	7	9	11
CNOT count	26	34	42	90	162	234
CNOT fidelity			99%			
approx. circ. fid.	77%	71%	66%	40%	20%	10%
CNOT fidelity			99.5%			
approx. circ. fid.	88%	84%	81%	64%	44%	31%



D. Paulson et al., PRX Quantum 2, 030334 (2021)

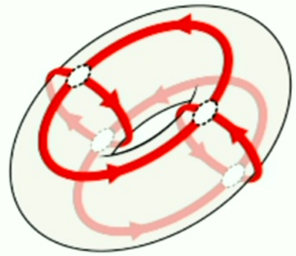
Experimental VQE results



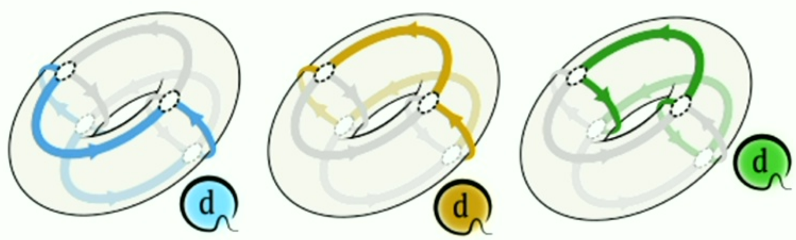
Can we see experimentally the improvements with refining the discretisation?

Pure gauge periodic plaquette

- Pure gauge system with periodic boundary condition



- 8 gauge fields
- 3 can be eliminated with Gauss Law
- 3 independent rotors describe the system's ground state



J. F. Haase et al., Quantum 5, 393 (2021)

Electric and magnetic representations

$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B$$

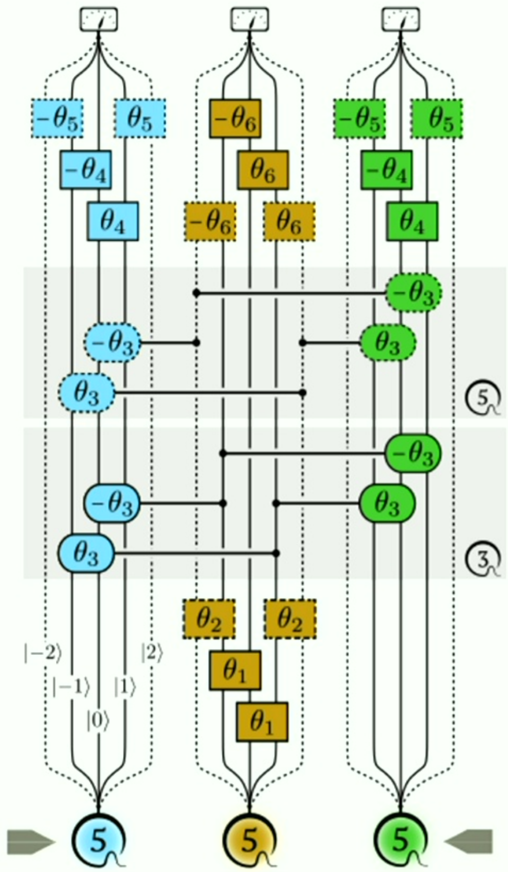
Two representation are used:

- electric basis
- magnetic basis

Each more efficient in different regimes of g^{-2}, but more truncation error in the opposite regime

J. F. Haase et al., Quantum 5, 393 (2021)

Example: variational circuit in electric basis

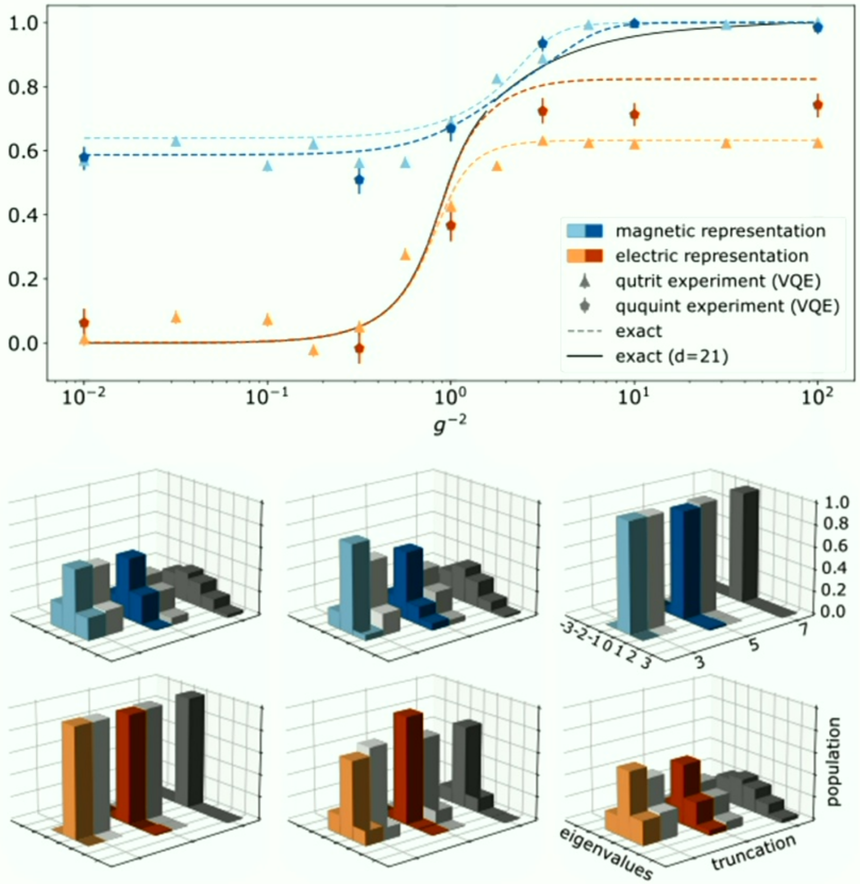


$$\hat{H}^{(\mathrm{e})} = g^2 \hat{H}_E^{(\mathrm{e})} + \frac{1}{g^2} \hat{H}_B^{(\mathrm{e})},$$

$$\hat{H}_E^{(\mathrm{e})} = 2\left[\hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 - \hat{R}_2\left(\hat{R}_1 + \hat{R}_3\right)\right],$$

$$\hat{H}_B^{(\mathrm{e})} = -\frac{1}{2}\left(\hat{P}_1 + \hat{P}_2 + \hat{P}_3 + \hat{P}_1\hat{P}_2\hat{P}_3 + \mathrm{H.c.}\right),$$

Ground state properties



Summary

- Successful experimental VQE with qutrits and ququints with trapped ions
- Qudits can be a natural implementation of LGT degrees of freedom
- Flexibility in realising systems with mixed dimensions, and in adjusting truncation

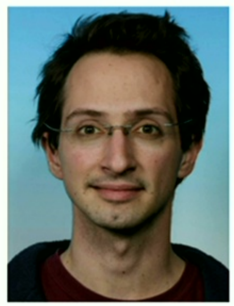
Outlook

- Going to (3+1)D system
- Going to sign-problem afflicted setting (time evolution)
- Qudits also available on other platforms (Rydberg atoms, microwave photons, superconducting architectures, ultracold atoms…)
- Extension of e.g. error mitigation techniques to qudit systems, efficient measurement protocols,…

Thank you for your attention!

universität innsbruck

Michael Meth



Martin Ringbauer

TQT Transformative Quantum Technologies

CANADA FIRST RESEARCH EXCELLENCE FUND

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