Title: Probing the limits of classical computing with arbitrarily connected quantum circuits

Speakers: Michael Foss-Feig

Collection: Foundations of Quantum Computational Advantage

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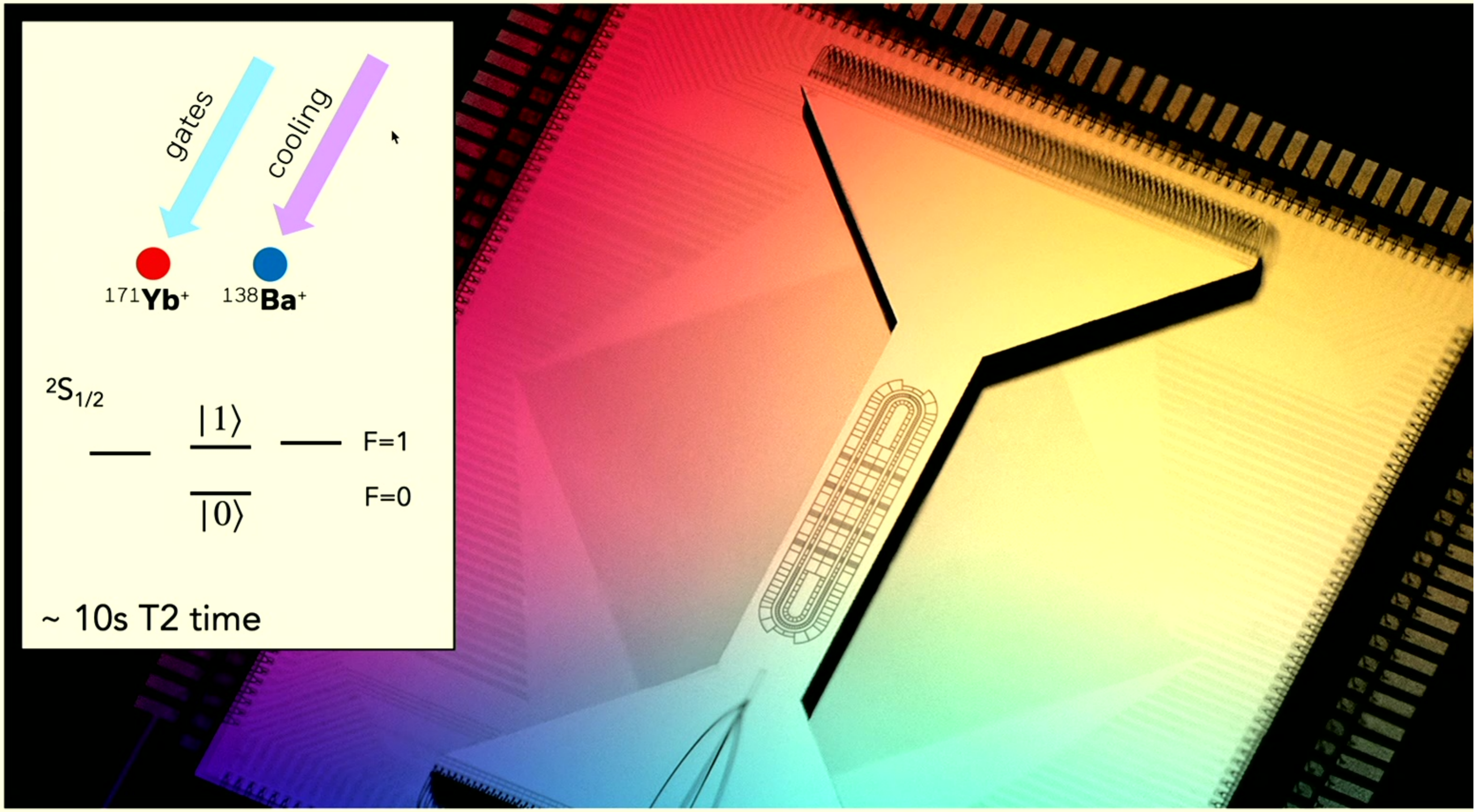
Abstract: Empirical evidence for a gap between the computational powers of classical and quantum computers has been provided by experiments that sample the output distribution of two-dimensional quantum circuits. Many attempts to close this gap have utilized classical simulations based on tensor network techniques, and their limitations shed light on the improvements to quantum hardware required to inhibit classical simulability. In particular, state of the art quantum computers having in excess of ~50 qubits are primarily vulnerable to classical simulation due to restrictions on their gate fidelity and their connectivity, the latter determining how many gates are required (and therefore how much infidelity is suffered) in generating highly-entangled states. Here, we describe numerical evidence for the difficulty of random circuit sampling in highly connected geometries.

SIMULATION COMPLEXITY OF CIRCUITS WITH RANDOM GEOMETRIES

Matt DeCross

Reza Haghshenas

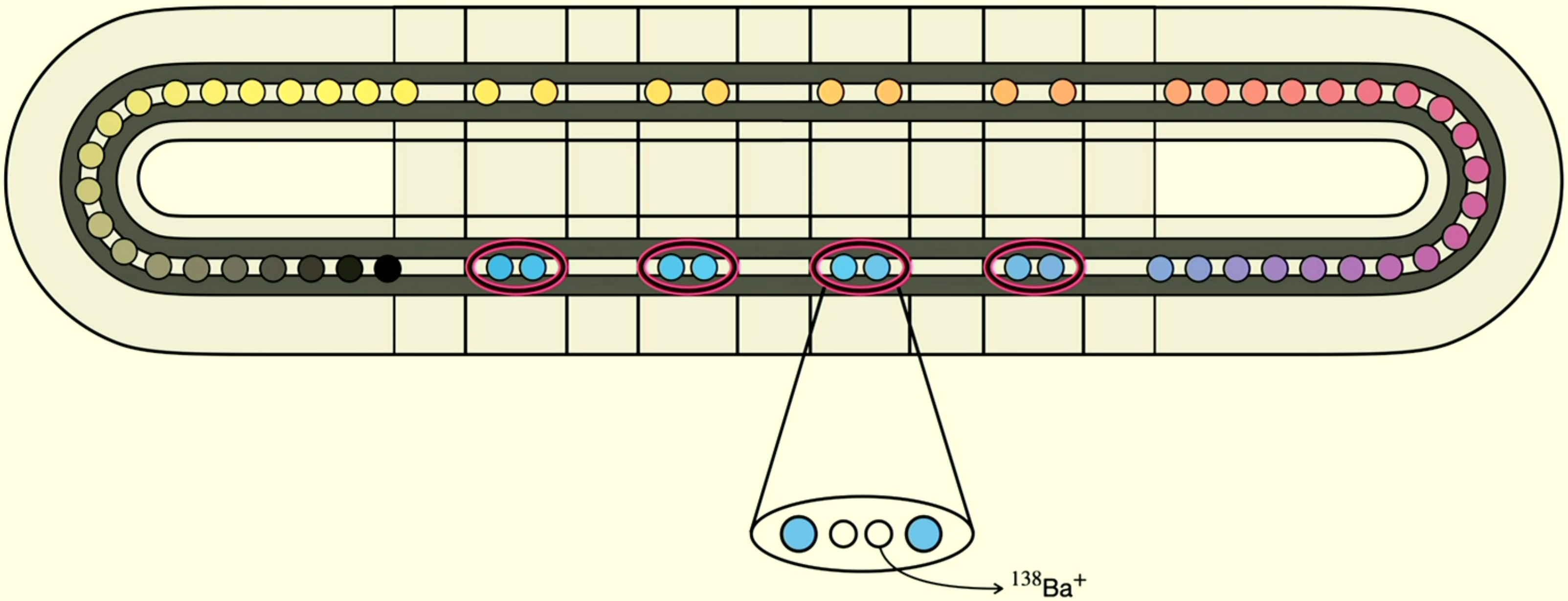


Michael Foss-Feig, Quantinuum

Overview of H2 quantum computer

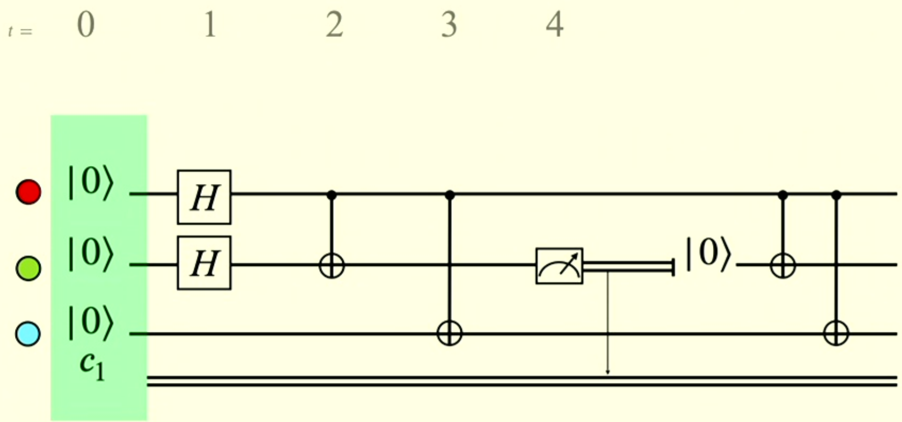
Launch configuration: 32 qubits, 4 operational gates zones

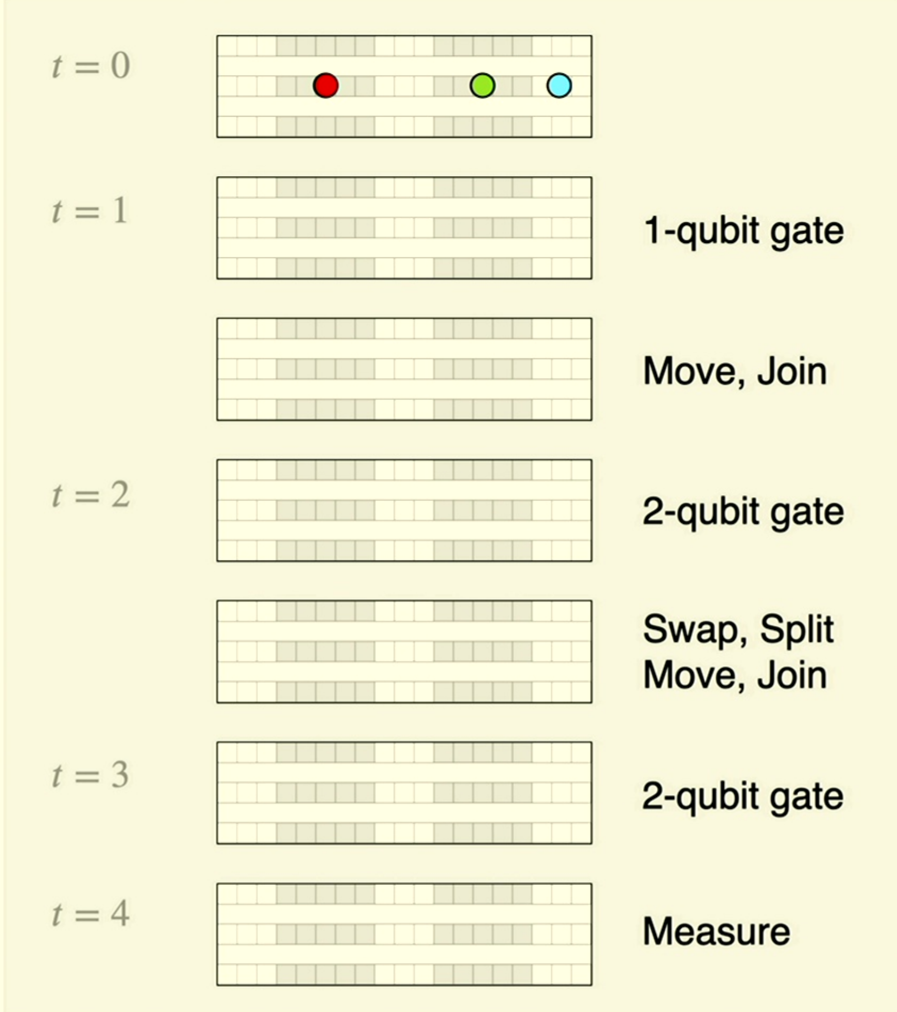
Will show preliminary data after loading 56 qubits (112 ions)



Overview of H2 quantum computer

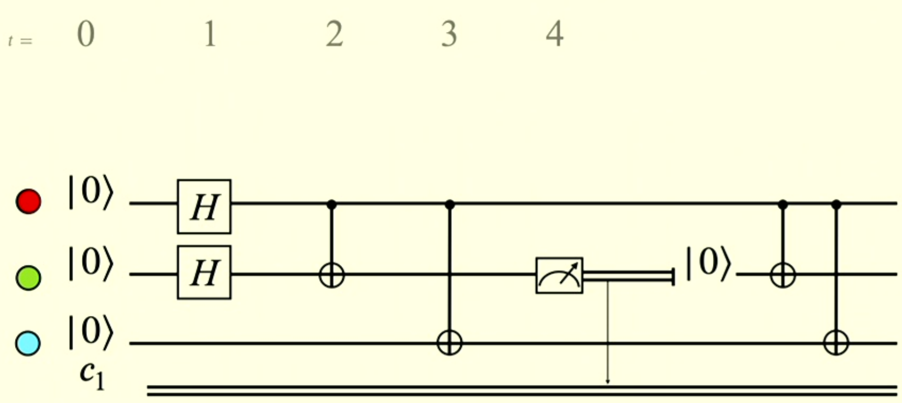
Quantum Circuit

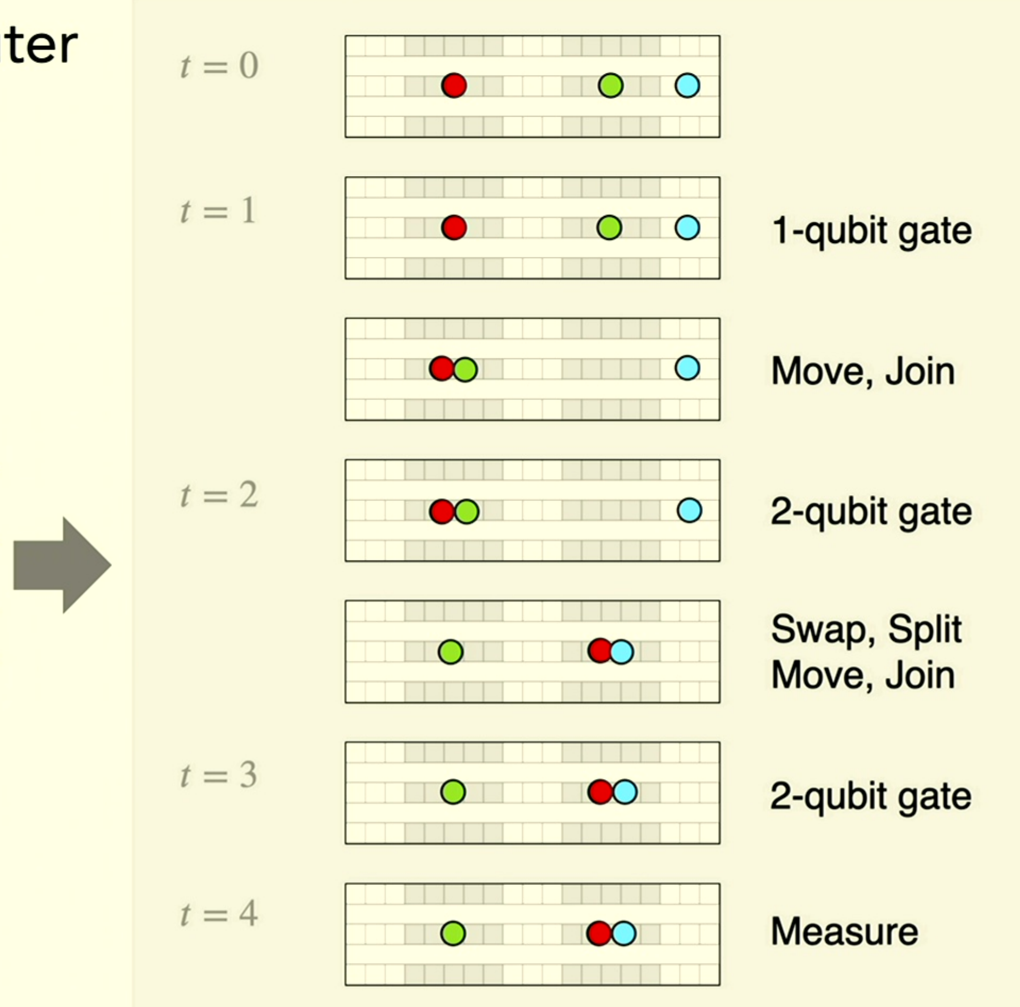




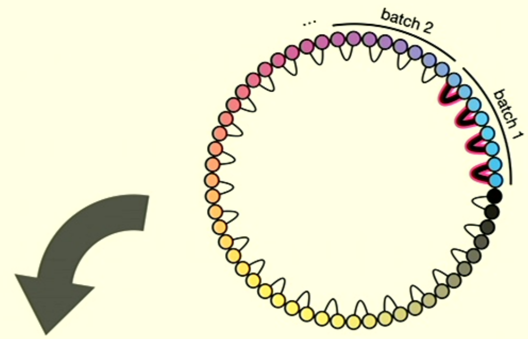
Overview of H2 quantum computer

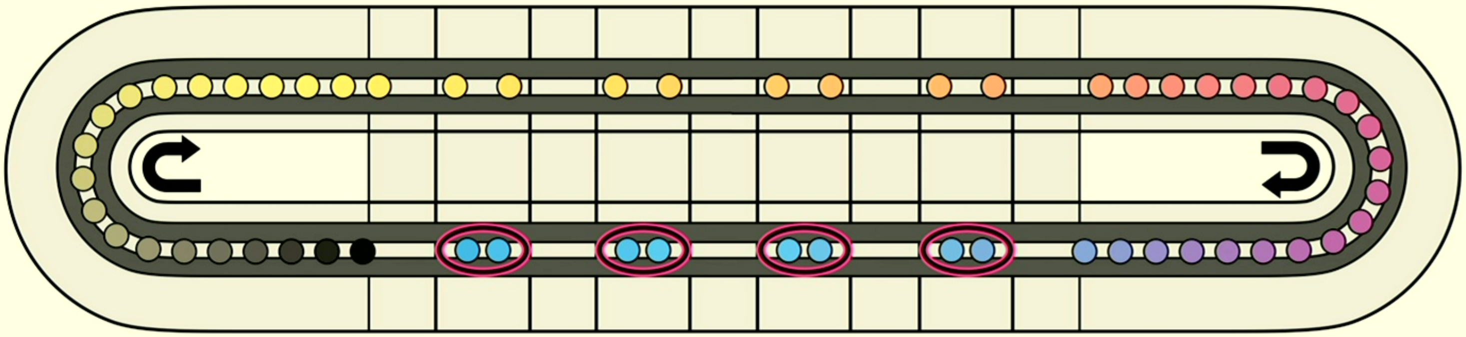
Quantum Circuit





Overview of the H2 quantum computer





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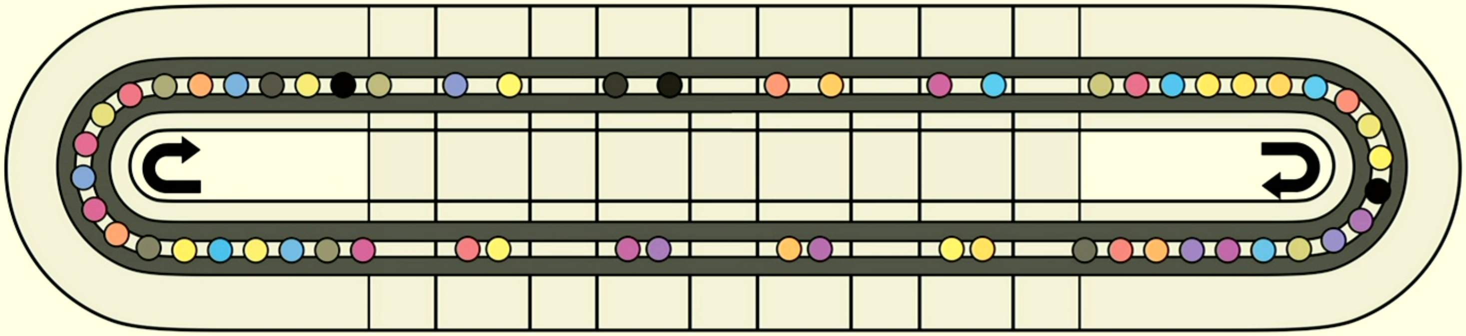
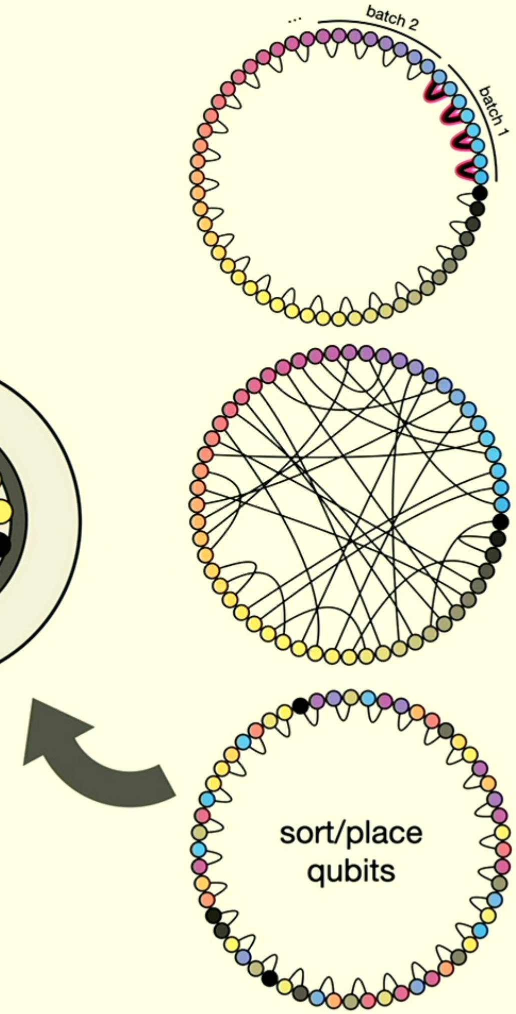
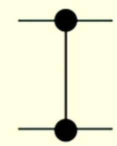


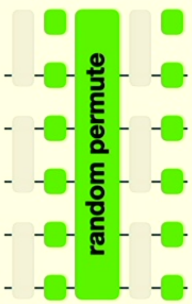


Overview of the H2 quantum computer
(32 qubits)

Test	Average infidelity $(\times 10^{-4})$
1Q RB	0.25(3)
1Q leakage	0.04(2)
2Q RB	18.3(5)
2Q leakage	3.9(2)
2Q SU(4) RB	41(1)
2Q parameterized RB	See Fig. 3
Transport 1Q RB	2.2(3)
Measurement crosstalk	0.045(6)
Reset crosstalk	0.038(6)
SPAM	16(1)



Overview of the H2 quantum computer
(32 qubits)



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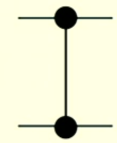


Overview of the H2 quantum computer
(32 qubits)

Errors in complex circuits:

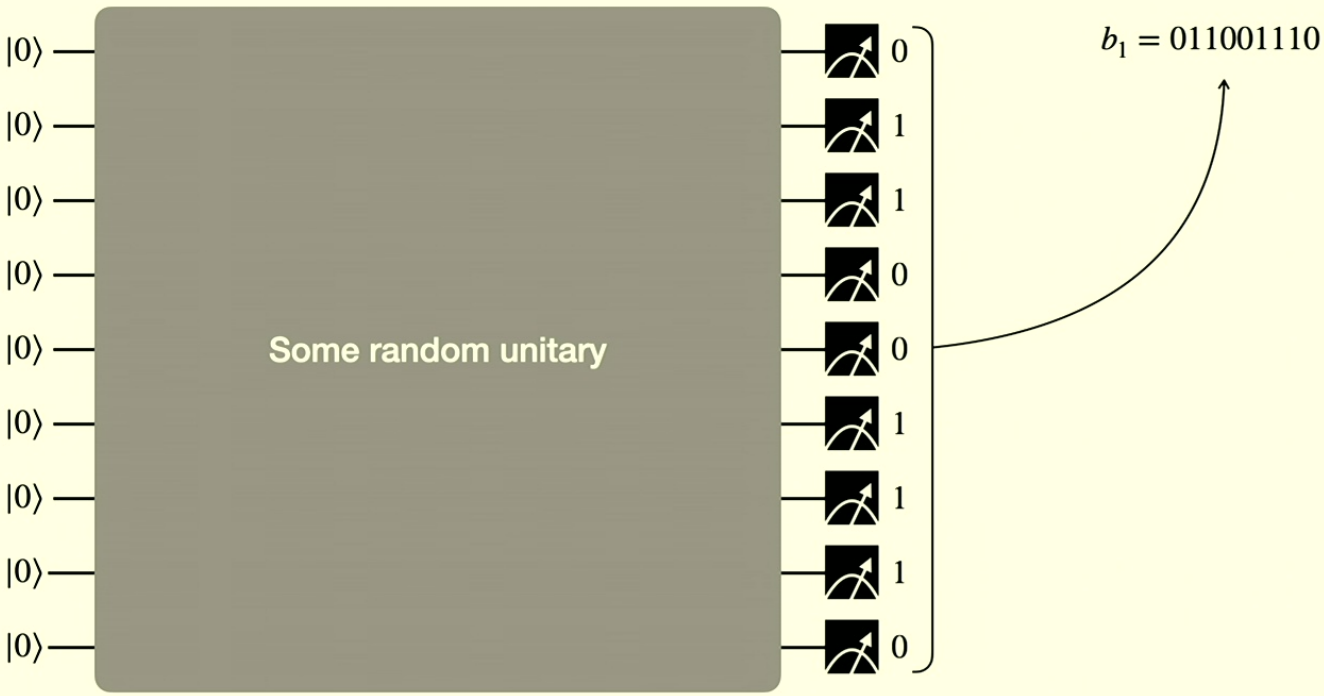
	Source	$\bar{\theta}$	$\epsilon_{\text{eff}}^{2Q}$ (inferred)	$\epsilon_{\text{eff}}^{2Q}$ (predicted)	
N=20,..,32	Mirror benchmarking	0.5π	2.6(2)	2.4(1)	Random connectivity
N=16	Quantum volume	0.35π	1.7(1)	1.9(1)	Random connectivity
N=12,..,32	Random circuit sampling	0.42π	1.9(2)	2.1(1)	2D connectivity

https://arxiv.org/pdf/2305.03828.pdf

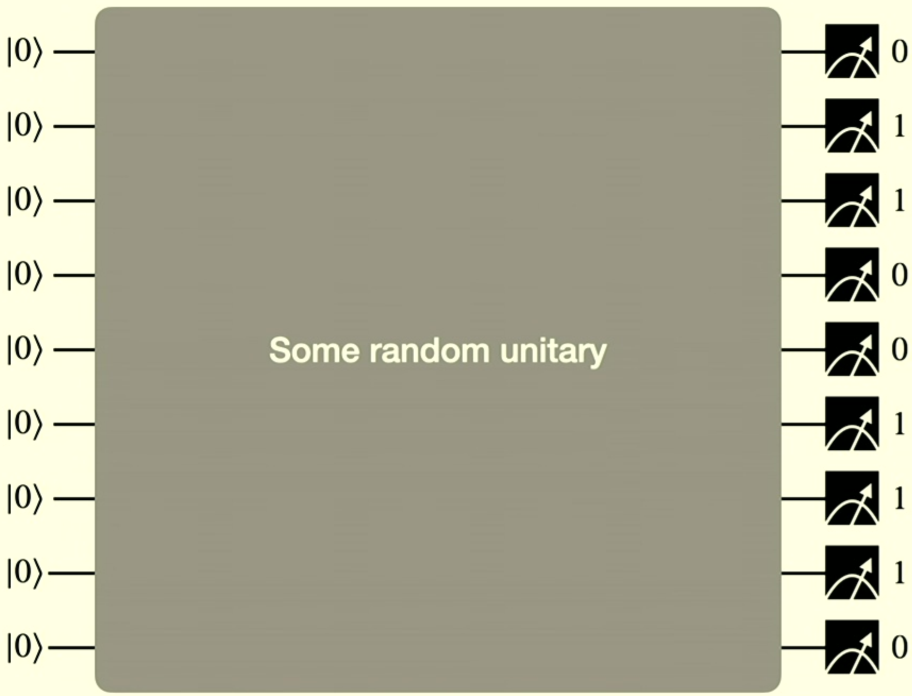
Effective error per two qubit gate:

$\epsilon_{\text{eff}}^{2Q}$ = **2Q RB** + **Transport 1Q RB** $\approx 2 \times 10^{-3}$

Random circuit sampling with random geometries



Random circuit sampling with random geometries



$b_1 = 011001110$

$b_2 = 101010111$

$b_3 = 111010000$

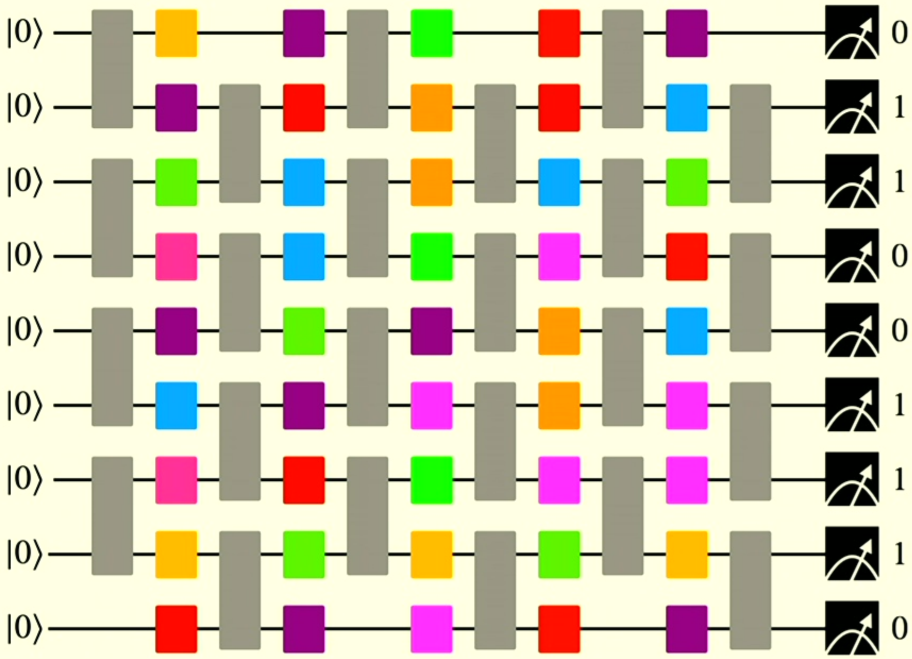
$\cdots$

$b_M = 011011100$

Data can be subjected to statistical tests to see if the more probable outcomes are more likely to be drawn (e.g. linear-XEB as in Google's work).

In practice, achieving very high scores on such a test appears to be about as hard as simulating the circuit.

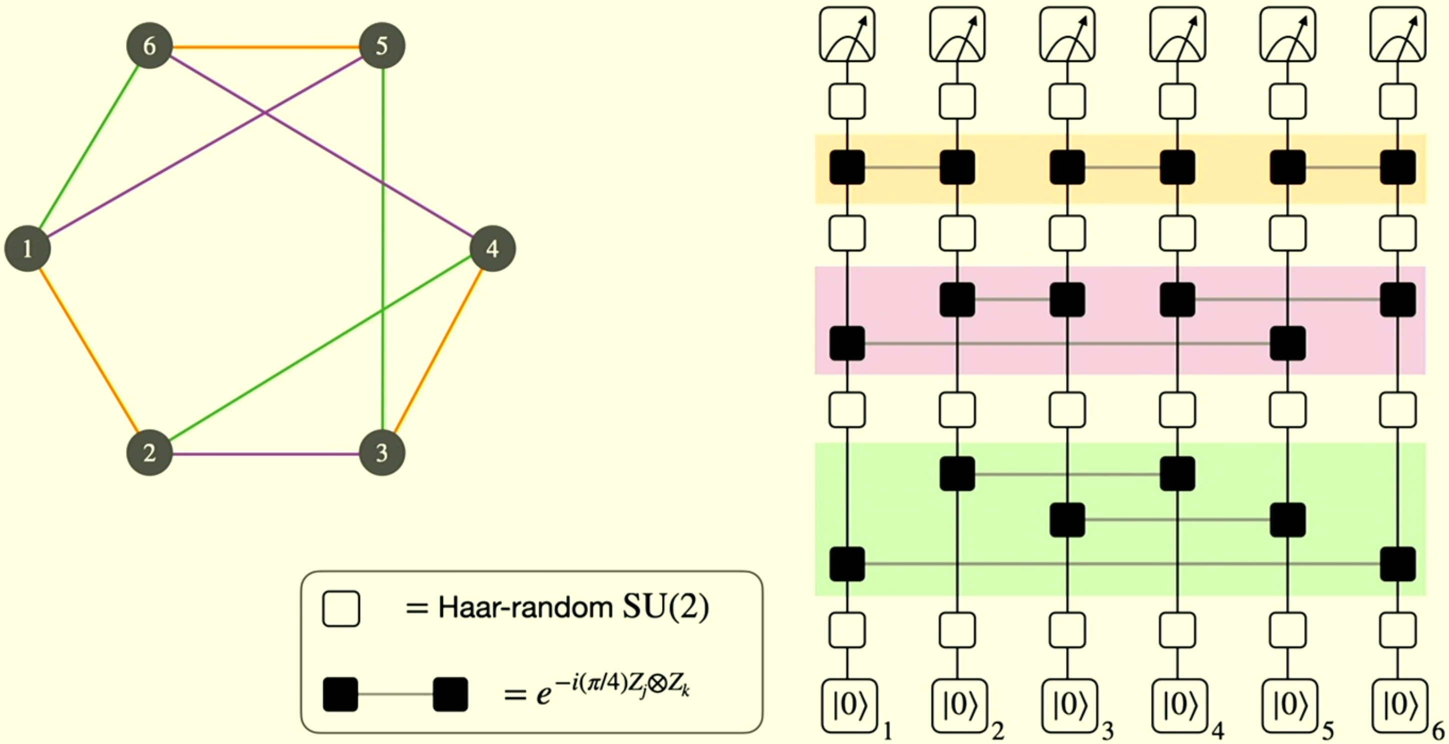
Random circuit sampling with random geometries



In practice, “random” has meant geometrically local circuits with random single-qubit gates (shown in 1D, though 2D is much better and also what has been)

Random circuit sampling with random geometries

$\mathcal{G}_{d,N}$ (random d-regular graph on N nodes) → $C_{d,N}$ (random depth-d circuit on N qubits)



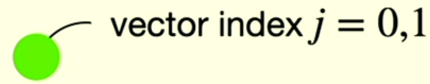
Cost of tensor network contraction

The best known general purpose method for simulating quantum circuits at sufficiently high fidelities is optimized tensor-network contraction.

Classically simulating these circuits (at the fidelity that H2 can run them) appears to be <u>nearly</u> impossible

Cost of tensor network contraction

The best known general purpose method for simulating quantum circuits at sufficiently high fidelities is optimized tensor-network contraction.



$$\vec{u} = (u_0, u_1)$$

Cost of tensor network contraction

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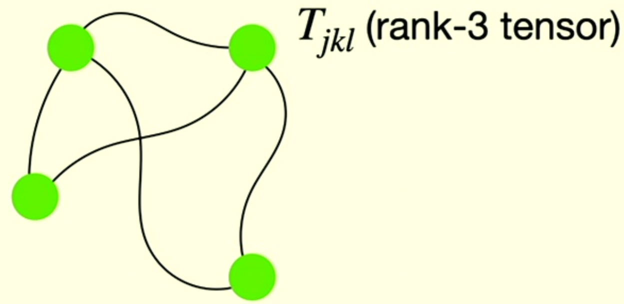

Cost of tensor network contraction

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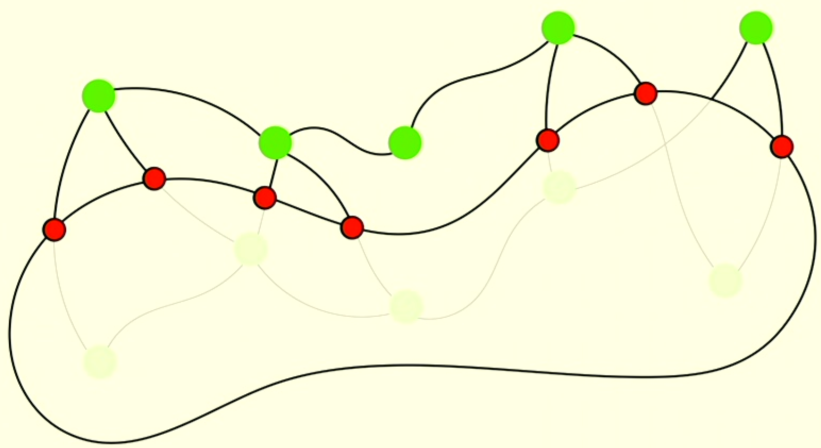
Contract bottom to top

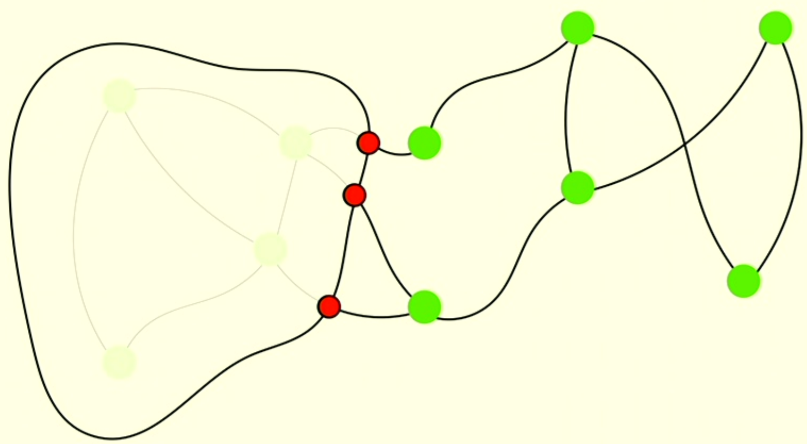


cost $> 2^{\#}$ ●

Cost of tensor network contraction

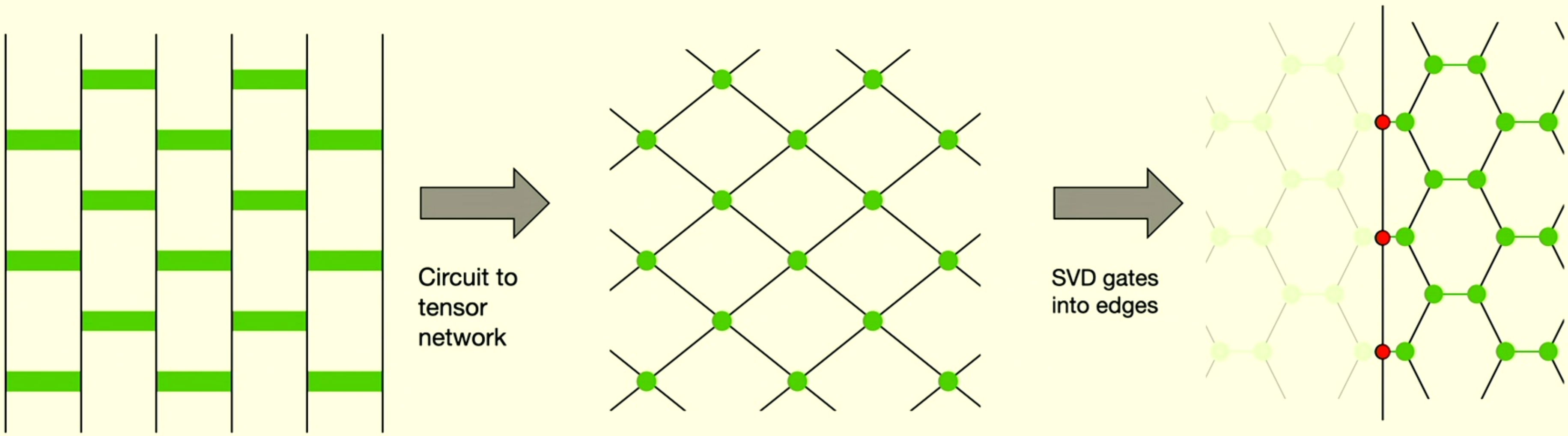
The best known general purpose method for simulating quantum circuits at sufficiently high fidelities is optimized tensor-network contraction.

Contract
left to right



$\text{cost} > 2^{\# \bullet}$

Cost of tensor network contraction

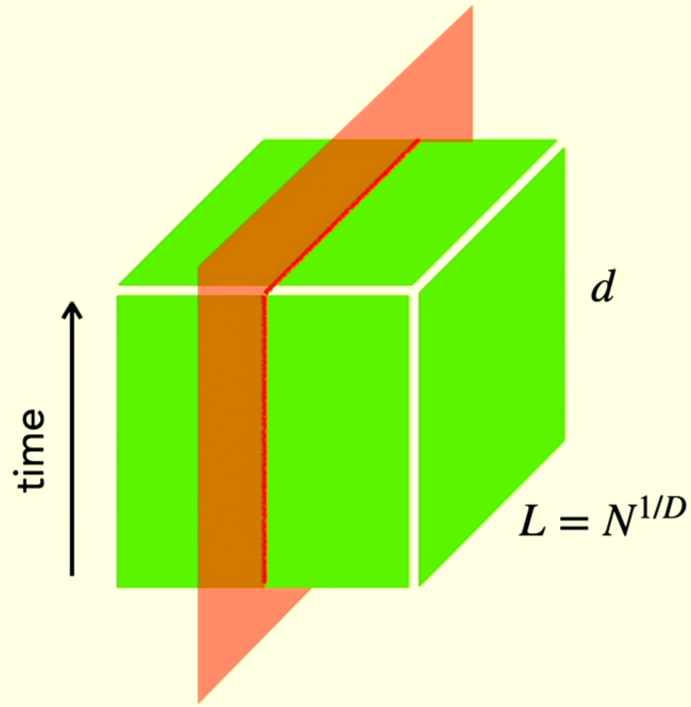


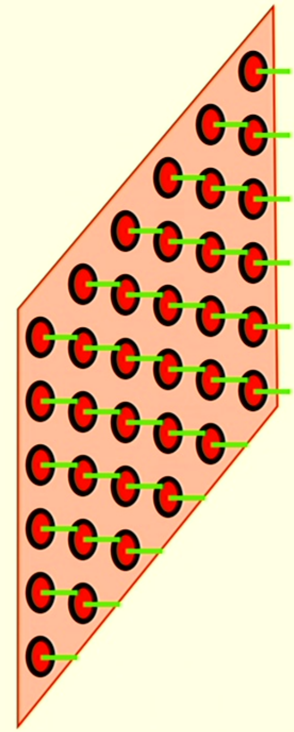
Contraction cost depends on contraction order.
Two extreme limits are to contract

- vertically (cutting qubit wires): cost $\sim \exp(N)$
- sideways (cutting gates): cost $\sim \exp(d \times \underbrace{N^{(D-1)/D}})$

Surface area of a bipartitioning hypersurface

Cost of tensor network contraction





- sideways (cutting gates): cost $\sim \exp(d \times \underbrace{N^{(D-1)/D}})$

Surface area of a bipartitioning hypersurface

Cost of tensor network contraction

Effective qubit number:

$$\mathcal{N} = \log_2\left(\frac{\text{cost in FLOPs}}{\text{\# of gates}}\right)$$

↓

$$\text{cost in FLOPs} = 2^{\mathcal{N}} \times (\text{\# of gates})$$

Circuit is as hard as the worst-case cost of simulating $\mathcal{N}$ qubits

For context, when $\mathcal{N} = 56$ one might expect 100s of seconds on Frontier as a conservative *lower bound*

Cost of tensor network contraction

Effective qubit number:

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Circuit is as hard as the worst-case cost of simulating $\mathcal{N}$ qubits

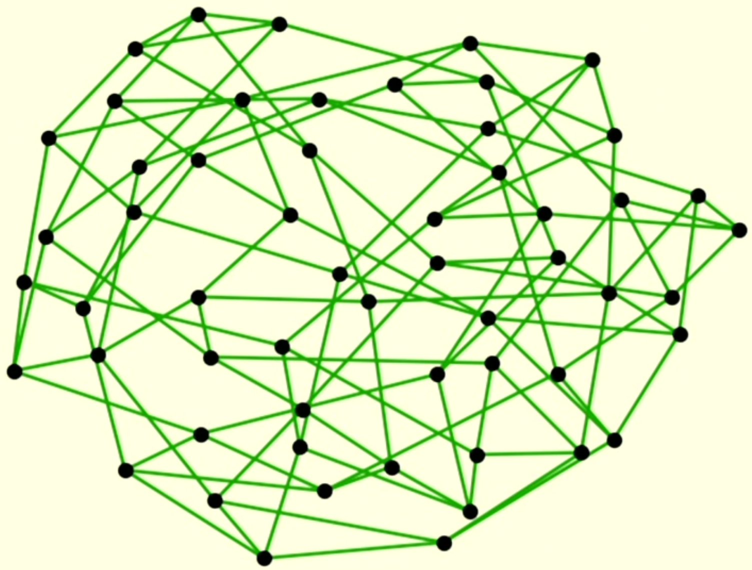
Complexity density:

$$\mathscr{C} = \mathcal{N}/N$$

$$\text{cost in FLOPs} = 2^{\mathscr{C} \times N} \times (\text{\# of gates})$$

$\mathscr{C}$ determines the *fraction* of qubits that contribute to the worst-case cost of simulation

Cost of tensor network contraction



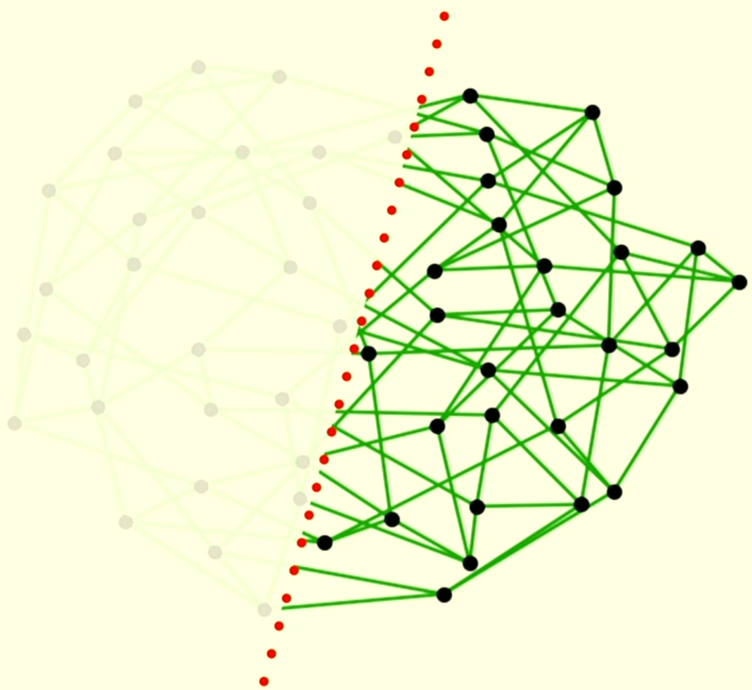
Complexity density:

$$\mathscr{C} = \mathscr{N}/N$$

$$\mathscr{C}_{d,N} \sim \min\left(1, d/N^{1/D}\right)$$

$\dfrac{\text{Edge boundary of a bipartition}}{N}$ ← $\dfrac{\textbf{Surface area of a spatial bipartition}}{N}$

Cost of tensor network contraction



Theorem (B. Bollobás)**:** The edge isoperimetric number of random regular graphs is bounded below by a constant (with high probability) as $N \to \infty$

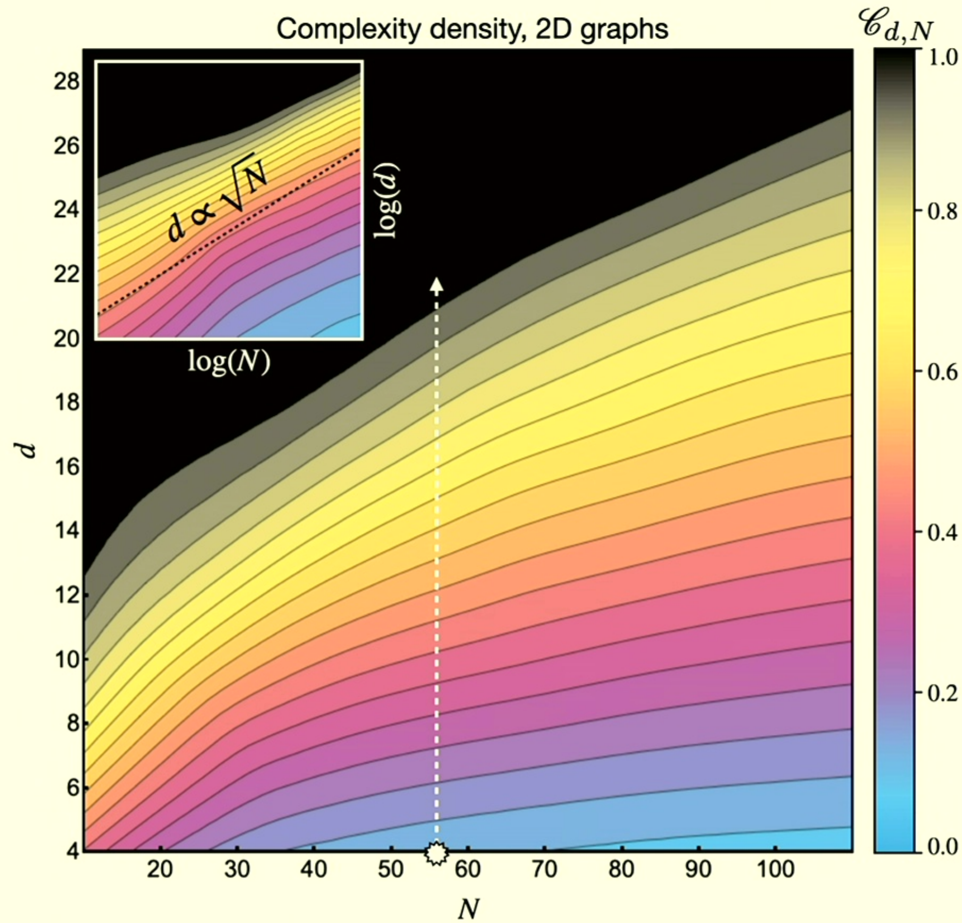
$$\mathscr{C}_{d,N} \gtrsim c > 0 \quad \text{as} \quad N \to \infty$$

$$\frac{\text{Edge boundary of a bipartition}}{N} = I(\mathscr{G}) \equiv \min_U |\partial U| / |U|$$

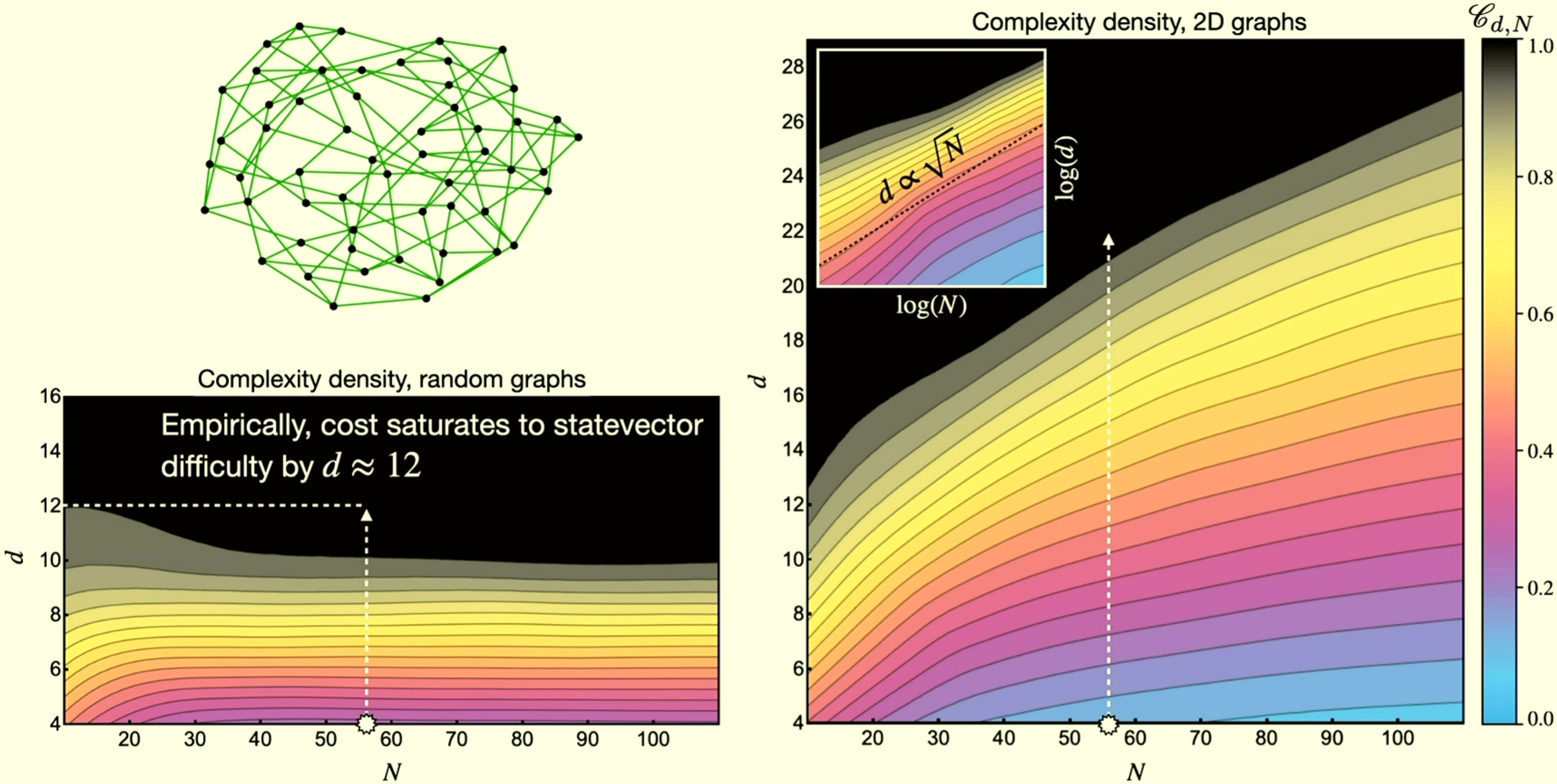
Cost of tensor network contraction

2D circuits:

$$\mathscr{C}_{d,N} \sim \min\left(1,\ d/N^{1/2}\right)$$

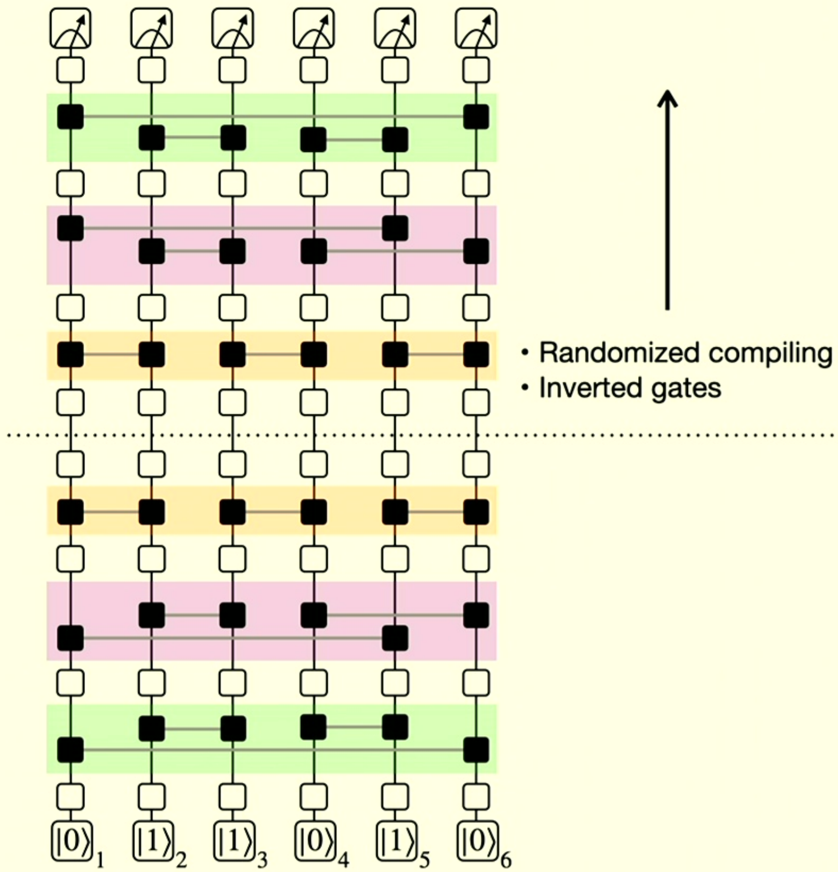


Cost of tensor network contraction



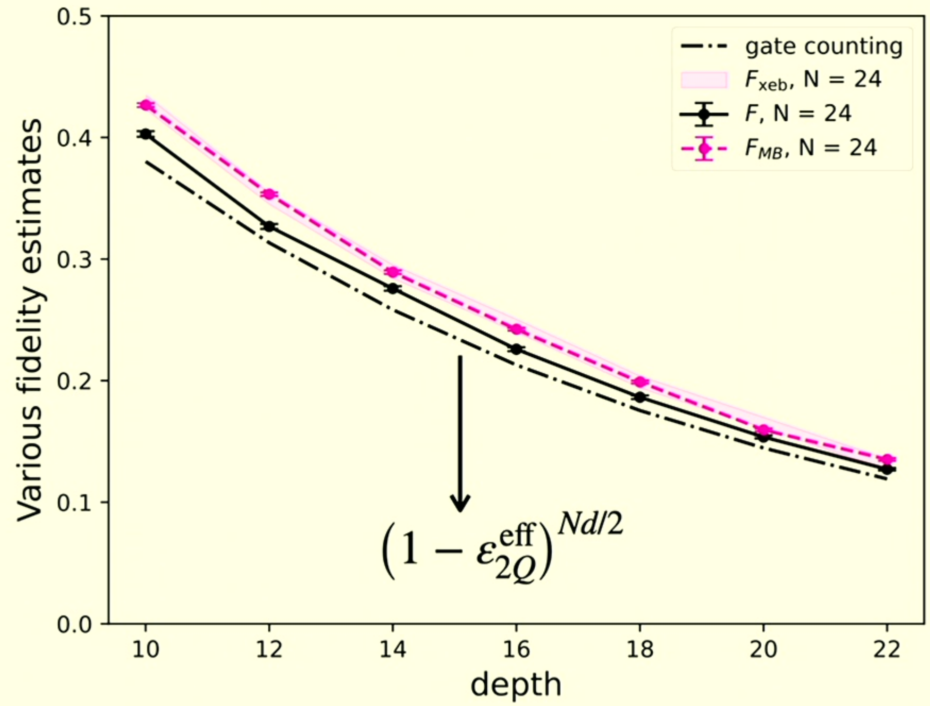
Circuit fidelity estimates from mirror benchmarking

$C_{d,N}$ (random depth-3 circuit)



Simulate using a detailed noise model including:

- Structured Pauli noise model from cycle benchmarking
- Coherent memory errors



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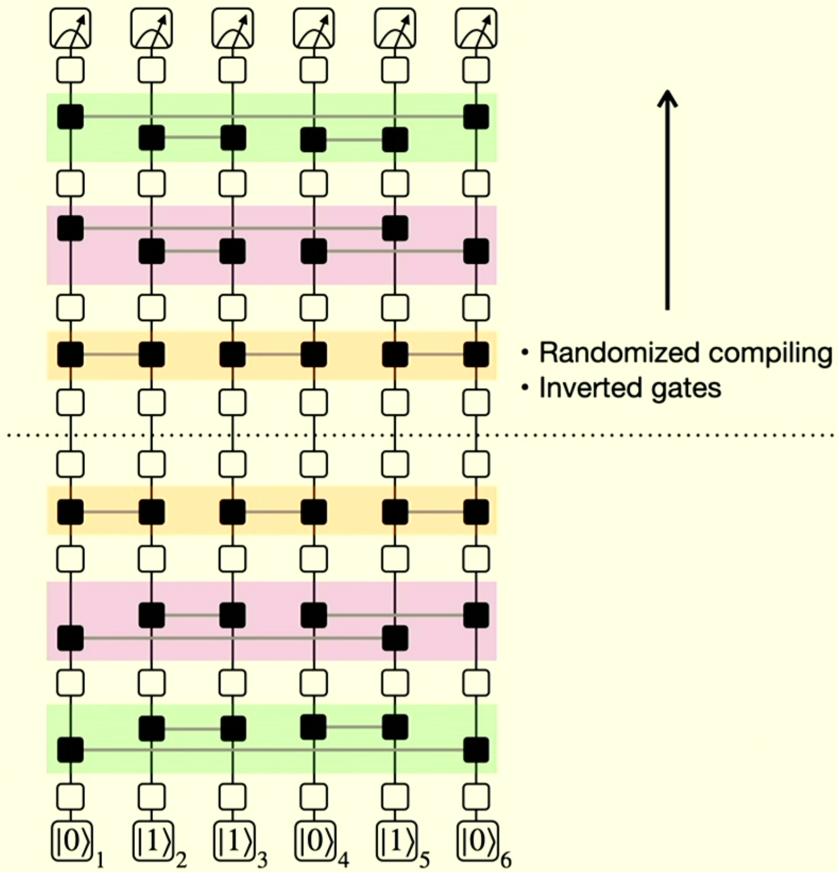

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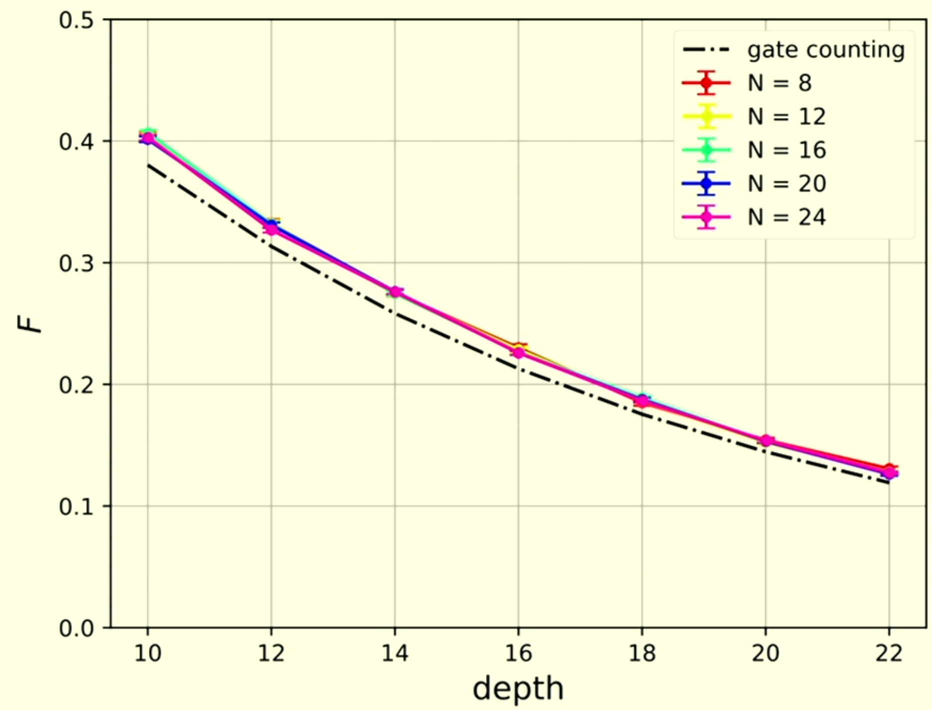

Performance on random-geometry circuits with 56 qubits