Title: Quantum Fields & Strings Seminar - TBA

Speakers: Horacio Casini

Series: Quantum Fields and Strings

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Abstract: Abstract TBA

Zoom link

ABJ anomaly as a U(1) symmetry, and Noether's theorem

Horacio Casini

Instituto Balseiro, CONICET, Centro Atómico Bariloche

Based on recent works with Valentin Benedetti and Javier Magán:

https://arxiv.org/abs/2309.03264
https://arxiv.org/abs/2205.03412

ABJ anomaly:

Massless fermions coupled to electromagnetic field, chiral symmetry $\psi \to e^{-i\alpha\gamma^5}\psi$

$j_5^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi$ would be Noether current, clasically conserved

$\partial_\mu j_5^\mu = \frac{1}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$ quantum non conservation

Necessary to explain neutral pion decay into photons in QCD

Several derivations in Lagrangian QFT: Feynman diagrams, non invariance of the path integral measure.

Is this still a symmetry, and if so in what sense?
Usually considered an "anomalous symmetry", not a «normal» symmetry, but not explicitly broken, different from spontaneously broken.

It has quite peculiar features:

1) If it is considered a continuous symmetry it does not have a conserved Noether current

2) However, if it is not considered a symmetry, still the Goldstone theorem works (pions)

3) Anomaly quantization: general anomaly coefficient is proportional to an integer (from Atiyah-Singer index theorem in the path integral computation, or WZW term in effective chiral model)

4) Anomaly matching (the coefficient of the anomaly matches between the UV and IR)

Idea of the talk:

ABJ anomaly as an ordinary internal U(1) symmetry that, however, messes with the Haag duality violations of the theory. This clarifies and unifies the origin of its main features in a perspective based on ordinary symmetry ideas

Haag duality violations / generalized symmetries (HDV)

Global symmetries versus HDV

An obstruction to the (strong form) of Noether's theorem

Classification of some simple possibilities: non compact/free, anomalous

Anomaly quantization, anomaly matching, and the absence of a Noether current

ABJ anomaly as a symmetry with charged HDV sectors: pion and QED realizations

«calculation» of the anomaly from Witten's effect

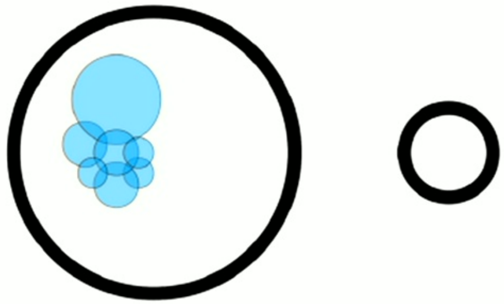
Some conjecture

Operator algebras and regions

QFT: R $\longrightarrow$ $\mathcal{A}(R)$ (net of von Neumann algebras)

Two different meanings of the world "local":

1) $\mathcal{A}(R) \equiv \bigvee_{B \text{ ball}, \cup B = R} \mathcal{A}(B)$ Additivity: operators constructed from local degrees of freedom



2) $\mathcal{A}(R) \subseteq (\mathcal{A}(R'))'$ Causality. $\mathcal{A}'$ Commutant. R': points spatially separated from R, causal complement

"Complete theory"

The local observable algebras formed additiviely with local degrees of freedom are the maximal ones compatible with causality.

$\mathcal{A}(R) = (\mathcal{A}(R'))'$ Haag duality of the additive algebra (for any R) H.C., J. Magán (2021)

Non complete theories?

H.C., M. Huerta, J. Magán, D. Pontello (2020)

$$\mathcal{A}_{\max}(R) \equiv (\mathcal{A}(R'))'$$

The maximal algebra compatible with causality corresponds to the smallest one for the complementary region

$$\mathcal{A}_{\max}(R) = \mathcal{A}(R) \vee \{a\}$$

$$\mathcal{A}_{\max}(R') = \mathcal{A}(R') \vee \{b\}$$

a,b, dual "non local" operators.
cannot choose the net to satisfy simultaneously duality and additivity

$$\begin{array}{ccccc} \mathcal{A}_{\max}(R) \equiv \mathcal{A}(R) \vee \{a\} & \supset & \mathcal{A}(R) & & \\ \updownarrow ' & & \updownarrow ' & & \\ \mathcal{A}(R') & \subset & \mathcal{A}(R') \vee \{b\} \equiv \mathcal{A}_{\max}(R') & & \end{array}$$

von Neumann theorem:

$$\boxed{\mathcal{A} = \mathcal{A}''}$$

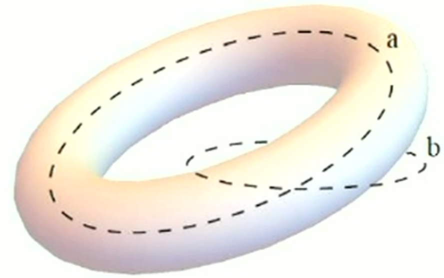
There are non local operator classes under the operation with local ones

Haag duality fails for R iff it fails for R' : non local operators come in dual sets.

Dual non local operators cannot commute (all) to each other, otherwise the maximal algebra would be included in the additive one. (Maximal algebras do not form a local net)

Topology: Haag duality holds for topological balls (all non local operators are ultimately additive considered in balls) + "transportability": non local operator classes (under the action of additive operators) are preserved under smooth deformations of the region.

Example: Regions with non trivial homotopy groups π_1 or π_{d-3}



Gauge theories: Wilson loops $W_r \equiv \mathrm{Tr}_r \mathcal{P}\, e^{i\oint_C dx^\mu A^r_\mu}$

Not all Wilson loops are non locally generated in a ring-like region:
for charged representations the loops can be broken in Wilson lines ended in charged operators

$\phi_r(x)\, Pe^{i\int_x^y dx^\sigma A_\sigma}\, \phi^\dagger_r(y)$ $\quad F_{\mu\nu}(x)\, Pe^{i\int_x^y dx^\sigma A_\sigma}\, F_{\alpha\beta(y)}$ $\longrightarrow$ Representations generated by the adjoint are locally generated

Non local classes of WL labelled by representations of the center of the gauge group

Dual operators are the 't Hooft loops labelled by elements of the center ´t Hooft (1978)

For SU(N) the dual groups of non local operators are Z_N

For the free Maxwell field these are exponentials of the magnetic and electric fluxes for any charges

$W_q = e^{i\,q\,\Phi_B} \quad T_q = e^{i\,q\,\Phi_E}$ gives $A=\mathbb{R}\,,B=\mathbb{R}\,,d\neq 4\,,\quad A=\mathbb{R}^2\,,B=\mathbb{R}^2\,,d=4$

$W_q\,W_{q'} = W_{q+q'}\,,\quad T_g\,T_{g'} = T_{g+g'} \qquad W_q\,T_g = e^{i\,q\,q}\,T_g\,W_q$

For QED: $A=U(1)\,,B=Z\,,d\neq 4\,,\quad A=U(1)\times\mathbb{Z}\,,B=\mathbb{Z}\times U(1)\,,d=4$

Remarks:

"Non local operator" is a relative notion. An operator (Wilson loop) can be non local in a region (ring) and is always additive on a different region (ball that contains the ring).

There are arguments indicating non local operators for pure ring sectors (and no two ball classes) form Abelian dual groups (d>3) and the commutation relations are fixed $a\,b = \chi_b(a)\,b\,a$

"Entropic order parameters for the phases of QFT"
H.C., M. Huerta, J. Magan, D. Pontello (2020)
H.C, J. Magan (in preparation)

In the standard HEP literature this subject is called "Generalized symmetries" (Gaiotto, Kapustin, Seiberg, Willett (2015))
Usual description do not use Haag duality violation but continues by putting the QFT in compact topologically non trivial spacetimes, ussually Euclidean description.

Global symmetry + HDV: obstructions to the strong form of Noether theorem

V. Benedetti, H.C., J. Magan (2022)

Global internal symmetry: automorphisms of the local algebras (a group by DHR theorem) $U(g)\,\mathcal{A}(R)\,U(g)^{-1} = \mathcal{A}(R)$

$\longrightarrow$ $U(g)\,\mathcal{A}(R)'\,U(g)^{-1} = \mathcal{A}(R)'\,, \qquad g \in G$ $\longrightarrow$ $U(g)\,\mathcal{A}_{\max}(R)\,U(g)^{-1} = \mathcal{A}_{\max}(R)$

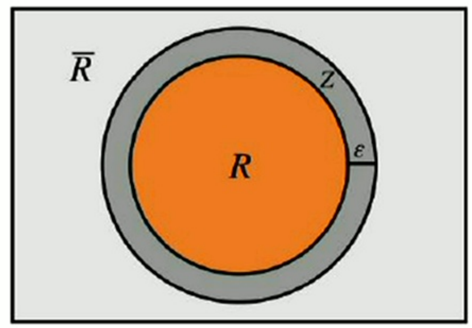
$A = \sum_{\lambda,s} O_{\lambda,s}\, a_\lambda\, \tilde{O}_{\lambda,s} \longrightarrow [a_\lambda]$ Non local classes can be charged or uncharged under the global symmetry

$\longrightarrow$ Global symmetry acts as a point-like transformation of class labels $[a_\lambda] \to [a_{\lambda'}]$

Twists of the global symmetry: produces the transformation of U(g) in R and do not transform outside RZ

Can always be constructed using modular theory (assuming split property): this general existence of twists is a weak form of Noether theorem, valid even for discrete symmetries. Doplicher, Longo (1983,1984), Buchholz, Doplicher, Longo (1986).

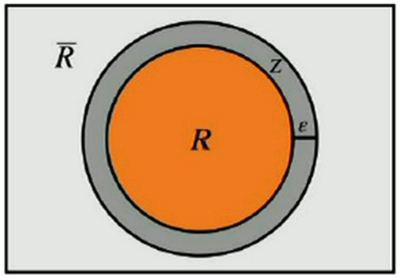
In general:
- $\tau_g(R,Z) \in \mathcal{A}_{\max}(R \cup Z)$
- Transforms in $\mathcal{A}(R)$



A twist is additive if belongs to $\mathcal{A}(R \cup Z)$

A twist is complete if transforms (as U(g)) in $\mathcal{A}_{\max}(R)$

Examples also constructed with non gauge invariant currents plus boundary terms



There are additive and complete twists for R

⟷ The non local classes of R are uncharged

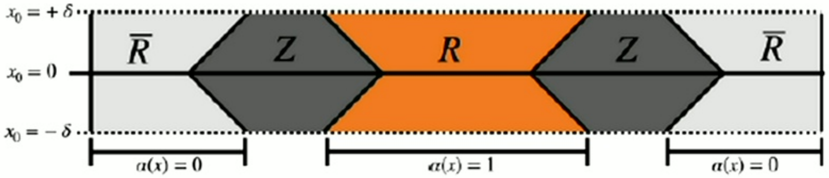
When a twist is complete and non local classes are charged the twist itself contains non local operators (can be pictured as boundary terms)

A Noether current allows the construction of additive charges:

$$\tau_\lambda(R,Z) = e^{i\lambda Q(R,Z)}\,, \qquad Q(R,Z) = \int d^dx\, \beta(x^0)\, \alpha(\vec{x})\, j_0(x)$$

The charge is also complete:

$$Q = Q(R,Z) + (Q - Q(R,Z))$$



If a continuos symmetry has a Noether current there cannot be charged non-local classes

Some examples of violations of the Noether theorem:

All known examples have charged non local sectors

Free graviton
No stress tensor.
Non local classes have Lorentz indices
Poincare symmetry mixes classes.

Two free Maxwell fields
No current for rotation symmetry.
Symmetry mixes Wilson loops.

} Weinberg-Witten theorem

Duality symmetry Maxwell field
Symmetry mixes electric and magnetic fluxes.

Maxwell field for dimension $d \neq 4$
Derivatives of free scalar $d>2$
No dilatation current.
Symmetry mixes classes with dimensionfull labels.

} Known counterexamples for DI implies CI

} Free and massless. Why?

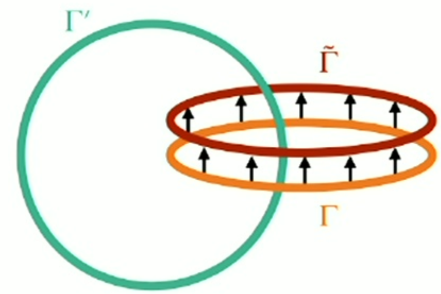
ABJ anomaly → Interacting case

An interpretation of Weinberg-Witten theorem:

V. Benedetti, H.C, J. magan (2021)

"There is no Noether current corresponding to a charged particle with helicity h=1 or greater, and there is no stress tensor for a theory with a massless particle with helicity h=2 or greater"

A (free) graviton in d=4 has closed two forms:



$$
\begin{aligned}
A_{\mu\nu} &= R_{(\mu\nu)(\alpha\beta)}\, a^{\alpha\beta}\,, \qquad a^{\alpha\beta} = -a^{\beta\alpha}\,,\\
B_{\mu\nu} &= R_{(\mu\nu)(\alpha\beta)}\, (x^{\alpha} b^{\beta} - x^{\beta} b^{\alpha})\,,\\
C_{\mu\nu} &= R_{(\mu\nu)(\alpha\beta)}\, c^{\alpha\beta\gamma} x_{\gamma}\,, \qquad c^{\alpha\beta\gamma} = -c^{\beta\alpha\gamma} = -c^{\alpha\gamma\beta}\,,\\
D_{\mu\nu} &= R_{(\mu\nu)(\alpha\beta)}\, (x^{\alpha} d^{\beta\gamma} x^{\gamma} - x^{\beta} d^{\alpha\gamma} x^{\gamma} + \frac{1}{2} d^{\alpha\beta} x^2)\,, \qquad d^{\alpha\beta} = -d^{\beta\alpha}
\end{aligned}
$$

$$
O_{\Gamma} = \int_{\Sigma} d\sigma^{\mu\nu}\, R_{\mu\nu\alpha\beta}(x)\, f^{\alpha\beta}(x)
$$

$$
f^{\alpha\beta}(x) = a^{\alpha\beta} + (x^{\alpha} b^{\beta} - x^{\beta} b^{\alpha}) + c^{\alpha\beta\gamma} x_{\gamma} + (x^{\alpha} d^{\beta\gamma} x^{\gamma} - x^{\beta} d^{\alpha\gamma} x^{\gamma} + \frac{1}{2} d^{\alpha\beta} x^2)
$$

$$
[O_{\Gamma}, O_{\Gamma'}] = i\left(a\cdot\cdot\tilde{d}^{*} + 2\, b\cdot\tilde{c}^{*} - 2\, c^{*}\cdot\tilde{b} - d^{*}\cdot\cdot\tilde{a} \right)
$$

There is a $\mathbb{R}^{20}$ group of generalized symmetries. The classes have Lorentz índices and transform non trivially as a linear representation of Poincare, conformal, and duality symmetries.

→ no stress tensor, no dilatation current, no duality current

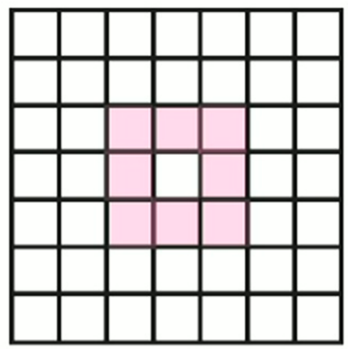
Generalizations of WW theorem to higher d and representations of Poincare group

Conjecture: (strong form of Noether's theorem)

If there are no non local classes non invariant under a continuous global symmetry then there is a Noether current.

This is the converse of our previous statement . (Given other standard assumptions :the split property plus time slice property).

Heuristic Idea: there are complete additive charges for arbitrary partitions of the t=0 surface that should concatenate to the global operator. Additive twists cannot concatenate when there are charged classes! (Due to boundary terms).



Importance

General existence of currents except some controlled (mostly free massless) cases

No QFT with gravitons: WW → no stress tensor → charged Poincare classes at all scales → free decoupled gravitons

A more formal application: Algebraic QFT → fields - Wightman QFT

What type of actions of a continuous 1-parameter symmetry on non local classes?

Classes and dual classes non invariant under a continuous symmetry both must form a continuum.
Lets assume Abelian group for the non local classes.

"One dimensional case": fusion $a_1 + a_2$ $b_1 + b_2$ Commutation relations $a\, b = e^{i\, a\, b}\, b\, a$

⟶ $a(\lambda) = e^{\lambda}\, a$ $b \to e^{-\lambda}\, b$

Implies dual classes are non-compact and continuous (group R). Example: dilatation symmetry for free Maxwell field d≠4

"Two dimensional case": $a = (a_1, a_2)$ $b = (b_1, b_2)$ $a\, b = b\, a\, e^{i a \cdot b}$

⟶ $a \to M(\lambda)\, a\,,$ $b \to (M(\lambda)^T)^{-1}\, b$

Apart from dilatation examples as the above one, there is rotation group U(1), for non compact sectors

$A = \mathbb{R} \times \mathbb{R}$ $B = \mathbb{R} \times \mathbb{R}$ Example: rotation between two free Maxwell fields, duality for Maxwell field

Reason for free massless models: non-compact sectors (or continuous dual sectors), independently of any global symmetry, lead to free massless theories: $\Phi_F = \int_{\Sigma_F} F\,,$ $\Phi_G = \int_{\Sigma_G} G$ $[\Phi_F, \Phi_G] = i$

It is necessary that the cross correlator is a "linking number term", a massless term that cannot renormalize

$$\langle F(x) G(0) \rangle = \int \frac{d^d p}{(2\pi)^{d-1}}\, \theta(p^0)\, \delta(p^2)\, e^{ipx}\, (P^{(k)} \tilde{*})(p) \qquad \Box \langle F(x) G(0) \rangle = 0$$

V. Benedetti, H.C, J. Magán (2022)

Additionaly,
there is only one possibility of U(1) group action that allows for compact sectors (and then for interacting theories)
"ABJ anomalous case"

$$A = \mathbb{Z} \times U(1) \qquad\qquad B = U(1) \times \mathbb{Z}.$$

$$(a_1, a_2) \to (a_1, a_2 + \lambda\, a_1)\,, \qquad (b_1, b_2) \to (b_1 - \lambda\, b_2, b_2)$$

$$a_1, b_2 \in \mathbb{Z}\,, \qquad a_2 \equiv a_2 + 2\pi\,, b_1 \equiv b_1 + 2\pi$$

In this case we would have:

1) A continuous global U(1) symmetry without Noether current (it changes classes)

2) Goldstone theorem: it follows from symmetry defined as automorphisms of local algebras and associated twists operators and does not require Noether current
(Buchholz, Doplicher, Longo, Roberts "a New look at Goldstone theorem", 1992)

3) "Quantization" of the group action: the compatibility of the cycle of the symmetry group $\lambda \equiv \lambda + \lambda_0$ with the one of the non-local sectors implies $\lambda_0 = 2\pi\, n$ (anomaly quantization)

4) If the non local operators exist in the IR limit then the symmetry has to exist in the IR: there must be massless local excitations charged under the symmetry and the rates of group action and group of non local operators must match between different scales (anomaly matching)

Pion electrodynamics (IR effective model, anomaly seen at the classical level)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \pi_0 \partial^\mu \pi_0 - \frac{1}{4\,e^2} F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\mu}\,\epsilon_{\mu\nu\rho\sigma}\,\pi_0 F^{\mu\nu}F^{\rho\sigma}$$

$$J^\mu = \mu\,\partial^\mu \pi_0\,, \qquad \partial_\mu J^\mu = \frac{1}{8}\,\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

anomaly equation

$$\tilde{J}^\mu = \mu\,\partial^\mu\pi_0 - \frac{1}{2}\,\tilde{F}^{\mu\nu}A_\nu\,, \quad \partial_\mu \tilde{J}^\mu = 0$$

New, conserved, but non gauge invariant current

$$\tilde{Q} = \int d^3x\,\tilde{J}^0(x) = \int d^3x\,\left(\mu\,\dot{\pi}_0(x) - \frac{1}{2}B^i(x)A_i(x)\right)$$

Corrected charge (gauge invariant action in infinite space)

$$\left[B^i(x), E^j(y)\right] = i\,e^2\,\epsilon^{ijk}\,\partial^x_k\,\delta(x-y)\,,$$
$$\left[p_0(x), E^i(y)\right] = \frac{i\,e^2}{\mu}B^i(y)\,\delta(x-y)\,,$$
$$\left[E^i(x), E^j(y)\right] = -\frac{i\,e^4}{\mu}\,\epsilon_{ijk}\left(\pi_0(y)\,\partial^k_y\delta(y-x) + \pi_0(x)\,\partial^k_x\delta(x-y)\right)$$

canonical commutation relations

$$\left[\tilde{Q}, \pi_0(x)\right] = -\mu\,i, \quad \left[\tilde{Q}, B_i(x)\right] = 0, \quad \left[\tilde{Q}, p_0(x)\right] = 0, \quad \left[\tilde{Q}, E_i(x)\right] = 0 \quad \left[\tilde{Q}, T^{\mu\nu}\right] = 0$$

$$U(\lambda)\,\pi_0(x)\,U(\lambda)^\dagger = \pi_0(x) + \lambda\,\mu\,, \qquad U(\lambda) = e^{i\lambda\tilde{Q}}\,.$$

The symmetry acts as an automorphism of the local algebras: only the pion is charged

What is special about this symmetry?

No (gauge invariant) Noether current: Are there non invariant non local classes?

$\partial_\nu \tilde{F}^{\mu\nu} = 0$ magnetic flux still conserved

$G^{\mu\nu} \equiv \frac{1}{e^2} F^{\mu\nu} - \frac{\pi_0}{\mu} \tilde{F}^{\mu\nu}, \quad \partial_\nu G^{\mu\nu} = 0$ redefined electric conserved flux: note it needs the pion

$\Phi_G = \int_\Sigma \star G, \quad \Phi_F = \int_{\tilde{\Sigma}} \star \tilde{F}$ $\quad [\Phi_G, \Phi_F] = i$ flux commutator

$W_q = e^{iq\Phi_F}, \quad T_g = e^{ig\Phi_G}$ $\longrightarrow$ generic non local operator is a dyon (d=4) $D_{(g,q)}$

$D^R_{(g,q)} D^{R'}_{(g',q')} = e^{i(q g' - q' g)} D^{R'}_{(g',q')} D^R_{(g,q)}$ commutation relations

The action of the symmetry $[\tilde{Q}, \Phi_G] = i\,\Phi_F, \quad [\tilde{Q}, \Phi_F] = 0$ $\longrightarrow$ $U(\lambda) D_{(g,q)} U^{-1}(\lambda) = D_{(g,q+\lambda g)}$

$$\begin{aligned} D^R_{(g_1,q_1+\lambda g_1)} D^R_{(g_2,q_2+\lambda g_2)} &= D^R_{(g_1+g_2,q_1+q_2+\lambda(g_1+g_2))}, \\ D^R_{(g,q+\lambda g)} D^{R'}_{(g',q'+\lambda g')} &= e^{i(q g' - q' g)} D^{R'}_{(g',q'+\lambda g')} D_{(g,q+\lambda g)}. \end{aligned}$$

respects the fusion rules
and commutation relations
Corresponds to the case discussed above

Since non local operators can be constructed additively in balls, this action on non local operators follows from the one on local operators. No freedom to choose the action of the group on non local operators here.

The same story applies for massless QED

$$S = \int d^4x \left[-\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}\, i\partial\!\!\!/\, \psi - \overline{\psi}\, A\!\!\!/\, \psi \right] \qquad J_5^\mu = \overline{\psi}\, \gamma^\mu \gamma^5\, \psi$$

$$\partial_\mu J_5^\mu = \frac{1}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \longrightarrow \tilde{J}_5^\mu = J_5^\mu - \frac{1}{4\pi^2} \tilde{F}^{\mu\nu} A_\nu \,, \qquad \partial_\mu\, \tilde{J}_5^\mu = 0$$

Adler 1969

$$\tilde{Q} = \int d^3x \left[\psi^\dagger(x) \gamma^5 \psi(x) - \frac{B^i(x) A_i(x)}{4\pi^2} \right]$$

Taking into account Schwinger terms $\left[J_5^0(x), E^i(y) \right] = \frac{ie^2}{2\pi^2} B^i(x)\, \delta(x-y)$ the charge generates an internal U(1) symmetry

$$\left[\tilde{Q}, A_i(x) \right] = 0 \,, \qquad \left[\tilde{Q}, E^i(x) \right] = 0 \qquad \left[\tilde{Q}, \bar{\psi} \left(\frac{1 \pm \gamma^5}{2} \right) \psi(x) \right] = \pm 2 \bar{\psi}(x) \left(\frac{1 \pm \gamma^5}{2} \right) \psi(x)$$

Again, it is an internal symmetry

How do we see the non trivial transformation of TL?: Witten effect

Chiral rotation of the fermion induces a topological theta term in the action from the measure of the path integral and this changes monopole to electric charge boundary conditions for the line operator

Quantization of the anomaly

(or how to compute the anomaly with a classical calculation plus non commutativity of dual non local operators)

$$\partial_\mu j^\mu = \beta\, \epsilon_{\alpha\beta\rho\sigma}\, F^{\alpha\beta}\, F^{\rho\sigma}$$

Take electric charges with minimal charge q_0 the (non local) WL charge have range $q \in [0, q_0)$

The non local TL have integer charges given by the Dirac quantization condition $g = \frac{2\pi}{q_0} k$

The current is normalized to have minimal charge equal to 1. Parameter $\lambda \in [0, 2\pi)$

Transformation with $\lambda(x)$ changes the action by $\delta S = \int (\partial_\mu j^\mu)\, \lambda(x) = \beta \int \lambda(x)\, \epsilon_{\mu\nu\rho\sigma}\, F^{\mu\nu}\, F^{\rho\sigma}$

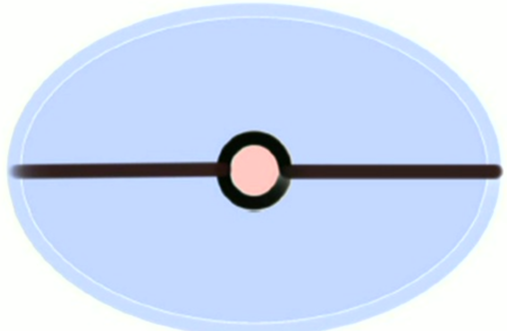
$$\longrightarrow \quad \nabla E = 8\,\beta\,(\nabla\lambda(x))\, B$$

Monopole boundary conditions (TL) get mixed with WL boundary conditions (Witten effect)

$$(g, q) \rightarrow (g, q + 8\,\beta\,(\delta\lambda) g)$$

$$\longrightarrow \quad 8\,\beta \times (2\pi) \times \left(\frac{2\pi}{q_0}\right) = n\, q_0$$

$$\longrightarrow \quad \partial_\mu \hat{j}^\mu = n\, \frac{q_0^2}{32\,\pi^2}\, \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$



QED then has the minimal possible value n=1 (given that the minimal gauge invariant chiral charge involves two electrons). QED with 1 chiral fermion does not make sense: no gauge invariant local chiraly charged operator (gauge and chiral charge are the same)

Final remarks

This anomalous action (U(1) that changes classes) required d=4 where we have magnetic and electric sectors in the same ring-like region, with the same topology.

For d=2 (Schwinger model) the symmetry is explicitly broken by instantons. There is no chiral symmetry.

For d=6 we have an analogous to the pion electrodynamics

$$S = \frac{1}{2}\int d\pi_0 \wedge \star d\pi_0 + \frac{1}{2c^2}\int F \wedge \star F + \frac{1}{\mu}\int \pi_0 F \wedge F \wedge F$$

$\pi_0 \to \pi_0 + \mathrm{cons}$ is a symmetry.

Again, this has a non gauge invariant current. However, TL and WL live in different topologies and the anomalous action does not hold, classes are invariant. Conjecture (from the strong form of Noether theorem): "An effective model with a continuous symmetry , no Noether current, and no non invariant HDV class cannot be UV completed"

There are several papers in the HEP literature that claim that this type of symmetry actions that mix with generalized symmetries are "non-invertible". However, the symmetry is a U(1) for the local physics. This invertibility is behind anomaly quantization.

The fact that the global internal symmetry is invertible (a group U(1)) is in accordance with the DHR theorem: It can be rephrased as "All 0-form symmetries come from a group".