Title: Concatenate codes, save qubits

Speakers: Hayata Yamasaki

Series: Perimeter Institute Quantum Discussions

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Abstract: The essential requirement for fault-tolerant quantum computation (FTQC) is the total protocol design to achieve a fair balance of all the critical factors relevant to its practical realization, such as the space overhead, the threshold, and the modularity. A major obstacle in realizing FTQC with conventional protocols, such as those based on the surface code and the concatenated Steane code, has been the space overhead, i.e., the required number of physical qubits per logical qubit. Protocols based on high-rate quantum low-density parity-check (LDPC) codes gather considerable attention as a way to reduce the space overhead, but problematically, the existing fault-tolerant protocols for such quantum LDPC codes sacrifice the other factors. Here we construct a new fault-tolerant protocol to meet these requirements simultaneously based on more recent progress on the techniques for concatenated codes rather than quantum LDPC codes, achieving a constant space overhead, a high threshold, and flexibility in modular architecture designs. In particular, under a physical error rate of 0.1%, our protocol reduces the space overhead to achieve the logical CNOT error rates 10^{-10} and 10^{-24} by more than 90% and 97%, respectively, compared to the protocol for the surface code. Furthermore, our protocol achieves the threshold of 2.4% under a conventional circuit-level error model, substantially outperforming that of the surface code. The use of concatenated codes also naturally introduces abstraction layers essential for the modularity of FTQC architectures. These results indicate that the code-concatenation approach opens a way to significantly save qubits in realizing FTQC while fulfilling the other essential requirements for the practical protocol design.

Zoom link

Concatenate codes, save qubits

Hayata Yamasaki
Graduate School of Science, The University of Tokyo
hayata.yamasaki@gmail.com @hayatayamasaki

10th April, 2024

References:

Hayata Yamasaki, Masato Koashi, arXiv:2207.08826 Nature Physics 2024

Satoshi Yoshida, Shiro Tamiya, Hayata Yamasaki, arXiv:2402.09606

About Me

Social implementation

Advance of IT society by quantum technology

↑

Theoretical foundation = my works

Useful quantum algorithm
Quantum machine learning with high speed/applicability

↑

Implementation of QC
Low-overhead/scalable fault-tolerant QC (FTQC)

↑

Efficient Q operations
Quantitative analysis of use of quantum resources

↑

Experimental foundation

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Low-overhead/scalable fault-tolerant QC (FTQC)

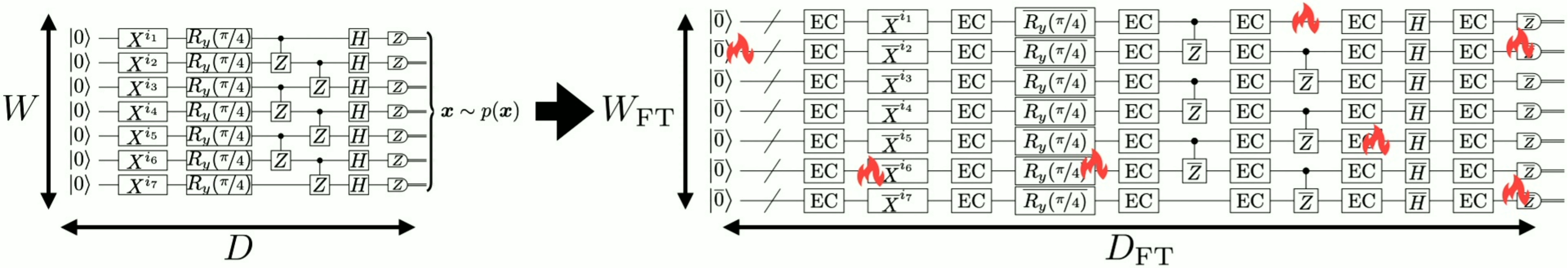
Efficient Q operations
Quantitative analysis of use of quantum resources

Experimental foundation — Advance of quantum technology

- **Provable quantum advantage** in solving machine learning tasks arXiv:2305.11212, arXiv:2312.03057
- **Quantum machine learning (QML) using exponential speedup** without sparse or low-rank matrices arXiv:2004.10756 (NeurIPS2020), arXiv:2106.09028 arXiv:2301.11936 (ICML2023)
- **Time-efficient constant-space-overhead FTQC** arXiv:2207.08826 (Nat.Phys.2024) arXiv:2402.09606
- Analysis of **GKP Code** arXiv:1910.08301 (PRA2020) arXiv:1911.11141 (PRR2020) arXiv:2006.05416
- Practical **testing** of entangled states arXiv:2201.11127 arXiv:2202.13131 (PRL2022)
- **Quantum resource theories** arXiv:2310.09154 (PRL2024) arXiv:2106.01372 (Quantum2022) etc.

Task of Fault-Tolerant Quantum Computation (FTQC)

Original circuit | **Noisy** circuit with **physical error rate** $p_0 > 0$



<u>Task</u>: Given ϵ and an original circuit, perform **the noisy circuit** to output $\boldsymbol{x} \sim \tilde{p}(\boldsymbol{x}) \approx_\epsilon p(\boldsymbol{x})$

(the fault-tolerant circuit)

→ Use **quantum error-correcting codes** that can suppress **logical error rate** arbitrarily

$$p_0 < p_{\mathrm{th}} \Rightarrow p_L \lesssim \frac{\epsilon}{WD}$$ for achieving the overall error $O(\epsilon)$

Below threshold

Width × Depth = Circuit size

Space overhead := $\frac{W_{\mathrm{FT}}}{W}$ Time overhead := $\frac{D_{\mathrm{FT}}}{D}$

Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024), Yoshida, Tamiya, Yamasaki, arXiv:2402.09606

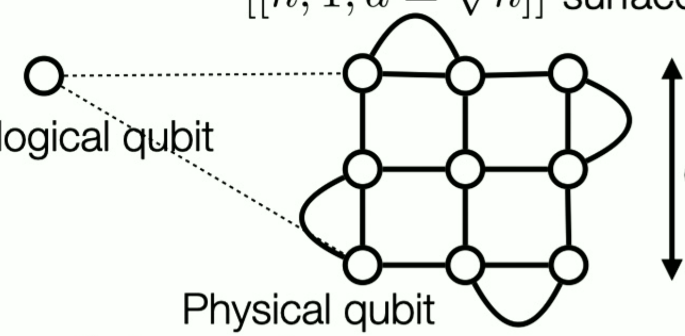
Obstacle: Overheads of FTQC

Two conventional approaches for FTQC to achieve $p_0 < p_{\rm th} \Rightarrow$ $p_L \lessapprox \frac{\epsilon}{WD}$

$[[n,k,d]]$ n #physical qubits, k #logical qubits, d distance

1: Quantum low-density parity-check (LDPC) code

$[[n,1,d=\sqrt{n}]]$ surface code

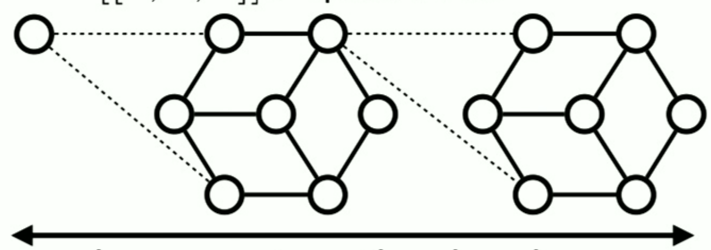


Increase **distance d**

$$n = d^2,\ p_L \lessapprox \left(\frac{p_0}{p_{\rm th}}\right)^d$$

2: Concatenated code

$[[7,1,3]]$ 7-qubit code



Increase **concatenation level L**

$$n = 7^L,\ p_L \lessapprox \left(\frac{p_0}{p_{\rm th}}\right)^{2^L}$$

Obstacle in realizing FTQC: Polylog overhead → diverging to infinity on large scales

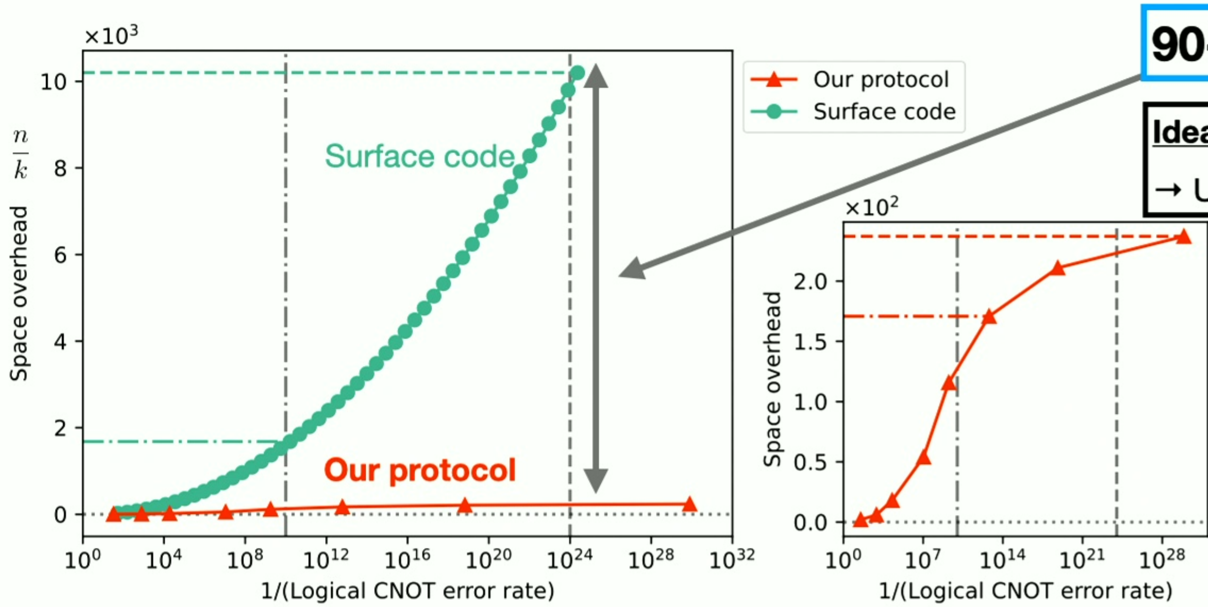
Space $\frac{W_{\rm FT}}{W} \approx \mathrm{polylog}\left(\frac{WD}{\epsilon}\right)$ Time $\frac{D_{\rm FT}}{D} \approx \mathrm{polylog}\left(\frac{WD}{\epsilon}\right)$

Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024), Yoshida, Tamiya, Yamasaki, arXiv:2402.09606

Summary of Main Results

Results: Concatenation of Q Hamming codes achieves **constant-space-overhead FTQC**

+ We construct an optimized protocol with **good performance in practical regimes**



90-97% saving of space overhead

Idea: Use small codes for **high threshold** at physical level

→ Use high-rate codes to **save qubits** at higher levels

Achieving constant space overhead

Switch to quantum Hamming codes

$[[15,7,3]], [[31,21,3]], [[63,51,3]], \ldots$ High-rate

Start with C4/C6 code

Knill, arXiv:quant-ph/0410199, Nature (2005)

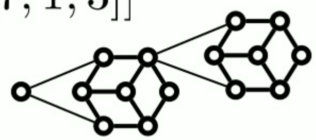
Merits: Full protocol + Saving qubits + High threshold 2.4% under circuit noise + Modularity

Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024), Yoshida, Tamiya, Yamasaki, arXiv:2402.09606 5

Concatenation of Quantum Hamming Codes

Steane's 7-qubit code

$[[7, 1, 3]]$

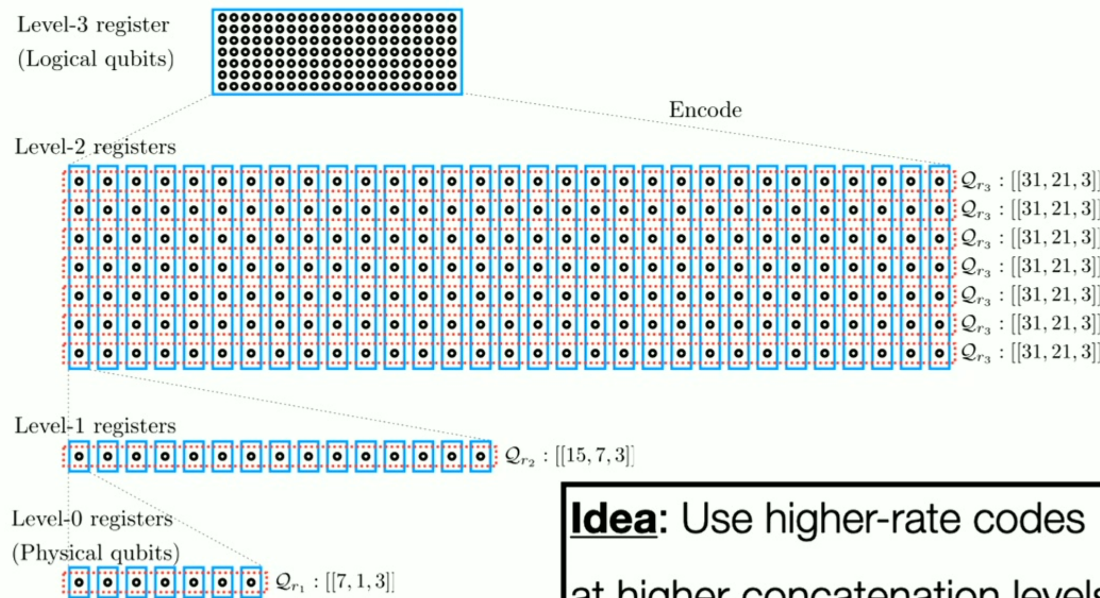


Generalization

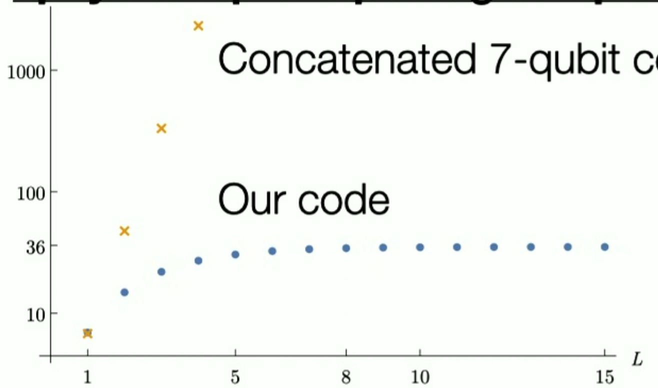
Quantum Hamming codes

$[[2^r - 1, 2^r - 2r - 1, 3]]$ Many logical qubits

$[[7, 1, 3]], [[15, 7, 3]], [[31, 21, 3]], [[63, 51, 3]], \ldots$



#physical qubits per logical qubit



Idea: Use higher-rate codes at higher concatenation levels

Concatenating quantum Hamming codes at growing rate yields a **non-vanishing overall rate**

Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024) 6

Efficient Decoder

Steane's 7-qubit code $[[7,1,3]]$

Generalization

Quantum Hamming codes

$[[2^r-1, 2^r-2r-1, 3]]$

To correct one error

Stabilizer generators

$$Z\ I Z\ I Z\ I Z$$
$$I Z Z\ I\ I Z Z$$
$$I\ I\ I Z Z Z Z$$

$$X\ I X\ I X\ I X$$
$$I X X\ I\ I X X$$
$$I\ I\ I X X X X$$

$$Z\ I Z\ I Z\ I Z\ I Z\ I Z\ I Z\ I Z$$
$$I Z Z\ I\ I Z Z\ I\ I Z Z\ I\ I Z Z$$
$$I\ I\ I Z Z Z Z\ I\ I\ I\ I Z Z Z Z$$
$$I\ I\ I\ I\ I\ I\ I Z Z Z Z Z Z Z Z$$
$$X\ I X\ I X\ I X\ I X\ I X\ I X\ I X$$
$$I X X\ I\ I X X\ I\ I X X\ I\ I X X$$
$$I\ I\ I X X X X\ I\ I\ I\ I X X X X$$
$$I\ I\ I\ I\ I\ I\ I X X X X X X X X$$

binary representation

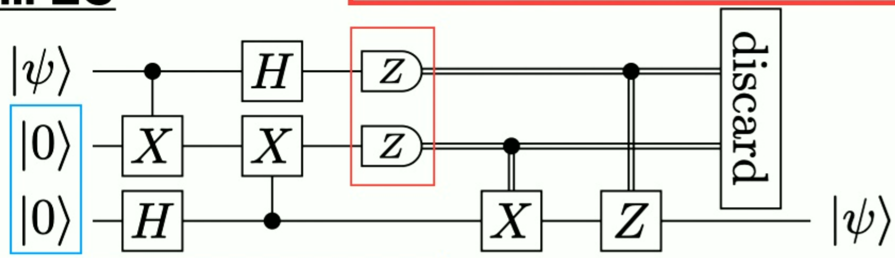
- Syndrome bitstring = the **binary representation** of the location of a single error
- Extraction of high-weight syndrome by **Knill or Steane QEC**, applicable to concatenated code

Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024)

Error Correction for Concatenated Codes

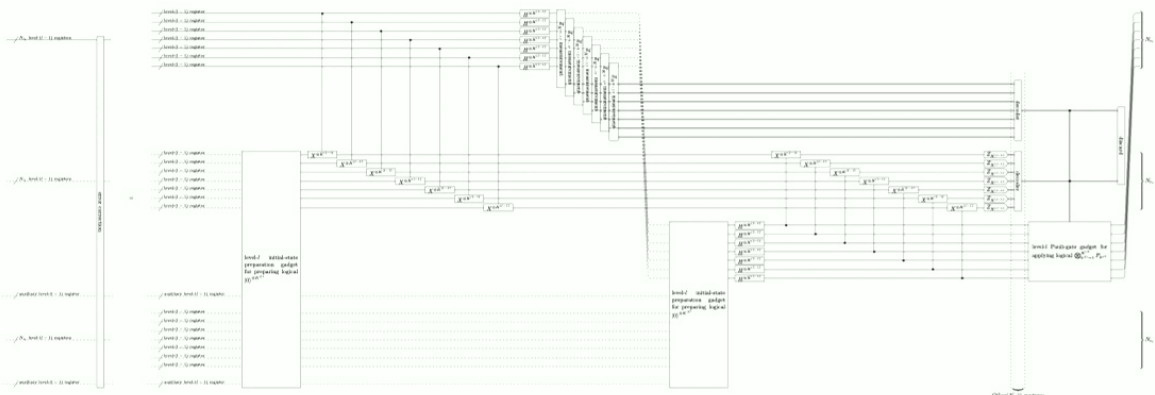
Knill EC

Extraction of all syndromes at once via measurements for **quantum teleportation**



Fault-tolerant state preparation for concatenated code

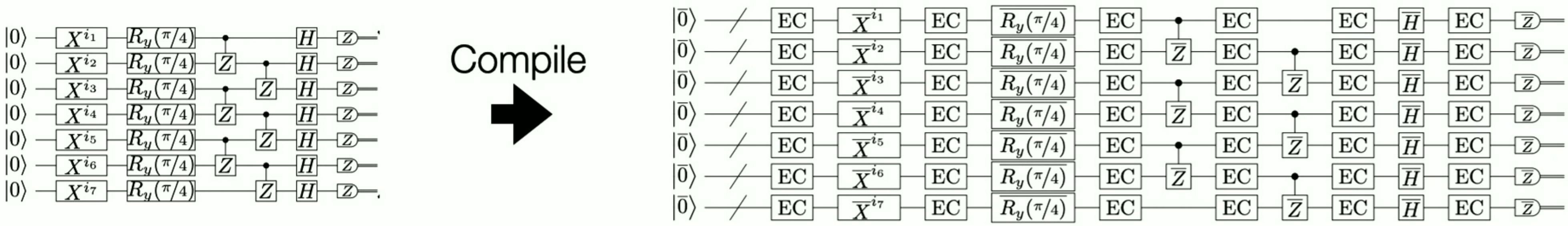
EC by **teleporting logical state into fresh auxiliary qubits**



Use Knill EC whenever possible to correct leakage errors

Full Construction of Fault-Tolerant Protocol

For l=L,L-1,...,1,



Level-l circuit

Level-(l-1) circuit to implement logical level-l circuit

<u>Contribution</u>: Explicit protocol (decoder/gate/compilation) & thorough runtime analysis

- Recursively replace operation with **gadget** & insert error correction in between
- For quantum Hamming codes, **transversal gates** are logical $H^{\otimes k}, \mathrm{CNOT}^{\otimes k}, \mathrm{Pauli}$
- All the other gates by **gate teleportation** with fault-tolerant preparation of auxiliary states

Construction of Gadgets for Operations

Transversal operations

Measurement

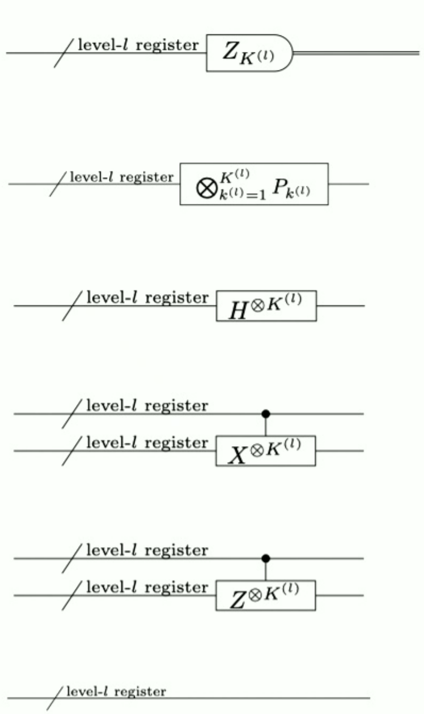
Pauli

Hadamard

CNOT

CZ

Wait

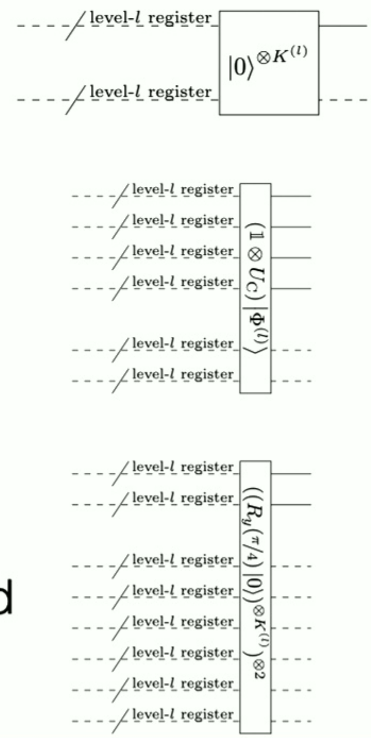


Fault-tolerant preparation by verification

Initial state

Clifford

Non-Clifford



$\left|\Phi^{(0)}\right\rangle \propto (|00\rangle + |11\rangle)^{\otimes 2}$

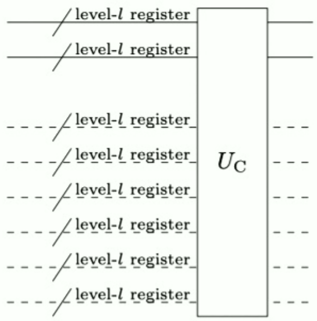
U_C : Two-register Clifford

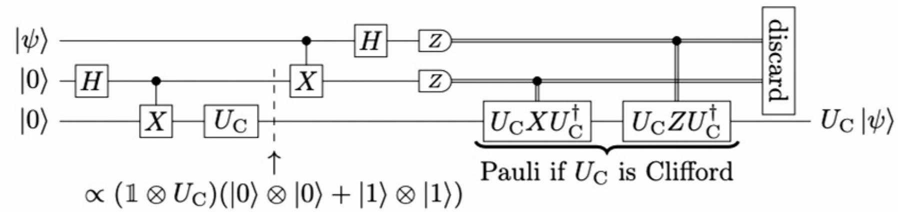
$$R_y(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

Explicit construction of **a full set of gadgets** for all operations required for FTQC

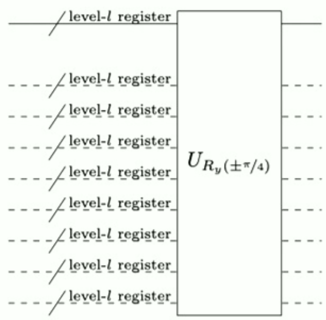
Gates by Gate Teleportation

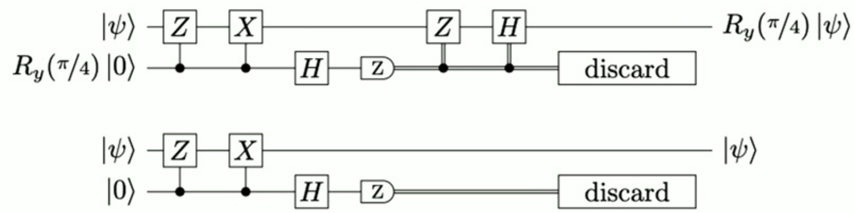
Two-register Clifford





Single-register non-Clifford



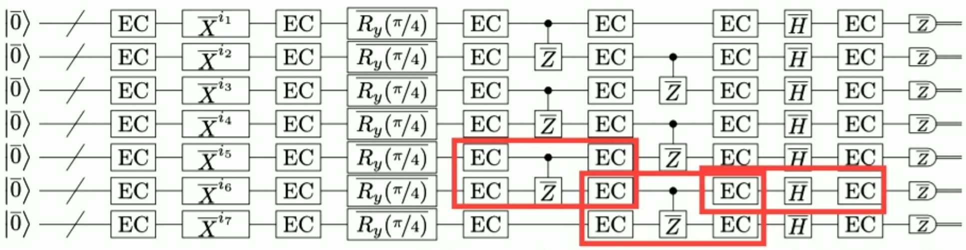


Universal gate set implemented by **gate teleportation** using fault-tolerant state preparation

Provable Existence of Threshold

<u>Error model</u>: Local stochastic error model

s different locations in a circuit of physical qubits may become faulty with probability $< p_0^s$



Extended rectangle (**ExRec**)

Level-(l-1) circuit to implement logical level-l circuit with **increasing parameter of quantum Hamming codes** linearly

$[[n_l = 2^r - 1, 2^r - 1 - 2r, 3]],\ r = l + 3$

We can show by induction for l=1,2,3…

Size of ExRec: growing polynomially in the code size

- Logical **level-l** circuit undergoes the local stochastic error model
- The **logical error rate** of the level-l circuit is $p_l \leqq \binom{\mathrm{poly}(n_l)}{2} p_{l-1}^2 \leqq 2^{\alpha l} p_{l-1}^2,\ (\alpha > 0)$

Maximum probability of having two errors in each ExRec

<u>Overall logical error rate:</u> the **same bound** as conventional concatenated codes

$$p_L \leqq 2^{\alpha L} p_{L-1}^2 \leqq 2^{\alpha(L 2^0 + (L-1) 2^1)} p_{L-2}^{2^2} \leqq \cdots \leqq 2^{\alpha(L\cdot 2^0 + (L-1)\cdot 2^1 + \cdots + 1\cdot 2^L)} p_0^{2^L} = O\left(\left(\frac{p_0}{p_{\mathrm{th}}}\right)^{2^L}\right)$$

Geometric sequence

Requirements for Practical FTQC

- Logical gates
- Low space overhead
- Short time overhead
- High threshold
- Modular implementation

↕ Covered by Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024)

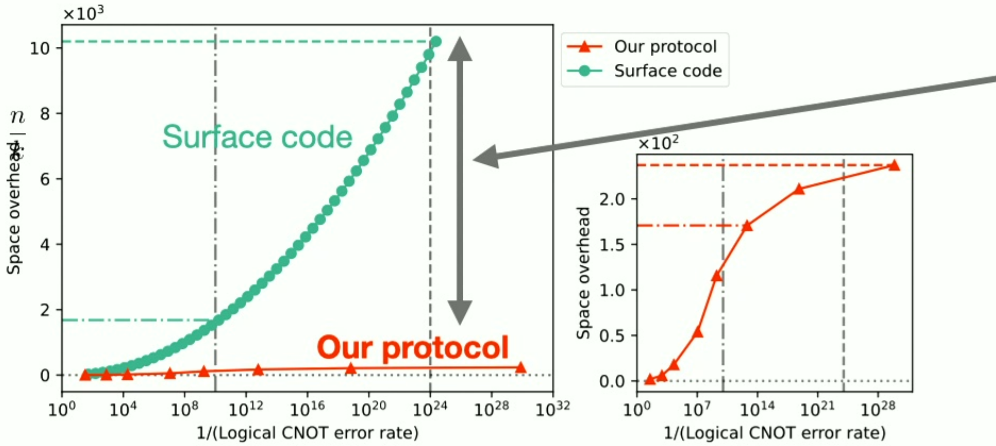
Problem

- Conventional protocols require large space overhead
- High-rate QLDPC codes sacrifice time, threshold, and modularity

→ Our new optimized protocol takes care of all essential requirements

Yoshida, Tamiya, Yamasaki, arXiv:2402.09606

Underlying Codes to Improve Threshold



90-97% saving of space overhead

- Threshold of the original protocol based on quantum Hamming codes: 10^{-5}
- Logical error rate of C4/C6 code < 10^{-7} with **a few hundred physical qubits**
- C4/C6 protocol has threshold 2.4%

	Quantum code	N	K	N/K
level-1	$C_4 (= [[4,2,2]])$	4	2	2
level-2	$C_6 (= [[6,2,2]])$	12	2	6
level-3	$C_6 (= [[6,2,2]])$	36	2	18
level-4	$C_6 (= [[6,2,2]])$	108	2	54
level-5	$\mathcal{Q}_4 (= [[15,7,3]])$	1.6×10^3	14	1.2×10^2
level-6	$\mathcal{Q}_5 (= [[31,21,3]])$	5.0×10^4	2.9×10^2	1.7×10^2
level-7	$\mathcal{Q}_6 (= [[63,51,3]])$	3.2×10^6	1.5×10^4	2.1×10^2
level-8	$\mathcal{Q}_7 (= [[127,113,3]])$	4.0×10^8	1.7×10^6	2.4×10^2

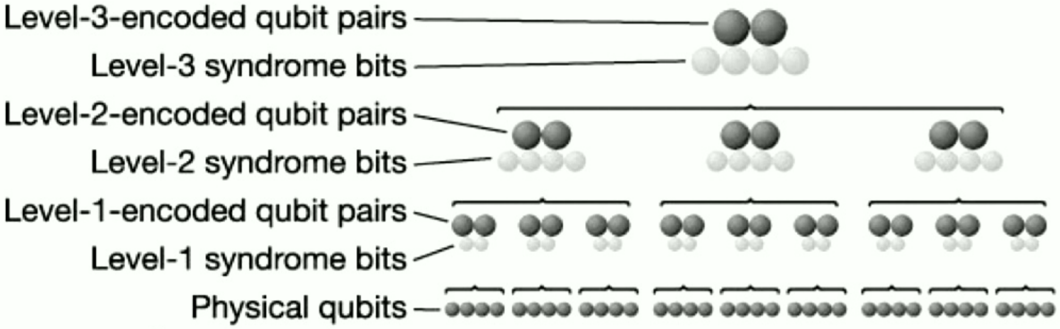
↕ Start with C4/C6 code (Next slide)
Knill, arXiv:quant-ph/0410199, Nature (2005)

↕ Switch to quantum Hamming codes
$[[15,7,3]], [[31,21,3]], [[63,51,3]], \ldots$ High-rate

→Achieving constant space overhead

C4/C6 Code

<u>Merit</u>: Threshold for protocol based on C4/C6 code : 2.4% vs Threshold for surface code: 0.3%



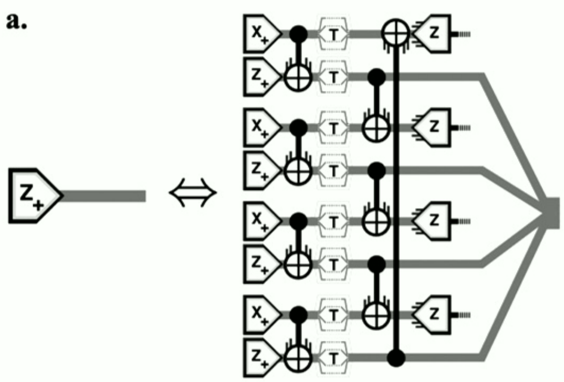
[[6, 2, 2]] code

[[6, 2, 2]] code Distance 2 for qubit pairs

[[4, 2, 2]] code

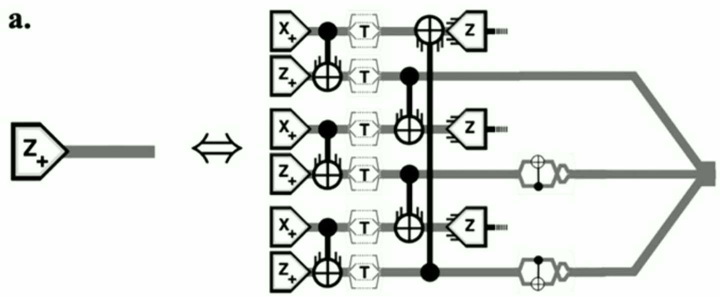
<u>Protocol</u>

8 qubits in 1D periodic arrangement

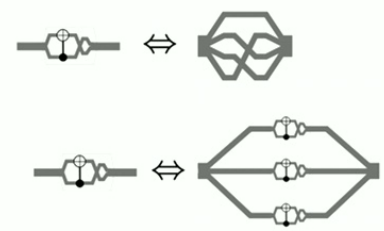


Preparation of [[4, 2, 2]] code

6 level-1 registers in 1D periodic arrangement



Preparation of [[6, 2, 2]] code



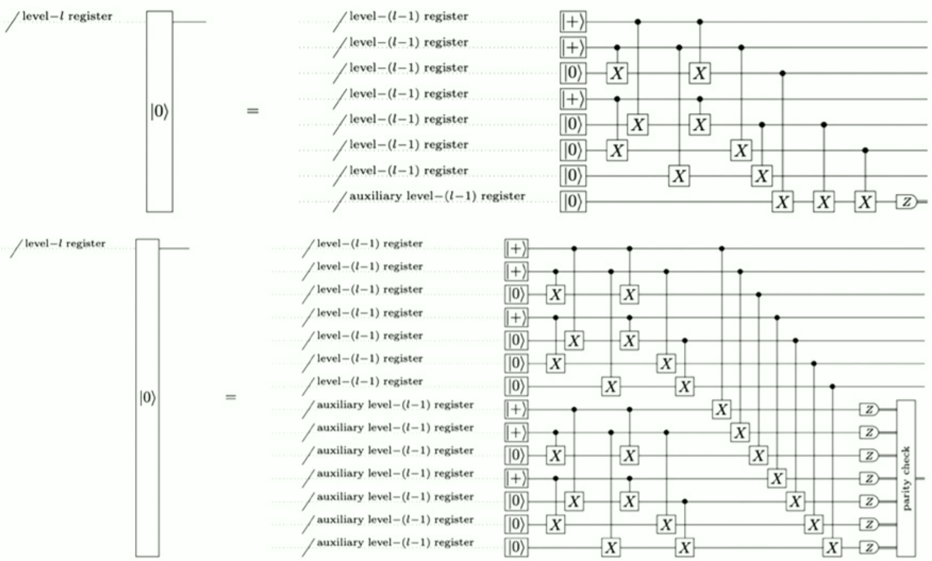
Threshold Analysis

	Threshold	Space overhead		
		$p = 0.01\%$	$p = 0.1\%$	$p = 1\%$
C_4/C_6 code	2.4%	18	**54**	**1458**
Surface code	0.31%	121	841	-
Steane code	0.030%	343	-	-
C_4/Steane code	0.15%	**14**	4802	-

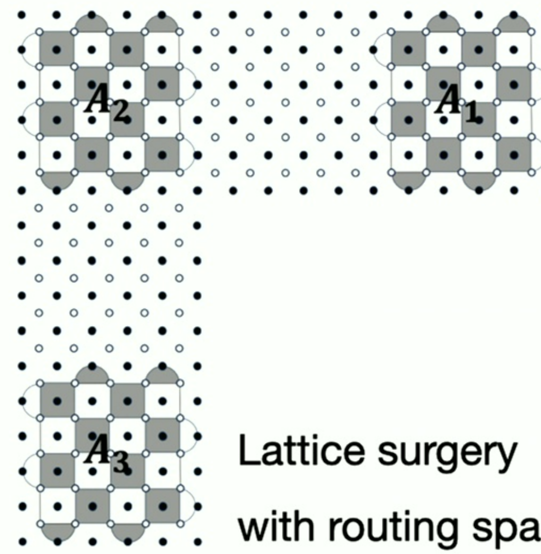
Our contribution: Systematic comparison

- Under circuit-level depolarizing noise
- Threshold for CNOT gate (not memory)
- Hard-decision decoder & MWPM decoder
- Numerical simulation in the same setting

Our paper includes…



Comparison of preparations of Steane code



Lattice surgery with routing space

Modularity

Error suppression by concatenated Q Hamming codes:

Size of ExRec: **Growing** polynomially in code size

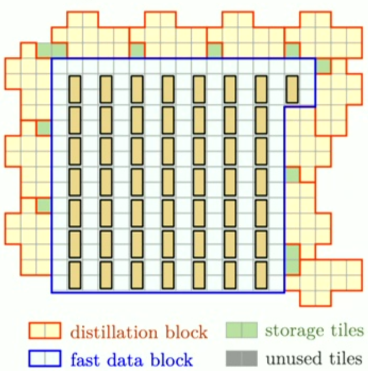
$$p_l \leq \binom{\mathrm{poly}(n_l)}{2} p_{l-1}^2 \leq 2^{\mathrm{poly}(l)} p_{l-1}^2$$

$$p_l \lesssim \left(\frac{p_0}{p_{\mathrm{th}}}\right)^{2^l}$$

Logical error rate: **Decaying much faster → More flexibility in architecture design on large scales**

Challenge: Full control of a large LDPC-code chip

Solution: Concatenation = Abstraction



Litinski, arXiv:1808.02892

Could this controllably scale up?

Unit at lower error rate

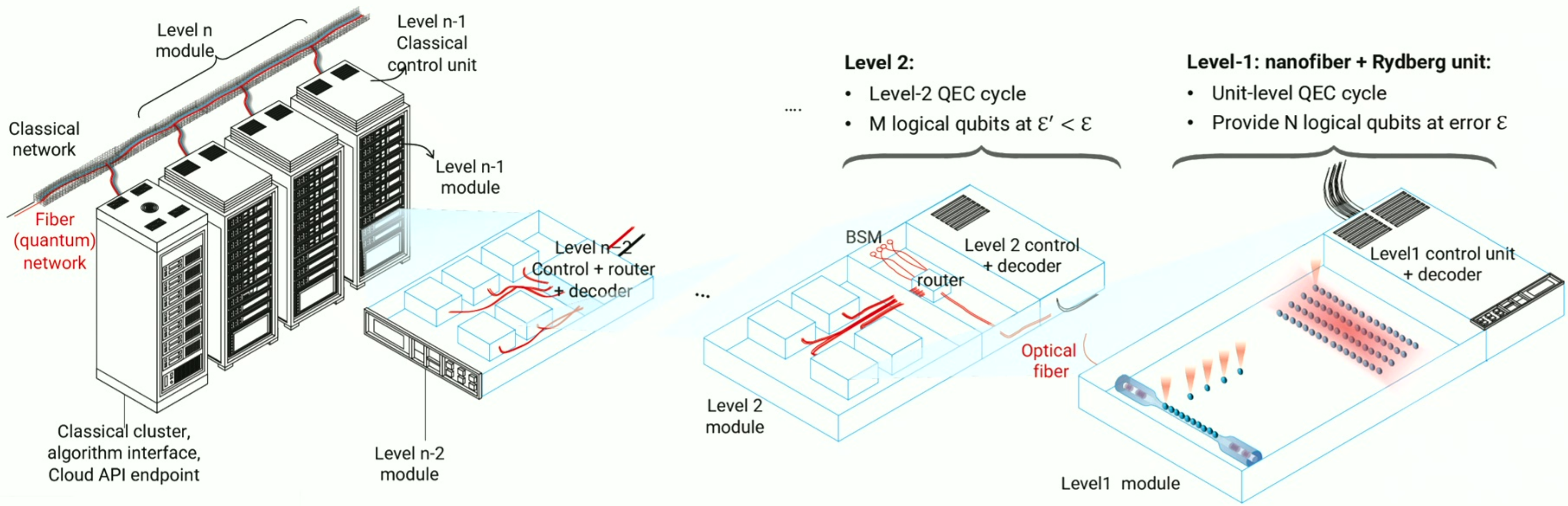
Concatenate the units to scale up

Opening a way of **low-overhead** FTQC **by concatenating small-size modular parts**

Yamasaki, Koashi, arXiv:2207.08826 (Nat. Phy. 2024), Yoshida, Tamiya, Yamasaki, arXiv:2402.09606

New criteria for physical implementation of quantum computation

Design principle: Finite technological requirement ＋ Threshold behavior ＋ Modularity



Develop finite-size fault-tolerant modules
Let the modules be equipped with communication interfaces
Goal：Abstract modules by QEC＋Inferfaces for scaling up

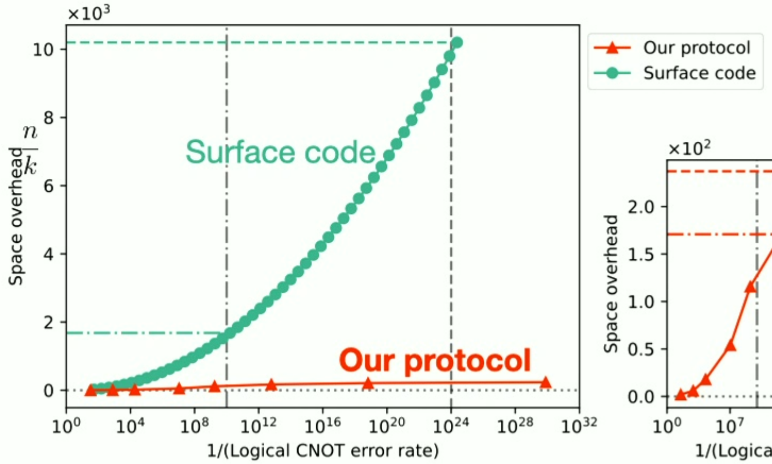
Combine finite-size modules by interfaces to make a concatenated code
Form a higher-level module at low error rate from lower-level modules
Repeat this procedure to suppress error rate, so we have more margin

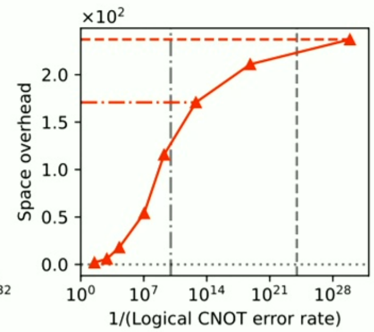
H. Yamasaki, S. Sunami, A. Goban, in preparation

Conclusion

Results: Concatenation of Q Hamming codes achieves **constant-space-overhead FTQC**

+ We construct an optimized protocol with **good performance in practical regimes**





We did not improve a protocol

What we need was a new approach

Merits: Full protocol + Saving qubits + High threshold 2.4% under circuit noise + Modularity

→ Concatenate codes, save qubits

Hayata Yamasaki hayata.yamasaki@gmail.com