

Title: From fluctuating gravitons to Lorentzian quantum gravity

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Series: Quantum Gravity

Date: April 11, 2024 - 2:30 PM

URL: <https://pirsa.org/24040086>

Abstract: I will review recent progress in the asymptotic safety approach to quantum gravity. This includes the computation of momentum-dependent graviton correlation functions, the structure of the Standard Model with asymptotically safe gravity, and the recent first computation directly in space-times with Lorentzian signatures via the spectral function of the graviton. Overall, I will display the progress towards the computation of the quantum effective action which encapsulates effective field theory in the IR limit and the asymptotic safety regime in the UV limit.

Zoom link

From fluctuating gravitons to Lorentzian quantum gravity

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Quantum Gravity Seminar, Perimeter Institute, 11. April 2024



Path integral for metric quantum gravity

- Assumptions
 - Metric carries fundamental degrees of freedom
 - Diffeomorphism invariance
- Path integral

$$\int \mathcal{D}\hat{g}_{\mu\nu} e^{-S[\hat{g}]} \quad \text{or} \quad \int \mathcal{D}\hat{g}_{\mu\nu} e^{iS[\hat{g}]}$$

Path integral for metric quantum gravity

- Assumptions
 - Metric carries fundamental degrees of freedom
 - Diffeomorphism invariance
- Path integral with gauge fixing, sources

$$Z[\bar{g}, J] \sim \int \mathcal{D}\hat{h}_{\mu\nu} e^{-S[\bar{g}+\hat{h}] - S_{\text{gf}}[\bar{g}, \hat{h}] - S_{\text{gh}}[\bar{g}, \hat{h}, \hat{c}, \hat{\bar{c}}] + \int_x \sqrt{\bar{g}} J^{\mu\nu}(x) \hat{h}_{\mu\nu}(x)}$$

- Metric split $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ required by gauge fixing and regulator
- Methods: Perturbation theory, lattice, functional methods, ...

Perturbative quantum gravity

Einstein-Hilbert gravity

$$S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_x \sqrt{\det g_{\mu\nu}} (2\Lambda - R(g_{\mu\nu}))$$

- Perturbatively non-renormalisable: $[G_{\text{N}}] = -2$
- Need infinitely many counter terms: No predictivity [’t Hooft, Veltmann '74; Goroff, Sagnotti '85]

Higher-derivative action

$$S_{\text{HD}} = S_{\text{EH}} + \int_x \sqrt{\det g_{\mu\nu}} \left(\frac{1}{2\lambda} C_{\mu\nu\rho\sigma}^2 - \frac{\omega}{3\lambda} R^2 \right)$$

- Perturbatively renormalisable: $[\omega] = [\lambda] = 0$
- Non-unitary [Stelle '74]

$$G_{\text{graviton}} \sim \frac{1}{p^2 + p^4/M_{\text{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

The functional renormalisation group

Non-perturbative renormalisation group equation [Wetterich '93]

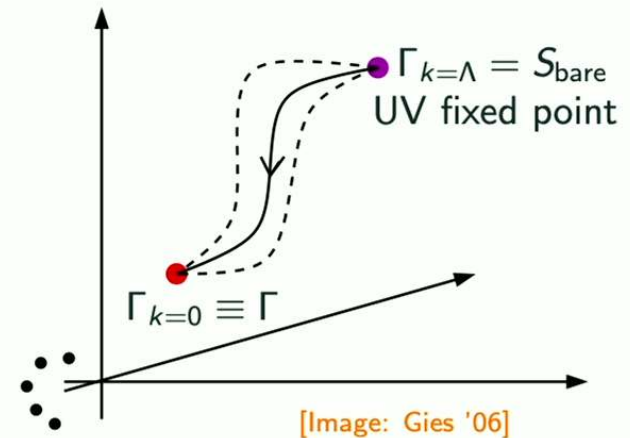
$$k\partial_k\Gamma_k = \frac{1}{2}\text{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k \right]$$

R_k = regulator

Γ_k = scale-dependent effective action

Interpolation between

- bare action / UV FP
- quantum effective action Γ
- Wilsonian integrating out of momentum modes

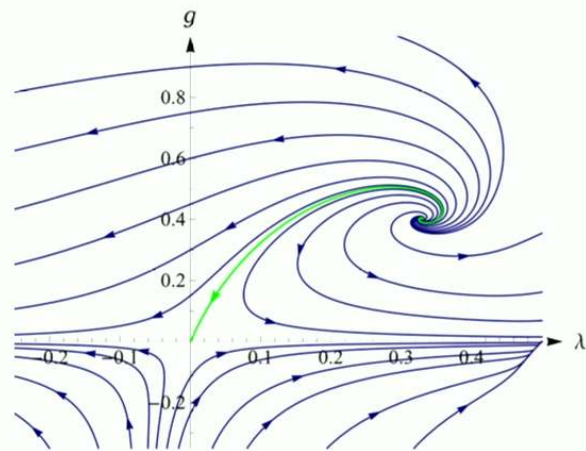


Asymptotically safe quantum gravity in $d = 4$

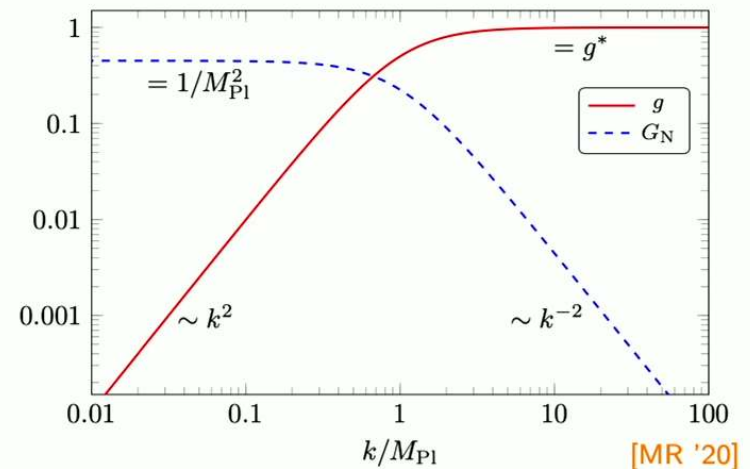
QG could be non-perturbatively renormalisable via an interacting UV FP

[Weinberg '76]

$$S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_{\text{x}} \sqrt{g} (2\Lambda - R)$$



[Reuter '96; Reuter, Saueressig '01]



[MR '20]

Expansion in curvature invariants

 Λ
 R

$$R^2 \quad R_{\mu\nu} R^{\mu\nu} \quad C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$$

$$R^3 \quad R \nabla^2 R \quad C_{\mu\nu\rho\sigma} \nabla^2 C^{\mu\nu\rho\sigma} \quad C_{\mu\nu}^{\kappa\lambda} C_{\kappa\lambda}^{\rho\sigma} C_{\rho\sigma}^{\mu\nu} \quad \dots$$

$$R^4 \quad R \nabla^4 R \quad C_{\mu\nu\rho\sigma} \nabla^4 C^{\mu\nu\rho\sigma} \quad \dots$$

 \vdots
 \vdots

[Gies, Knorr, Lippoldt, Saueressig '16, Falls, Litim, Schröder '18; Knorr, Ripken, Saueressig '19; Kluth, Litim '20; Falls, Ohta, Percacci '20; Knorr '21; Baldazzi, Falls, Kluth, Knorr '23; ...]

Fluctuation approach – expansion in momentum dependent graviton correlations

- Treat \bar{g} and h independently, resolve fluctuation correlation functions

$$\Gamma_k[\bar{g}, h] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(0, h_{a_1} \dots h_{a_n})}[\bar{g}, 0] \cdot h_{a_1} \dots h_{a_n}$$

- Flat background $\bar{g} = \delta$ allows for momentum-space techniques

$$\begin{aligned} \partial_t \Gamma_k &= \frac{1}{2} \text{[Diagram 1]} - \text{[Diagram 2]} \\ \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{[Diagram 3]} + \text{[Diagram 4]} - 2 \text{[Diagram 5]} \\ \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{[Diagram 6]} + 3 \text{[Diagram 7]} - 3 \text{[Diagram 8]} + 6 \text{[Diagram 9]} \\ \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \text{[Diagram 10]} + 3 \text{[Diagram 11]} + 4 \text{[Diagram 12]} - 6 \text{[Diagram 13]} - 12 \text{[Diagram 14]} + 12 \text{[Diagram 15]} - 24 \text{[Diagram 16]} \end{aligned}$$

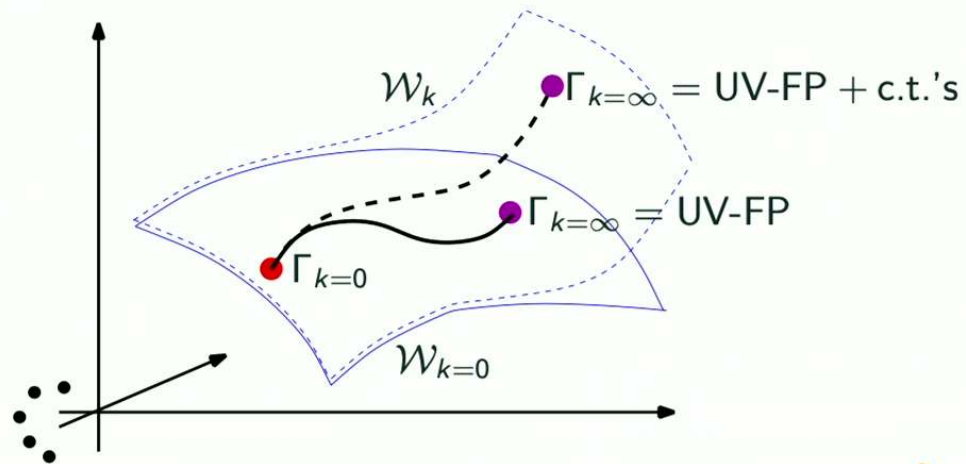
The diagrams represent various Feynman diagrams for graviton loops and vertices. Diagrams with red dashed lines and arrows represent ghost loops and vertices, while blue solid lines and circles represent graviton loops and vertices. The diagrams are arranged in two columns, with the left column showing the expansion of $\partial_t \Gamma_k$ and the right column showing the expansion of $\partial_t \Gamma_k^{(h)}$ and $\partial_t \Gamma_k^{(c\bar{c})}$.

[Christiansen, Knorr, Meibohm, Pawłowski, MR '15; Denz, Pawłowski, MR '16; Pawłowski, MR '20; '23; ...]

Controlling the diffeomorphism symmetry

- Background metric $\bar{g}_{\mu\nu}$ and fluctuation field $h_{\mu\nu}$ are treated independently
- Diffeomorphism symmetry is governed by non-trivial Ward identity

$$\mathcal{W}_k = \mathcal{G}\Gamma_k + \mathcal{G}S_{\text{regulator}} - \langle \mathcal{G}(S_{\text{gf}} + S_{\text{gh}} + \Delta S_{\text{regulator}}) \rangle = 0$$

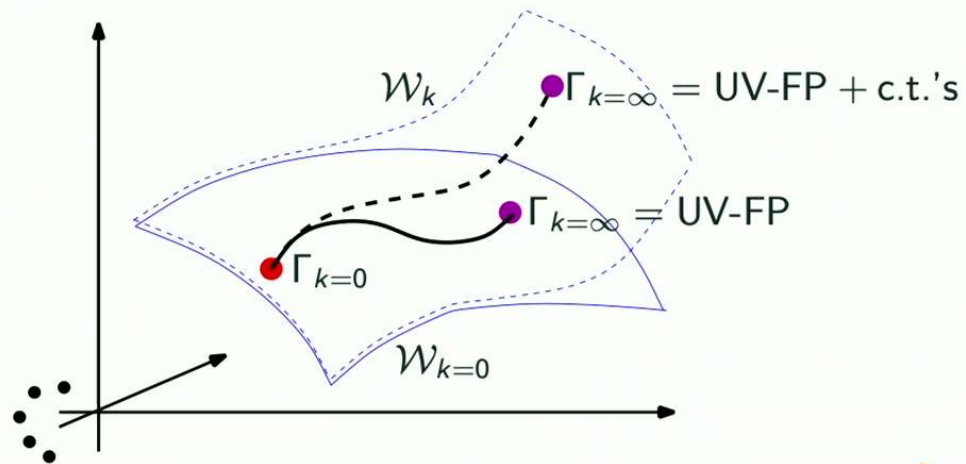


[Image: Gies '06]

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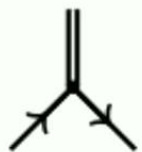


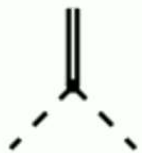
[Image: Gies '06]

Avatars of couplings


 $\longrightarrow G_3(p_1, p_2, p_3)$


 $\longrightarrow G_c(p_1, p_2, p_3)$


 $\longrightarrow G_\psi(p_1, p_2, p_3)$


 $\longrightarrow G_\varphi(p_1, p_2, p_3)$

...

- Momentum dependent couplings
- Related by symmetry identities
- Reduce to G_N + higher-order terms for $k \rightarrow 0$

Matching to curvature invariants

Form factor action

[Knorr, Ripken, Saueressig '22; ...]

$$\Gamma[\bar{g}, h] = \int_x \left(\frac{2\Lambda - R}{16\pi G_N} + Rf_R(\square)R + C_{\mu\nu\rho\sigma}f_C(\square)C^{\mu\nu\rho\sigma} + \dots \right) + S_{\text{gf}} + S_{\text{gh}}$$

Fully determined by momentum dependence of propagator ($g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_N} h_{\mu\nu}$)

[Knorr, Schiffer '21; Pawłowski, MR '23]

$$G_{h_{tt}h_{tt}} = \frac{32\pi}{p^2 + 32\pi G_N p^4 f_C(p^2)} \quad G_{h_s h_s} = \frac{-16\pi}{p^2 - 96\pi G_N p^4 f_R(p^2)}$$

Three-point Newton coupling relates to Goroff-Sagnotti form factor

$$\int_x f_{C^3}(\nabla_1, \nabla_2, \nabla_3) C_{\mu\nu\rho\sigma} C^{\rho\sigma\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$$

Lorentzian computations

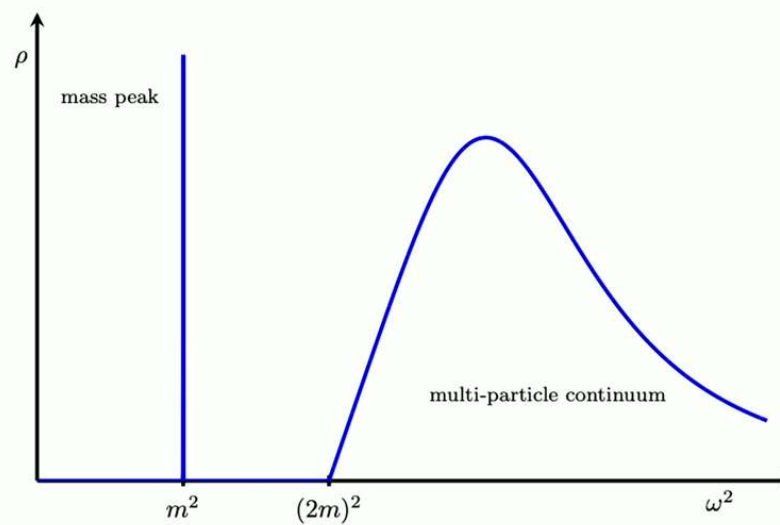
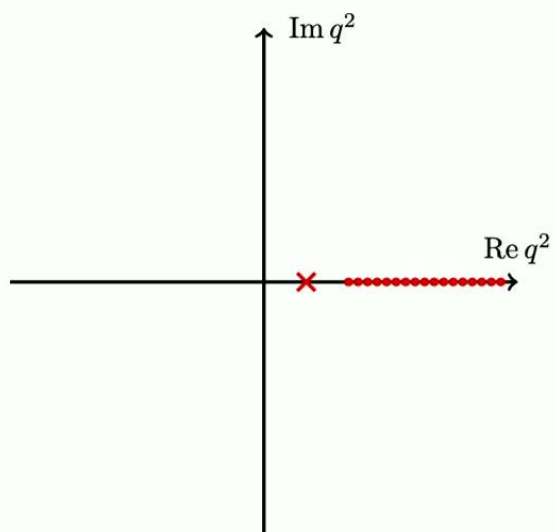
Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{d\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2}$$

with

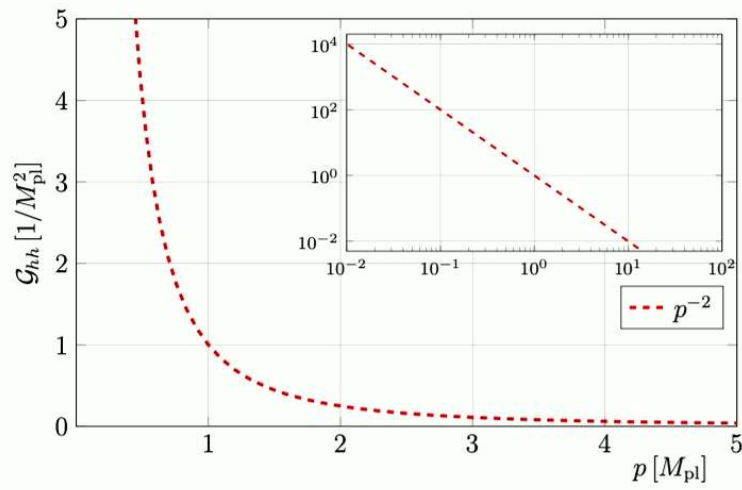
$$\rho(\omega^2) = - \lim_{\varepsilon \rightarrow 0} \text{Im} G(\omega^2 + i\varepsilon)$$



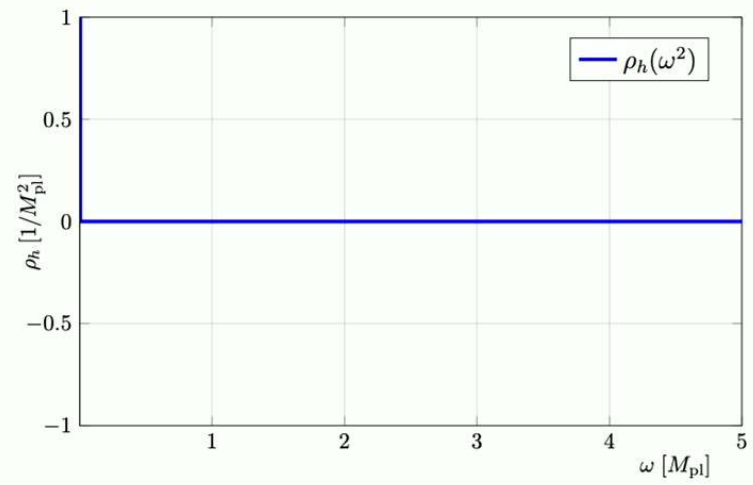
Classical graviton spectral function

$$\text{Einstein-Hilbert action: } S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_x \sqrt{g} (2\Lambda - R)$$

$$\text{Flat Minkowski background: } g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



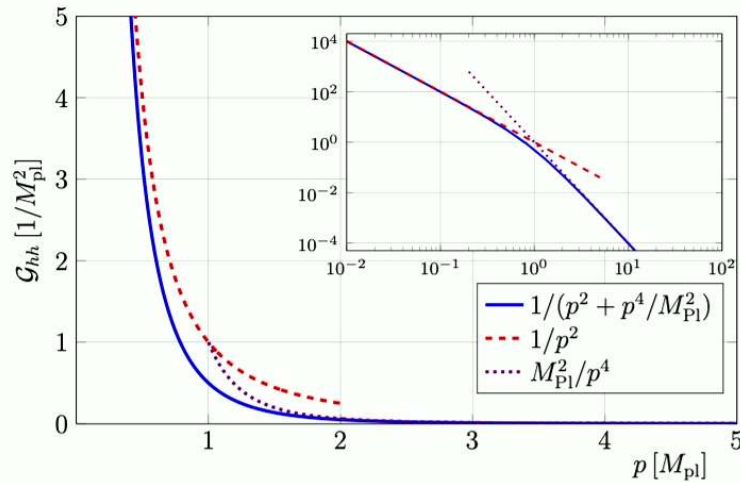
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$$



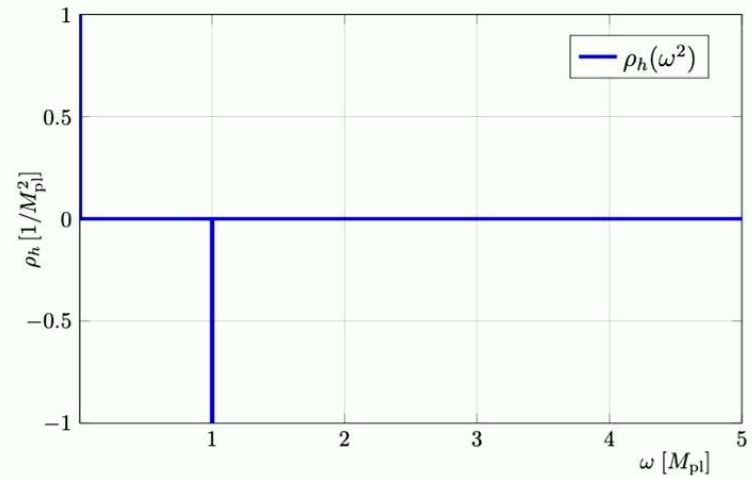
$$\rho_h(\omega^2) \sim \delta(\omega^2)$$

Classical graviton spectral function

Higher-derivative action: $S_{\text{HD}} = S_{\text{EH}} + \int_x \sqrt{g} (aR^2 + bC_{\mu\nu\rho\sigma}^2)$



$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

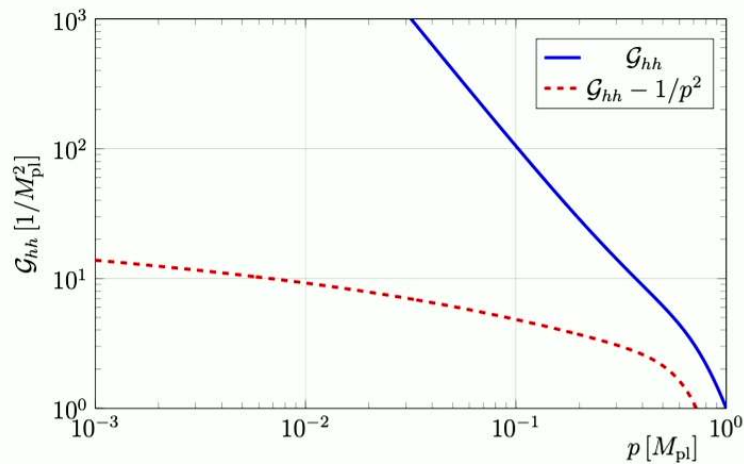


$$\rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\text{Pl}}^2)$$

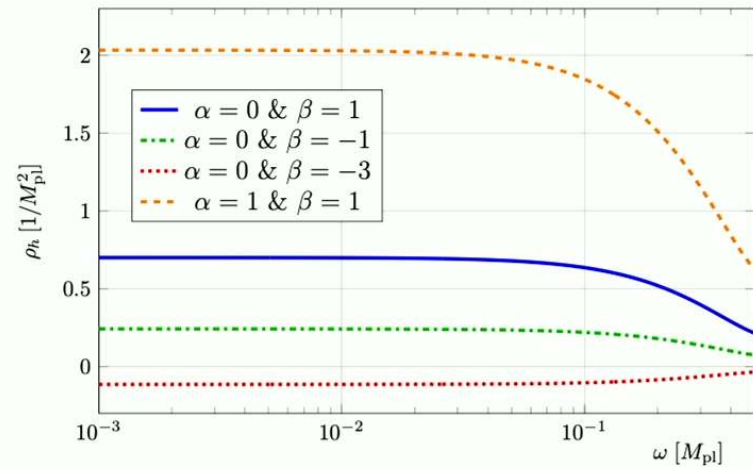
EFT graviton spectral function

One-loop effective action: $\Gamma_{1\text{-loop}} = S_{\text{EH}} + \int_x \sqrt{g} (c_1 R \ln(\square) R + c_2 C_{\mu\nu\rho\sigma} \ln(\square) C^{\mu\nu\rho\sigma}) + \dots$

Gauge-fixing $S_{\text{gf}} = \frac{1}{\alpha} \int_x F_\mu^2$ with $F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4} \bar{\nabla}_\mu h^\nu{}_\nu$



$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2)p^4}$$

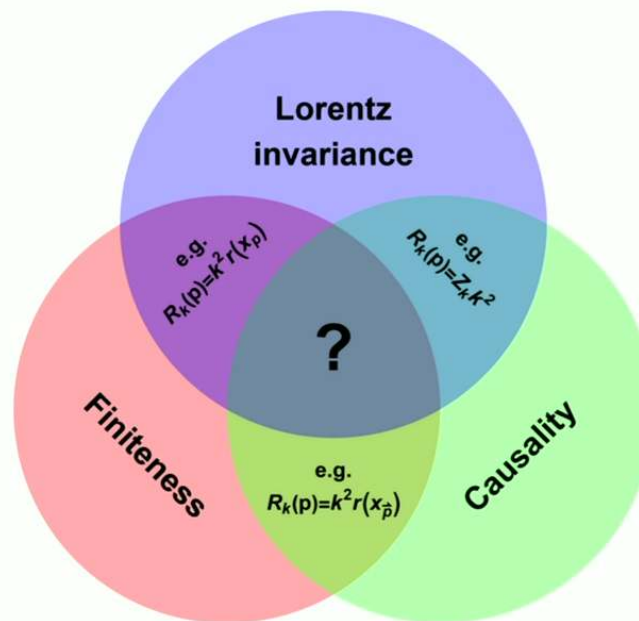
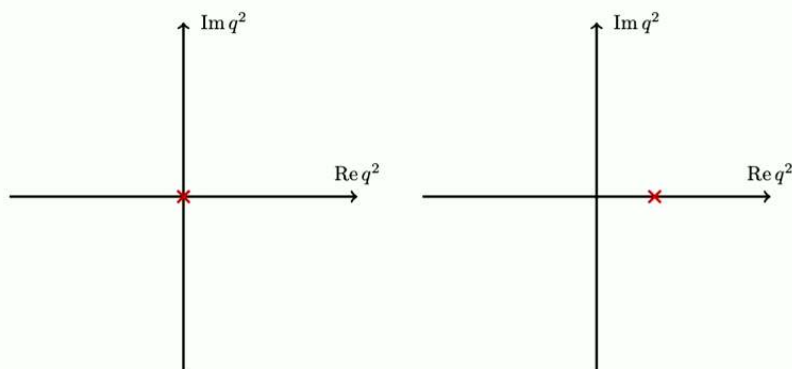


[Pawlowski, MR '23]

$$\rho_h(\omega^2) \sim \delta(\omega^2) + \text{const}$$

Regulator $R_k = k^2 r_k(x)$

- $r_k = r_k(p^2/k^2)$ breaks causality
- $r_k = r_k(\vec{p}^2/k^2)$ breaks Lorentz invariance
- $r_k = 1$ provides no UV regularisation



- Callan-Symanzik cutoff uniquely preserves causality and Lorentz invariance

$$R_k = Z_\phi k^2$$

- Finite flow equation with counterterms

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \mathcal{G}_k \partial_t R_k - \partial_t S_{\text{ct},k}$$

- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$ possible
- Finitely many counter terms if gravity is asymptotically safe
- Similar flow as in [D'Angelo, Drago, Pinamonti, Rejzner '22; D'Angelo Rejzner '23; Banerjee, Niedermaier '22]

- Expansion about flat Minkowski background
- Direct flow of ρ_h with $m_h^2 = k^2(1 + \mu)$ and $Z_h = Z_h(p^2 = -m_h^2)$

$$\rho_h = \frac{1}{Z_h} \left[2\pi \delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda) \right]$$

- Use ρ_h in flow diagrams

$$\partial_t \rho_h \propto \text{diagram} + \dots \quad \text{with} \quad \mathcal{G}_h(q^2) = \int_0^\infty \frac{d\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}$$

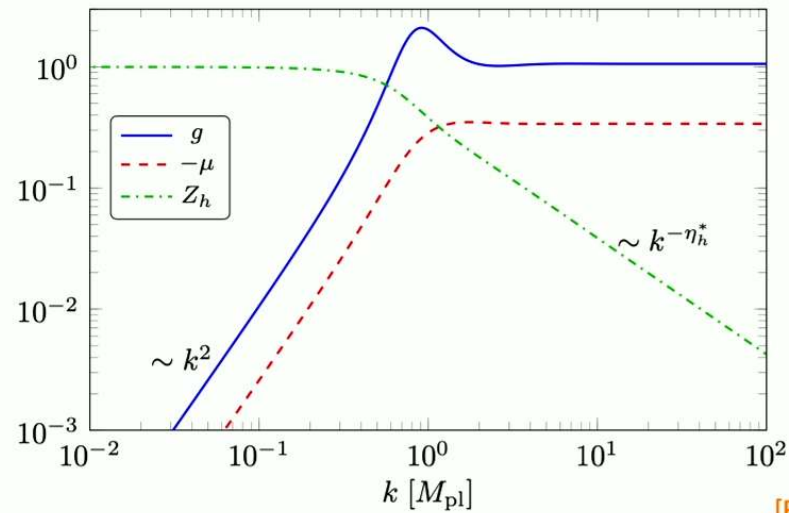

- Approximation: neglect feedback from f_h in diagrams
- Flow of $g = G_N k^2$ from three-graviton vertex at $p = 0$

[Christiansen, Knorr, Meibohm, Pawłowski, MR '15; Denz, Pawłowski, MR '16]

Lorentzian UV-IR trajectories

$$(g, \eta_h, \mu)|_* = (1.06, 0.96, -0.34)$$

$$\theta = 2.49 \pm 3.17 i$$

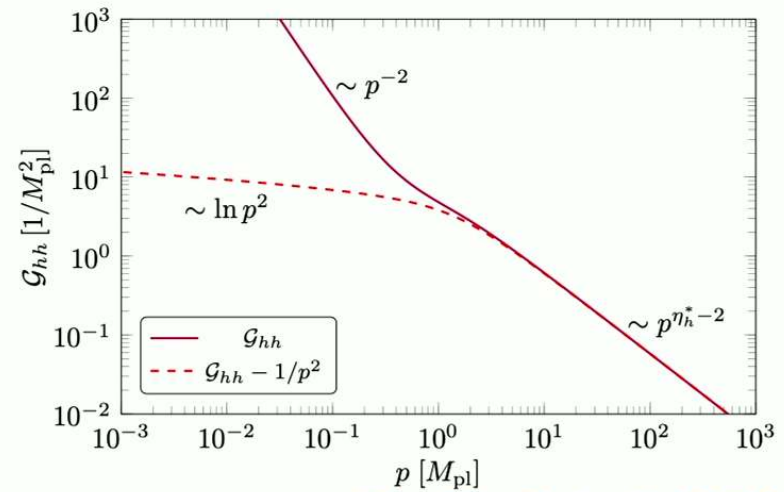
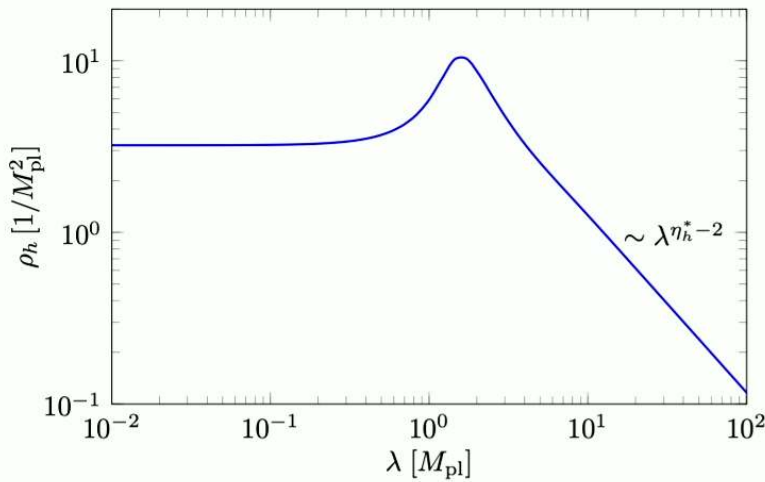


[Fehre, Litim, Pawłowski, MR '21]

$$G_{\text{N}}(k) = g(k)/k^2 \xrightarrow{k \rightarrow 0} G_{\text{N}}$$

$$-2\Lambda(k) = k^2 \mu(k) \xrightarrow{k \rightarrow 0} -2\Lambda = 0$$

Graviton spectral function

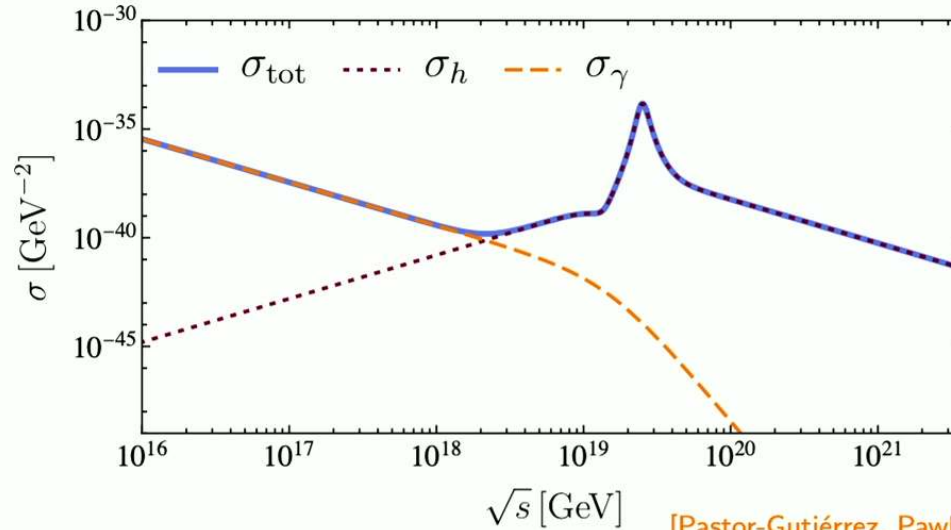
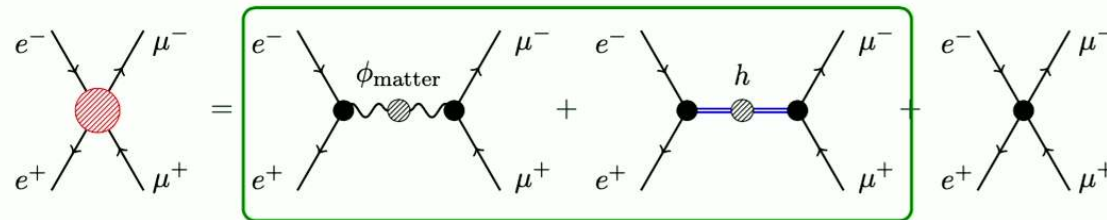


[Fehre, Litim, Pawłowski, MR '21]

- Massless graviton delta-peak with multi-graviton continuum
- Non-normalisable spectral function $\int \rho_h d\lambda = \infty$
- No ghosts and no tachyons \rightarrow no indications for unitarity violation
- Good agreement with reconstruction results

[Bonanno, Denz, Pawłowski, MR '21]

Towards graviton-mediated scattering cross-sections



[Pastor-Gutiérrez, Pawłowski, MR, Ruisi (in prep)]

Summary

- Expansion of quantum effective action in graviton correlation functions
- Restored diffeomorphism invariance at $k = 0$
- Momentum dependent correlation function link to curvature expansion
- Direct Lorentzian computation of graviton spectral function with spectral fRG
- Well-behaved spectral function without ghost or tachyonic instabilities
- Key step towards scattering processes and unitarity

Thank you for your attention!

$$d\mathcal{L} = \underline{\mathbb{E}} + d \mathbb{I}$$

$\mathbb{I} \in \Omega^{\text{top}, 1}$, (pre)symplectic pot. current.
 source $\Omega^{\text{top}, 1}$ form, "Euler Lagrange"
 $\underline{\mathbb{E}} \in \Omega^{\text{top}, 0}$ "Lagrangian"

$$\mathcal{L} = \mathcal{L}(x, \varphi, \partial\varphi) \in$$

$$\mathbb{I} = d\varphi^I \left(\underbrace{\frac{\partial \mathcal{L}}{\partial \varphi^I} - \partial_e \frac{\partial \mathcal{L}}{\partial \partial_e \varphi^I}}_{\text{com}} \right) \in$$

$$\mathbb{I} = d\varphi^I \left(\frac{\partial \mathcal{L}}{\partial \partial_e \varphi^I} \right) \in \mathfrak{a}$$

\mathcal{F} , current.

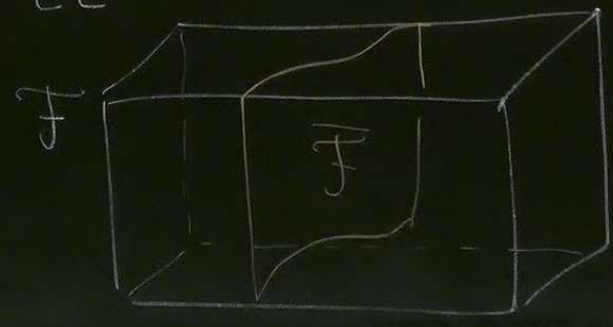
$$\underline{\epsilon}_a = i \gamma_a \underline{\epsilon}$$
$$\mathcal{L}_\Sigma^* \underline{\epsilon}_a = N_a \underline{\epsilon}_\Sigma$$

On-shell

DEF Euler Lagrange locus

$$\bar{\mathcal{F}} := \{ \varphi \in \mathcal{F} : E|_\varphi = 0 \}$$

$$\mathcal{L}_{EL} : \bar{\mathcal{F}} \hookrightarrow \mathcal{F}$$



On-shell

to go on shell \equiv pulling back to \bar{F}

[\neq "evaluating at a $\varphi \in \bar{F}$]

Ex: \mathbb{R}^4 foliated by const. time slices, $\Sigma_0 = \{t=0\}$

$$\alpha = 3dt$$

$$\alpha|_{\Sigma_0} = 3dt$$

$$\int_{\Sigma_0}^* \alpha = 0$$

ocus

$$E|_{\varphi=0}$$

\mathcal{F}



On-shell

to go on shell \equiv pulling back to $\overline{\mathcal{F}}$

[\neq "evaluating at a $\varphi \in \overline{\mathcal{F}}$ "]

eg: \mathbb{R}^4 foliated by const. time slices, $\Sigma_0 = \{t=0\}$



$$\alpha = 3dt$$

$$\alpha|_{\Sigma_0} = 3dt$$

$$\int_{\Sigma_0}^* \alpha = 0$$

Notation $\underline{\alpha} \approx 0$ iff $\int_{\mathcal{E}}^* \alpha = 0$

DEF [Jacobi fields]

Solutions of the linearized EoM.

Rmk $X \in T_{\bar{\varphi}} \bar{F}$ is a Jacobi field:

$$0 = \mathbb{L}_X \underline{E}|_{\bar{\varphi}} = \int \underbrace{d(\delta_X \varphi)}_{=0} \underline{E}|_{\bar{\varphi}} + d\varphi \left(\frac{\delta E}{\delta \varphi} \delta_X \varphi \right) |_{\bar{\varphi}}$$

Assumption:
Converse is also true

$$= \int d\varphi \left(\underbrace{\frac{\partial E}{\partial \varphi} |_{\bar{\varphi}} \delta_X \varphi + \frac{\partial E}{\partial (\partial_\alpha \varphi)} |_{\bar{\varphi}} \partial_\alpha (\delta_X \varphi) + \dots}_{\text{EoM linearized at } \bar{\varphi}} \right)$$

LOCALITY

• $\mathbb{F} = \Gamma(M, F)$

$\delta\varphi(x)$ is whatever I want,
not constrained

\Rightarrow "local"

• $\overline{\mathbb{F}} \neq$ space of sections of a bundle over M .

$\delta\varphi \in T\overline{\mathbb{F}} \Rightarrow$ must satisfy $LE \circ M$

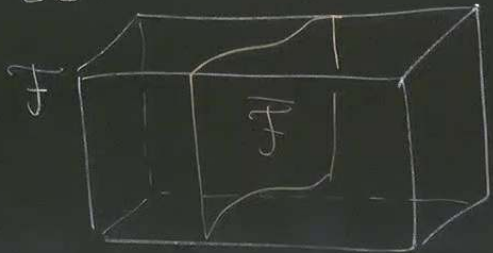
\Rightarrow not local over M

Consequence: Takens does not hold over $\overline{\mathbb{F}}$!

$\frac{\delta E}{\delta \varphi} \left(\frac{\delta \varphi}{\delta x} \right) \Big|_{\overline{\mathbb{F}}}$



$$\mathcal{L}_{EL}: \mathcal{F} \rightarrow \mathcal{F}$$



$$\alpha = 3dt$$

$$\alpha|_{\Sigma_0} = 3dt$$

$$\mathcal{L}_{\Sigma_0}^* \alpha = 0$$

Notation

$$\underline{\alpha} \approx 0 \text{ iff } \mathcal{L}_{EL}^* \underline{\alpha} = 0$$

LOCALITY

$$\mathcal{F} = \Gamma(M, F)$$

$\mathcal{S}\mathcal{P}(x)$ is whatever I want,
not constrained

\Rightarrow "local"

$\overline{\mathcal{F}} \neq$ space of sections of a bundle over M .

$\mathcal{S}\mathcal{P} \in T\overline{\mathcal{F}} \Rightarrow$ must satisfy LEM

\Rightarrow not local over M

Consequence: Takens does not hold over $\overline{\mathcal{F}}$!

\mathcal{F} can still be isomorphic
to a local space, just not
one over M .
Usually: one over Σ .

If 1-to-1 correspondence btw
(unconstrained) initial conds at Σ
and $\overline{\mathcal{F}} \in \overline{\mathcal{F}}$ \uparrow "canonical ph. sp."

DEF (pre)symplectic current
 $\underline{\Omega} = d\underline{\Theta} \in \Omega^{\text{top-1}, 2}(M \times \mathcal{F})$

THM $\underline{\Omega}$ is conserved on-shell
 $d\underline{\Omega} \approx 0$

Pf: $0 \equiv d^2 \underline{L} = d(\underline{E} + d\underline{\Theta})$
 $= d\underline{E} - d\underline{\Omega}$
??
0 by def \square

If M globally hyp, ∂
 $\underline{\Omega}_\Sigma = \int_\Sigma \underline{\Omega}$ does
depend on choice of Σ

If M globally hyp, $\partial\Sigma = \emptyset$

$$\Omega_\Sigma = \int_\Sigma \underline{\Omega} \quad \text{does not}$$

depend on choice of Σ , on-shell.

$\partial\Sigma \neq \emptyset$

Ω_{fin}

$-\Omega_{\text{in}}$

$$\approx \int_B \underline{\Omega}$$

sympl. flux
through

$$B = C \times (t_{\text{in}}, t_{\text{fin}})$$

\Rightarrow

$$\left(\overline{\mathcal{F}}, \int_{\Sigma}^* \Omega_\Sigma \right) \equiv$$

"covariant ph. sp."

EXAMPLES

1) Particle

$$\underline{L} = \left(\frac{1}{2} \dot{q}^2 - V(q) \right) dt$$

$$dL = dq(t) \text{ (can)} + \underbrace{\frac{dt}{dt} \frac{\partial}{\partial t}}_{=d} \left(\underbrace{dq}_{\in \Omega} \dot{q} \right) \xrightarrow{\text{top-1}} \downarrow \mathbb{R} (M \times F)$$

$$\begin{aligned} \underline{\Omega}(t) &= d\dot{q} \wedge dq \\ &= \partial_t dq(t) \wedge dq(t) \end{aligned}$$

$$\Sigma = \{t_0\} \hookrightarrow M = \mathbb{R}$$

$$\underline{\Omega}_\Sigma = d\dot{q}(t_0) \wedge dq(t_0)$$

$$\text{At } t=t_0, \quad \begin{cases} x = q(t_0) \\ p = \dot{q}(t_0) \end{cases}$$

can be specified freely & independently

$$(x,p) \in \mathcal{F} = \overset{\uparrow \{t_0\}}{C^\infty}(\Sigma, \mathbb{R}^2) = \mathbb{R}^2$$

finite dim

$$(\overline{\mathcal{F}}, \overline{\omega} \in \Omega_{\mathbb{R}^2})$$

\mathbb{R}^2

$$(\overline{\mathcal{F}}, \overline{\omega}) = T^*\mathbb{R}$$

uphase sp. of point particle

$L_{\Sigma} \equiv$ action

)
 $T^*\mathbb{R}$
of
de
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2) Massive KG field

$$\underline{L} = -\frac{1}{2} (d\varphi \wedge *d\varphi + m^2 \varphi * \varphi)$$

$$\underline{E} = d\varphi \wedge (dx d\varphi - m^2 * \varphi) = dx \wedge (\square \varphi - m^2 \varphi)$$

$$\underline{\Omega} = d\varphi \wedge (*d\varphi) \quad \text{top form on } \Sigma$$

$$\Omega = \int d\varphi \wedge d(\nabla^* \varphi) n_e$$

$$\Omega_{\Sigma} = \int_{\Sigma} d\varphi \wedge d(*d\varphi)$$

$$= - \int_{\Sigma} d\varphi \wedge d\dot{\varphi} \in, \quad \dot{\varphi} = n^0 \partial_0 \varphi$$

$$\begin{aligned} \phi(x \in \Sigma) &= \varphi(x \in \Sigma) \\ \pi(x) &= (n^0 \partial_0 \varphi)(x \in \Sigma) \end{aligned}$$

$$(P, \omega), \quad \omega = \int_{\Sigma} d\phi \wedge d\pi$$

$$\downarrow \sum = \int \dot{q}(t_0) \wedge \dot{q}(t_0)$$

$(\dot{q}(t_0), \dot{q}(t_0))$
 $\{t_0\}$

finite dim

NOETHER 1

