Title: From fluctuating gravitons to Lorentzian quantum gravity

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Series: Quantum Gravity

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Abstract: I will review recent progress in the asymptotic safety approach to quantum gravity. This includes the computation of momentum-dependent graviton correlation functions, the structure of the Standard Model with asymptotically safe gravity, and the recent first computation directly in space-times with Lorentzian signatures via the spectral function of the graviton. Overall, I will display the progress towards the computation of the quantum effective action which encapsulates effective field theory in the IR limit and the asymptotic safety regime in the UV limit.

Zoom link

From fluctuating gravitons to Lorentzian quantum gravity

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Quantum Gravity Seminar, Perimeter Institute, 11. April 2024

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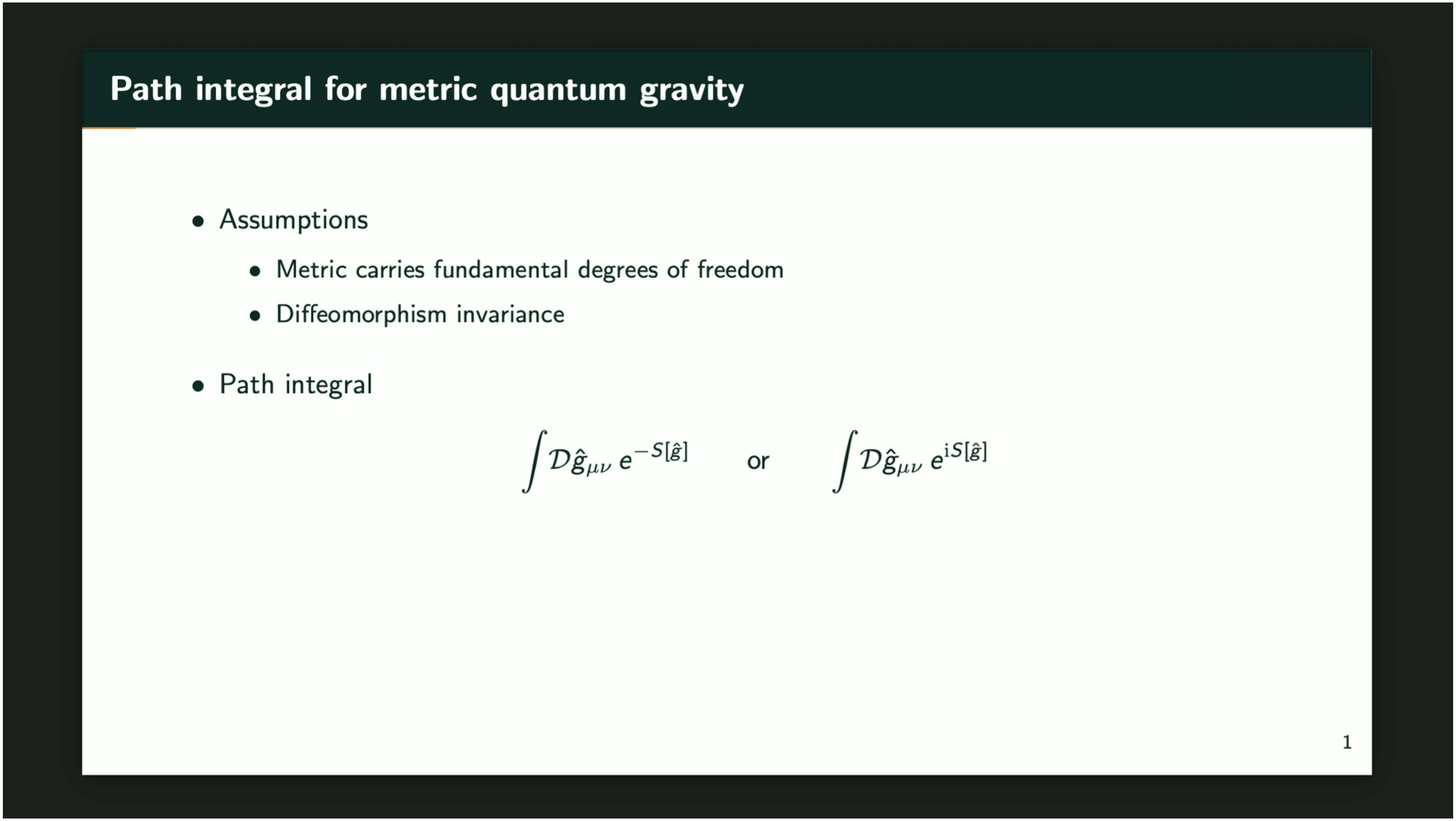
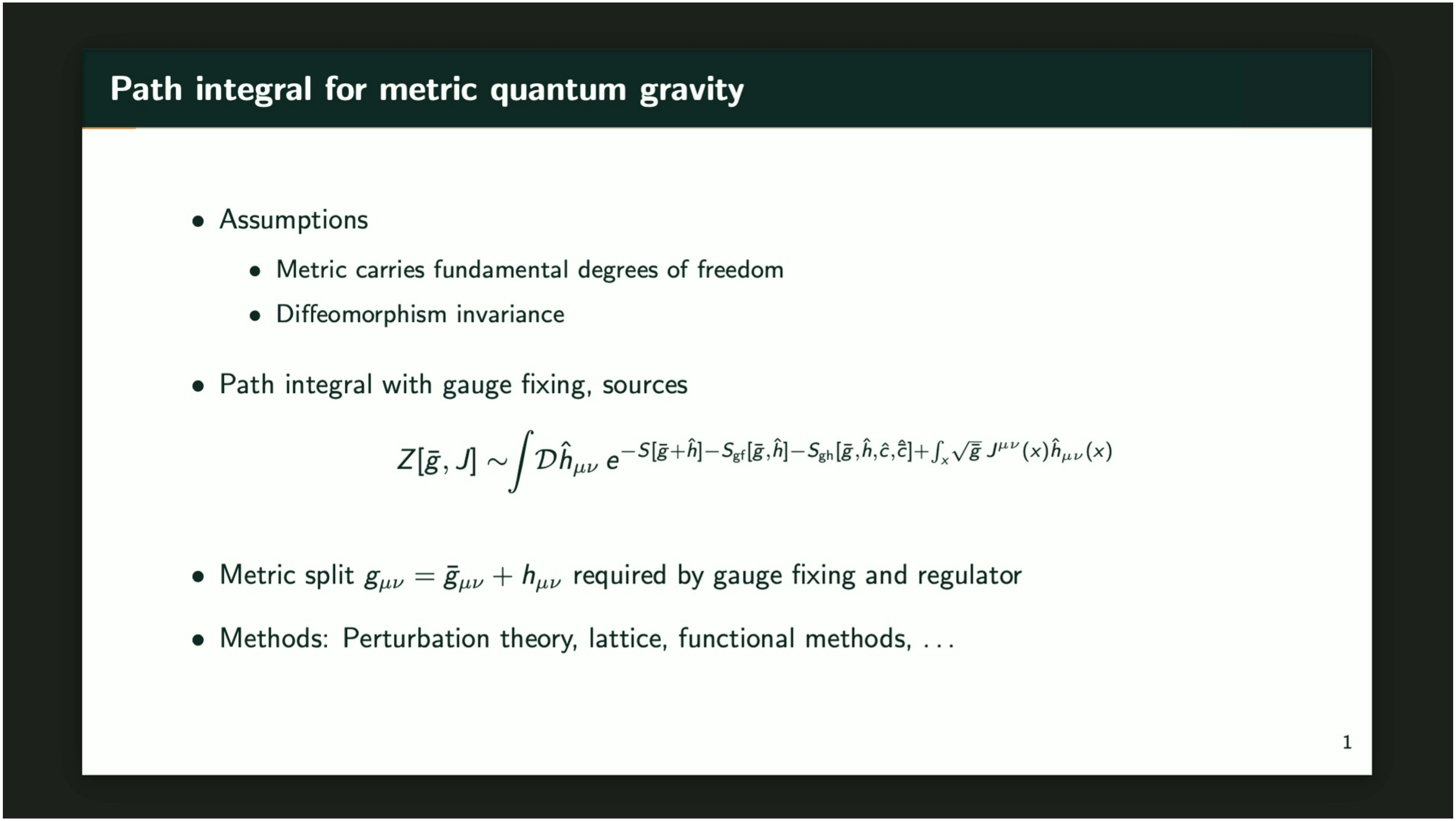


Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}]} \qquad \text{or} \qquad \int \mathcal{D}\hat{g}_{\mu\nu}\, e^{\mathrm{i}S[\hat{g}]}$$

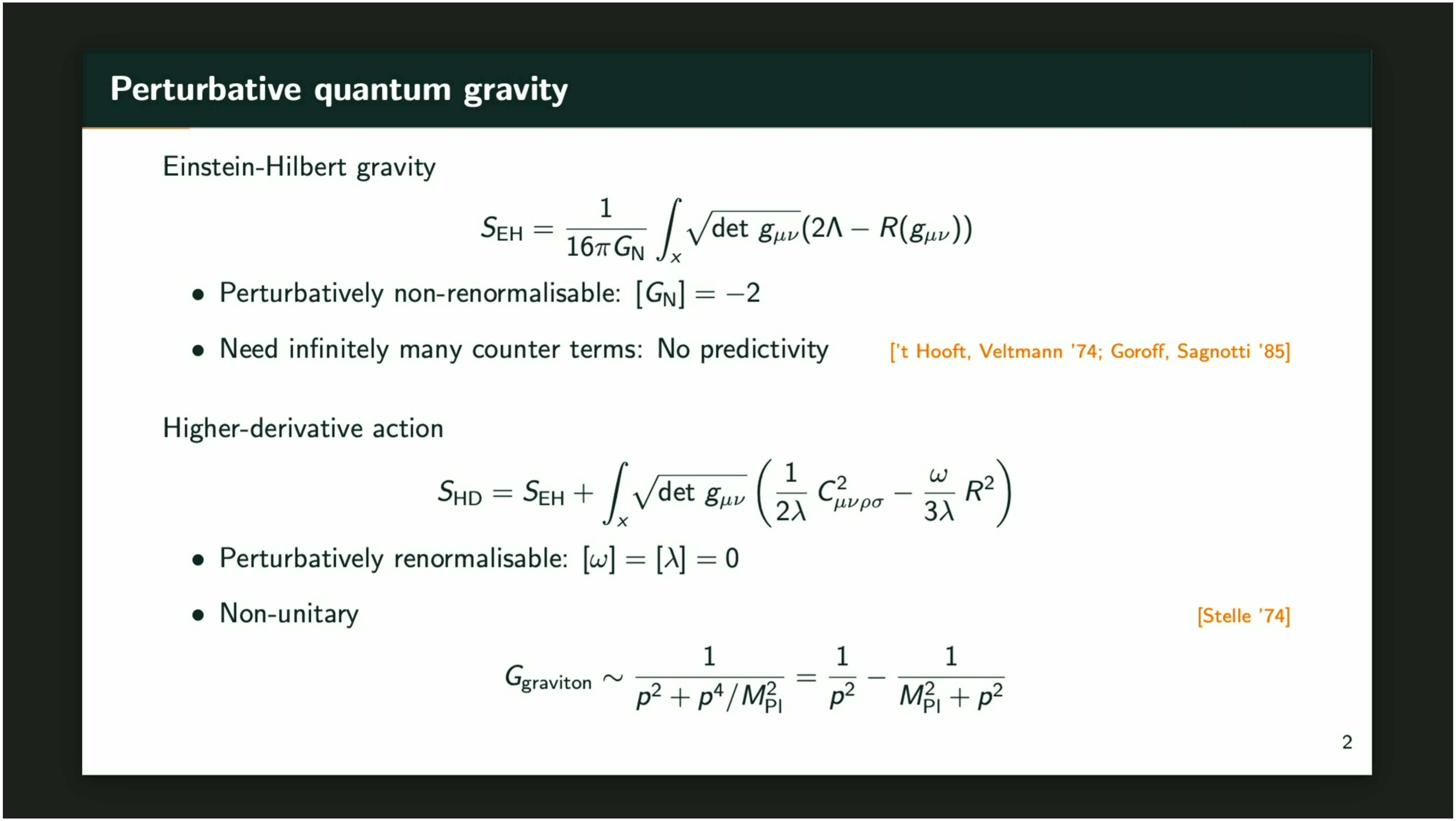


Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral with gauge fixing, sources

$$Z[\bar{g}, J] \sim \int \mathcal{D}\hat{h}_{\mu\nu}\, e^{-S[\bar{g}+\hat{h}]-S_{\mathrm{gf}}[\bar{g},\hat{h}]-S_{\mathrm{gh}}[\bar{g},\hat{h},\hat{c},\hat{\bar{c}}]+\int_x \sqrt{\bar{g}}\, J^{\mu\nu}(x)\hat{h}_{\mu\nu}(x)}$$

- Metric split $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ required by gauge fixing and regulator
- Methods: Perturbation theory, lattice, functional methods, ...



Perturbative quantum gravity

Einstein-Hilbert gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{\det g_{\mu\nu}} \left(2\Lambda - R(g_{\mu\nu})\right)$$

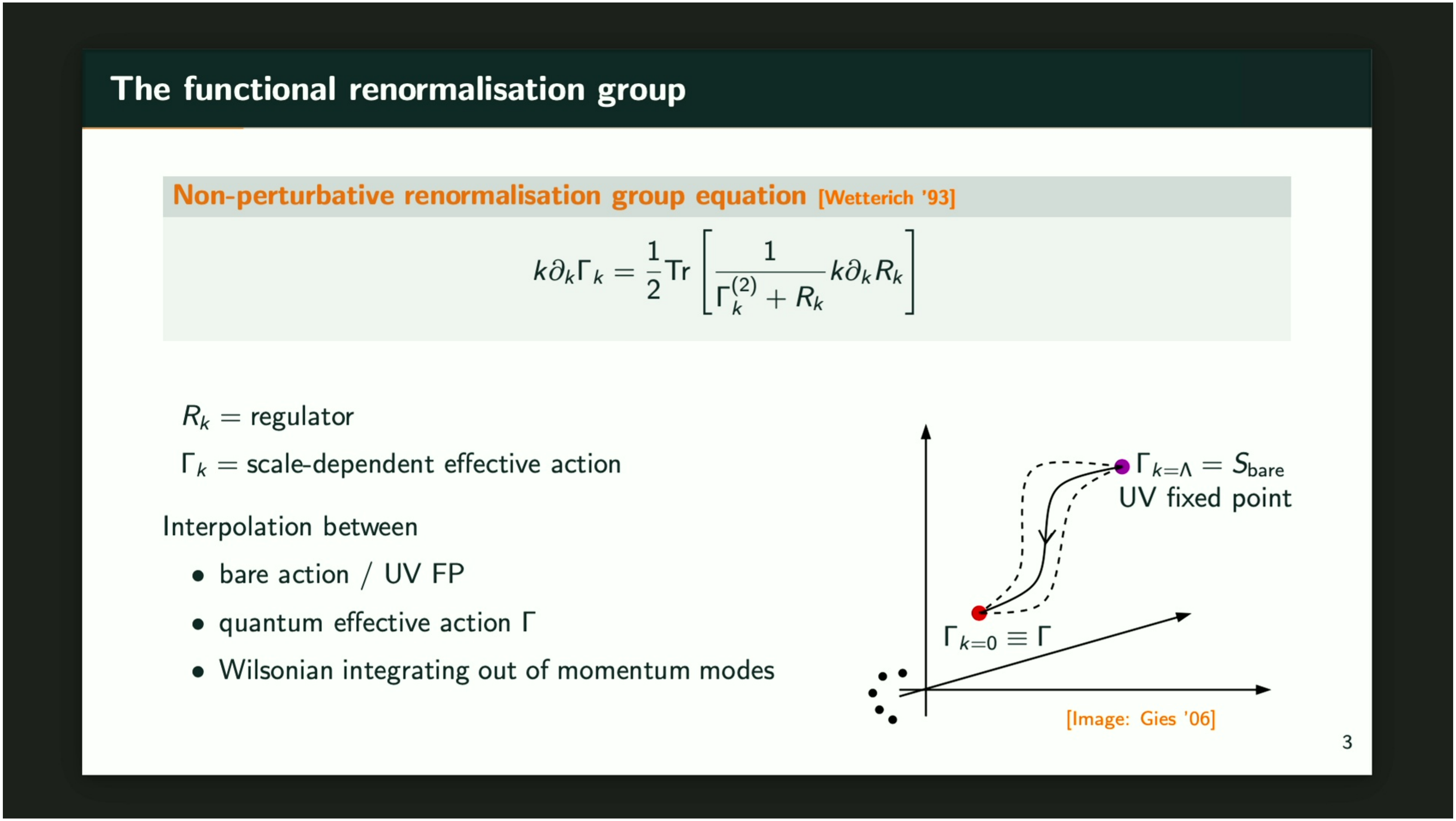
- Perturbatively non-renormalisable: $[G_{\mathrm{N}}] = -2$
- Need infinitely many counter terms: No predictivity ['t Hooft, Veltmann '74; Goroff, Sagnotti '85]

Higher-derivative action

$$S_{\mathrm{HD}} = S_{\mathrm{EH}} + \int_x \sqrt{\det g_{\mu\nu}} \left(\frac{1}{2\lambda} C^2_{\mu\nu\rho\sigma} - \frac{\omega}{3\lambda} R^2 \right)$$

- Perturbatively renormalisable: $[\omega] = [\lambda] = 0$
- Non-unitary [Stelle '74]

$$G_{\text{graviton}} \sim \frac{1}{p^2 + p^4/M_{\mathrm{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\mathrm{Pl}}^2 + p^2}$$



The functional renormalisation group

Non-perturbative renormalisation group equation [Wetterich '93]

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k\right]$$

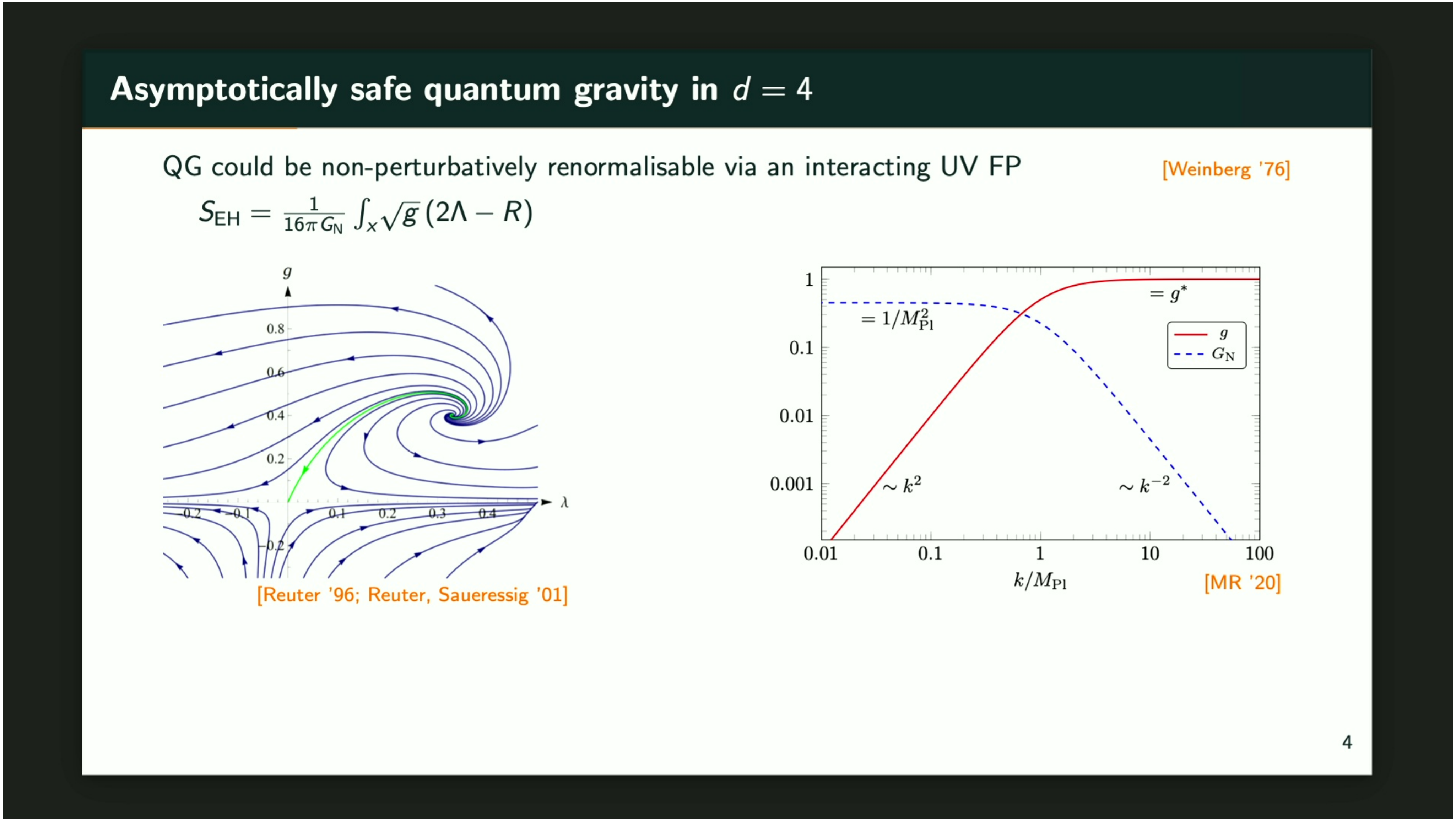
R_k = regulator

Γ_k = scale-dependent effective action

Interpolation between

- bare action / UV FP
- quantum effective action Γ
- Wilsonian integrating out of momentum modes

[Image: Gies '06]



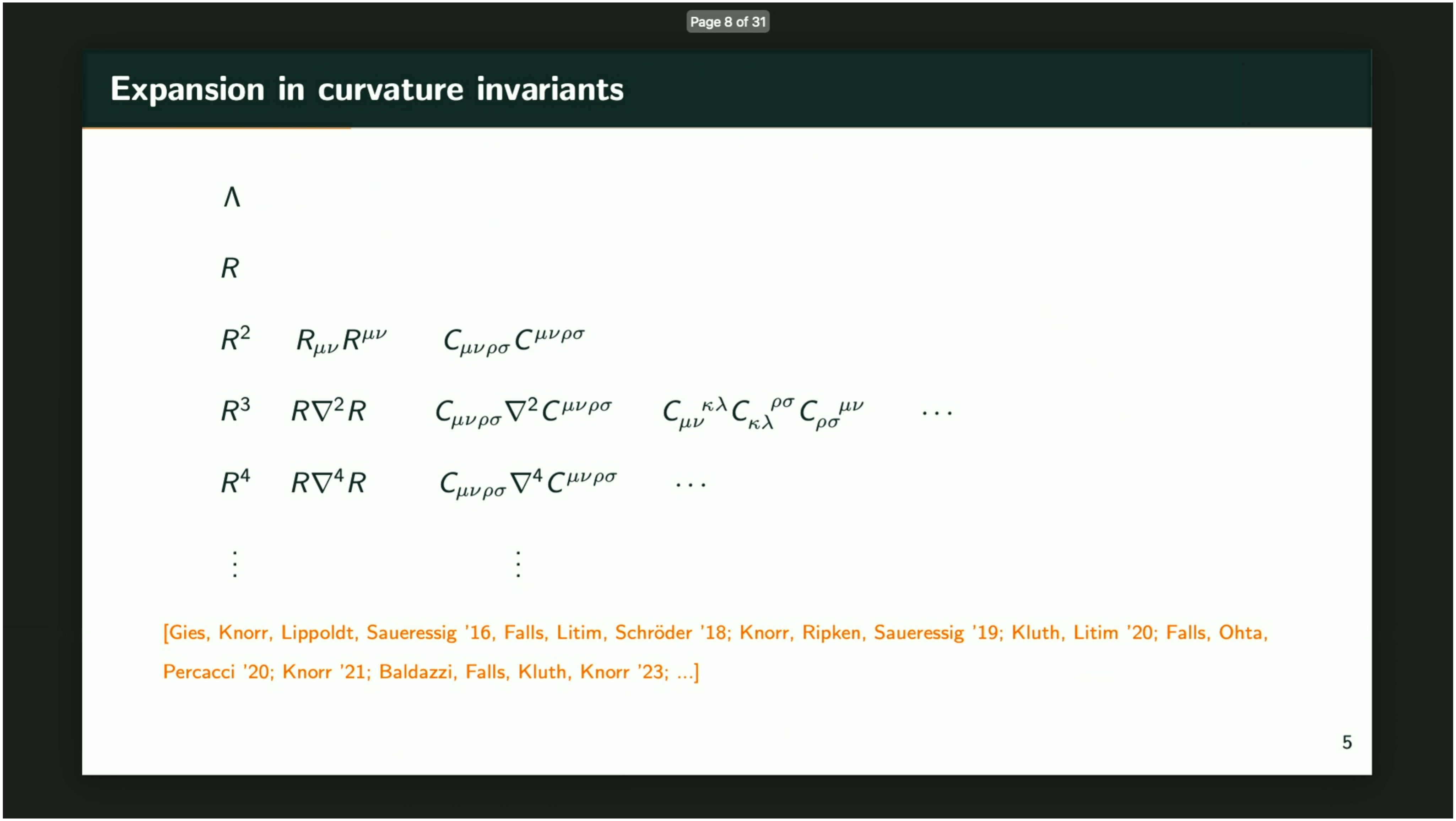
Asymptotically safe quantum gravity in $d = 4$

QG could be non-perturbatively renormalisable via an interacting UV FP [Weinberg '76]

$$S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}}\int_x \sqrt{g}\,(2\Lambda - R)$$

[Reuter '96; Reuter, Saueressig '01]

[MR '20]



Expansion in curvature invariants

$$\Lambda$$

$$R$$

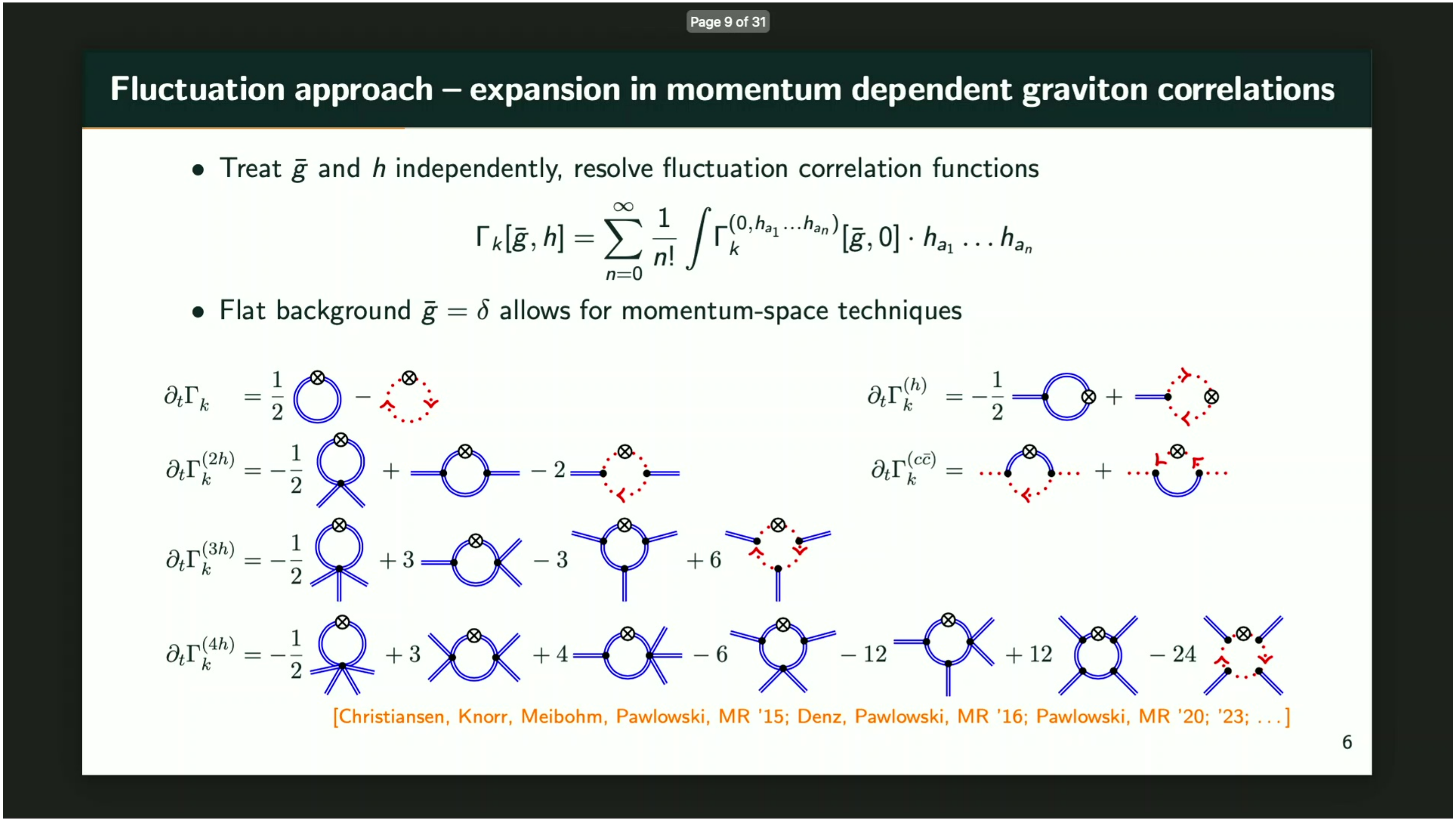
$$R^2 \qquad R_{\mu\nu}R^{\mu\nu} \qquad C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

$$R^3 \qquad R\nabla^2 R \qquad C_{\mu\nu\rho\sigma}\nabla^2 C^{\mu\nu\rho\sigma} \qquad C_{\mu\nu}{}^{\kappa\lambda}C_{\kappa\lambda}{}^{\rho\sigma}C_{\rho\sigma}{}^{\mu\nu} \qquad \cdots$$

$$R^4 \qquad R\nabla^4 R \qquad C_{\mu\nu\rho\sigma}\nabla^4 C^{\mu\nu\rho\sigma} \qquad \cdots$$

$$\vdots \qquad\qquad\qquad \vdots$$

[Gies, Knorr, Lippoldt, Saueressig '16, Falls, Litim, Schröder '18; Knorr, Ripken, Saueressig '19; Kluth, Litim '20; Falls, Ohta, Percacci '20; Knorr '21; Baldazzi, Falls, Kluth, Knorr '23; ...]



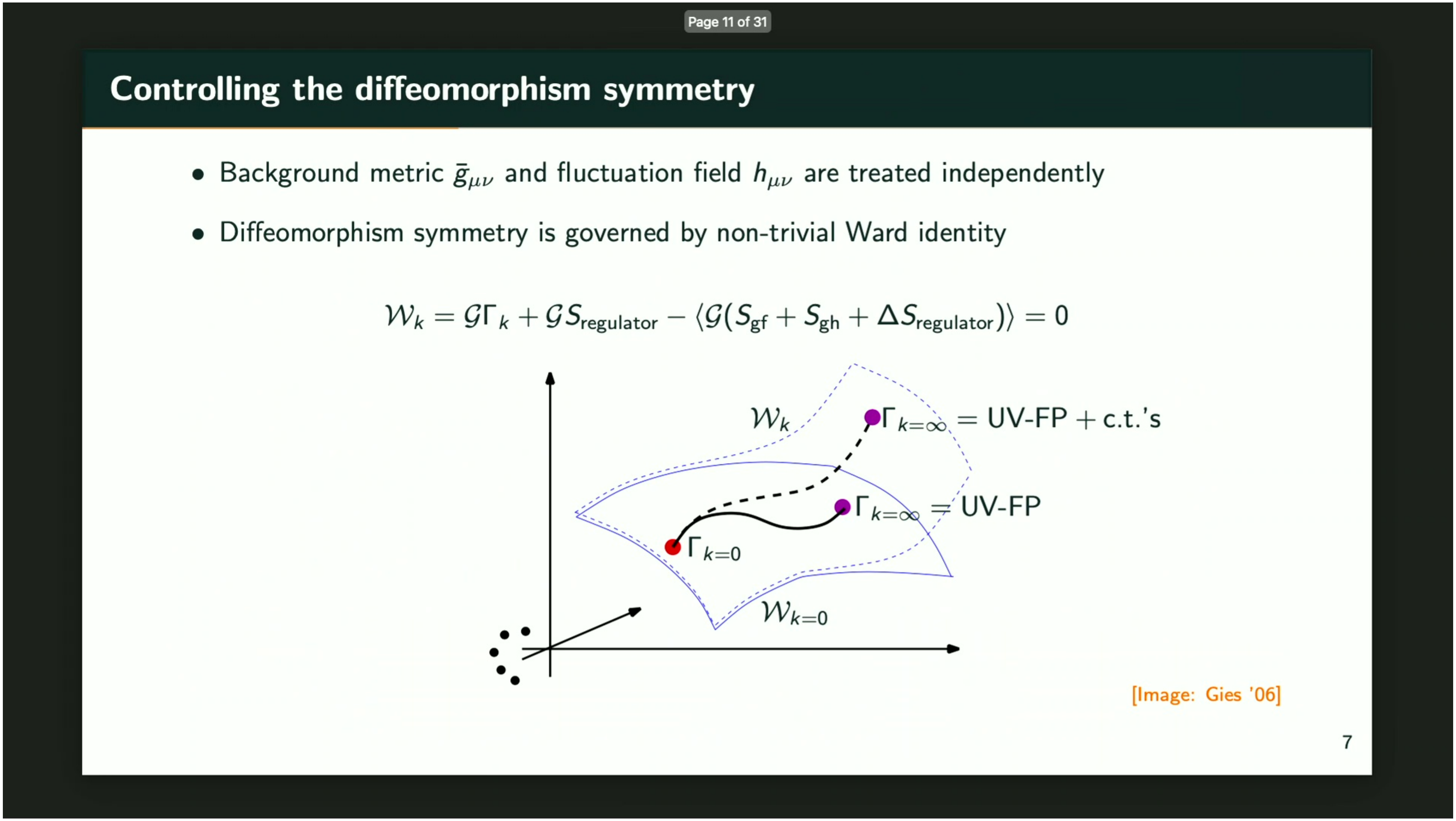
Fluctuation approach – expansion in momentum dependent graviton correlations

- Treat $\bar{g}$ and h independently, resolve fluctuation correlation functions

$$\Gamma_k[\bar{g},h] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(0,h_{a_1}\dots h_{a_n})}[\bar{g},0] \cdot h_{a_1} \dots h_{a_n}$$

- Flat background $\bar{g} = \delta$ allows for momentum-space techniques

[Christiansen, Knorr, Meibohm, Pawlowski, MR '15; Denz, Pawlowski, MR '16; Pawlowski, MR '20; '23; ...]

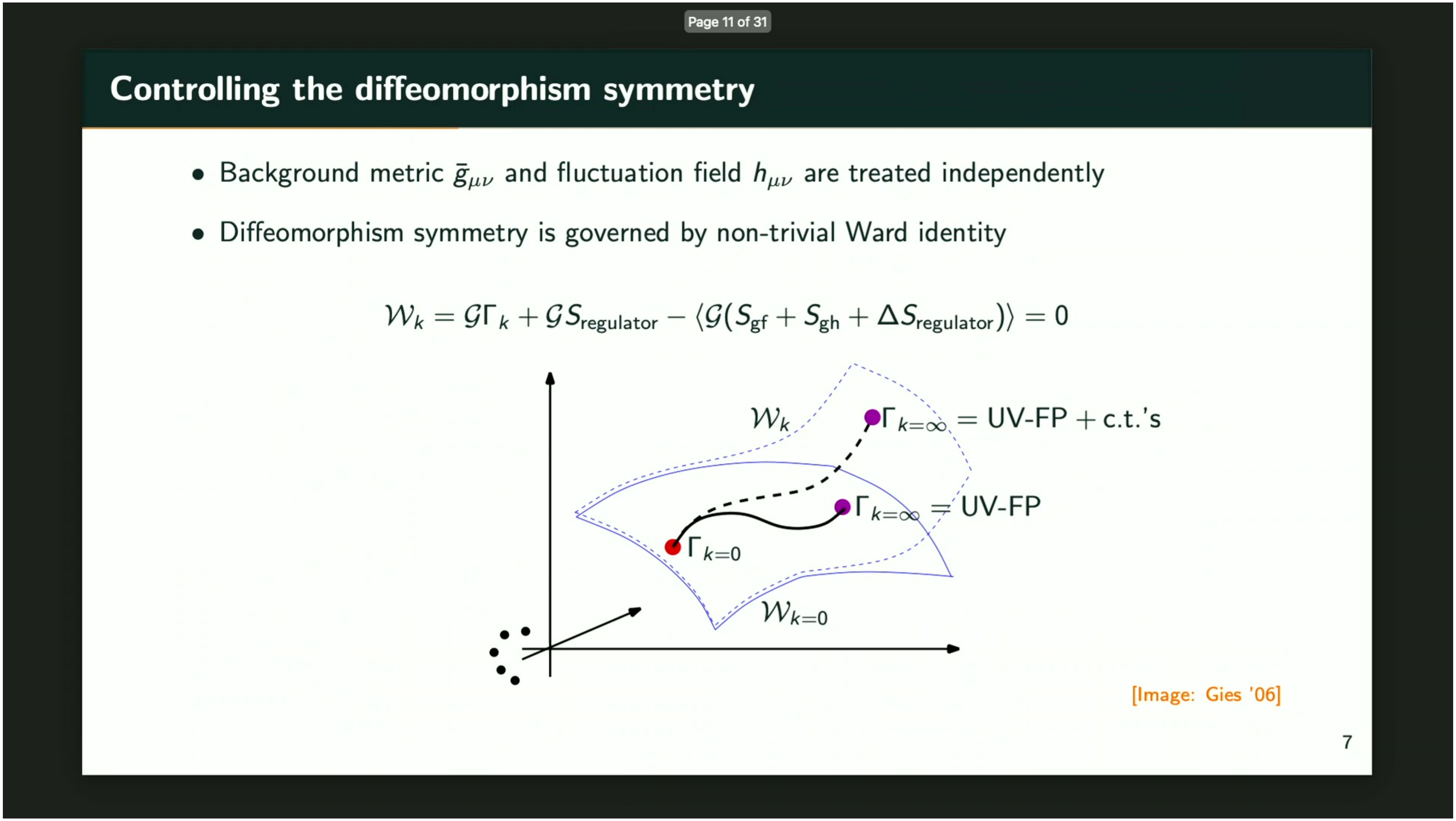


Controlling the diffeomorphism symmetry

- Background metric $\bar{g}_{\mu\nu}$ and fluctuation field $h_{\mu\nu}$ are treated independently
- Diffeomorphism symmetry is governed by non-trivial Ward identity

$$\mathcal{W}_k = \mathcal{G}\Gamma_k + \mathcal{G}S_{\text{regulator}} - \langle \mathcal{G}(S_{\text{gf}} + S_{\text{gh}} + \Delta S_{\text{regulator}}) \rangle = 0$$

[Image: Gies '06]



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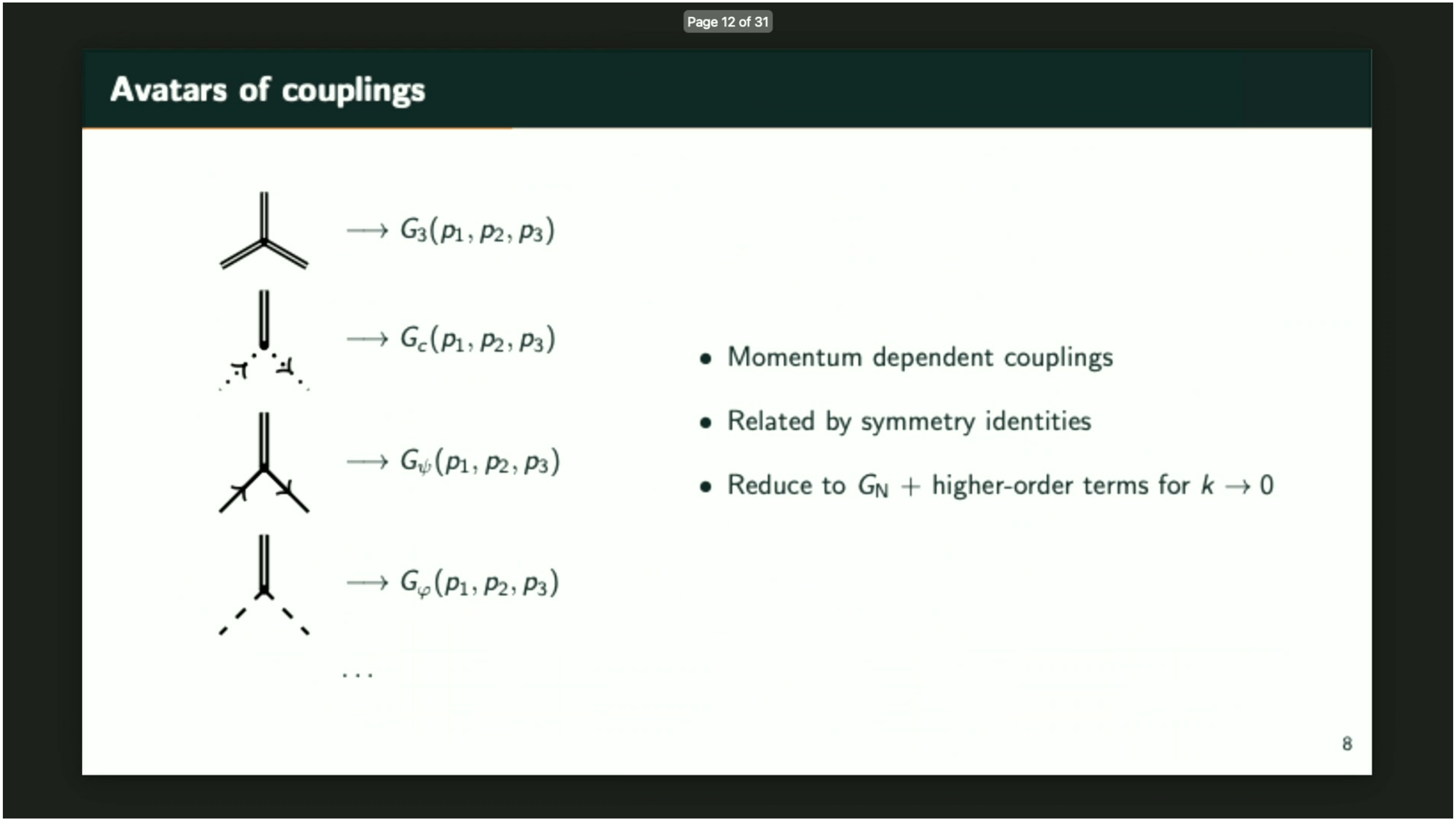


Avatars of couplings

$\longrightarrow G_3(p_1, p_2, p_3)$

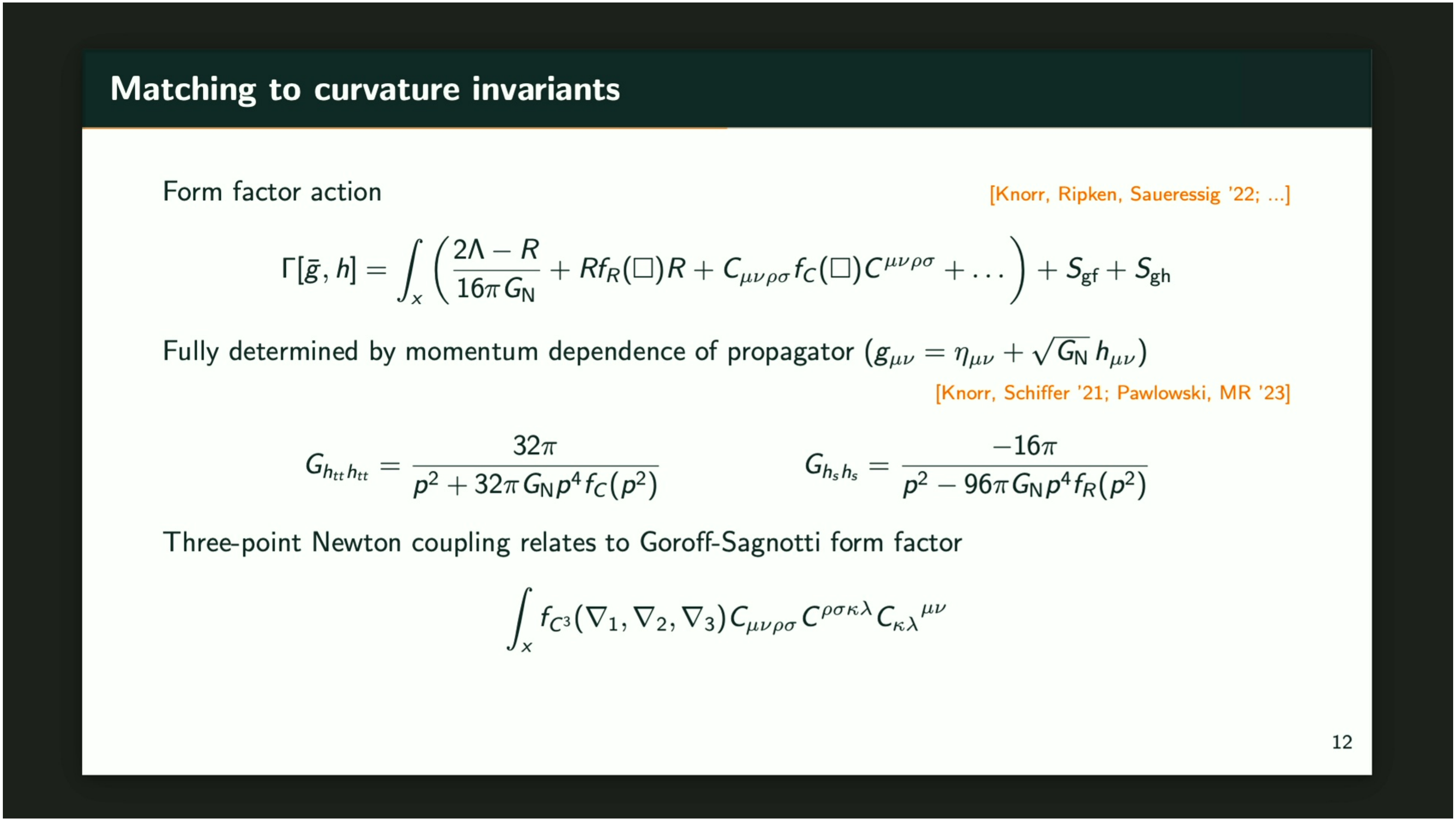
$\longrightarrow G_c(p_1, p_2, p_3)$

$\longrightarrow G_\psi(p_1, p_2, p_3)$

$\longrightarrow G_\varphi(p_1, p_2, p_3)$

$\dots$

- Momentum dependent couplings
- Related by symmetry identities
- Reduce to G_N + higher-order terms for $k \to 0$



Matching to curvature invariants

Form factor action [Knorr, Ripken, Saueressig '22; ...]

$$\Gamma[\bar{g}, h] = \int_x \left(\frac{2\Lambda - R}{16\pi G_\mathrm{N}} + R f_R(\Box) R + C_{\mu\nu\rho\sigma} f_C(\Box) C^{\mu\nu\rho\sigma} + \dots \right) + S_\mathrm{gf} + S_\mathrm{gh}$$

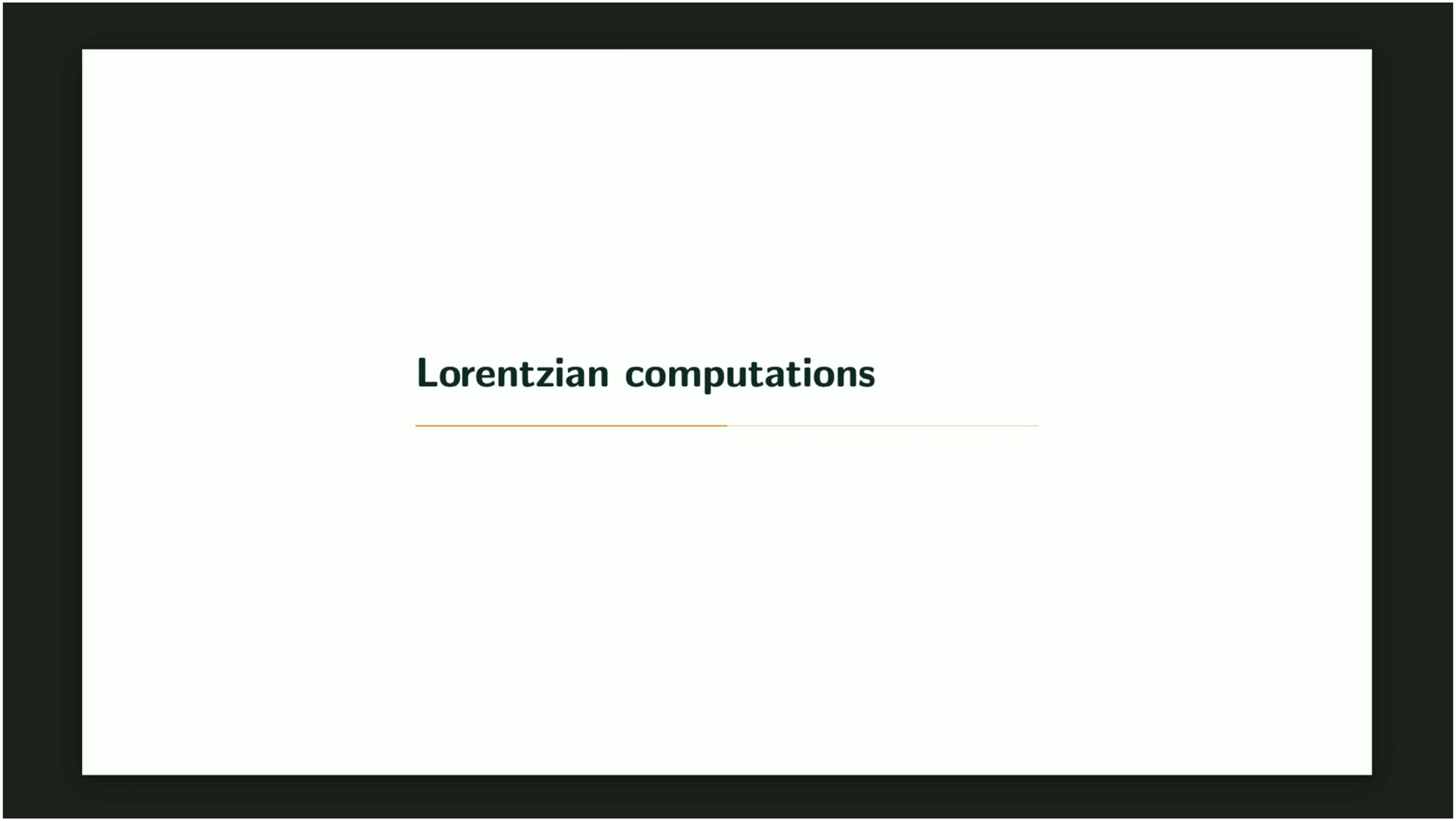
Fully determined by momentum dependence of propagator ($g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_\mathrm{N}}\, h_{\mu\nu}$)

[Knorr, Schiffer '21; Pawlowski, MR '23]

$$G_{h_{tt}h_{tt}} = \frac{32\pi}{p^2 + 32\pi G_\mathrm{N} p^4 f_C(p^2)} \qquad G_{h_s h_s} = \frac{-16\pi}{p^2 - 96\pi G_\mathrm{N} p^4 f_R(p^2)}$$

Three-point Newton coupling relates to Goroff-Sagnotti form factor

$$\int_x f_{C^3}(\nabla_1, \nabla_2, \nabla_3) C_{\mu\nu\rho\sigma} C^{\rho\sigma\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$$

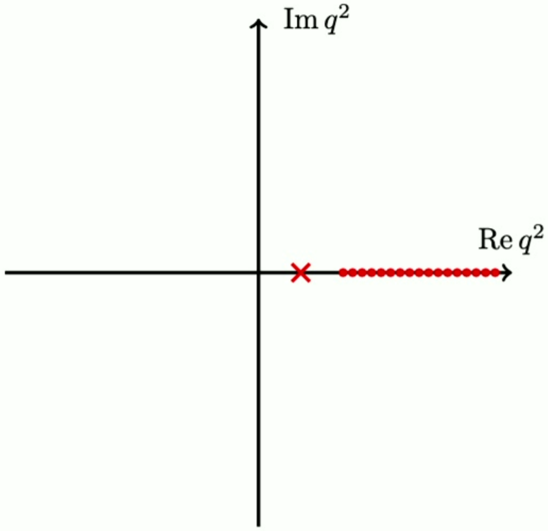


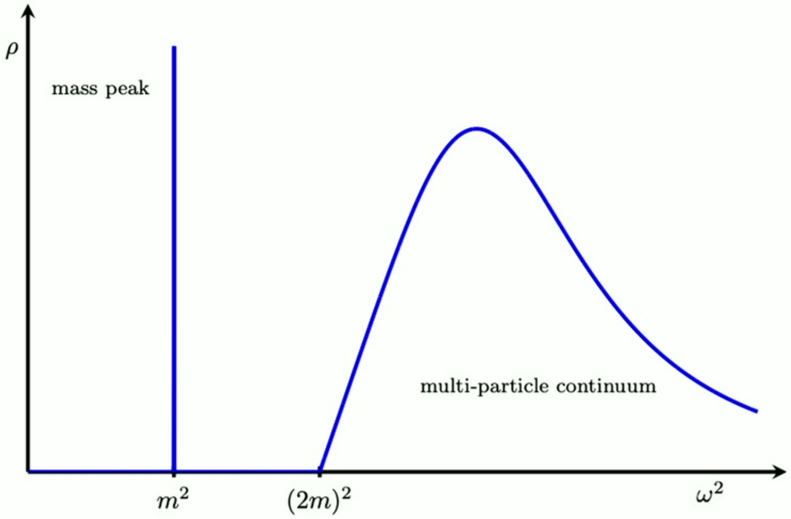
Lorentzian computations

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$

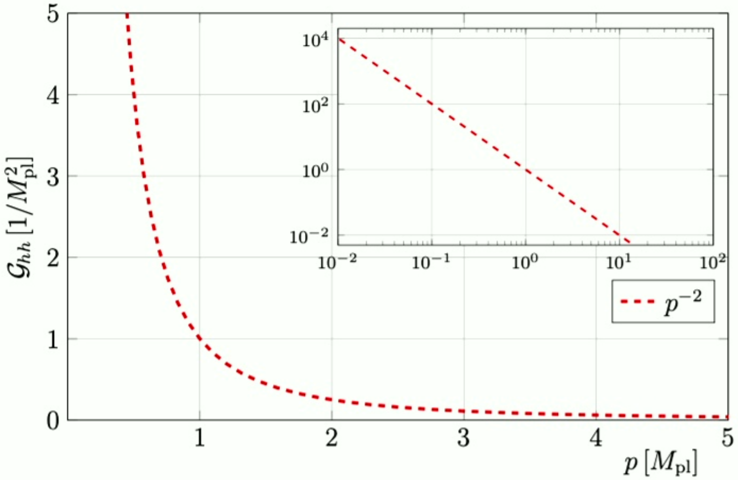


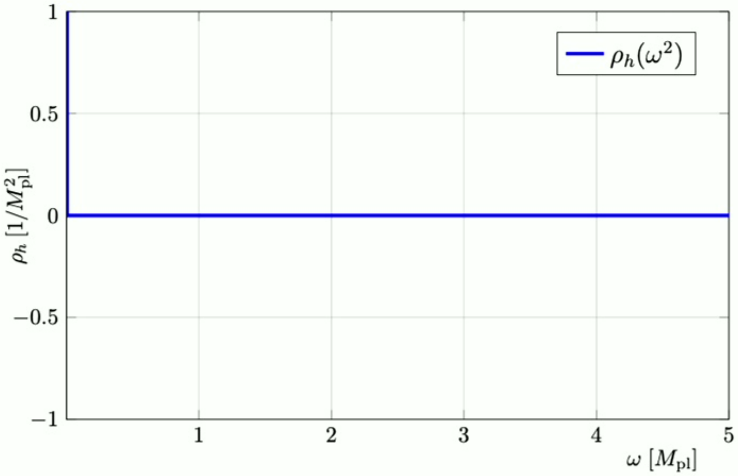


Classical graviton spectral function

Einstein-Hilbert action: $S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$

Flat Minkowski background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



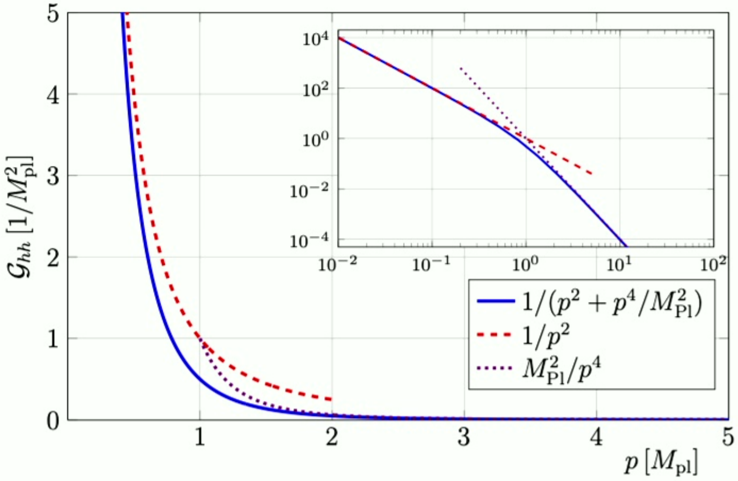


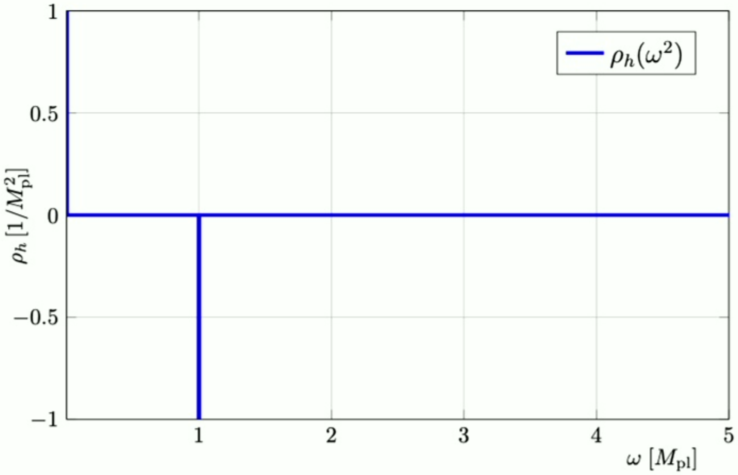
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2)$$

Classical graviton spectral function

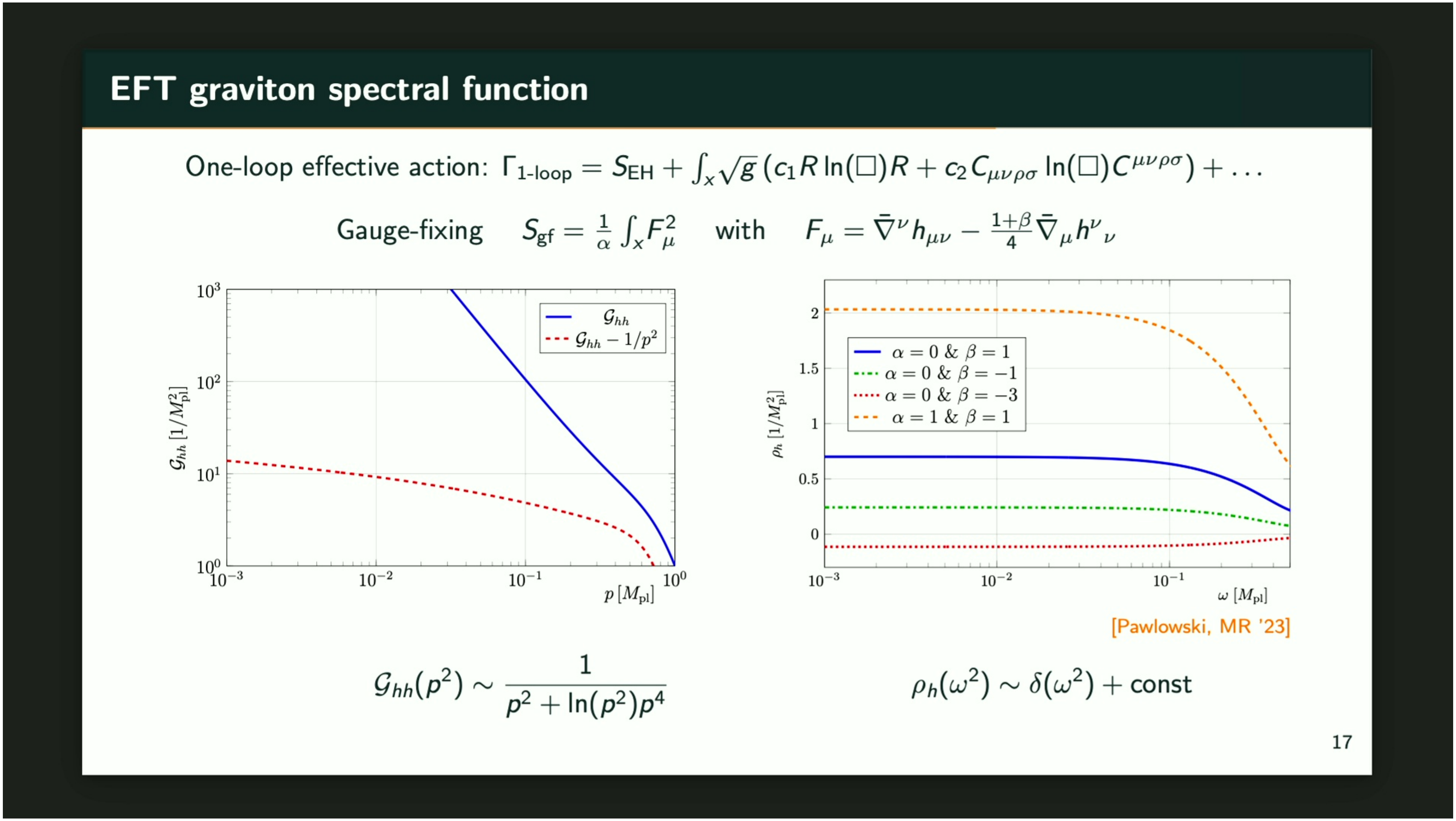
Higher-derivative action: $S_{\mathrm{HD}} = S_{\mathrm{EH}} + \int_x \sqrt{g}\,(aR^2 + bC^2_{\mu\nu\rho\sigma})$





$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\mathrm{Pl}}^2 + p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\mathrm{Pl}}^2)$$



EFT graviton spectral function

One-loop effective action: $\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\,(c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma}) + \dots$

Gauge-fixing $\quad S_{\text{gf}} = \frac{1}{\alpha}\int_x F_\mu^2 \quad$ with $\quad F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\mu h^\nu{}_\nu$

[Pawlowski, MR '23]

$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2)p^4}$$

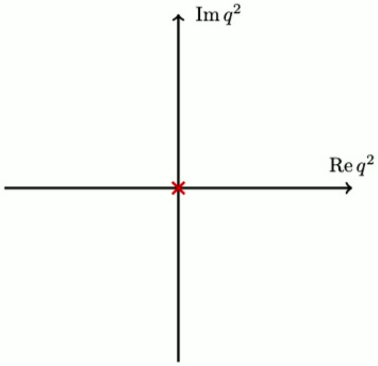
$$\rho_h(\omega^2) \sim \delta(\omega^2) + \text{const}$$

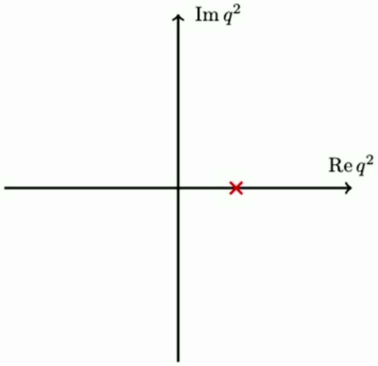
Spectral Renormalisation Group

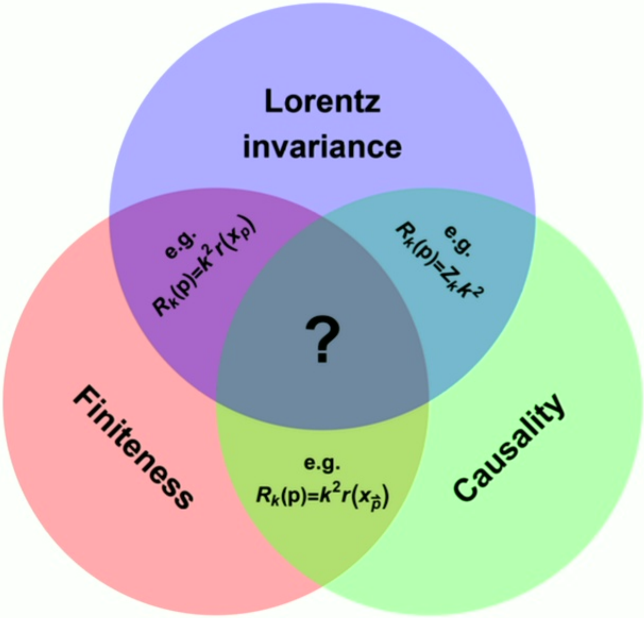
[fQCD, MR '22]

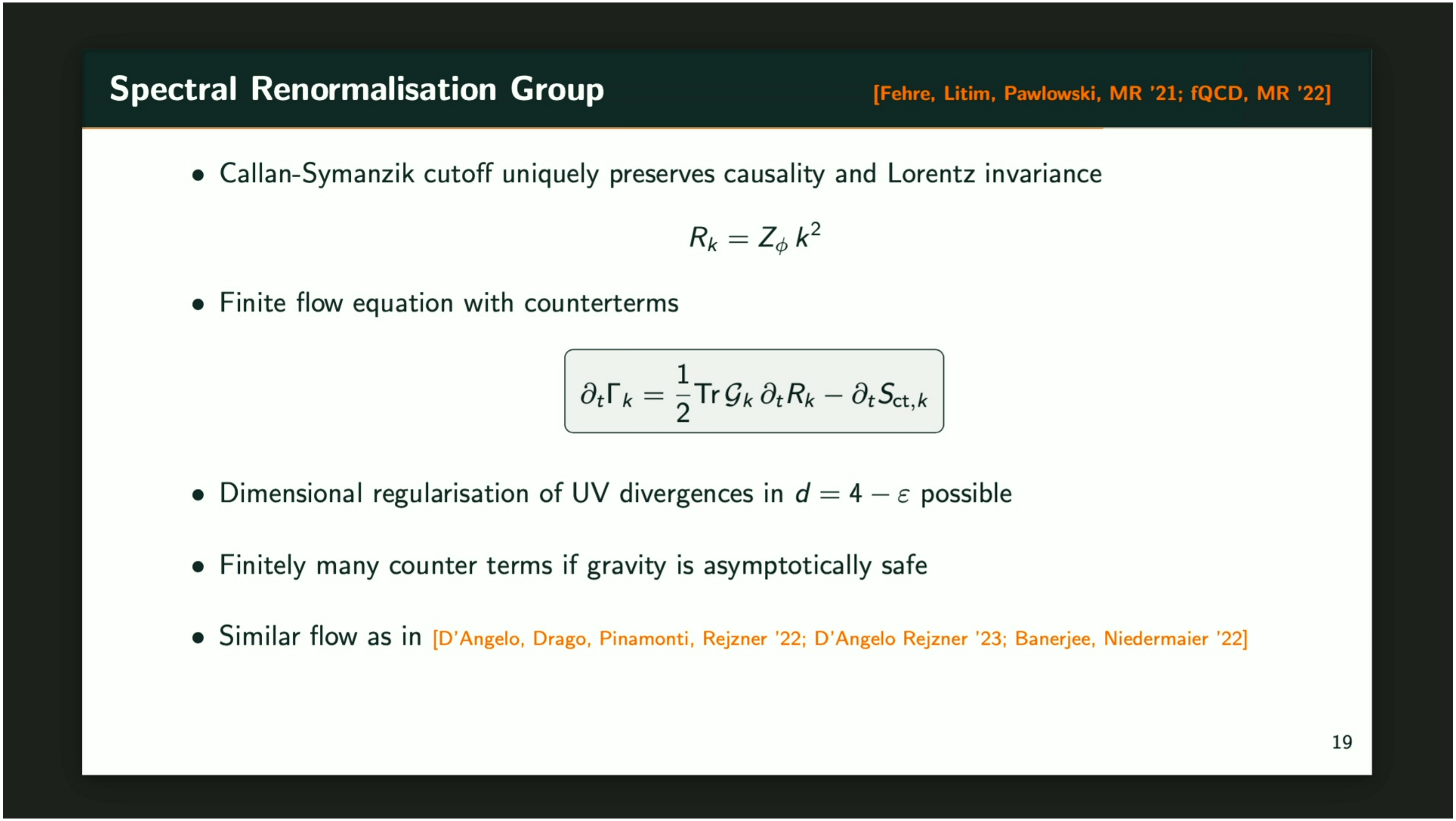
Regulator $R_k = k^2 r_k(x)$

- $r_k = r_k(p^2/k^2)$ breaks causality
- $r_k = r_k(\vec{p}^{\,2}/k^2)$ breaks Lorentz invariance
- $r_k = 1$ provides no UV regularisation









Spectral Renormalisation Group

[Fehre, Litim, Pawlowski, MR '21; fQCD, MR '22]

- Callan-Symanzik cutoff uniquely preserves causality and Lorentz invariance

$$R_k = Z_\phi \, k^2$$

- Finite flow equation with counterterms

$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \mathcal{G}_k \, \partial_t R_k - \partial_t S_{\mathrm{ct},k}}$$

- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$ possible
- Finitely many counter terms if gravity is asymptotically safe
- Similar flow as in [D'Angelo, Drago, Pinamonti, Rejzner '22; D'Angelo Rejzner '23; Banerjee, Niedermaier '22]

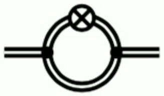
Lorentzian setup

[Fehre, Litim, Pawlowski, MR '21]

- Expansion about flat Minkowski background
- Direct flow of ρ_h with $m_h^2 = k^2(1+\mu)$ and $Z_h = Z_h(p^2 = -m_h^2)$

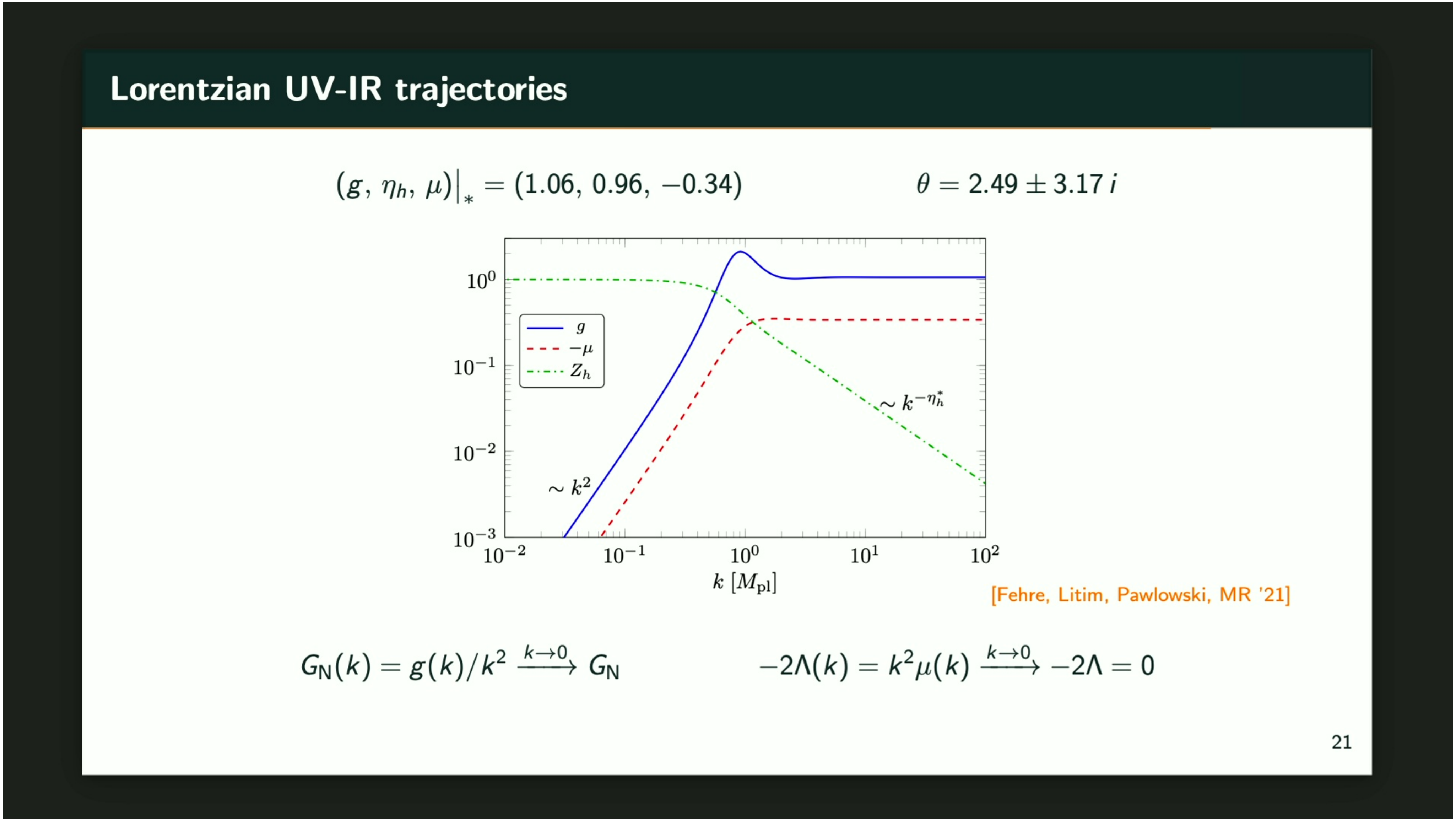
$$\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2)f_h(\lambda)\Big]$$

- Use ρ_h in flow diagrams

$\partial_t\rho_h \propto$  $+ \dots$ with $\mathcal{G}_h(q^2) = \int\limits_0^\infty \frac{\mathrm{d}\lambda^2}{\pi}\,\frac{\rho_h(\lambda^2)}{q^2-\lambda^2}$

- Approximation: neglect feedback from f_h in diagrams
- Flow of $g = G_\mathrm{N}k^2$ from three-graviton vertex at $p = 0$

[Christiansen, Knorr, Meibohm, Pawlowski, MR ' 15; Denz, Pawlowski, MR '16]

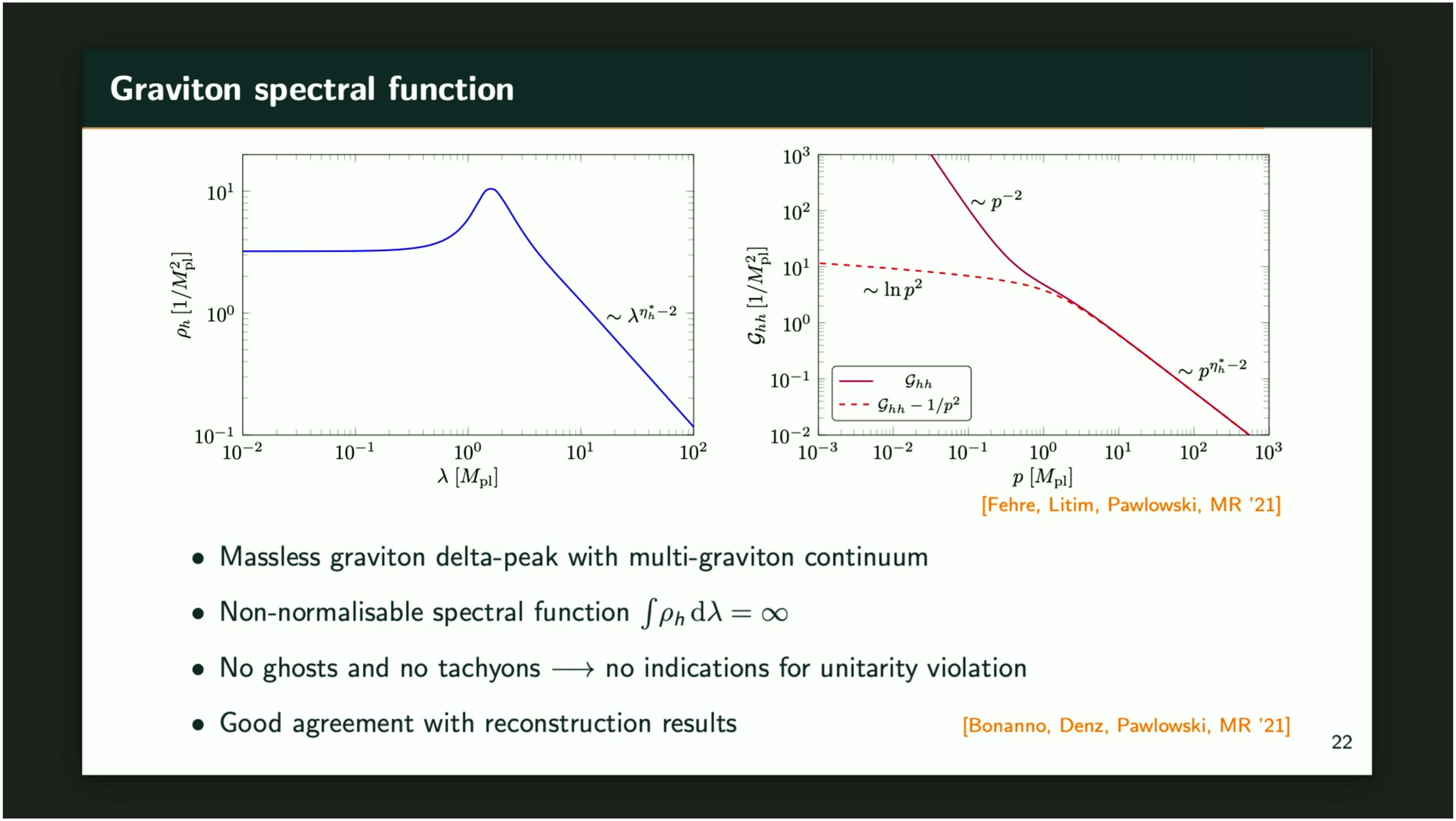


Lorentzian UV-IR trajectories

$$(g,\,\eta_h,\,\mu)\big|_* = (1.06,\,0.96,\,-0.34) \qquad \theta = 2.49 \pm 3.17\,i$$

[Fehre, Litim, Pawlowski, MR '21]

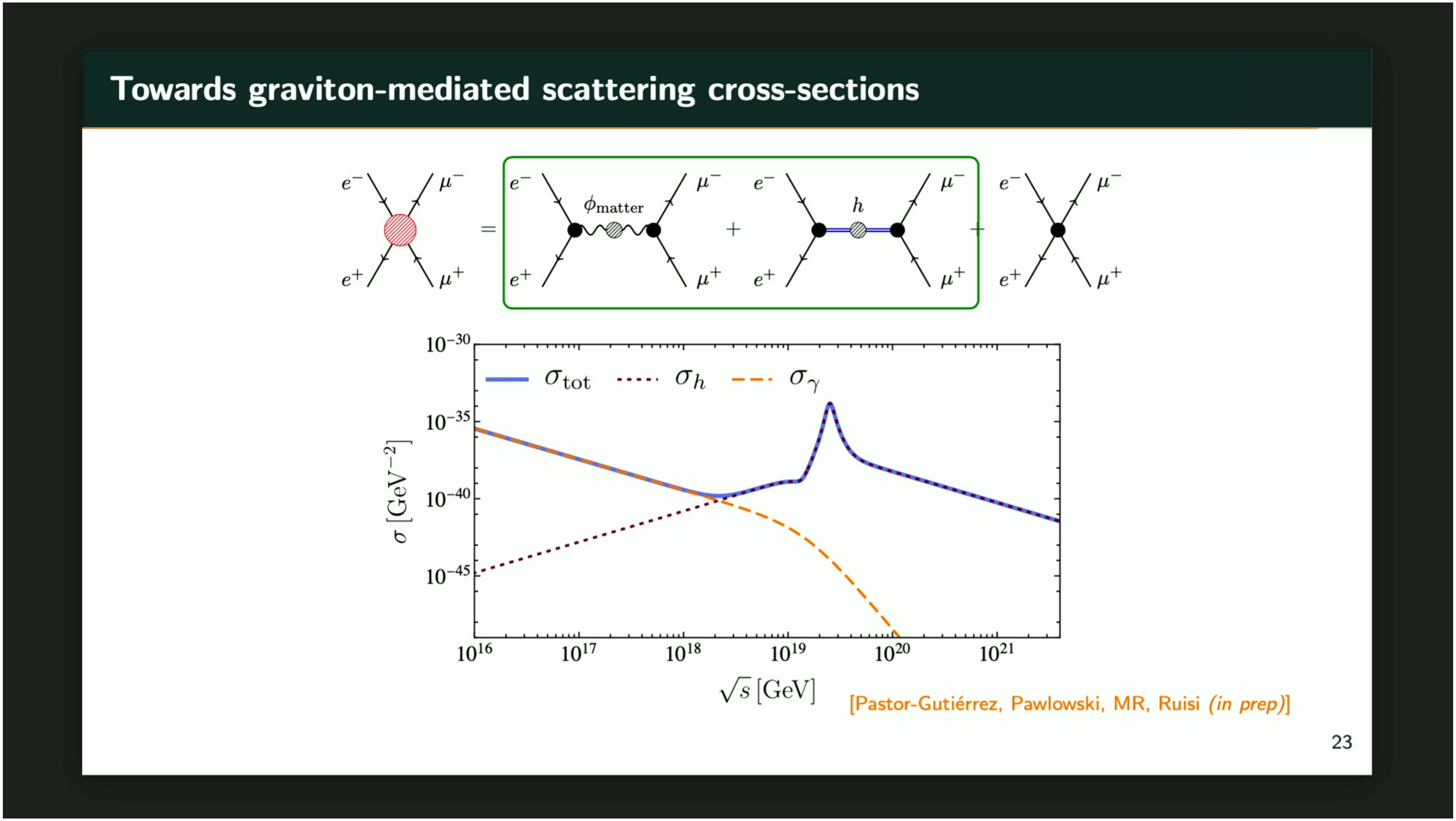
$$G_N(k) = g(k)/k^2 \xrightarrow{k\to 0} G_N \qquad -2\Lambda(k) = k^2\mu(k) \xrightarrow{k\to 0} -2\Lambda = 0$$



Graviton spectral function

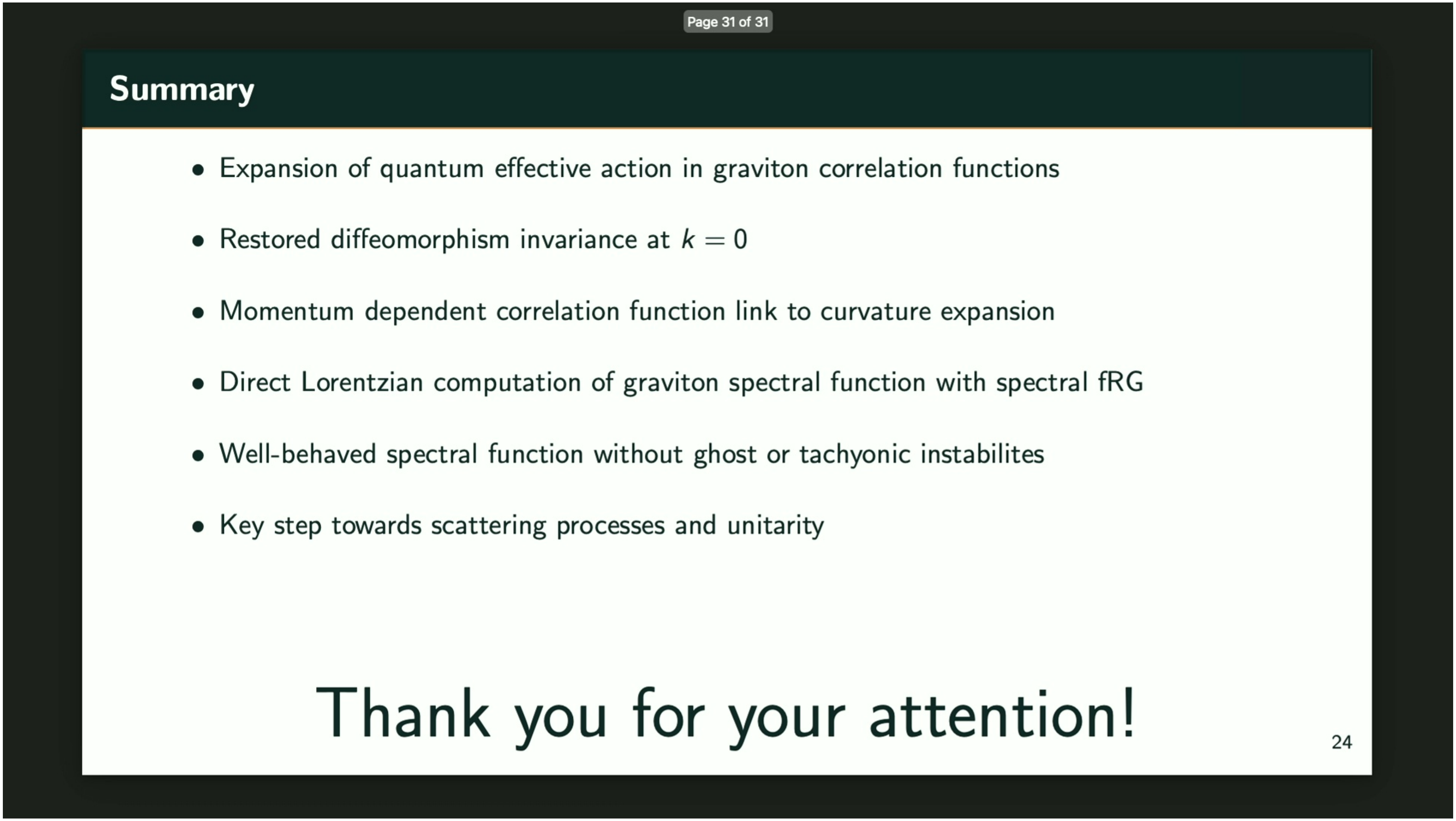
[Fehre, Litim, Pawlowski, MR '21]

- Massless graviton delta-peak with multi-graviton continuum
- Non-normalisable spectral function $\int \rho_h\,\mathrm{d}\lambda = \infty$
- No ghosts and no tachyons ⟶ no indications for unitarity violation
- Good agreement with reconstruction results [Bonanno, Denz, Pawlowski, MR '21]



Towards graviton-mediated scattering cross-sections

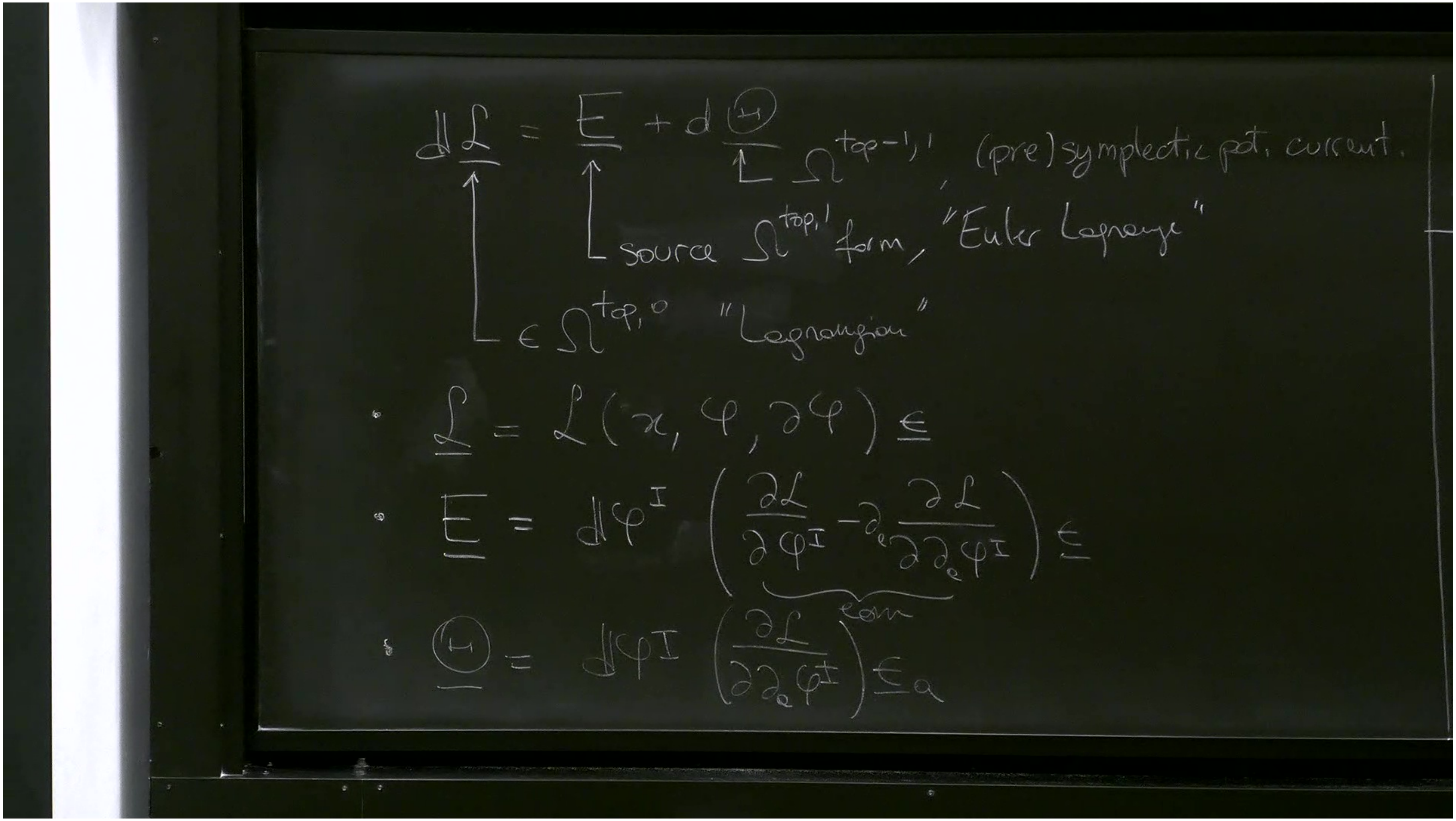
[Pastor-Gutiérrez, Pawlowski, MR, Ruisi *(in prep)*]



Summary

- Expansion of quantum effective action in graviton correlation functions
- Restored diffeomorphism invariance at $k = 0$
- Momentum dependent correlation function link to curvature expansion
- Direct Lorentzian computation of graviton spectral function with spectral fRG
- Well-behaved spectral function without ghost or tachyonic instabilites
- Key step towards scattering processes and unitarity

Thank you for your attention!



$$\mathbb{d}\underline{\mathcal{L}} = \underline{E} + d\,\underline{\Theta}$$

$\in \Omega^{top-1,1}$, (pre)symplectic pot. current.

source $\Omega^{top,1}$ form, "Euler Lagrange"

$\in \Omega^{top,0}$ "Lagrangian"

- $\underline{\mathcal{L}} = \mathcal{L}(x, \varphi, \partial\varphi)\,\underline{\epsilon}$
- $\underline{E} = \mathbb{d}\varphi^I \left(\underbrace{\frac{\partial \mathcal{L}}{\partial \varphi^I} - \partial_a \frac{\partial \mathcal{L}}{\partial \partial_a \varphi^I}}_{eom}\right)\underline{\epsilon}$
- $\underline{\Theta} = \mathbb{d}\varphi^I \left(\frac{\partial \mathcal{L}}{\partial \partial_a \varphi^I}\right)\underline{\epsilon}_a$

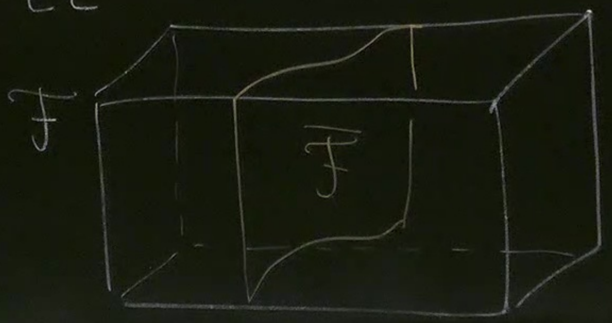
current.

$$\boxed{\begin{aligned} \underline{\epsilon}_a &= i_{\partial_a} \underline{\epsilon} \\ \iota_\Sigma^* \underline{\epsilon}_a &= n_a \underline{\epsilon}_\Sigma \end{aligned}}$$

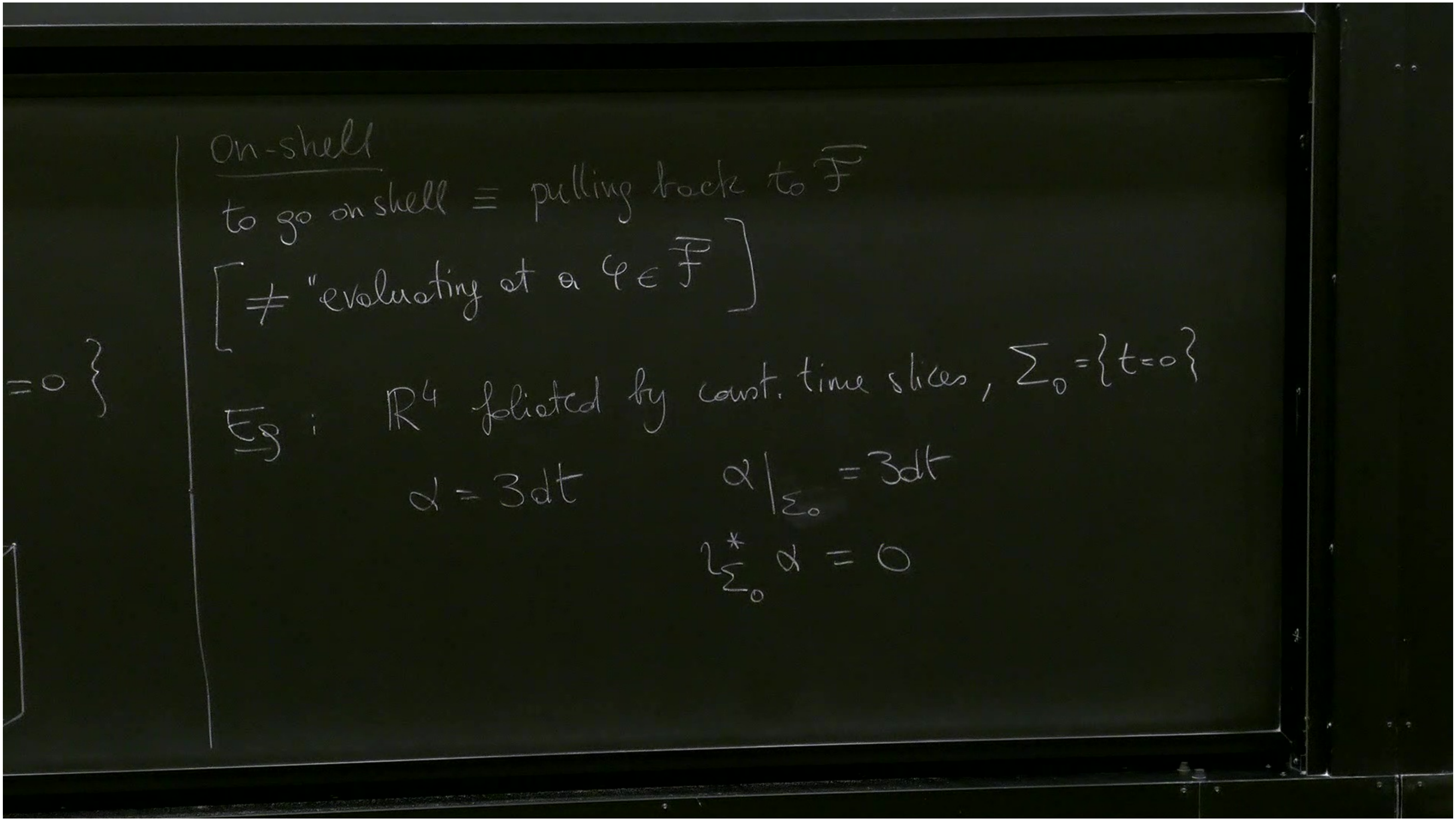
DEF Euler Lagrange locus

$$\overline{\mathcal{F}} := \{ \varphi \in \mathcal{F} : \underline{E}|_\varphi = 0 \}$$

$$\iota_{EL} : \overline{\mathcal{F}} \hookrightarrow \mathcal{F}$$



On-shell



$= 0 \}$

On-shell

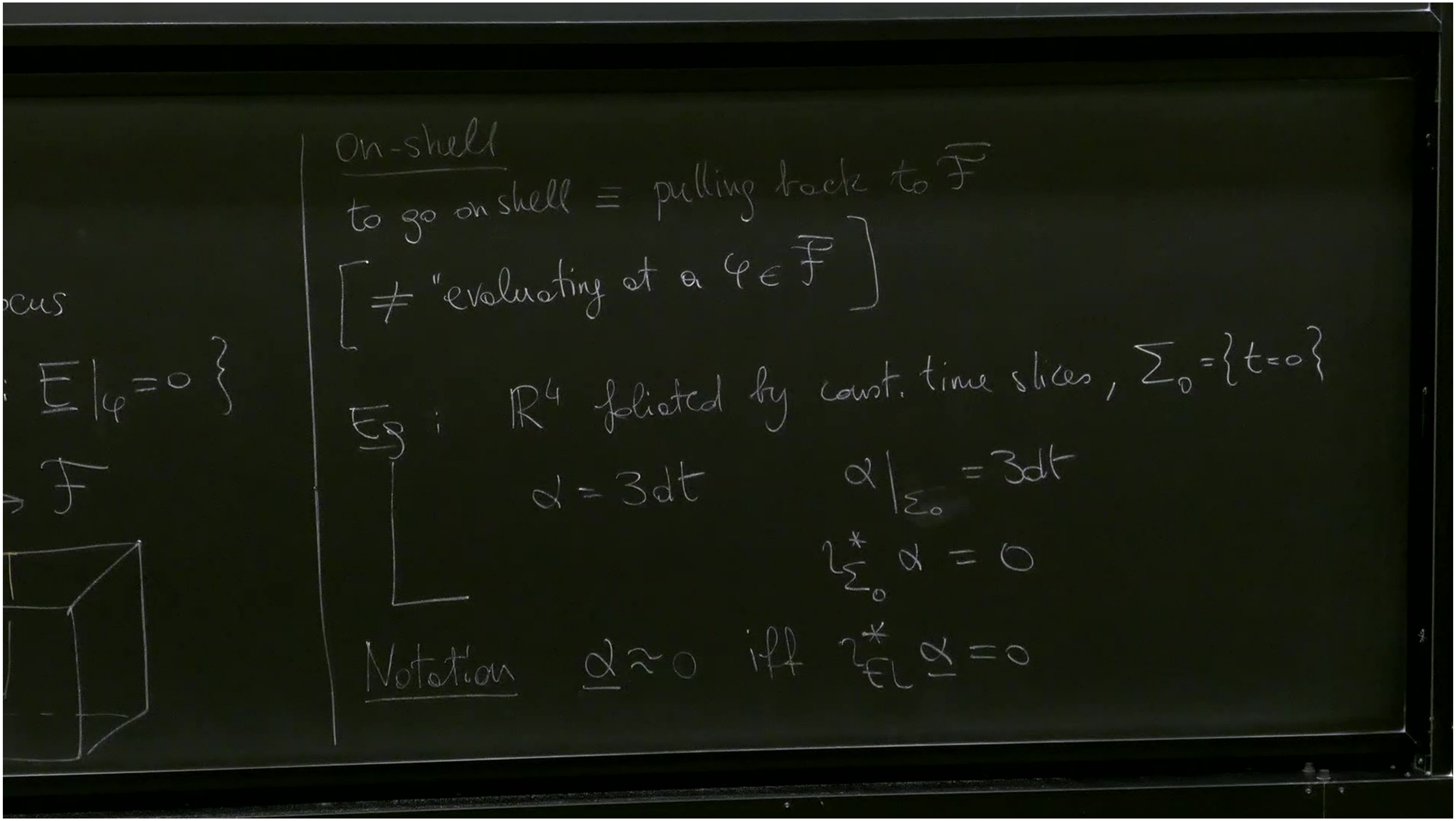
to go on shell $\equiv$ pulling back to $\mathcal{F}$

[$\neq$ "evaluating at a $\varphi \in \mathcal{F}$"]

Eg : $\mathbb{R}^4$ foliated by const. time slices, $\Sigma_0 = \{t=0\}$

$\alpha = 3dt$ $\qquad$ $\alpha|_{\Sigma_0} = 3dt$

$\iota^*_{\Sigma_0} \alpha = 0$



ocus

: $\underline{E}|_{\varphi} = 0 \}$

$\to \mathcal{F}$

On-shell

to go on shell $\equiv$ pulling back to $\mathcal{F}$

$[\neq$ "evaluating at a $\varphi \in \mathcal{F}$ $]$

Eg : $\mathbb{R}^4$ foliated by const. time slices, $\Sigma_0 = \{t=0\}$

$\alpha = 3dt \qquad \alpha|_{\Sigma_0} = 3dt$

$\iota^*_{\Sigma_0} \alpha = 0$

Notation $\underline{\alpha} \approx 0$ iff $\iota^*_{EL} \underline{\alpha} = 0$

$$\underline{\Theta} = \mathbb{d}\varphi^I \left(\frac{\quad}{\partial\partial_a\varphi^I}\right)\underline{\epsilon}_a$$

DEF [Jacobi fields]

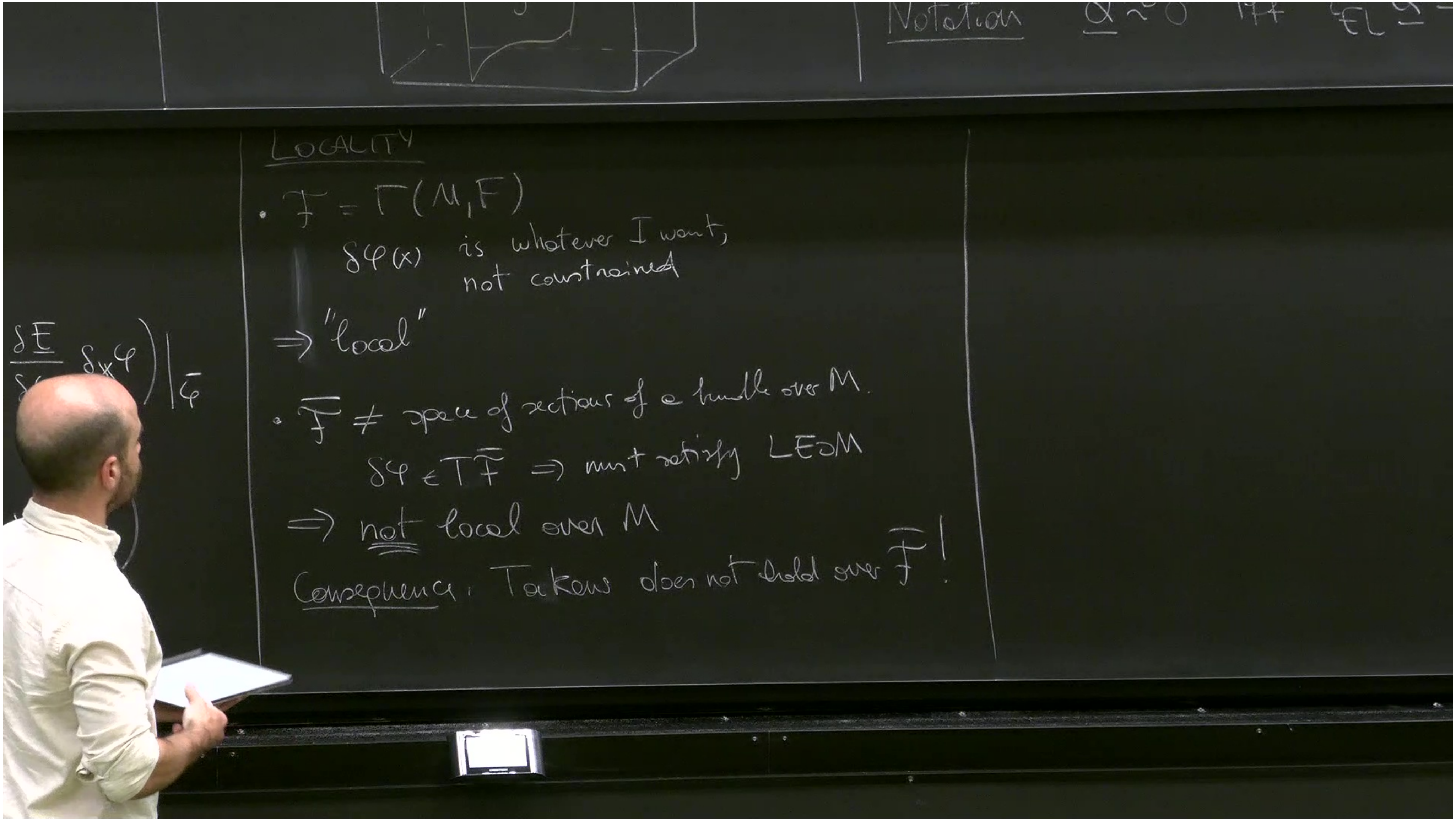
Solutions of the linearized EoM.

Rmk $\mathbb{X} \in T_{\bar\varphi}\mathcal{F}$ is a Jacobi field:

$$0 = \mathbb{L}_{\mathbb{X}}\, \underline{E}\big|_{\bar\varphi} = \int \mathbb{d}(\delta_{\mathbb{X}}\varphi)\; \underbrace{\underline{E}\big|_{\bar\varphi}}_{=0} + \mathbb{d}\varphi\left(\frac{\delta E}{\delta\varphi}\,\delta_{\mathbb{X}}\varphi\right)\Big|_{\bar\varphi}$$

$$= \int \mathbb{d}\varphi\left(\underbrace{\frac{\partial E}{\partial\varphi}\Big|_{\bar\varphi}\delta_{\mathbb{X}}\varphi + \frac{\partial E}{\partial(\partial_a\varphi)}\Big|_{\bar\varphi}\partial_a(\delta_{\mathbb{X}}\varphi)}_{\text{EoM linearized at } \bar\varphi} + \cdots\right)$$

Assumption: converse is also true



Notation

$\frac{\delta E}{\delta\varphi}\delta_X\varphi\Big)\Big|_{\bar\varphi}$

LOCALITY

- $\mathcal{F} = \Gamma(M, F)$

  $\delta\varphi(x)$ is whatever I want, not constrained

$\Rightarrow$ "local"

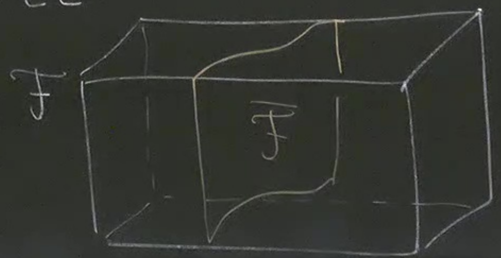
- $\bar{\mathcal{F}} \neq$ space of sections of a bundle over M.

  $\delta\varphi \in T\bar{\mathcal{F}} \Rightarrow$ must satisfy LEoM

$\Rightarrow$ not local over M

Consequence: Takens does not hold over $\bar{\mathcal{F}}$!

$\iota_{EL} : \bar{\mathcal{F}} \hookrightarrow \mathcal{F}$



$\alpha = 3dt$ $\qquad \alpha\big|_{\Sigma_0} = 3dt$

$\iota^*_{\Sigma_0} \alpha = 0$

<u>Notation</u> $\underline{\alpha} \approx 0$ iff $\iota^*_{EL} \underline{\alpha} = 0$

$)\big|_{\bar\varphi}$

<u>LOCALITY</u>

- $\mathcal{F} = \Gamma(M, F)$

  $\delta\varphi(x)$ is whatever I want, not constrained

$\Rightarrow$ "local"

- $\boxed{\bar{\mathcal{F}}} \neq$ space of sections of a bundle over M.

  $\delta\varphi \in T\bar{\mathcal{F}} \Rightarrow$ must satisfy LEOM

$\Rightarrow$ <u>not</u> local over M

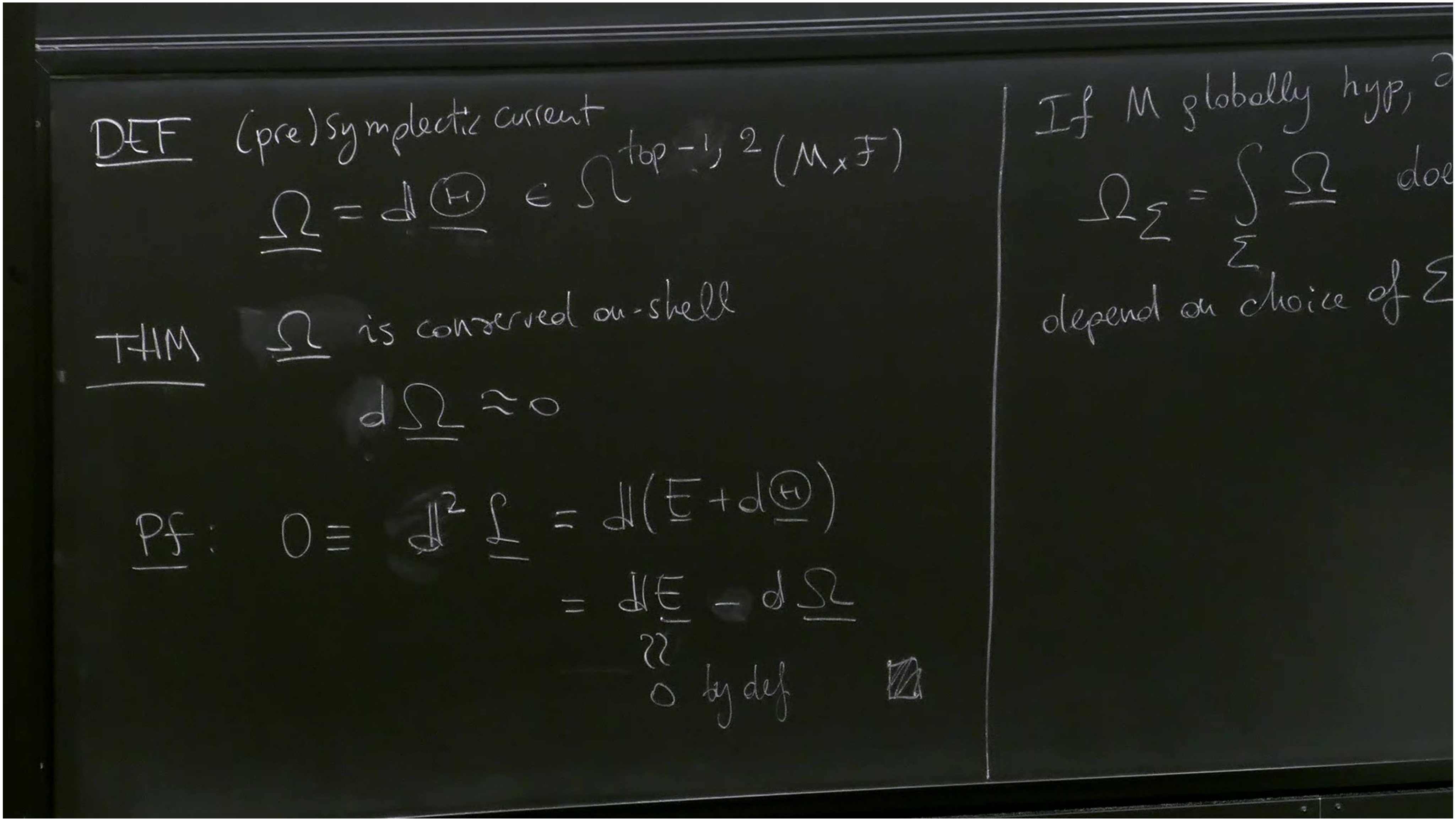
<u>Consequence</u>: Takens does not hold over $\bar{\mathcal{F}}$!

$\bar{\mathcal{F}}$ can still be isomorphic to a local space, just not one over M.

Usually: one over Σ.

If 1-to-1 correspondence btw (unconstrained) initial conds at Σ and $\bar\varphi \in \bar{\mathcal{F}}$

$\uparrow$ "canonical ph. sp."



<u>DEF</u> (pre)symplectic current

$$\underline{\Omega} = d\underline{\Theta} \in \Omega^{top-1,2}(M \times \mathcal{F})$$

<u>THM</u> $\underline{\Omega}$ is conserved on-shell

$$d\underline{\Omega} \approx 0$$

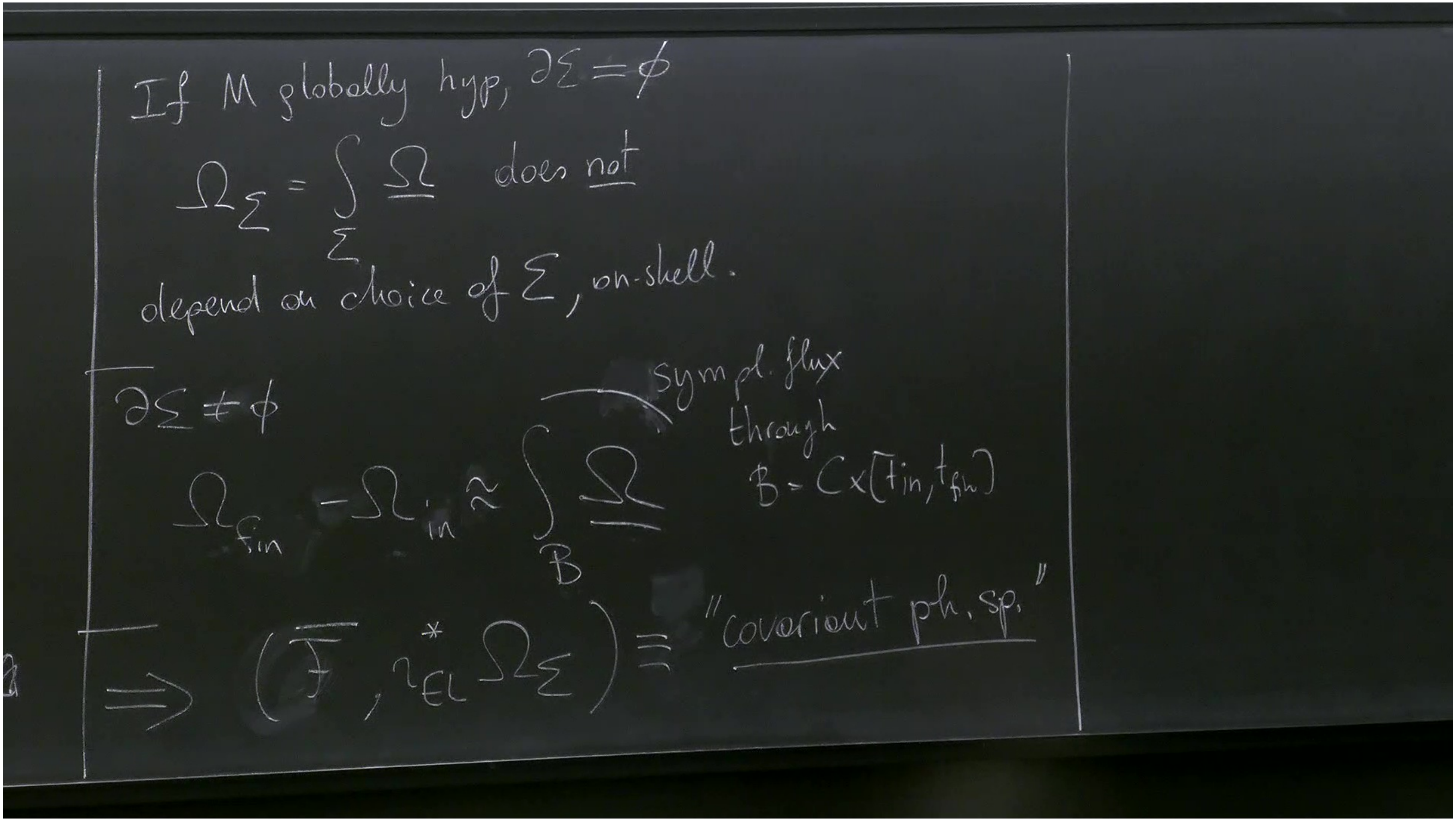
<u>Pf</u>: $0 \equiv d^2 \underline{L} = d(\underline{E} + d\underline{\Theta})$

$$= \underset{\underset{0 \text{ by def}}{\wr\wr}}{d\underline{E}} - d\underline{\Omega} \qquad ▨$$

If M globally hyp, ∂

$$\Omega_\Sigma = \int_\Sigma \underline{\Omega} \quad \text{doe}$$

depend on choice of Σ



If M globbally hyp, $\partial\Sigma = \phi$

$$\Omega_\Sigma = \int_\Sigma \underline{\Omega}$$ does <u>not</u>

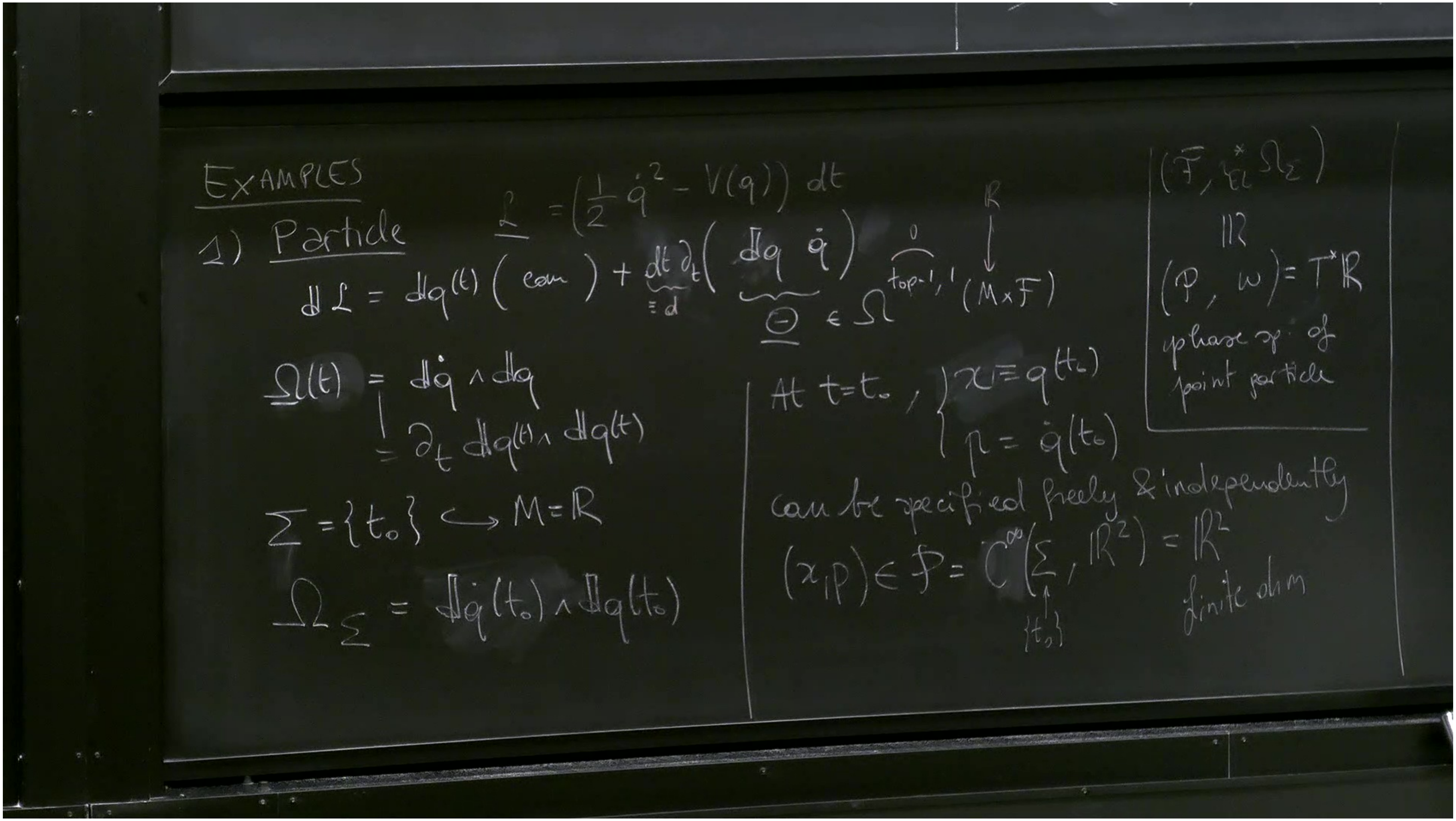
depend on choice of Σ, on-shell.

$\partial\Sigma \neq \phi$

$$\Omega_{fin} - \Omega_{in} \approx \int_B \underline{\Omega}$$

sympl. flux through $B = C \times [t_{in}, t_{fin}]$

$$\Rightarrow \left(\mathcal{F}, \imath^*_{EL}\Omega_\Sigma\right) \equiv$$ "<u>covariant ph. sp.</u>"



EXAMPLES

1) Particle $\underline{L} = \left(\frac{1}{2}\dot{q}^2 - V(q)\right) dt$

$$\mathbb{d} L = \mathbb{d}q^{(t)} \left(\text{eom} \right) + \underbrace{dt\,\partial_t}_{\equiv d} \left(\underbrace{\mathbb{d}q \ \dot{q}}_{\underline{\Theta}} \right)$$

$$\underline{\Theta} \in \Omega^{\overset{0}{\widehat{\text{top}-1}},1}\left(\overset{\overset{\mathbb{R}}{\downarrow}}{M} \times F\right)$$

$$\underline{\Omega}(t) = \mathbb{d}\dot{q} \wedge \mathbb{d}q \overset{!}{=} \partial_t \mathbb{d}q^{(t)} \wedge \mathbb{d}q^{(t)}$$

$$\Sigma = \{t_0\} \hookrightarrow M = \mathbb{R}$$

$$\underline{\Omega}_\Sigma = \mathbb{d}\dot{q}(t_0) \wedge \mathbb{d}q(t_0)$$

At $t = t_0$, $\begin{cases} x \equiv q(t_0) \\ p = \dot{q}(t_0) \end{cases}$

can be specified freely & independently

$$(x, p) \in \mathcal{P} = C^\infty(\underset{\overset{\uparrow}{\{t_0\}}}{\Sigma}, \mathbb{R}^2) = \mathbb{R}^2$$

finite dim

$(\mathcal{F}, \iota_\Sigma^* \Omega_\Sigma)$

$\Vert$

$(\mathcal{P}, \omega) = T^*\mathbb{R}$

phase sp. of point particle

$\Sigma) \equiv$ covariant

$T^*\mathbb{R}$

of

de

ntly

2) Massive KG field

$$\underline{L} = -\frac{1}{2}\left(d\varphi \wedge * d\varphi + m^2 \varphi * \varphi\right)$$

$$\underline{E} = \mathbb{d}\varphi \wedge \left(d * d\varphi - m^2 * \varphi\right) = \cancel{\mathbb{d} *}\left(\Box\varphi - m^2\varphi\right)$$

$$\underline{\textcircled{L}} = \mathbb{d}\varphi \wedge (* d\varphi) \quad \underset{\sim}{\text{top}} \text{ form on } \Sigma$$

$$\Omega = \int \mathbb{d}\varphi \wedge \mathbb{d}(\nabla^a \varphi)\, n_a$$

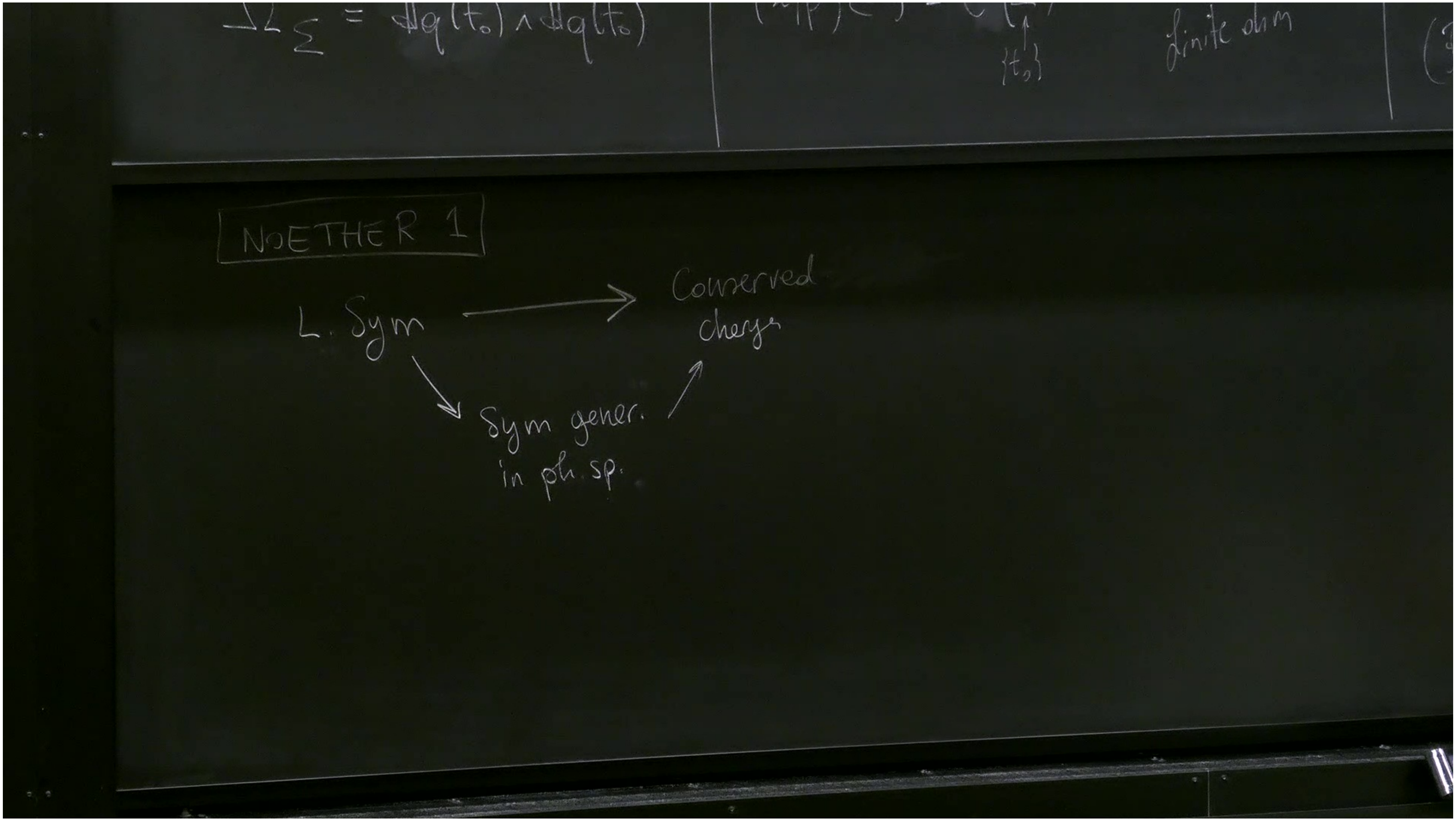
$$\Omega_\Sigma = -\int_\Sigma \mathbb{d}\varphi \wedge \mathbb{d}(* d\varphi)$$

$$= -\int_\Sigma \mathbb{d}\varphi \wedge \mathbb{d}\dot{\varphi}\, \underline{\epsilon}\ , \quad \dot{\varphi} = n^a \partial_a \varphi$$

$$\phi(x \in \Sigma) = \varphi(x \in \Sigma)$$

$$\pi(\quad) = (n^a \partial_a \varphi)(x \in \Sigma)$$

$$(\mathcal{P}, \omega), \quad \omega = \int_\Sigma \mathbb{d}\phi \wedge \mathbb{d}\pi$$



$\Omega_\Sigma = dq(t_0) \wedge dq(t_0)$

$\{t_0\}$

finite dim

$$\boxed{\text{NOETHER 1}}$$

$$\begin{tikzcd}
\text{L. Sym} \arrow[rr] \arrow[dr] & & \text{Conserved charge} \\
& \text{Sym gener. in ph. sp.} \arrow[ur] &
\end{tikzcd}$$