Title: The Unruh effect and its connection to classical radiation (virtual)

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Series: Particle Physics

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Abstract: Particle production can occur in a curved spacetime like, for example, in the case of thermal emission of particles from black holes, otherwise known as Hawking radiation. The Unruh effect is a quantum field theory result that is closely connected to Hawking radiation. It states that accelerated observers associate a thermal bath of particles to the vacuum state of inertial observers. The Unruh effect has been given special attention because contrary to black hole evaporation, it is a prediction made in a flat (Minkowski) spacetime and therefore can be, in principle, tested in the laboratory. Recently, we have investigated the connection between the Unruh effect and classical radiation for a uniformly accelerated particle. This link seems counter-intuitive since the former is a purely quantum effect while the latter is a classic one. Nonetheless, we find that using a full quantum field treatment of the radiation exchanged by an accelerated charge with the surrounding Unruh thermal bath, the resultant power reduces at tree-level to the usual Larmor formula. The results are also consistent with the observation made by Unruh and Wald which states that the emission of a photon in the inertial frame corresponds to the emission or absorption of a photon in the accelerated frame. The fact that the derivation makes the link between the Unruh effect and the Larmor radiation from a uniformly accelerated charged particle clearer will perhaps help in resolving some of the controversies that have surrounded the Unruh effect since its discovery.

Zoom link

The Unruh effect and its connection to classical radiation

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Overview

A. Motivations

B. QFT in curved space

C. The Unruh effect

Overview

A. Motivations

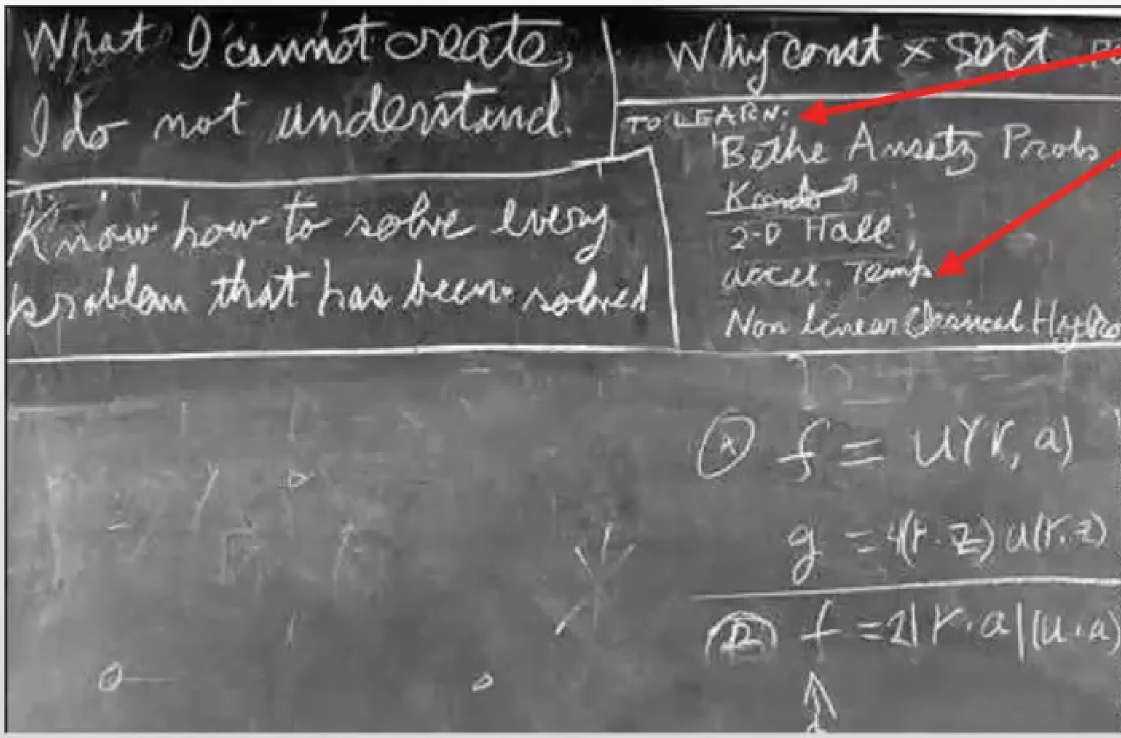
B. QFT in curved space

C. The Unruh effect

D. The Unruh effect and classical radiation

E. Proposed experiments

Feyman’s last blackboard, 1988



From Caltech Photographs

To learn:
Accel. Temp.

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Particle creation in curved space

- Particle creation in the early universe
- Hawking radiation from black holes, 1974
- Link between HR and the Unruh effect

(Part of the) Literature on the Unruh effect

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Papers in agreement with the Unruh effect

- Fulling, 1973
- Davies, 1975
- Unruh, 1976
- ...
- Fulling, Unruh, 2004

Papers questioning the Unruh effect

- Belniskii *et al.*, 1997
- Fedotov *et al.* 1999
- Oriti, 2000
- Narozhny *et al.*, 2002 & 2004

B. QFT in curved spacetime

[1] The Unruh effect and its applications L. C. B. Crispino *et al*, 2008
[2] Quantum fields in curved space, N. D. Birrell and P.C.W. Davies, 1982

QFT in Minkowski spacetime

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$$\left(\partial^\mu \partial_\mu + m^2\right)\phi(x) = 0$$

- Set of solutions: plane waves $f_{\mathbf{k}}(x) \propto e^{i\mathbf{k}\cdot\mathbf{x} - ik_0 t}$, if $k_0 = \sqrt{k^2 + m^2}$
- $f_{\mathbf{k}}$ are said to be positive frequency if $\partial_t f_{\mathbf{k}} = -ik_0 f_{\mathbf{k}}, \quad k_0 > 0$

Annihilation operator

$$\hat{\phi} = \sum_k \left[\hat{a}_k f_k(x) + \hat{a}_k^\dagger f_k^*(x)\right]$$

Creation operator

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Multiple vacua

- In Minkowski spacetime, all inertial observers agree on the same vacuum. In a general spacetime, no reason to expect this.

- Each set of modes defines a vacuum.

- Preferred vacuum if spacetime is static $ds^2 = (N(\mathbf{x}))^2 dt^2 - G_{ij}(\mathbf{x}) dx^i dx^j$. Then $f_i \propto e^{-i\omega_i t}$ and ω_i is the energy.

Bogoliubov transformations

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- Consider two sets $\{f_i^{(1)}\}$ and $\{f_I^{(2)}\}$ which define two vacua $|0_{(1)}\rangle$ and $|0_{(2)}\rangle$

$$f_I^{(2)} = \sum_i \left[\alpha_{Ii}\, f_i^{(1)} + \beta_{Ii}\, f_i^{(1)*}\right]$$

$$\downarrow$$

$$\hat{a}_I^{(2)} = \sum_i \left[\alpha_{Ii}^*\, \hat{a}_i^{(1)} - \beta_{Ii}^*\, \hat{a}_i^{(1)\dagger}\right]$$

Then, $\langle 0_{(1)} | \hat{a}_I^{(2)\dagger} \hat{a}_I^{(2)} | 0_{(1)} \rangle = \sum_i |\beta_{Ii}|^2$

↑ Number operator

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C. The Unruh effect

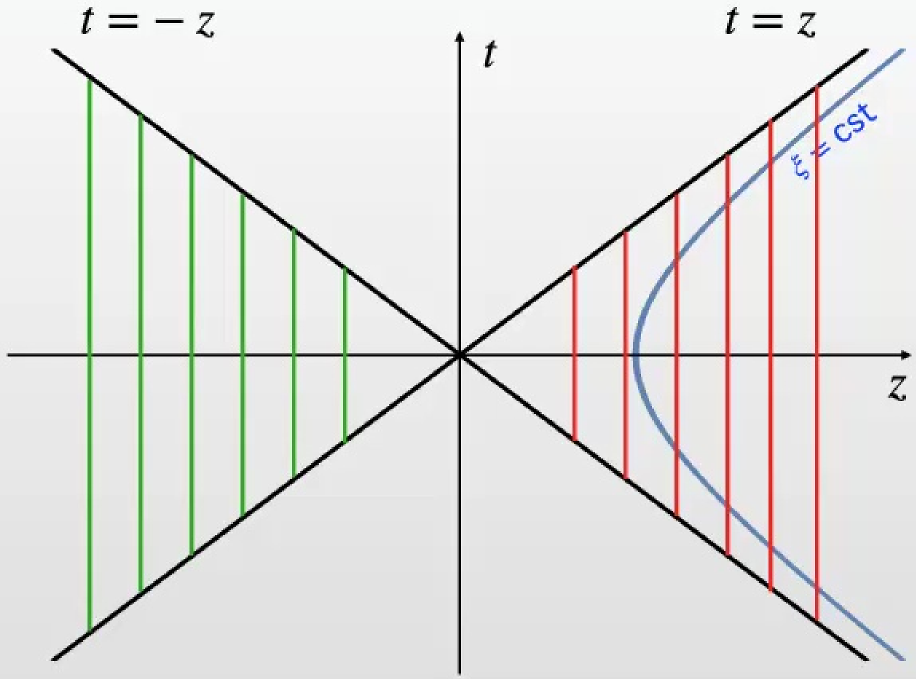
[1] The Unruh effect and its applications L. C. B. Crispino *et al*, 2008
[2] Quantum fields in curved space, N. D. Birrell and P.C.W. Davies, 1982

Rindler wedges

LRW

$t = a^{-1}e^{a\bar{\xi}} \cosh a\bar{\tau}$

$z = -a^{-1}e^{a\bar{\xi}} \cosh a\bar{\tau}$



RRW

$t = a^{-1}e^{a\xi} \sinh a\tau$

$z = a^{-1}e^{a\xi} \cosh a\tau$

Static spacetime with time translation generated by $z\partial_t + t\partial_z \rightarrow |0_R\rangle$

Minkowski and Rindler spaces

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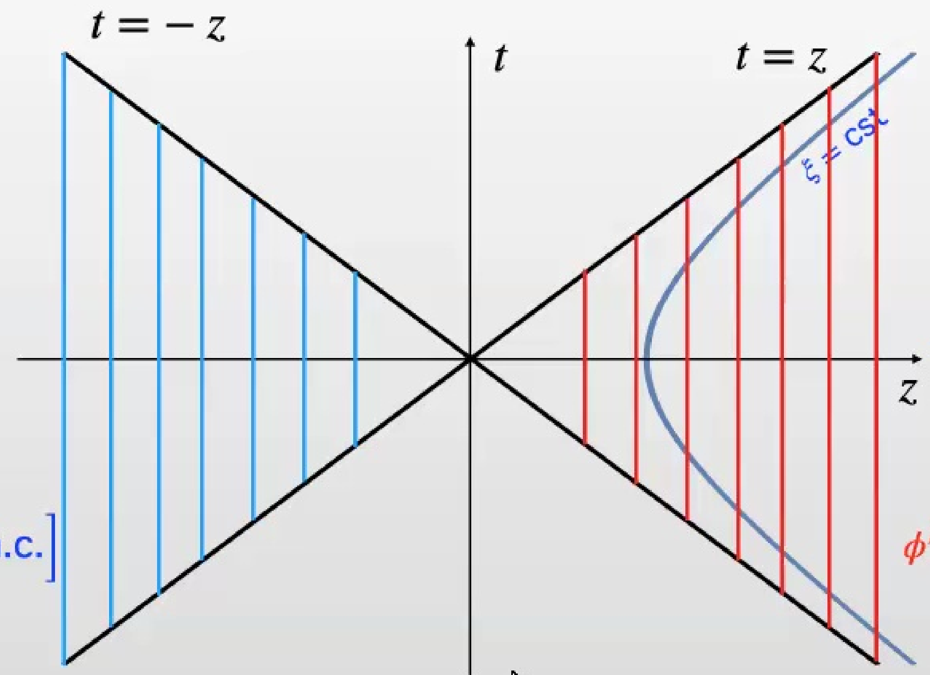
Minkowski

- EoM $\left[\partial_t^2 - \nabla^2\right]\phi = 0$
- $\partial_t, \partial_x, \partial_y, \partial_z$ are Killing vectors
- $f_{\mathbf{k}} = \left[(2\pi)^3 2k_0\right]^{-1/2} e^{i\mathbf{k}\cdot\mathbf{x} - ik_0 t}$
- Vacuum state $|0_M\rangle$

Rindler

- EoM $\left[\partial_\tau^2 - \partial_\xi^2 - e^{2a\xi}(\partial_x^2 + \partial_y^2)\right]\phi = 0$
- $\partial_\tau, \partial_x, \partial_y$ are Killing vectors
- $v^R_{\omega\mathbf{k}_\perp} = \sqrt{\frac{\sinh \pi\omega/a}{4\pi^4 a}} K_{i\omega/a}\left(\frac{k_\perp}{a} e^{a\xi}\right) e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp - i\omega\tau}$
- Vacuum state $|0_R\rangle$

Rindler wedges (II)



$$\phi^L = \int d^2\mathbf{k}_\perp \int_0^{+\infty} d\omega \left[a^L_{\omega\mathbf{k}_\perp} v^L_{\omega\mathbf{k}_\perp} + \text{h.c.} \right]$$

$$\phi^R = \int d^2\mathbf{k}_\perp \int_0^{+\infty} d\omega \left[a^R_{\omega\mathbf{k}_\perp} v^R_{\omega\mathbf{k}_\perp} + \text{h.c.} \right]$$

$$\phi = \phi^L + \phi^R$$

Bogoliubov coeff.

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To derive the Unruh effect we link the Rindler modes to the Minkowski ones

$$v^R_{\omega \mathbf{k}_\perp} \propto \int d\tilde{k}_z \left[\alpha^R_{\omega \mathbf{k}}\, e^{-ik_0 t + ik_z z} + \beta^R_{\omega \mathbf{k}}\, e^{ik_0 t - ik_z z}\right]$$

$$v^L_{\omega \mathbf{k}_\perp} \propto \int d\tilde{k}_z \left[\alpha^L_{\omega \mathbf{k}}\, e^{-ik_0 t + ik_z z} + \beta^L_{\omega \mathbf{k}}\, e^{ik_0 t - ik_z z}\right]$$

Then we find, $\beta^R_{\omega \mathbf{k}} = -\, e^{-\pi\omega/a} \alpha^{L*}_{\omega \mathbf{k}}$ and $\beta^L_{\omega \mathbf{k}} = -\, e^{-\pi\omega/a} \alpha^{R*}_{\omega \mathbf{k}}$

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Positive frequency modes

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Purely positive frequency modes

$$w_{-\omega\mathbf{k}_\perp} = \frac{v^R_{\omega\mathbf{k}_\perp} + e^{-\pi\omega/a} v^{L*}_{\omega-\mathbf{k}_\perp}}{\sqrt{1-e^{-2\pi\omega/a}}}$$

$$w_{+\omega\mathbf{k}_\perp} = \frac{v^L_{\omega\mathbf{k}_\perp} + e^{-\pi\omega/a} v^{R*}_{\omega-\mathbf{k}_\perp}}{\sqrt{1-e^{-2\pi\omega/a}}}$$

$$\longrightarrow \quad w_{\pm\omega\mathbf{k}_\perp} \propto \int dk_z f(\omega, k_z)\, e^{i\mathbf{k}\cdot\mathbf{x} - ik_0 t}$$

Then, the operators annihilate the Minkowski vacuum

$$(a^R_{\omega\mathbf{k}_\perp} - e^{-\pi\omega/a} a^{L\dagger}_{\omega-\mathbf{k}_\perp})|0_M\rangle = 0$$

$$(a^L_{\omega\mathbf{k}_\perp} - e^{-\pi\omega/a} a^{R\dagger}_{\omega-\mathbf{k}_\perp})|0_M\rangle = 0$$

Number operator $\langle 0_M | a^{R\dagger}_{\omega\mathbf{k}_\perp} a^R_{\omega'\mathbf{k}'_\perp} | 0_M\rangle = \dfrac{1}{e^{2\pi\omega/a} - 1}\delta(\omega-\omega')\delta^{(2)}(\mathbf{k}_\perp - \mathbf{k}'_\perp)$

Summary of the UE

- *The Unruh effect is the fact that the Minkowski vacuum restricted to the RRW (LRW) is a thermal state with temperature proportional to the acceleration.*

- Connection to Hawking radiation $T_U = \dfrac{\hbar a}{2\pi c k_B} \leftrightarrow T_H = \dfrac{\hbar g}{2\pi c k_B}$

- Also valid for other types of fields

D. The Unruh effect and classical radiation

[1] Bremsstrahlung and Fulling-Davies-Unruh thermal bath, A. Higuchi *et al*, 1992
[2] The classical Larmor formula through the Unruh effect for uniformly accelerated electrons, G. Vacalis *et al*, 2023

Quantization of the EM field in the RRW

The EM Lagrangian $\frac{\mathscr{L}}{\sqrt{-g}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \underbrace{\frac{1}{2}(\nabla_\alpha A^\alpha)^2}_{\text{gauge fixing}}$

$$\nabla^\mu \nabla_\mu A_\nu = 0$$

$$\hat{A}^R_\mu = \int d^2\mathbf{k}_\perp \int_0^{+\infty} d\omega \sum_{\lambda=1}^{4} \left[a^R_{(\lambda,\omega,\mathbf{k}_\perp)} A_\mu^{R(\lambda,\omega,\mathbf{k}_\perp)} + \text{h.c.} \right]$$

Only 2 physical

Adding an interacting detector

- The Unruh effect is independent of detectors and the type of particle
- Consider a structureless detector accelerating and making a current which interacts with the thermal bath
- The detector is accelerated for a finite time

The charged source

- The interaction is $\mathscr{L}_{int} = -\sqrt{-g}j^\mu A_\mu$
- For a classical charge $j^\mu = q\dfrac{dx^\mu(\tau)}{dt}\delta^{(3)}(\mathbf{x} - \mathbf{x}(\tau)) \longrightarrow$

$$j^\tau = q\delta(\xi)\delta(x)\delta(y)$$

$$j^\xi = j^x = j^y = 0$$

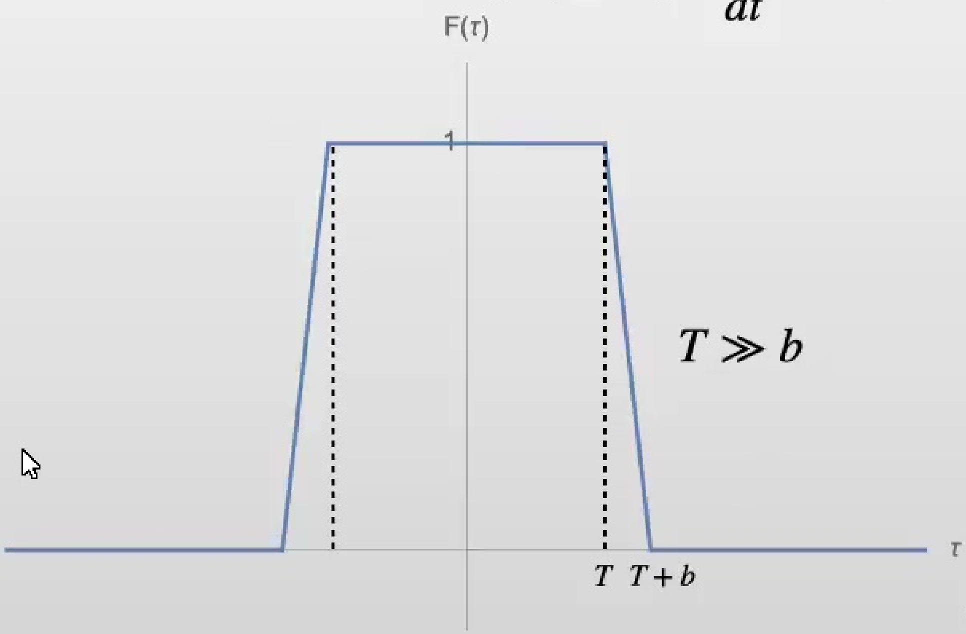
$$\downarrow$$

$$j^\tau = qF(\tau)\delta(\xi)\delta(x)\delta(y)$$

$$j^\xi = -qF'(\tau)e^{-2a\xi}\theta(\xi)\delta(x)\delta(y)$$

$$j^x = j^y = 0$$

$$\nabla_\mu j^\mu = 0$$

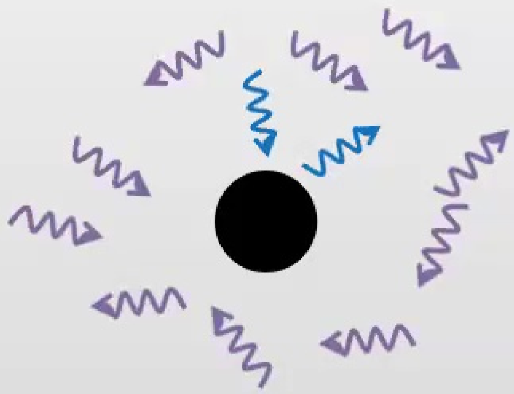


$T \gg b$

Interaction between the charge and the thermal bath

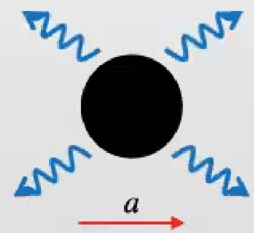
- Unruh, Wald 1984 : Emission in the inertial frame = emission or absorption in acc. frame
- Higuchi *et al*. 1992, explicitly verified the absorption and emission rates

In the acc. frame



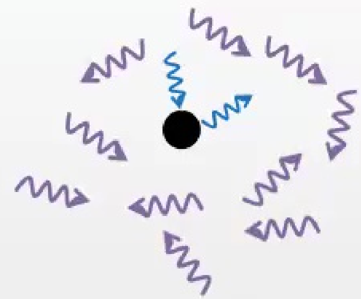
Emission and absorption
to an from the thermal bath

In the inertial frame



Only emission in the
inertial frame

Interaction between the charge and the thermal bath



Emission and absorption
to an from the thermal bath

- Emission amplitude $\mathscr{A}^{e}_{\omega,\mathbf{k}_\perp} = i\langle \mathrm{I\!I}, \omega, \mathbf{k}_\perp | S_I | 0_R \rangle$
- Absorption amplitude $\mathscr{A}^{a}_{\omega,-\mathbf{k}_\perp} = i\langle 0 | S_I | \mathrm{I\!I}, \omega, -\mathbf{k}_\perp \rangle$

Interaction probability $P_{tot} = \int d^2\mathbf{k}_\perp \, d\omega \left[\frac{|\mathscr{A}^{a}_{\omega,-\mathbf{k}_\perp}|^2}{e^{2\pi\omega/a} - 1} + |\mathscr{A}^{e}_{\omega,\mathbf{k}_\perp}|^2 \left(\frac{1}{e^{2\pi\omega/a} - 1} + 1 \right) \right]$

Induced + spontaneous

Response rate and power

Response rate $R(k_\perp) = \frac{P(k_\perp)}{2T} = \frac{q^2}{4\pi^3 a}\left|K_1\left(\frac{k_\perp}{a}\right)\right|^2$ known from Higuchi *et al*. 1992

We calculated independently $R(k_\perp) = \int d\bar{k}_z\, F(k_\perp, \bar{k}_z)$

$$\downarrow \times \bar{k}_0 = \sqrt{k_\perp^2 + \bar{k}_z^2}$$

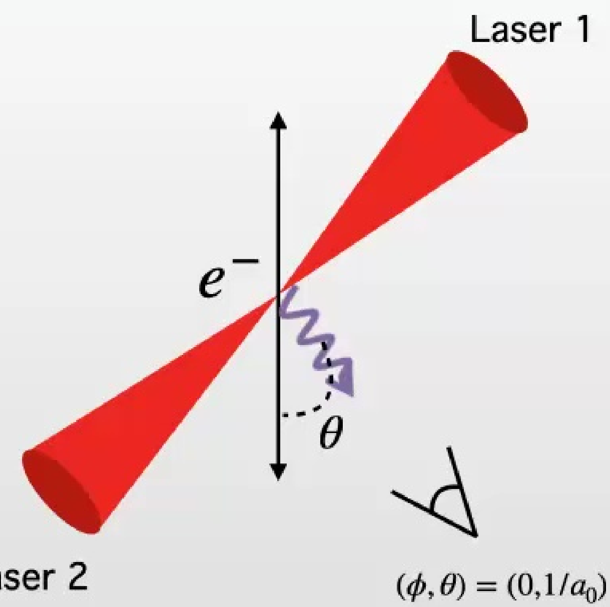
Power in the rest frame $S_{acc,inst} = \frac{q^2 a^2}{6\pi}$

Partial Conclusions

- Quantum calculation with Minkowski modes in inertial frame→ Larmor formula
- Valid at tree level. Corrections include scattering, radiation reaction
- Unruh effect reducing to Larmor formula was also found in Portales-Oliva *et al*., 2019, Landulfo *et al*. 2022, Lynch *et al*. 2021 in the inertial frame.

E. Proposed experiments

Unruh "radiation" and lasers



$$E_z = E_0(\cos\omega(t-x) + \cos\omega(t+x))$$
$$B_y = -E_0(\cos\omega(t-x) - \cos\omega(t+x))$$

- Chen and Tajima, PRL, 1999, proposed to use two lasers to accelerate an electron
- Radiation reaction can be treated more easily in the rest frame of the electron
- At small angles: $\dfrac{dS_U}{d\Omega} \propto \dfrac{1}{(1+a_0^2\,\theta^2)^3}$ and

$$\frac{dS_L}{d\Omega} \propto \left[1 - \frac{4a_0^2\,\theta^2(1-\phi^2)}{(1+a_0^2\,\theta^2)}\right]$$

The UE in agreement with Maxwell's eqs.

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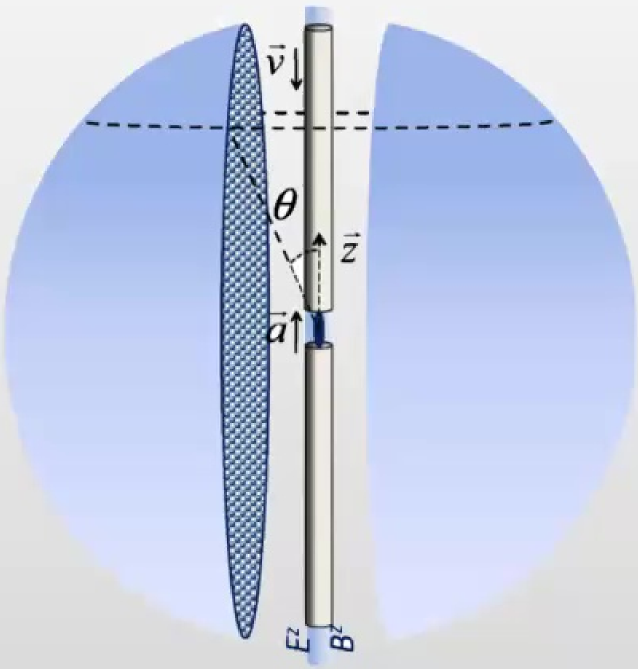
Cozzella *et al*. PRL, 2017



Found by in. observers — Free Parameter

1. Cozzella *et al.* (2017) : $\dfrac{dN^M_{k_\perp}}{dk_\perp} = \Delta\tau \cdot \dfrac{dR^R_{k_\perp}}{dk_\perp}(T)$

2. Using CE, they show $\dfrac{dN^M_{k_\perp}}{dk_\perp} = \Delta\tau \cdot \dfrac{dR^R_{k_\perp}}{dk_\perp}\Bigg|_{T=T_U}$

Conclusions

- The Unruh effect is a QFT result that has been the subject of controversies. It states that *the Minkowski vacuum restricted to the RRW (LRW) is a thermal state.*

- Absorption+emission in acc. frame = emission in the inertial frame

- At tree-level, the Unruh effect reduces to Larmor's formula for radiation

- Results available at arxiv:2310.06127