Title: Quantum Fields and Strings Seminar - TBA

Speakers: Javier Martinez Magan

Series: Quantum Fields and Strings

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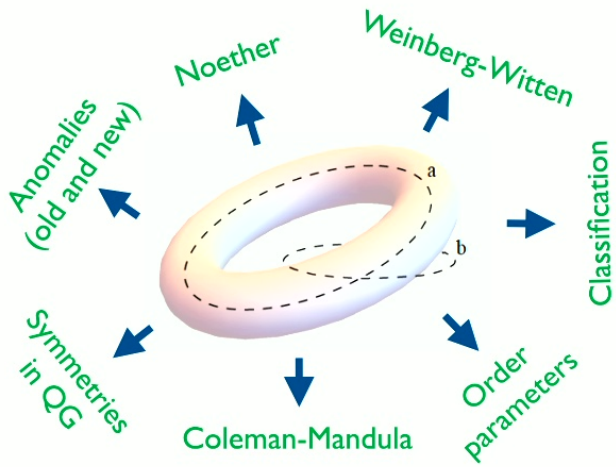
Abstract: Abstract TBA

Zoom link

Local aspects of generalized symmetries



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In collaboration with Horacio Casini, Marina Huerta, Valentín Benedetti, Diego Pontello, Pedro J. Martínez, Pablo Bueno

Context

- Landau paradigm
- Classification of general phases of matter: generalized order parameters
- Intrinsic (non-Lagrangian) approaches
- Quantum Information
- Constraints/classification of the zoo of possibilities
- Confinement, mass gap, etc
- Bootstrap and modular invariance
- Folk theorems in quantum gravity

Generalized Global Symmetries

Important progress in these directions was made by considering the notion of generalized global symmetries

[Gaiotto, Kapustin, Seiberg, Willett, 2014]

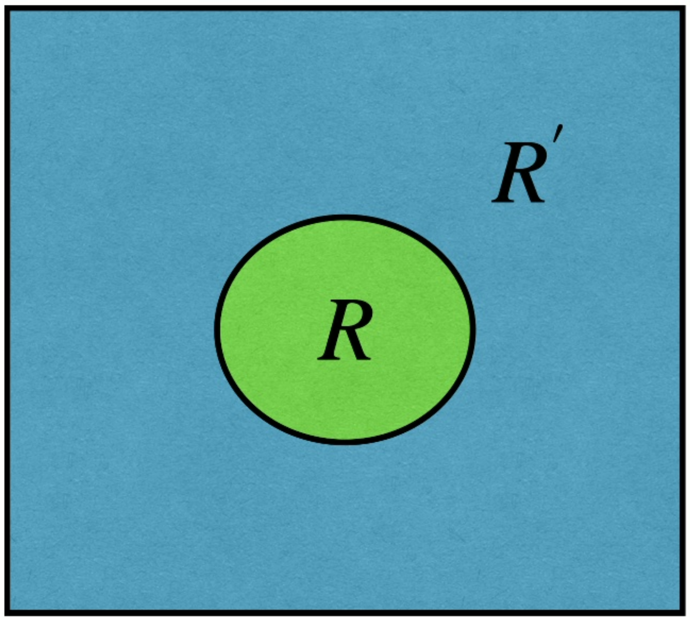
Briefly, this proposal can be described by a certain classification of operators

- Topological operators (generators of the symmetry)
- Charged operators (genuine lines for example)
- Global gauge group (algebra of charged operators)

Plan of the talk

- Generalized order parameters and completeness in QFT
- Examples
- Entropic order parameters and the phases of QFT
- Universal charged density of states
- Discussion and outlook

Generalized order parameters and completeness in QFT



Haag-Kastler approach to QFT

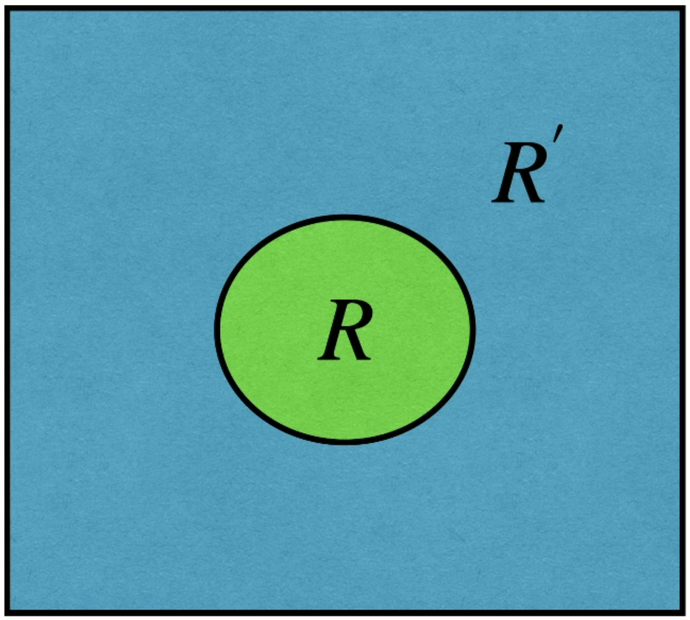
Locality

$$R_{ball} \to \mathscr{A}(R_{ball})$$

Einstein causality is written as

$$\mathscr{A}(R_{ball}) \subset \mathscr{A}(R'_{ball})'$$

Generalized order parameters and completeness in QFT



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Minimalistic approach

Wightman implies Haag.
Other way around is an open problem.

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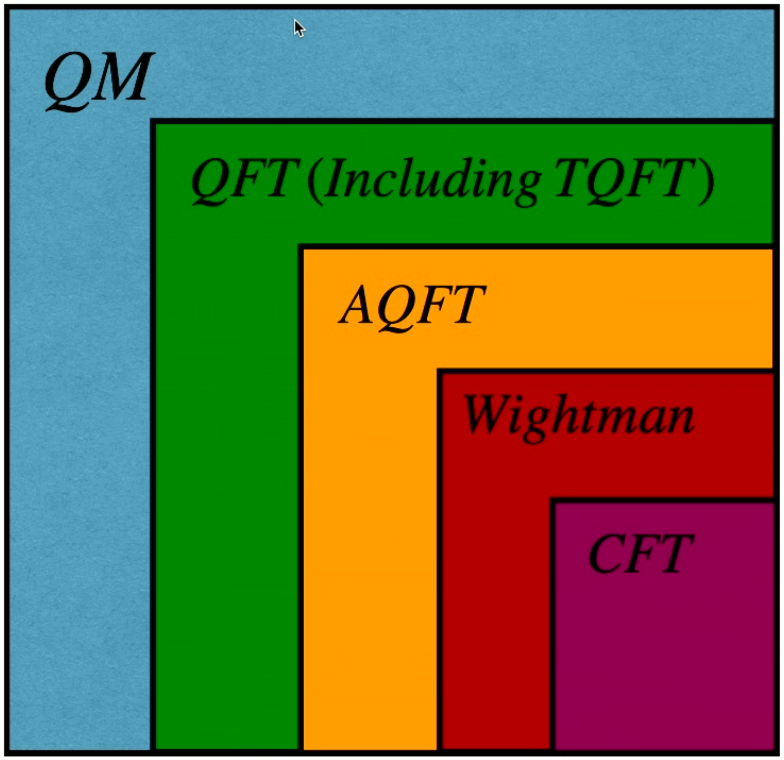

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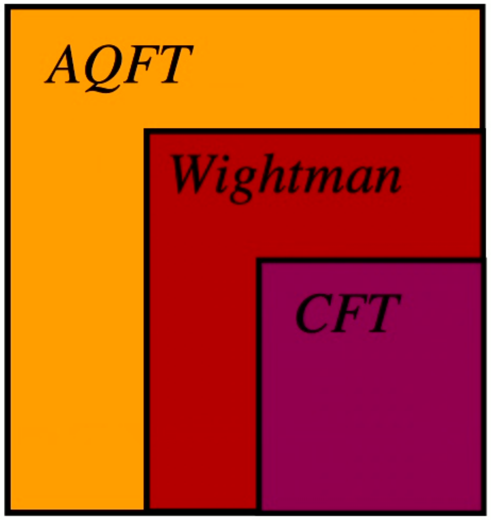


Haag-Kastler approach to QFT

Locality

$$R_{ball} \rightarrow \mathscr{A}(R_{ball})$$

Einstein causality is written as

$$\mathscr{A}(R_{ball}) \subset \mathscr{A}(R_{ball}^{'})^{'}$$

One typically 'assumes' Haag duality

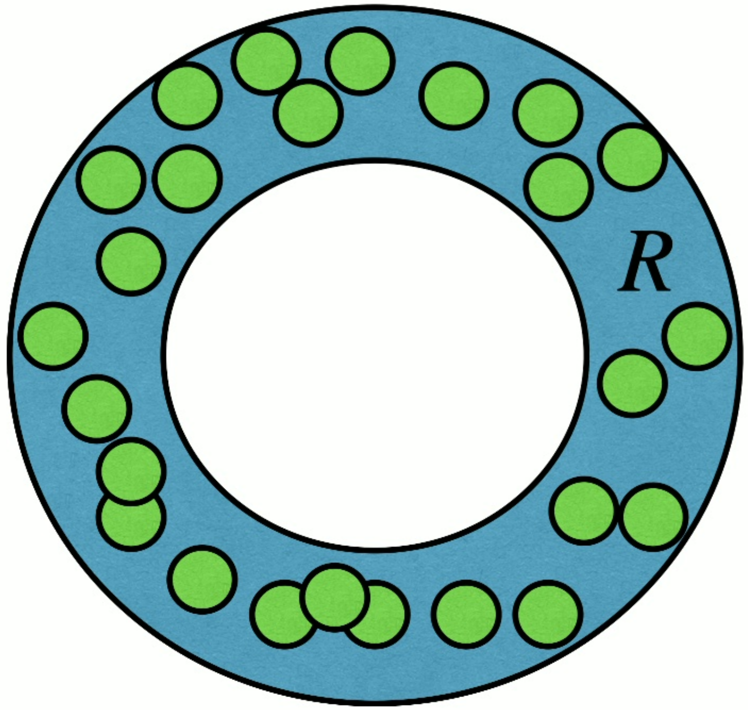
$$\mathscr{A}(R_{ball}) = \mathscr{A}(R_{ball}^{'})^{'}$$

Valid in Rindler wedge through
Bisognano-Wichmann theorem.

Valid in CFT in balls

Generalized order parameters and completeness in QFT

Exploring the Haag-Kastler approach for topologically non-trivial regions



Does the Haag-Kastler approach assign an unambiguous algebra to these regions?

Yes! 'The additive algebra'

$$\mathscr{A}_{add}(R) \equiv \bigvee_{B\, ball\, ,\cup B = R} \mathscr{A}(B)$$

Yields the additive net

Key question: Does the additive algebra satisfy duality?

$$\mathscr{A}_{add}(R) \ ? = ? \ \mathscr{A}_{add}(R')'$$

Generalized order parameters and completeness in QFT

A simple intrinsic definition for generalized order parameters in QFT was proposed in

[Casini, Huerta, J.M.M, Pontello 2020]
[Casini, J.M.M 2021]

'A generalized order parameter in the space of AQFT is an operator that violates duality in a certain region. An order parameter is a Haag-duality-violating (HDV) operator.'

This also suggests an intrinsic definition of completeness in QFT

"A QFT is complete if for an arbitrary region of the space the observable algebra generated by the local degrees of freedom is the maximal one compatible with causality."

Equivalently

"Completeness $\equiv$ uniqueness of the net $\equiv$ duality for the additive net $\equiv$ no order parameter."

Generalized order parameters and completeness in QFT

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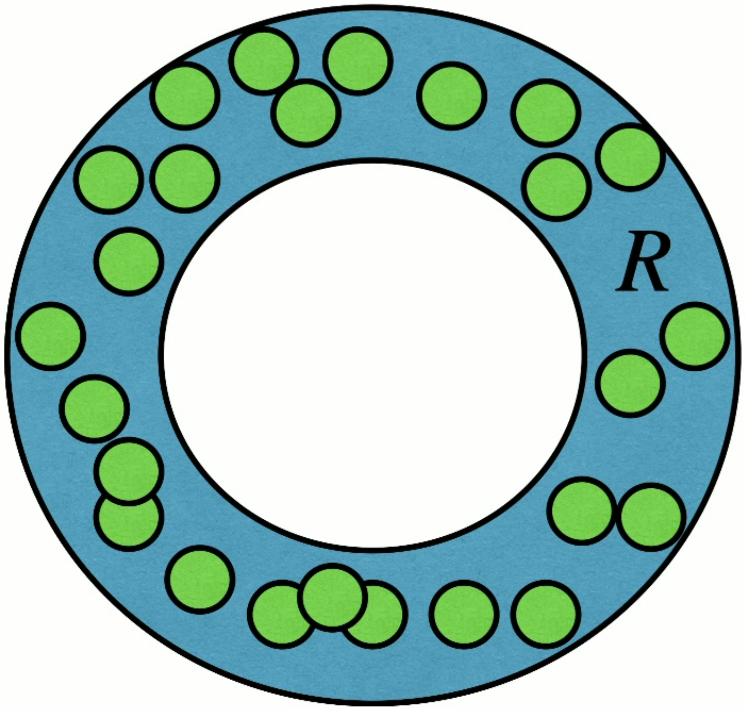

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Equivalently

"Completeness ≡ uniqueness of the net ≡ duality for the additive net ≡ no order parameter."

Generalized order parameters and completeness in QFT

What can we say about non-complete theories?

$$\mathscr{A}_{add}(R) \subsetneq \mathscr{A}_{add}(R')' \equiv \mathscr{A}_{max}(R)$$

First, it must be the case that

$$\mathscr{A}_{max}(R) = \mathscr{A}_{add}(R) \vee \{a\}$$

For some set of HDV operators $\{a\}$, the order parameters.

HDV operators define classes of operators in the regions

$$\tilde{a} \in [a] \to \tilde{a} = \sum_{\lambda} O_1^{\lambda}\, a\, O_2^{\lambda} \qquad O_i^{\lambda} \in \mathscr{A}_{add}(R)$$

Important consequence: If duality is violated in R, it is also violated in R'

$$\mathscr{A}_{add}(R) \subsetneq \mathscr{A}_{max}(R) \to \mathscr{A}_{add}(R') \subsetneq \mathscr{A}_{max}(R')$$

And it implies the existence of further order (or disorder) parameters and classes in the QFT

$$\mathscr{A}_{max}(R') = \mathscr{A}_{add}(R') \vee \{b\}$$

The dual a's and b's do not commute with each other (quantum complementarity)

$$\sigma_a^2\, \sigma_b^2 \sim \hbar$$

Generalized order parameters and completeness in QFT

The dual a's and b's do not commute with each other (quantum complementarity)

$$\sigma_a^2 \, \sigma_b^2 \sim \hbar$$

The HDV operators generate local generalized symmetry transformations.
Unified view of order parameters and generalized symmetry transformations.

"Non-completeness implies the existence of a generalized symmetry."

"Completeness $\equiv$ uniqueness of the net $\equiv$ duality for unique net $\equiv$ absence of generalized symmetries."

[Casini, J.M.M 2021]

Unification of previous folk theorems in quantum gravity

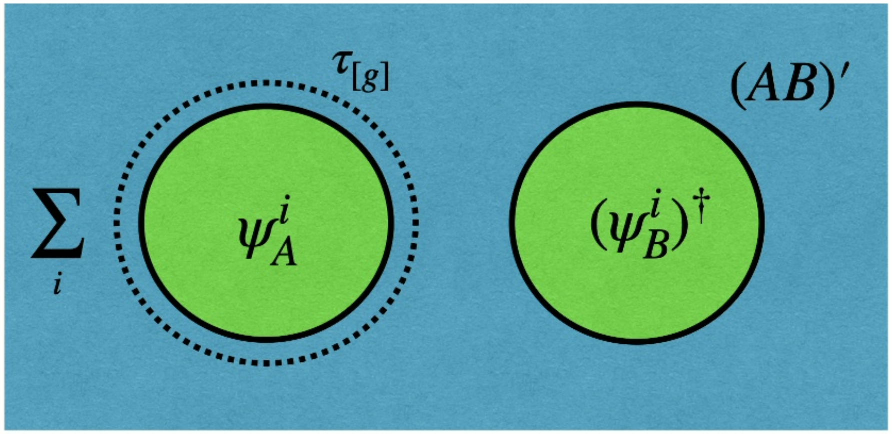
[Polchinski, 2003] **[Seiberg, Banks, 2010]**

Final important remark: Haag duality for balls $\mathscr{A}(R_{ball}) = \mathscr{A}(R'_{ball})'$ implies all HDV operators, namely all generalized order parameters, are generated by local operators in the QFT.

Examples

Global symmetries

Theories with global symmetries display violation of duality in regions with non trivial $\pi_0(R)$ and $\pi_{d-2}(R')$



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The generalized order parameters satisfy the following fusion rules

$$\tau_g \tau_{g'} = \tau_{gg'}$$

$$\tau_{[g]} = \sum_{g \in [g]} \tau_g \longrightarrow \tau_{[g]} \tau_{[g']} = \sum_{[g'']} \tilde{N}_{[g][g']}^{[g'']} \tau_{[g'']}$$

$$\mathcal{I}_r = \sum_i \psi_{1r}^i (\psi_{2r}^i)^\dagger \longrightarrow \mathcal{I}_r \mathcal{I}_{r'} = \sum_{[g'']} N_{rr'}^{r''} \mathcal{I}_{r''}$$

Examples

Global symmetries

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The HDV operators can be seen to be locally generated since

$$\mathcal{I}_r(x,y)\mathcal{I}_r(y,z) = \sum_i \psi^i_{1r}(x)\,(\psi^i_{2r})^\dagger(y)\,\sum_j \psi^j_{1r}(y)\,(\psi^j_{2r})^\dagger(z) = \mathcal{I}_r(x,z)$$

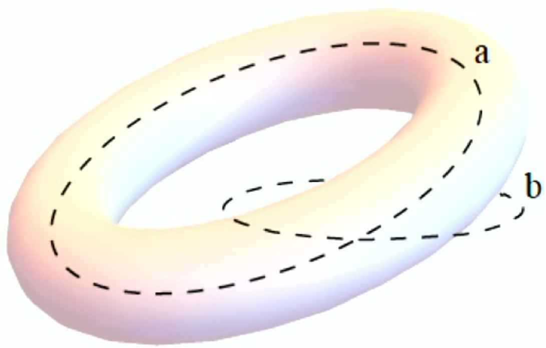
The 'locality' of the twist follows easily if there is a Noether current.

If there is no Noether current, the locality of the twist follows from the one of the intertwiner and Von Neumann double commutant theorem

Examples

Symmetries in gauge theories:

Gauge theories with generalized symmetries violate of duality in regions with non trivial $\pi_1(R)$ and $\pi_{d-3}(R')$



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Symmetries in gauge theories:

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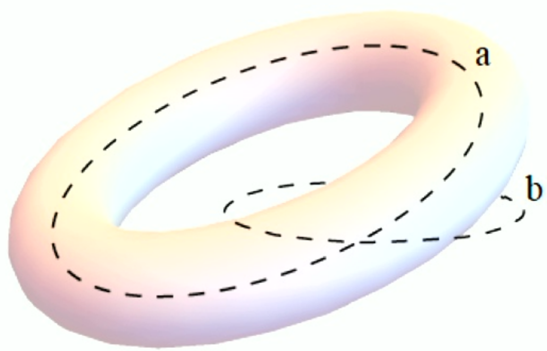
Non-abelian gauge theory

$$a_r \equiv W_r \equiv Tr_r \mathscr{P} e^{i\oint_C dx^\mu A_\mu^r} \; ???$$



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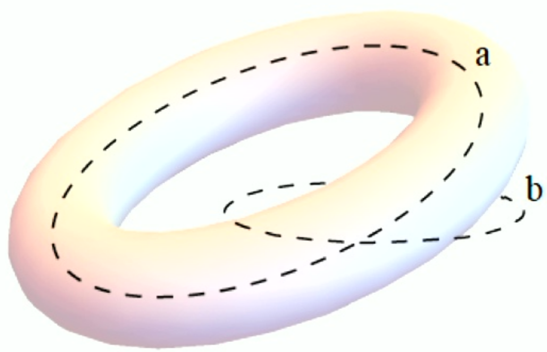

Maxwell

$$a_q \equiv e^{iq\Phi_B} \qquad b_g \equiv e^{ig\Phi_E}$$

They violate duality due to generalized current conservation

$$dF = 0 \qquad d * F = 0$$

The algebra is that of $\mathbb{R} \times \mathbb{R}$

$$a_q a_{q'} = a_{q+q'} \qquad b_g b_{g'} = b_{g+g'}$$

Together with

$$a_q b_g a_{-q} = e^{iqg} b_g$$

Generalized symmetry transformation

Examples

Symmetries in gauge theories:

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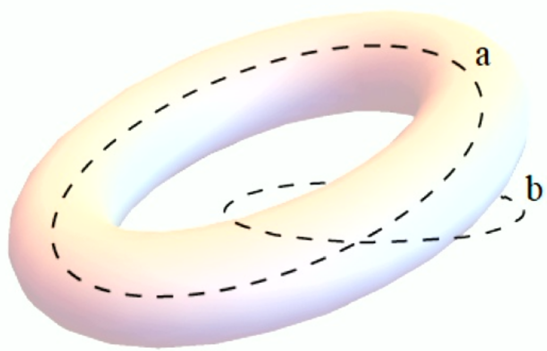

Non-abelian gauge theory

$$a_r \equiv W_r \equiv Tr_r \mathscr{P} e^{i \oint_C dx^\mu A_\mu^r} ???$$

WL in the adjoint can be broken

$$F_{\mu\nu}(x)\ Pe^{i\int_x^y dx^\sigma A_\sigma}\ F_{\alpha\beta}(y)$$

The HDV operators are labeled as

$$a_z \sim \Lambda_\omega / \Lambda_{root} \sim \Lambda_Z$$

The dual ones are given by electric fluxes over the center of the gauge group

Examples

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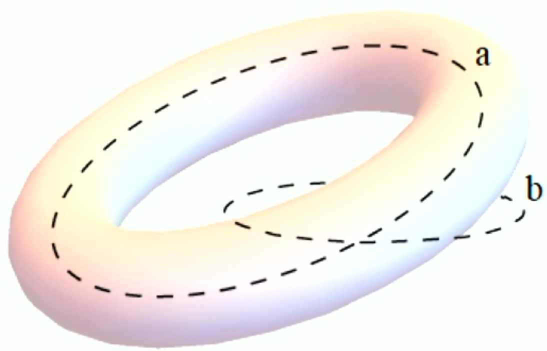

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Showing the WL to be a local operator in a ball can be done explicitly in the lattice.

In the continuum follows from locality of the electric flux plus Von Neumann double commutant theorem

Examples

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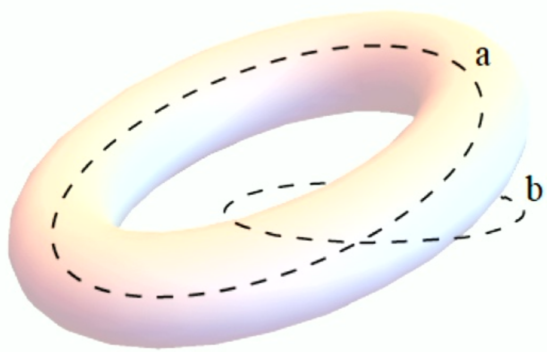


The additive net does not satisfy duality.

↓

Add maximal number of HDV operators compatible with causality such that duality is restored

↓

Haag-Dirac nets (Global gauge group)

↓

Local physics is constrained by the symmetries of all global gauge groups

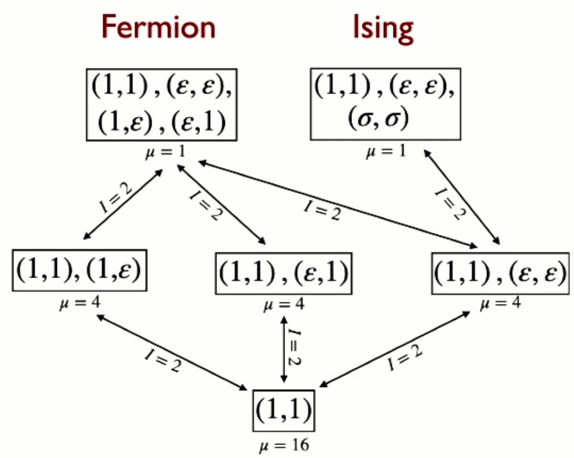
Examples

Symmetries in 2d CFTs:

Like global symmetries (violate duality in regions with non trivial π_0)

Like symmetries in gauge theories (many different completions)

CFT's with $c = 1/2$



The Doplicher-Haag-Roberts reconstruction theorem does not apply in two dimensions

Entropic order parameters and the phases of QFT

Quantum complementarity is nicely described as follows

$$\begin{array}{ccc} \mathscr{A}_{add}(R) \vee \{a\} & \xrightarrow{\varepsilon} & \mathscr{A}_{add}(R) \\ {\scriptstyle \prime}\updownarrow & & \updownarrow{\scriptstyle \prime} \\ \mathscr{A}_{add}(R') & \xleftarrow{\varepsilon'} & \mathscr{A}_{add}(R') \vee \{b\} \end{array}$$

Longo-Rehren type inclusion

[Longo, Rheren, 1995]

The $\mathcal{E}$ are called conditional expectations. They are projections from the maximal algebra to the additive algebra (they 'integrate out' the non-local operators).

This suggests defining the following entropic order parameters

[Casini, Huerta, J.M.M, Pontello 2020]

$$S_{\mathscr{A}_{max}(R)}(\omega \mid \omega \circ \varepsilon) \qquad \textit{and} \qquad S_{\mathscr{A}_{max}(R')}(\omega \mid \omega \circ \varepsilon')$$

Entropic Order — Entropic Disorder

These are relative entropies measuring the difference between the actual state and a state in which we have integrated out the non-local operators

Entropic order parameters and the phases of QFT

A certainty relation can be proven between the dual entropic order parameters

$$S_{\mathscr{A}_{max}(R)}(\omega \mid \omega \circ \varepsilon) + S_{\mathscr{A}_{max}(R')}(\omega \mid \omega \circ \varepsilon') = \log \lambda_\varepsilon$$

[Casini, Huerta, J.M.M, Pontello 2019]

[J.M.M, D. Pontello, 2020]

Generalization to type III algebras in **[S. Hollands, 2020]** **[Feng Xu, 2018]**

$\lambda_\varepsilon = \lambda_{\varepsilon'}$ is the algebraic index of the conditional expectation.

[Jones, 1983] **[Kosaki, 1986]** **[Longo, 1989]**

It measures the size of the dual generalized symmetries

Entropic order parameters and the phases of QFT

When characterizing phases we typically observe

Conformal phase	$\langle W \rangle \sim e^{-P}$	$\longleftrightarrow$	$\langle T \rangle \sim e^{-P}$
Disorder symmetry breaking (Higgs)	$\langle T \rangle \sim e^{-A}$	$\longleftrightarrow$	$\langle W \rangle = constant$
Order symmetry breaking (Confinement)	$\langle W \rangle \sim e^{-A}$	$\longleftrightarrow$	$\langle T \rangle = constant$

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This follows as well for any generalized symmetry in any dimension.

$$\langle a \rangle = constant \quad \longleftrightarrow \quad \langle b \rangle \sim e^{-R^{d-1-i}}$$

$$\langle b \rangle = constant \quad \longleftrightarrow \quad \langle a \rangle \sim e^{-R^{i+1}}$$

Similar scalings can be observed with the entropic order parameters.

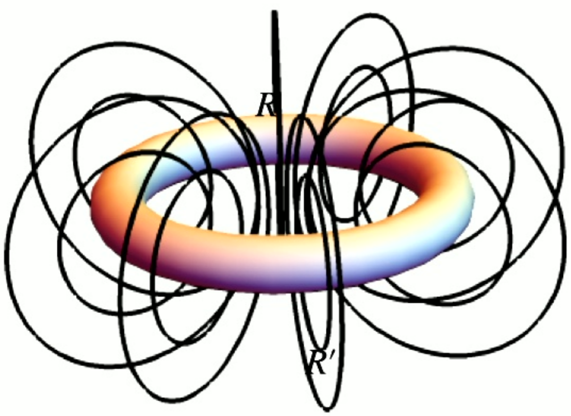
Entropic order parameters and the phases of QFT

Operator order/disorder parameters are typically considered as independent

But the certainty relation tells us they are two faces of the same coin

$$S_{\mathscr{A}_{max}(R)}(\omega \mid \omega \circ \varepsilon) + S_{\mathscr{A}_{max}(R')}(\omega \mid \omega \circ \varepsilon') = \log \lambda_\varepsilon$$

We should be able to derive the area law from the constant law and viceversa.
The heuristic path goes as follows



The entropic disorder parameter is bounded from above by

$$S_{\mathscr{A}_{max}(R')}(\omega \mid \omega \circ \varepsilon) \leq \log Z - S_{\mathscr{A}(R)}(\omega \mid \omega \circ \varepsilon')$$

Where $\mathscr{A}(R) \subset \mathscr{A}_{max}(R)$ is arbitrary.

Chose a set of WL in region R (black lines) and define

$$\mathscr{A}(R) = \vee_i W_i^A$$

Assuming constant expectation values for WL it follows

$$S_{\mathscr{A}_{max}(R')}(\omega \mid \omega \circ \varepsilon) \lesssim e^{-cA} \longrightarrow \langle T \rangle \sim e^{-cA}$$

Entropic order parameters and the phases of QFT

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Similar scalings can be observed with the entropic order parameters.

Summary and discussion

- Generalized order parameters are HDV operators.
- Constraints/classification of the zoo of possibilities
- Obstructions to mixing of symmetries
- Anomalies, Weinberg-Witten and Noether's theorem
- Relation between Wightman and algebraic approaches
- Confinement, mass gap, etc
- Generalization of modular invariance to high dimensions