Title: Anomalously fun: aspects of many-body quantum kinematics

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Series: Colloquium

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Abstract: A fundamental result in solid-state physics asserts that a crystalline material cannot be insulating unless the number of electrons per unit cell is an integer. Statements of this nature are immensely powerful because they are sensitive only to the general structure of the system and not to the microscopic details of the interactions. Such "kinematic constraints" have been extensively generalized in contemporary times, commonly under the term "quantum anomaly". In this colloquium, I will first review some basic aspects of anomaly constraints in many-body quantum physics. Subsequently, I will demonstrate, through several recent examples, the significant role of quantum anomaly in constraining, understanding, and even unveiling novel quantum phases of matter.

Zoom link

Anomalously fun: aspects of many-body quantum kinematics

Chong Wang
Perimeter Institute

Perimeter Colloquium
April 10, 2024

The quantum many-body world

Interacting electrons/atoms

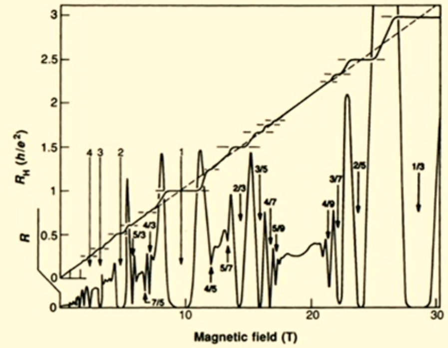
Weakly interacting

Li

Perturbation theory works,
e.g. alkali metals

Strongly interacting

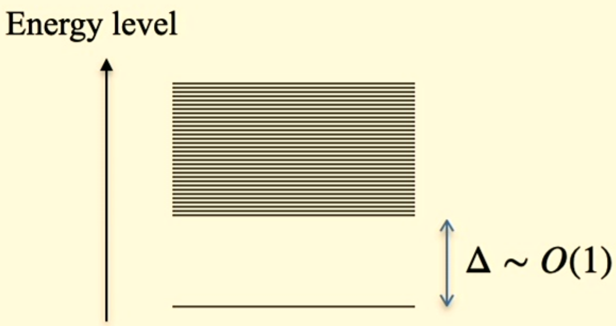
Perturbation theory may not work
— **need more tools!**
e.g. high temperature superconductors, fractional quantum Hall effects…



Quantum Hall effect

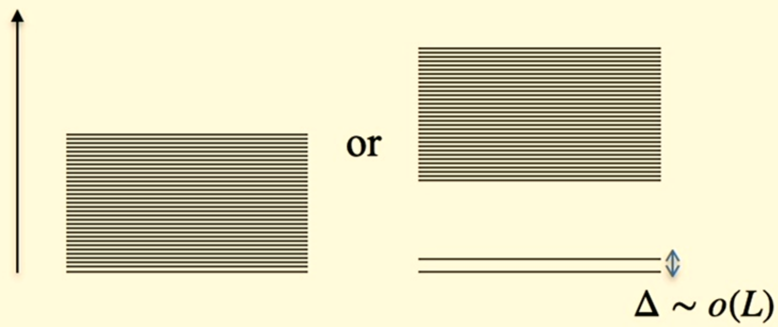
Quantum phases of matter: a dichotomy

Degrees of freedom at low energy



"Nothing":
unique gapped ground state

Example: ordinary insulator



"Something"

Examples:
metal, topological orders,
spontaneous symmetry breaking…

Even just deciding "Nothing" vs. "Something" is in general very(!) hard

Short vs. long range entanglement

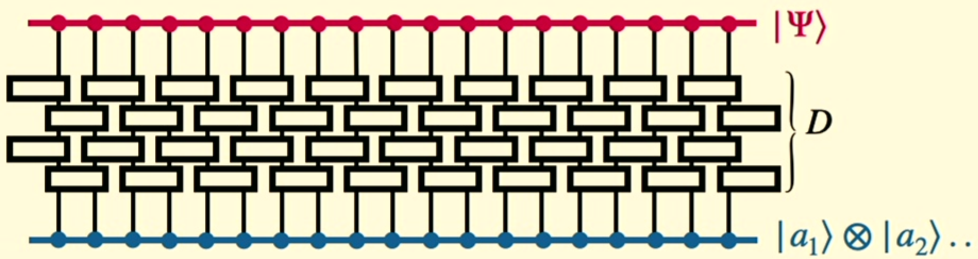
- Ground state wavefunction already knows a lot about low energy physics

- "Nothing" ≈ *short-range entangled* ground state:

$$|\Psi\rangle = U_{FD}|a_1\rangle \otimes |a_2\rangle \otimes \ldots \otimes |a_L\rangle$$

U_{FD}: a local unitary circuit with finite depth ($D \sim O(1)$)

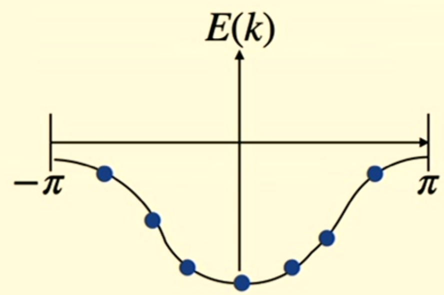
- "Something" ≈ *long-range entangled* ground state: $\nexists U_{FD}$



Chen, Gu, Liu, Wen

Kinematic constraint: from solid-state 101 to Lieb-Schultz-Mattis

Electrons in a crystal can form an insulator ("nothing") only if *number of electrons per unit cell* $\in \mathbb{Z}$



- Derived in textbook for free fermions

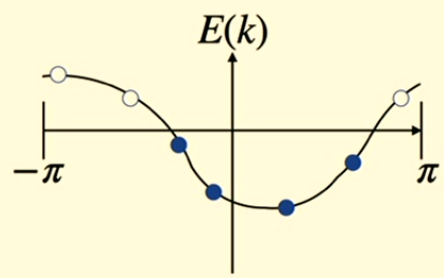
- But the statement is true as long as

A. $U(1) \times \mathbb{Z}^d$ symmetry (charge conservation, lattice translation) is unbroken

B. Hamiltonian is local:

$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + \sum_{\langle ijkl \rangle} V_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \dots$$

Lieb, Schultz, Mattis; Oshikawa; Hasting…



Such kinematic constraints are (loosely) called "anomaly"

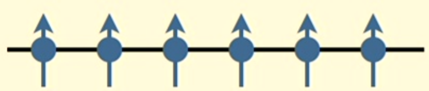
Lieb-Schultz-Mattis anomaly for spins

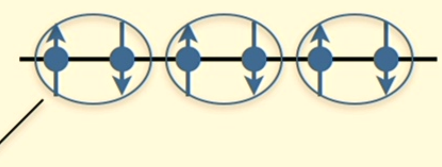
- System: lattice of spins with $SO(3)$ spin-rotation and $\mathbb{Z}^d$ translation symmetry. Spin per unit cell $S \in \mathbb{Z} + 1/2$. Hamiltonian is local.

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

- Theorem (Lieb-Shultz-Mattis): ground state cannot be gapped and unique, i.e. must be long-range entangled

- Has many generalizations, e.g. $SO(3) \to G$, "half-integer S" $\to$ projective representation of G



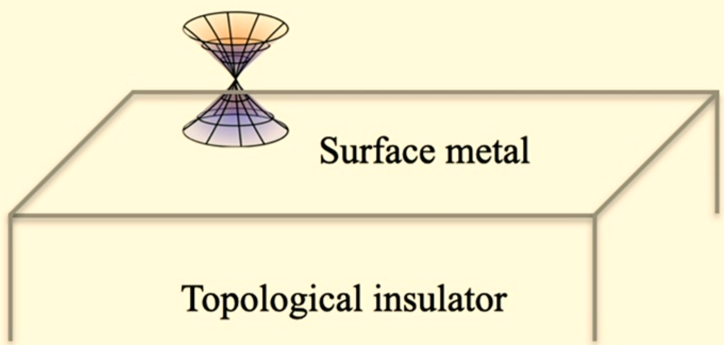


$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Another example: boundary of topological insulator

- Bulk is insulating, i.e. "nothing" at low energy

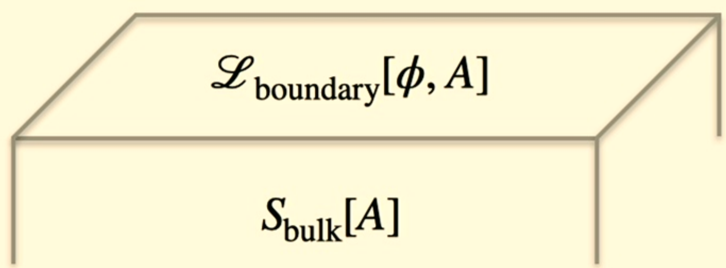
- But boundary is forced to be nontrivial, as long as symmetry (e.g. $U(1)$ and time-reversal) is preserved



’t Hooft anomaly

“anomaly” ≡ kinematic obstruction of trivial state
≈ obstruction of coupling to gauge field (a.k.a. ’t Hooft anomaly)

- Provides a unified view and powerful calculation tool
- But will not be emphasized for this talk
- Instead, I will focus on some new examples of kinematic constraints



Cheng, Zaletel, Barkeshli, Vishwanath, Bonderson;
And many others…

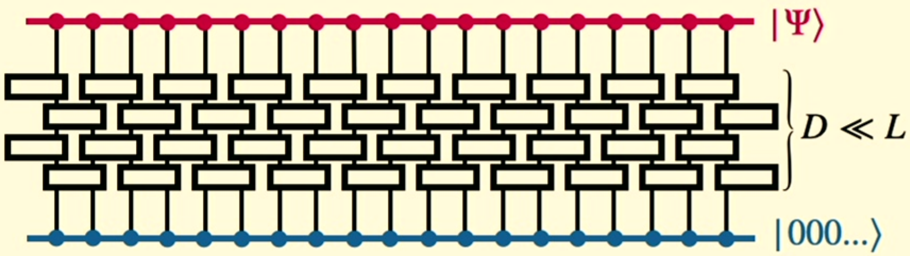
Outline

- Kinematic constraint from translation alone
- Disordered systems: average symmetry and average anomaly
- Discovering new quantum phases through anomaly

Theorem

Lei Gioia Yang, CW
PRX (2022)

- System: lattice spins (or fermions), $d \geq 1$
- Short-range entangled quantum state $|\Psi\rangle = U_{circ}^{D}|00...\rangle$ with $D \ll L$
- If $T_x|\Psi\rangle = e^{iP}|\Psi\rangle$, then $e^{iP} = 1$ ($e^{iP} = \pm 1$ for fermions)



"Nonzero momentum requires long-range entanglement"

Intuition (spins)

- Think about a product state:
$$T_x : |a_1\rangle \otimes |a_2\rangle \otimes \ldots |a_L\rangle \rightarrow |a_2\rangle \otimes \ldots \otimes |a_L\rangle \otimes |a_1\rangle$$

- The only translation eigenstate has trivial momentum
$$|a\rangle \otimes |a\rangle \otimes \ldots |a\rangle$$

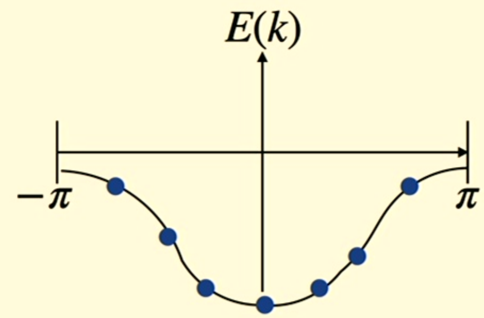
- So a product state cannot have nontrivial momentum

- A cat state (GHZ) can have nontrivial momentum, e.g.
$$|0101...\rangle - |1010...\rangle$$

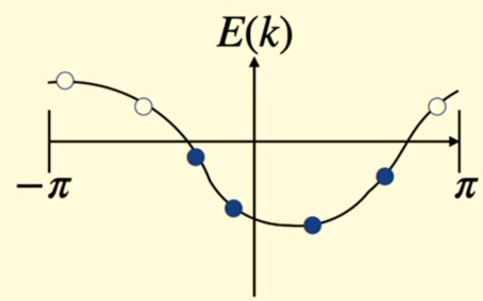
- But GHZ is long-range entangled, requires circuit depth $D \sim L$

Intuition (fermions)

- Band insulator: $P = \sum_{B.Z.} k \in \{0,\pi\}$



- Non-centrosymmetric Fermi surface: arbitrary $P = \sum_{k_{F,L}<k<k_{F,R}} k$



Application: Lieb-Schultz-Mattis

- System: $1d$ lattice with $U(1) \times \mathbb{Z}$ symmetry, charge per unit cell $Q/L \notin \mathbb{Z}$
- LSM: Symmetric state $|\Psi\rangle$ must be long-range entangled (LRE)
- A simple proof: if $P \neq 0$, then LRE from our theorem
- If $P = 0$, consider

$$|\Psi'\rangle = \prod_x \exp(\frac{i2\pi x\hat{Q}(x)}{L})|\Psi\rangle$$

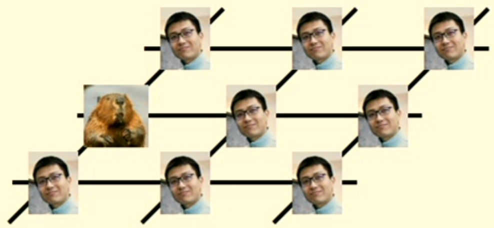
- $|\Psi'\rangle$ has momentum $P' = 2\pi Q/L \neq 0 \pmod{2\pi}$, must be LRE from our theorem
- $|\Psi\rangle$ and $|\Psi'\rangle$ only differ by a depth-1 circuit, so $|\Psi\rangle$ also LRE

Outline

- Kinematic constraint from translation alone
- **Disordered systems: average symmetry and average anomaly**
- Discovering new quantum phases through anomaly

Disordered system

- In real materials, exact translation symmetry is a lie (impurities, lattice defects…)



- But we can still talk about *average translation symmetry*
- *Q:* do average symmetries offer nontrivial anomaly and kinematic constraints?
- Broader motivation: extending the scope of *symmetries* in quantum matter

Disordered spin system

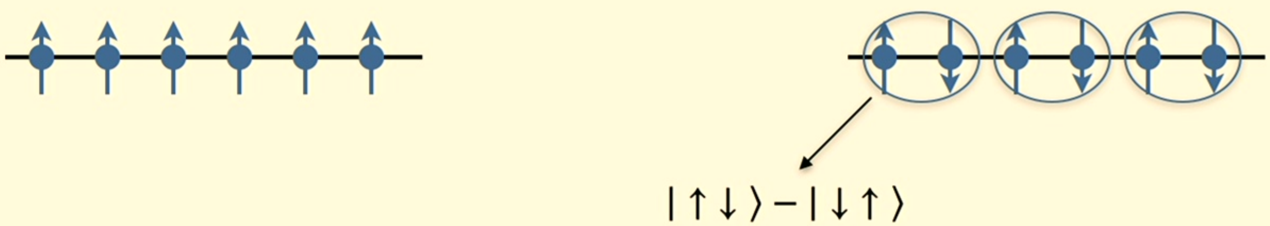
- Lattice of spins with $SO(3)$ spin-rotation symmetry, spin $S = 1/2$ per unit cell

$$H = \sum_{\langle ij \rangle} J_{i,j} \vec{S}_i \cdot \vec{S}_j + \dots$$

- $J_{i,j}$ are drawn randomly from a probability distribution. The distribution has *average* translation symmetry

$$P[J_{i,j}] = P[J_{i+r,j+r}]$$

- For each disorder realization $\{J_{i,j}\}$, ground state $|\Psi\rangle$ not translationally symmetric



Average Lieb-Schultz-Mattis

Ruochen Ma, CW
PRX (2023)

$\forall$ finite ξ_0, $P(\xi > \xi_0)$, the probability for a state $|\Psi\rangle$ to have correlation length* $\xi > \xi_0$, goes to 1 as system size $L \to \infty$

Average LSM $\Rightarrow$ "almost certainly long-range entangled"

More generally: 't Hooft anomaly can be defined for average symmetries
Nontrivial average anomaly $\Rightarrow$ "almost certainly long-range entangled"

Ma, CW 23; Ma, Zhang, Bi, Cheng, CW 24

*: defined as the minimum circuit depth needed to create $|\Psi\rangle$ from $|00...\rangle$

Outlook

- Disordered quantum critical phenomena: ubiquitous experimentally (quantum Hall transition, superconductor-insulator transition, metal-insulator transitions), but extremely challenging theoretically

- Instead of directly studying the dynamics, perhaps we can gain some milage from the kinematics, i.e. symmetries & anomalies?

Outline

- Kinematic constraint from translation alone
- Disordered systems: average symmetry and average anomaly
- **Discovering new quantum phases through anomaly**

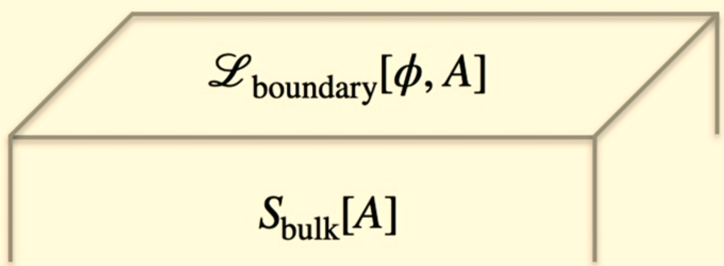
Anomaly-matching

UV system (field theory; lattice system…)
Symmetry G with anomaly $S[A]$

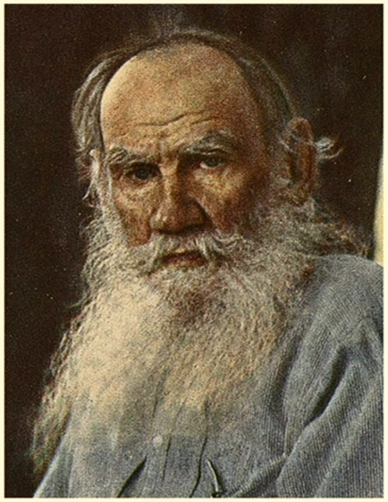
↓

IR theory (field theory; ground state wave function $|\psi\rangle$; or even a density matrix ρ)
Symmetry G with the same anomaly $S[A]$

Strategy for discovering new phases:
find IR theory that matches the anomaly!

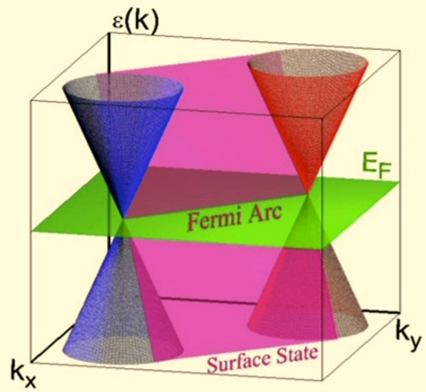


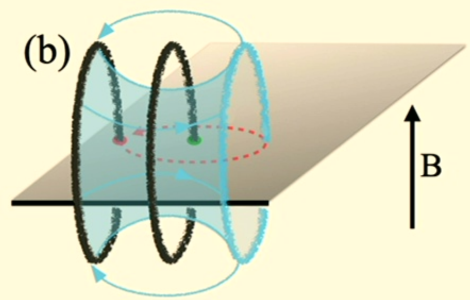
"All trivial states are alike; each nontrivial state is nontrivial in its own way."



Leo Tolstoy

Example 1: Weyl semimetal



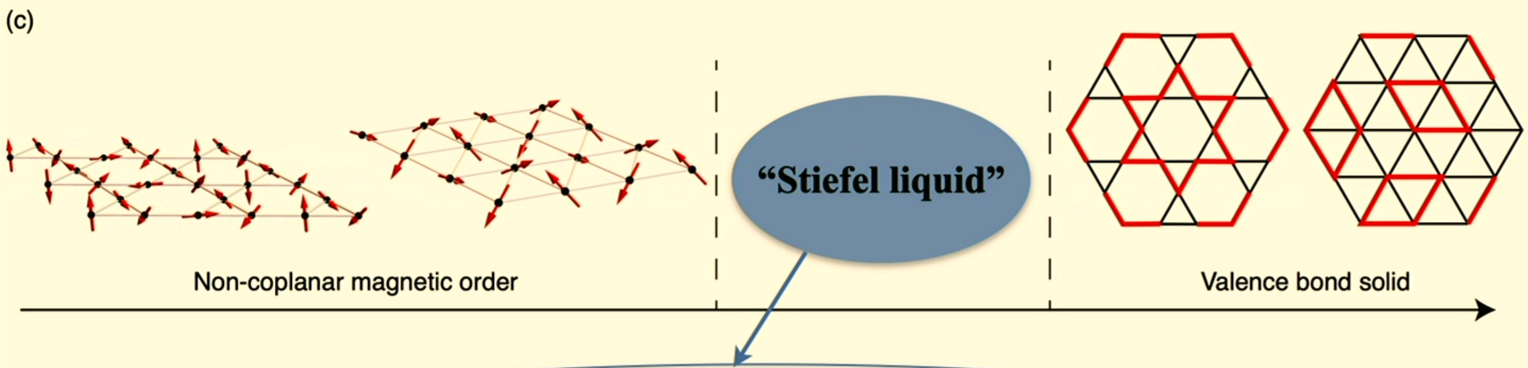


$$\psi_L^\dagger(\mathbf{k}-\mathbf{k}_L)\cdot\vec{\sigma}\,\psi_L - \psi_R^\dagger(\mathbf{k}-\mathbf{k}_R)\cdot\vec{\sigma}\,\psi_R$$

$3d$ analogue of fractional quantum Hall effect:Topological order with nontrivial loop excitations

CW, Gioia, Anton Burkov
PRL (2019)

Example 2: Quantum magnetism



A gapless long-range entangled spin state (a.k.a. "critical spin liquid")
with exotic properties

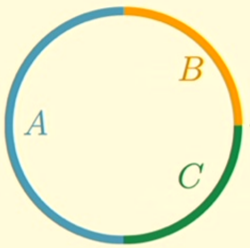
Liujun Zou, Yin-Chen He, CW, PRX (2021)
Weicheng Ye, Guo, He, CW, Zou, Scipost (2022)

Example 3: Long-range entangled mixed states

In a qubit chain: $\rho \propto I + \prod_i X_i \prod_i CZ_{i,i+1}$ is long-range entangled

This is due to $g \equiv \prod_i X_i \prod_i CZ_{i,i+1}$ being an anomalous $\mathbb{Z}_2$ symmetry

Chen, Liu, Wen; Else, Nayak



From anomaly $\Rightarrow \rho$ is tripartite non-separable

$$\rho \neq \sum_i P_i U_{FD,i} |\psi_i^A\rangle\langle\psi_i^A| \otimes |\psi_i^B\rangle\langle\psi_i^B| \otimes |\psi_i^C\rangle\langle\psi_i^C| U_{FD,i}^\dagger$$

But ρ is bipartite separable for any bipartition

$$\rho = \sum_i P_i |\psi_i^A\rangle\langle\psi_i^A| \otimes |\psi_i^{\bar{A}}\rangle\langle\psi_i^{\bar{A}}|$$

ρ: intrinsically mixed long-range entangled state

Leonardo Lessa, Meng Cheng, CW (2024)

Summary

1. Kinematic constraints from anomaly — states guaranteed to be interesting!

2. Many applications:

   - Nonzero momentum requires long-range entanglement
   - Average LSM ⇒ states guaranteed to be "almost always nontrivial"
   - Discovering new phases using anomaly: "gapped Weyl semimetal", "Stiefel liquids", long-range entangled mixed states…

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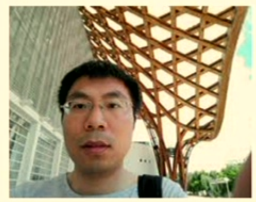

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Thank you!