Title: Type I von Neumann algebras from gravitational path integrals: Ryuâ€“Takayanagi as entropy without holography

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Series: Quantum Gravity

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Abstract: We show that the Ryu-Takayanagi (RT) formula, originally introduced to compute the entropy of a holographic boundary CFT, can be interpreted as entropy of an algebra of bulk gravitational observables. In particular, we show that any Euclidean gravitational path integral satisfying a simple and familiar set of axioms defines type I von Neumann algebras of bulk observables acting on closed codimension-2 asymptotic boundaries. The entropies associated to these algebras, defined via the gravitational path integral, can be written in terms of standard density matrices and standard Hilbert space traces, and in appropriate semiclassical limits are computed by the RT formula with quantum corrections. Our work thus provides a bulk state-counting interpretation of the Ryu-Takayanagi entropy. Since our axioms do not severely constrain UV bulk structures, they may be expected to hold equally well for successful formulations of string field theory, spin-foam models, or any other approach to constructing a UV-complete theory of gravity.

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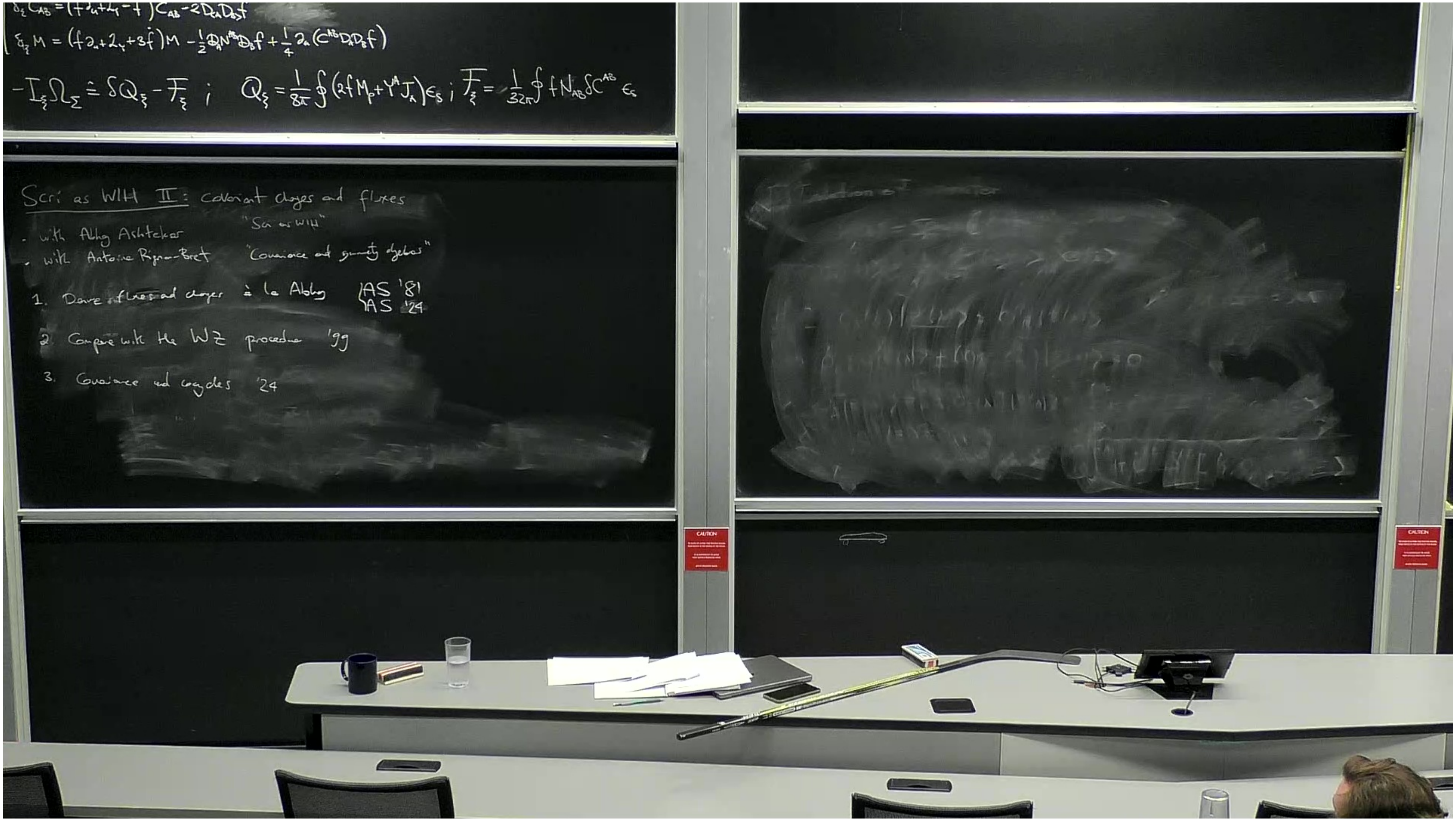
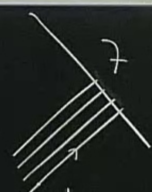
Zoom link TBA


$\delta_\xi M = (f\partial_u + 2_Y + 3\dot f)M - \frac{1}{2}\mathcal{D}_A N^{AB} \mathcal{D}_B f + \frac{1}{4}\partial_u (C^{AB} \mathcal{D}_A \mathcal{D}_B f)$
$-I_\xi \Omega_\Sigma \hat{=} \delta Q_\xi - \mathcal{F}_\xi$ ; $Q_\xi = \frac{1}{8\pi}\oint (2f M_p + Y^A J_A)\epsilon_S$ ; $\mathcal{F}_\xi = \frac{1}{32\pi}\oint f N_{AB}\delta C^{AB} \epsilon_S$
Scri as WIH II: Covariant charges and fluxes
"Scri as WIH"
- with Abhay Ashtekar
- with Antoine Rignon-Bret "Covariance and symmetry algebras"
1. Derive fluxes and charges à la Abhay
IAS '81
IAS '24
2. Compare with the WZ procedure '99
3. Covariance and cocycles '24

$$\begin{cases} g_{uu} = -\frac{R}{2} + \frac{2M}{r} + \ ; \quad g_{ur} = -1 - \frac{2\beta}{r^2} + \dots, \quad \beta = -\frac{1}{32} C_{AB}{}^2 \\ g_{uA} = -U_A + \frac{2}{3r}\left(J_A + \partial_A \beta - \frac{1}{2} C_{AB} U^B\right) + \dots, \quad U_A = -\frac{1}{2} \mathcal{D}_B C_A{}^B \\ g_{AB} = r^2 q_{AB} + r C_{AB} + \dots, \quad q_{AB} \to \mathcal{D}_A, R \end{cases}$$

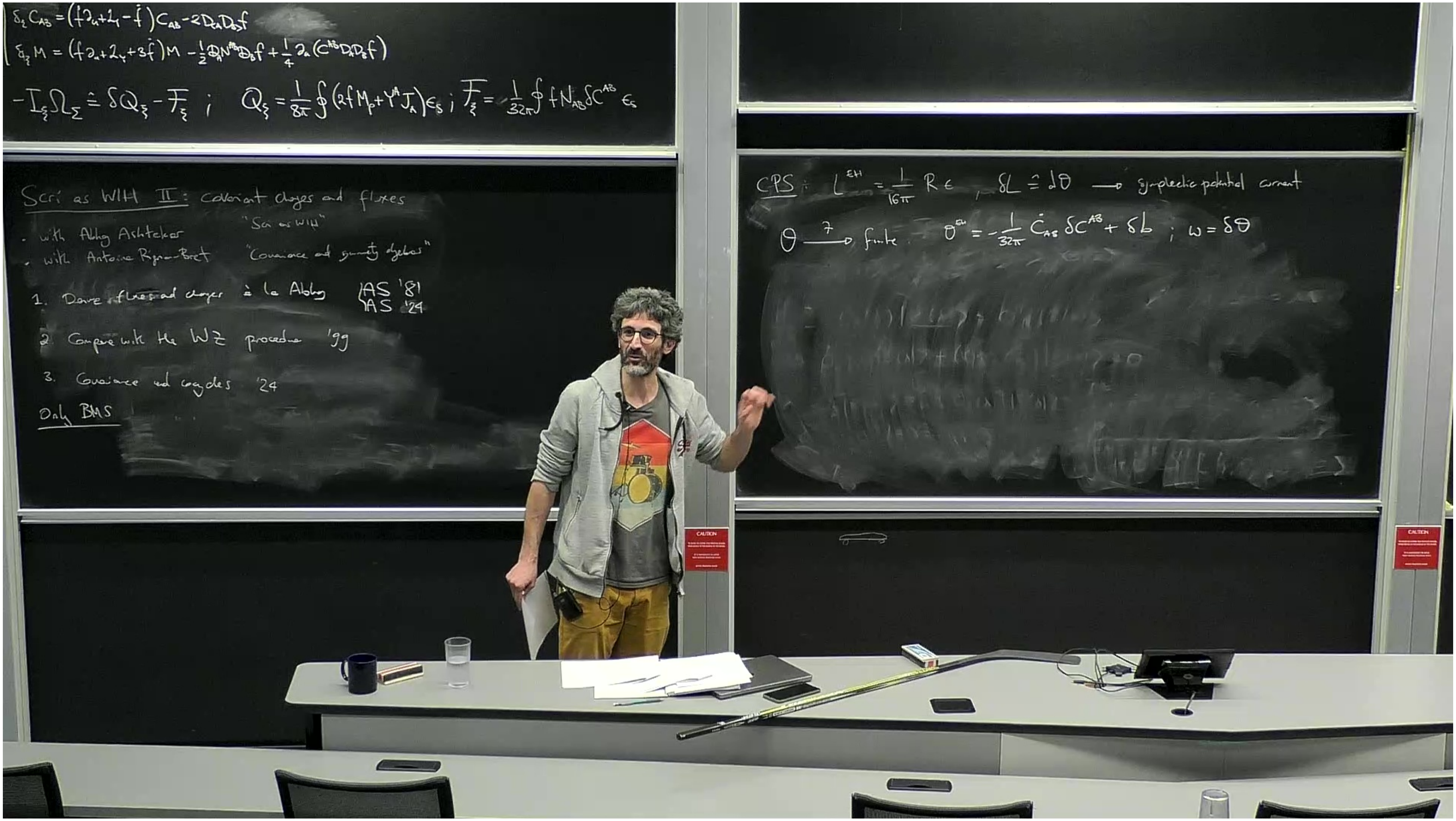



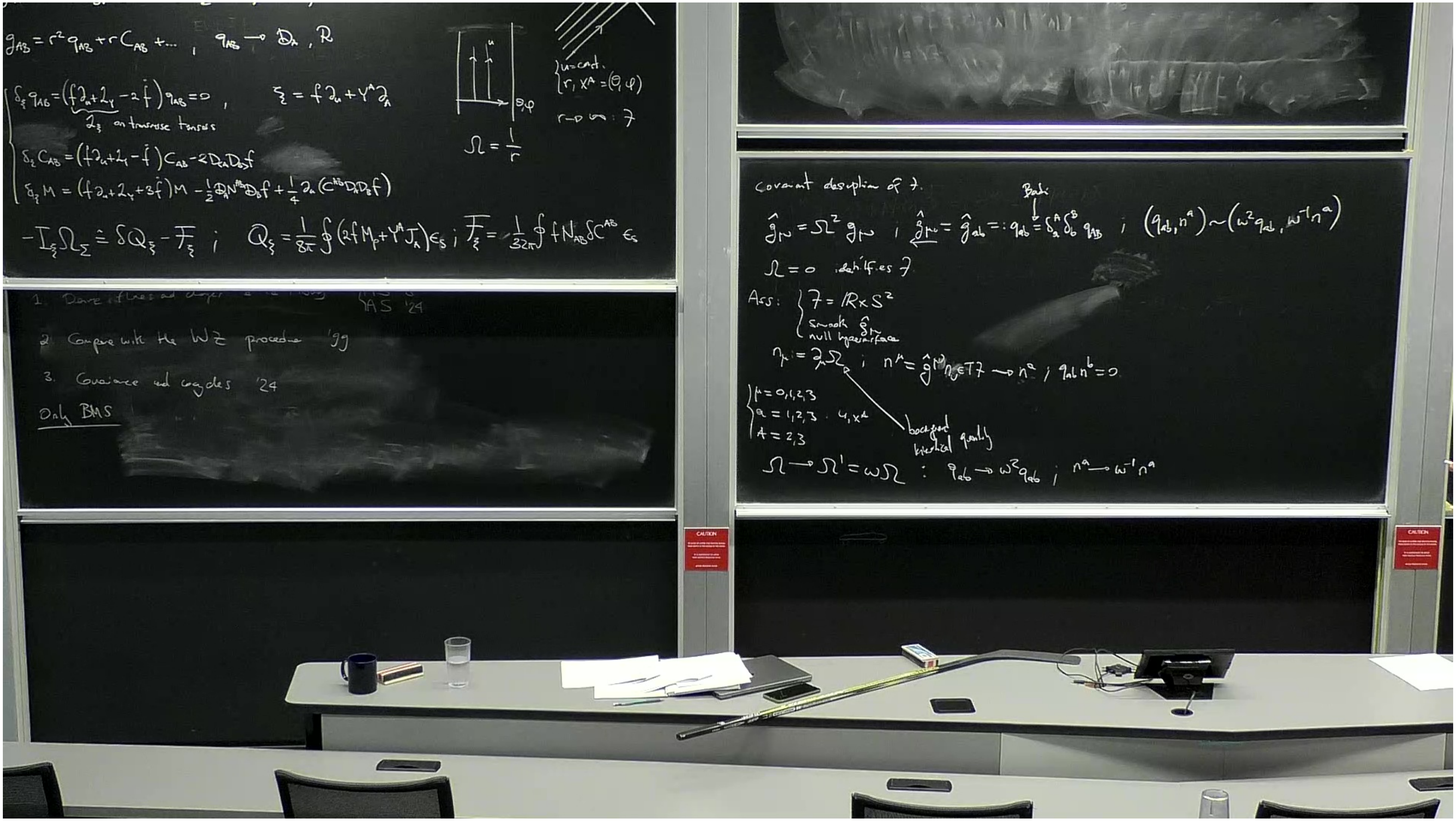

$\begin{cases} u = \text{cnst.} \\ r, x^A = (\theta, \varphi) \end{cases}$

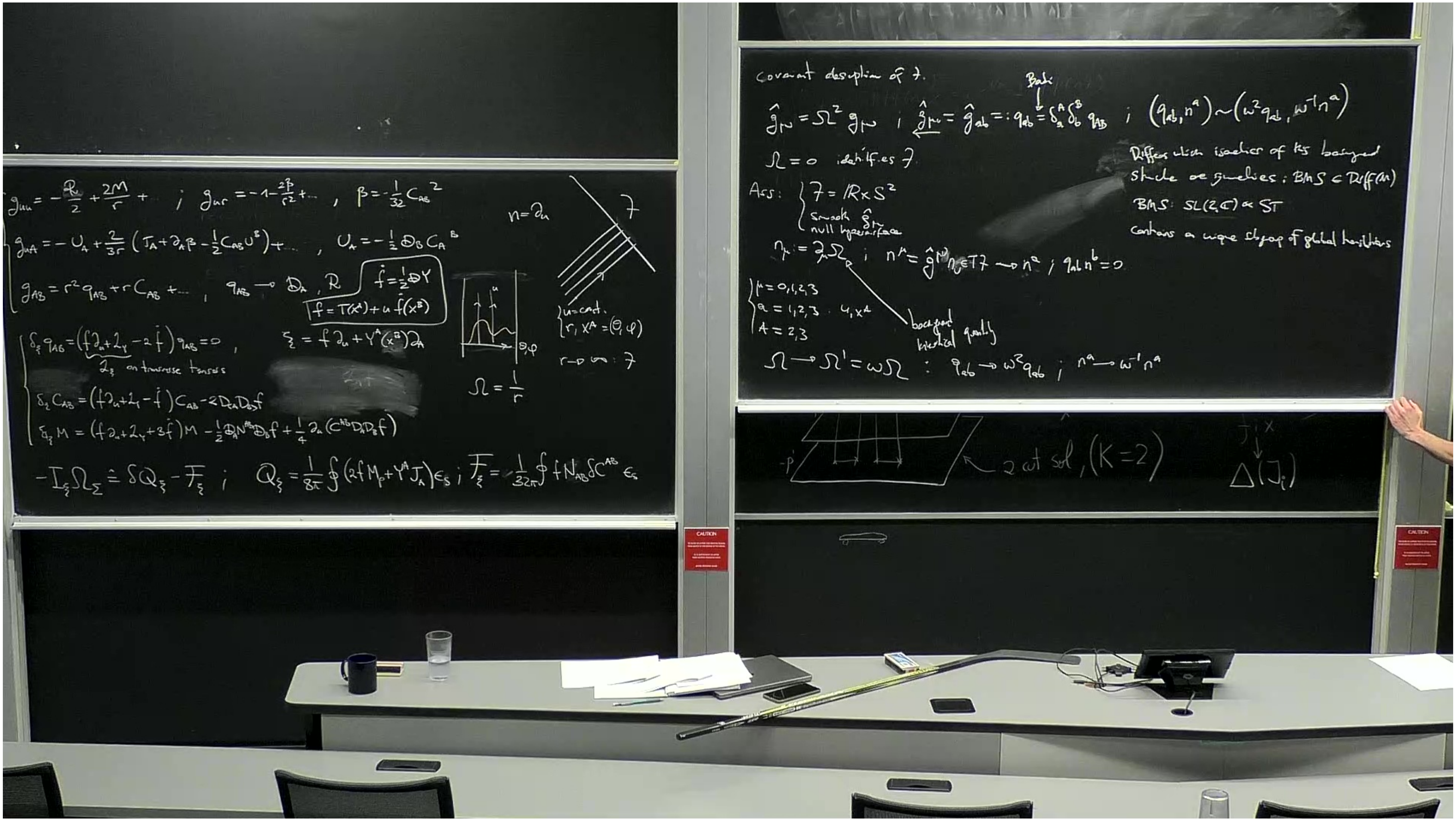
$r \to \infty : \mathcal{I}$

$$\begin{cases} \delta_\xi q_{AB} = (\underbrace{f\partial_u + \mathcal{L}_Y}_{\mathcal{L}_\xi \text{ on transverse tensors}} - 2\dot{f}) q_{AB} = 0, \quad \xi = f\partial_u + Y^A \partial_A \\ \delta_\xi C_{AB} = (f\partial_u + \mathcal{L}_Y - \dot{f}) C_{AB} - 2 \mathcal{D}_{\langle A} \mathcal{D}_{B \rangle} f \\ \delta_\xi M = (f\partial_u + \mathcal{L}_Y + 3\dot{f}) M - \frac{1}{2} \mathcal{D}_A N^{AB} \mathcal{D}_B f + \frac{1}{4} \partial_u (C^{AB} \mathcal{D}_A \mathcal{D}_B f) \end{cases}$$

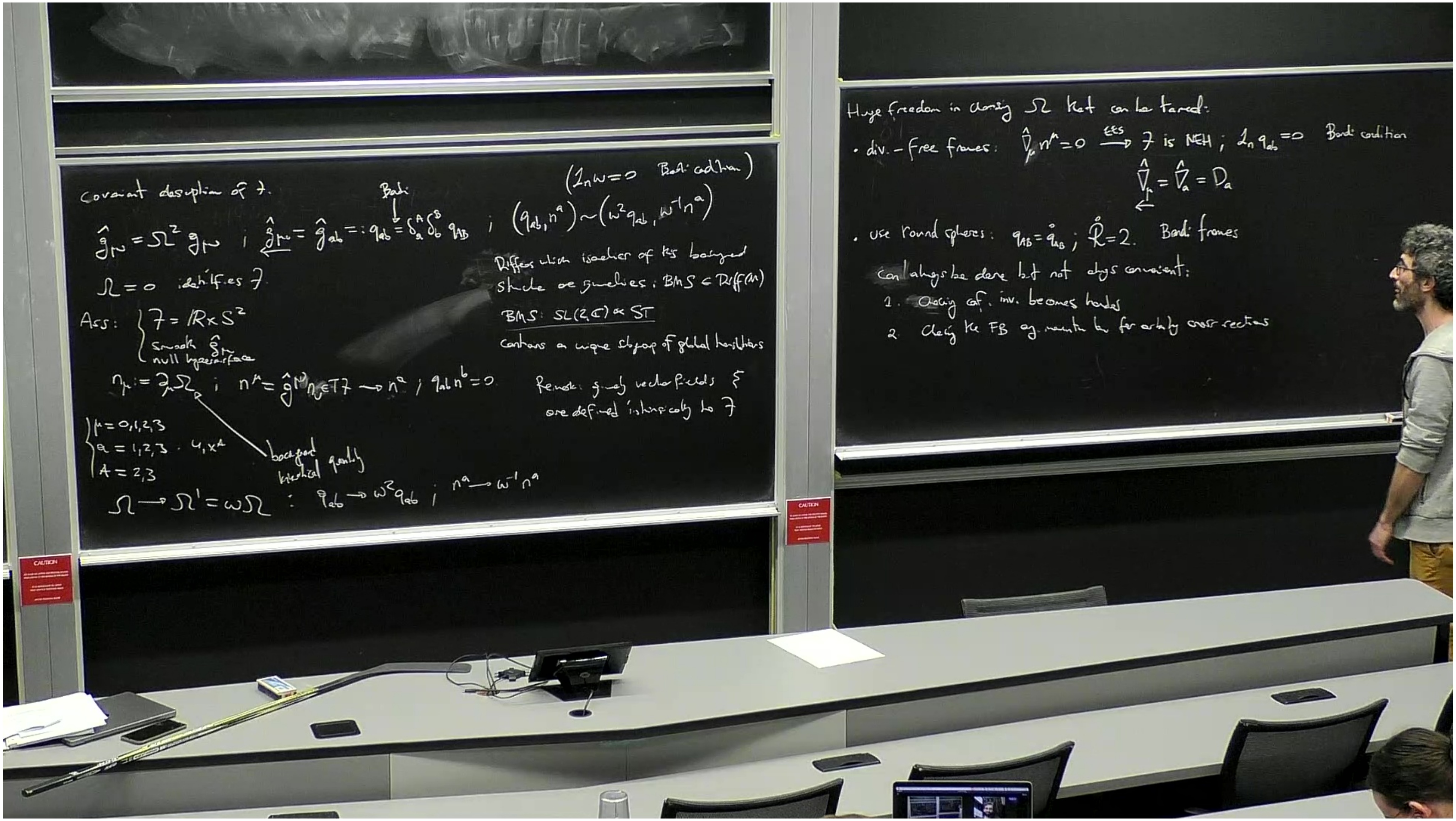
$$-I_\xi \Omega_\Sigma \doteq \delta Q_\xi - \mathcal{F}_\xi \ ; \quad Q_\xi = \frac{1}{8\pi} \oint (2f M_p + Y^A J_A) \epsilon_S \ ; \quad \mathcal{F}_\xi = \frac{1}{32\pi} \oint f N_{AB} \delta C^{AB} \epsilon_S$$

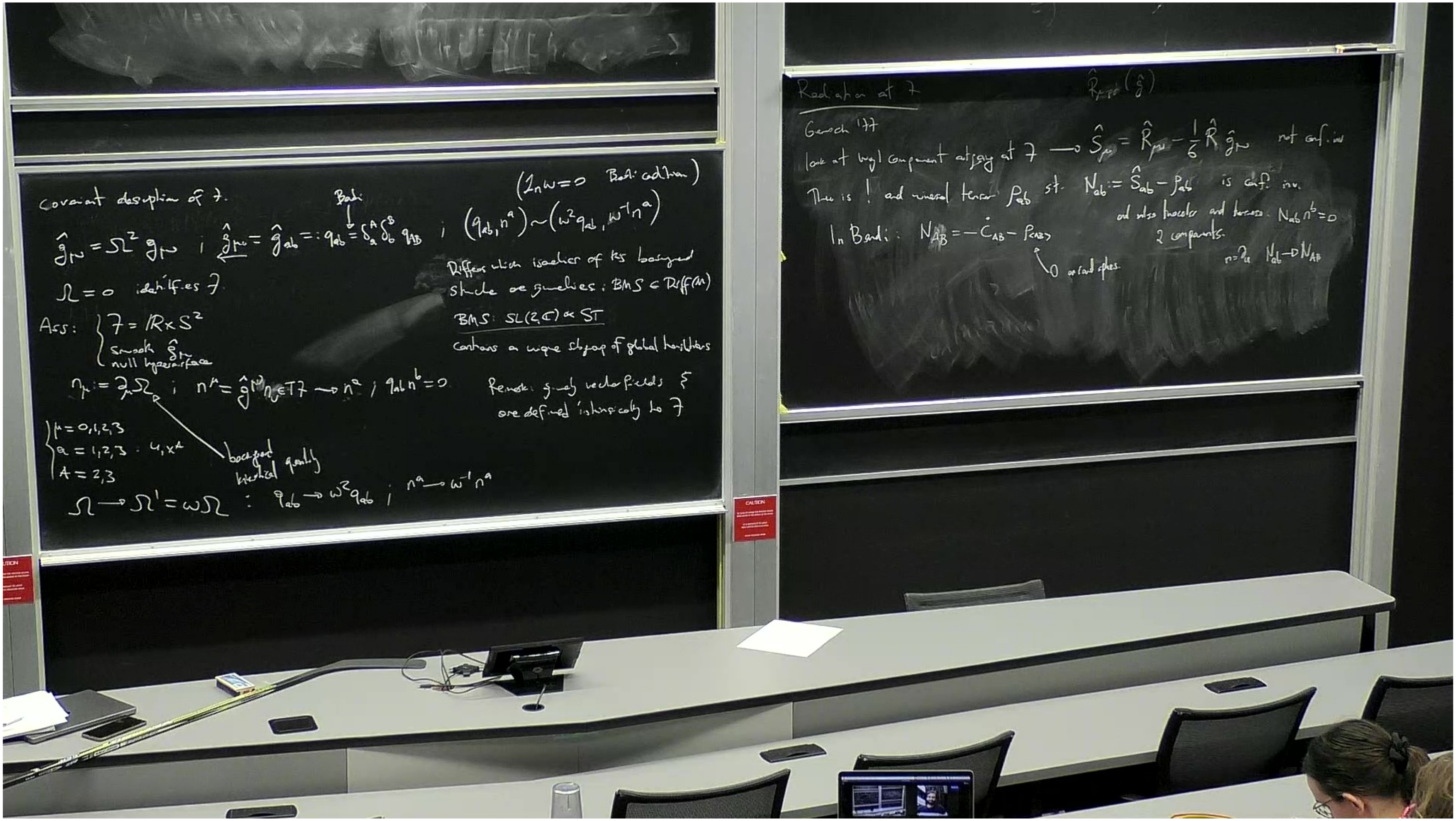

$\delta_\xi C_{AB} = (f\partial_u + \mathcal{L}_Y - \dot{f})C_{AB} - 2D_A D_B f$
$\delta_\xi M = (f\partial_u + \mathcal{L}_Y + 3\dot{f})M - \frac{1}{2}D_A N^{AB} D_B f + \frac{1}{4}\partial_u (C^{AB} D_A D_B f)$
$-I_\xi \Omega_\Sigma \doteq \delta Q_\xi - \mathcal{F}_\xi \; ; \; Q_\xi = \frac{1}{8\pi}\oint (2f M_p + Y^A J_A)\epsilon_S \; ; \; \mathcal{F}_\xi = \frac{1}{32\pi}\oint f N_{AB}\delta C^{AB} \epsilon_S$
Scri as WIH II: Covariant charges and fluxes
. with Abhay Ashtekar "Scri as WIH"
. with Antoine Rignon-Bret "Covariance and symmetry algebras"
1. Derive fluxes and charges à la Abhay IAS '81 IAS '24
2. Compare with the WZ procedure '99
Only BMS
CPS: $L^{EH} = \frac{1}{16\pi} R\epsilon \; , \; \delta L \doteq d\Theta$ → symplectic potential current
$\Theta \xrightarrow{?}$ finite $\Theta^{EH} = -\frac{1}{32\pi}\dot{C}_{AB}\delta C^{AB} + \delta b \; ; \; \omega = \delta\Theta$

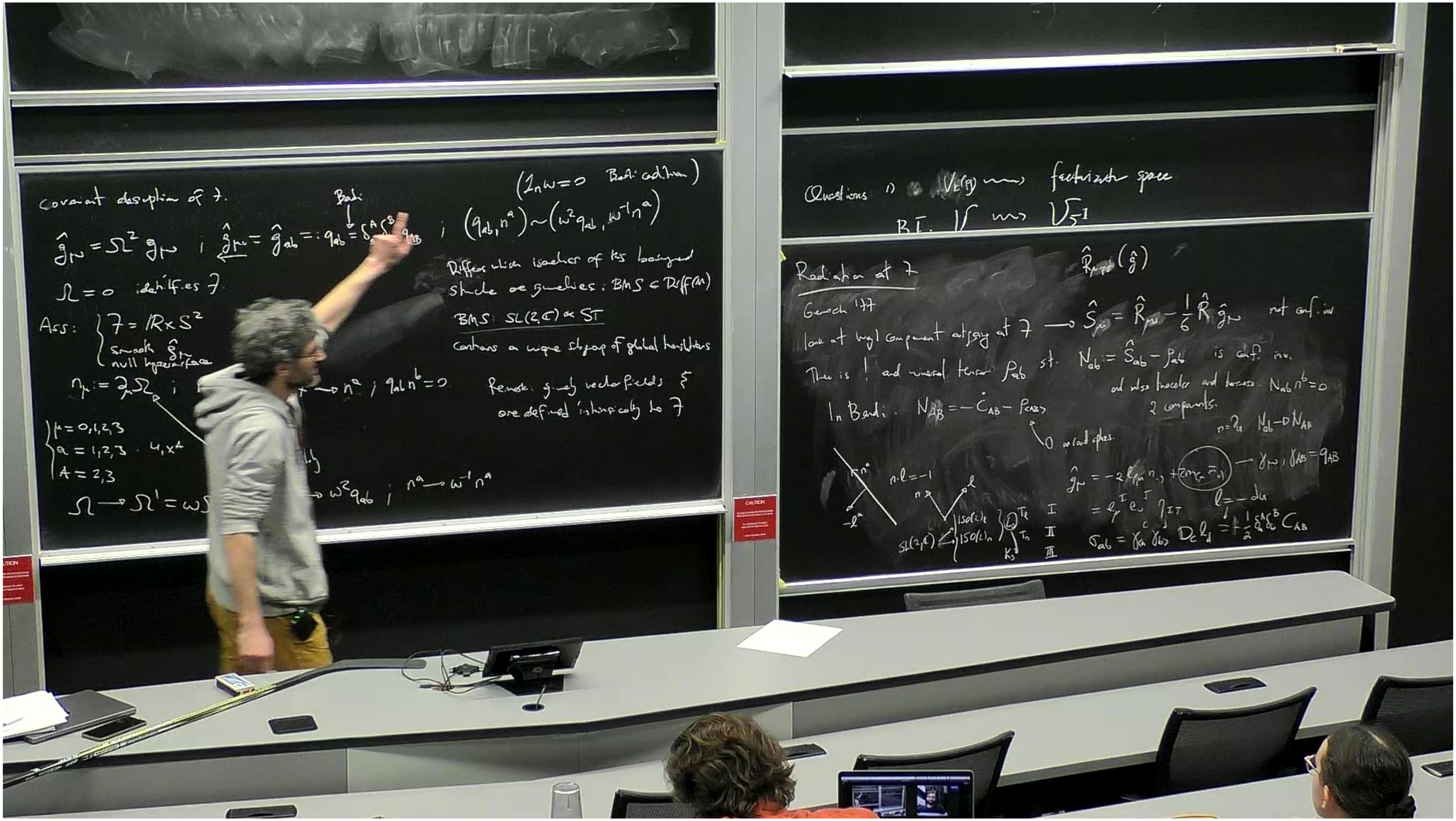

$g_{AB} = r^2 q_{AB} + r C_{AB} + \dots , \quad q_{AB} \to D_A , R$
$\delta_\xi q_{AB} = (f\partial_u + \mathcal{L}_Y - 2\dot f) q_{AB} = 0 , \quad \xi = f\partial_u + Y^A\partial_A$
$\mathcal{L}_\xi$ on transverse tensors
$\delta_\xi C_{AB} = (f\partial_u + \mathcal{L}_Y - \dot f) C_{AB} - 2 D_A D_B f$
$\delta_\xi M = (f\partial_u + \mathcal{L}_Y + 3\dot f) M - \frac{1}{2} D_A N^{AB} D_B f + \frac{1}{4}\partial_u (C^{AB} D_A D_B f)$
$-I_\xi \Omega_\Sigma \doteq \delta Q_\xi - \mathcal{F}_\xi \; ; \; Q_\xi = \frac{1}{8\pi}\oint (2f M_p + Y^A J_A)\epsilon_S \; ; \; \mathcal{F}_\xi = \frac{1}{32\pi}\oint f N_{AB}\delta C^{AB} \epsilon_S$
$u = $ const.
$r, x^A = (\theta, \varphi)$
$r \to \infty : \mathcal{I}$
$\Omega = \frac{1}{r}$
YAS '24
2. Compare with the WZ procedure '99
3. Covariance and corners '24
Only BMS
Covariant description of $\mathcal{I}$.
$\hat g_{\mu\nu} = \Omega^2 g_{\mu\nu} \; ; \; \hat g_{\mu\nu} \leftarrow \hat g_{ab} =: q_{ab} \doteq \delta_a^A \delta_b^B q_{AB} \; ; \; (q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
$\Omega = 0$ identifies $\mathcal{I}$
Ass: $\mathcal{I} = \mathbb{R} \times S^2$, smooth $\hat g_{\mu\nu}$, null hypersurface
$n_\mu = \partial_\mu \Omega \; ; \; n^\mu = \hat g^{\mu\nu} n_\nu \in T\mathcal{I} \to n^a \; ; \; q_{ab} n^b = 0$
$\mu = 0,1,2,3$
$a = 1,2,3 : u, x^A$
$A = 2,3$
background quantity
$\Omega \to \Omega' = \omega\Omega : q_{ab} \to \omega^2 q_{ab} \; ; \; n^a \to \omega^{-1} n^a$

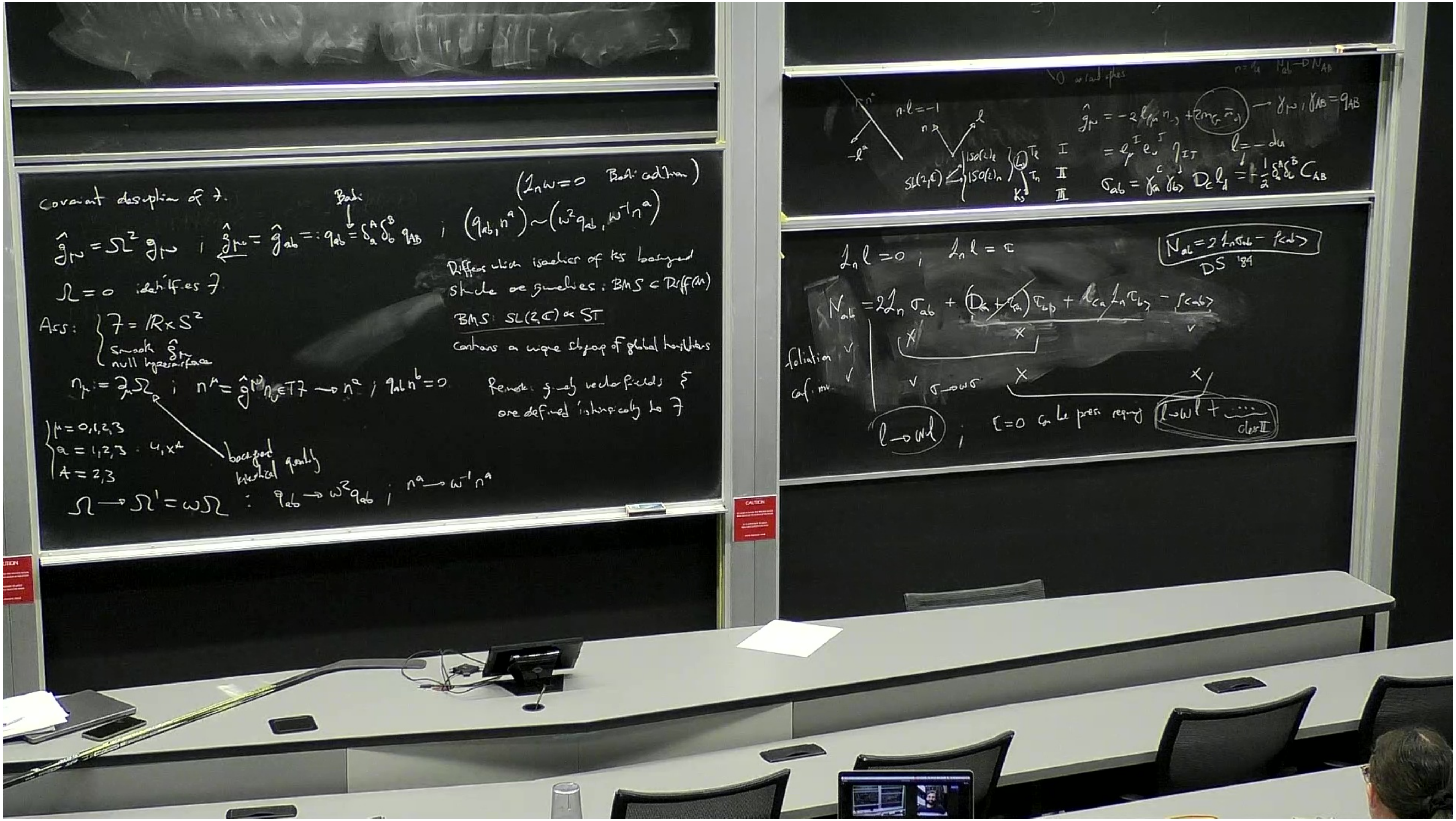


$g_{uu} = -\frac{R}{2} + \frac{2M}{r} + \dots$ ; $g_{ur} = -1 - \frac{2\beta}{r^2} + \dots$ , $\beta = -\frac{1}{32} C_{AB}{}^2$
$g_{uA} = -U_A + \frac{2}{3r}\left(J_A + \partial_A\beta - \frac{1}{2}C_{AB}U^B\right) + \dots$ , $U_A = -\frac{1}{2} D_B C_A{}^B$
$g_{AB} = r^2 q_{AB} + r C_{AB} + \dots$ , $q_{AB} \to D_A, R$
$\dot{f} = \frac{1}{2} D Y$
$f = T(x^A) + u \dot{f}(x^B)$
$\delta_\xi q_{AB} = (f\partial_u + \mathcal{L}_Y - 2\dot{f}) q_{AB} = 0$ , $\xi = f\partial_u + Y^A(x^B)\partial_A$
$\mathcal{L}_\xi$ on transverse tensors
$\delta_\xi C_{AB} = (f\partial_u + \mathcal{L}_Y - \dot{f}) C_{AB} - 2 D_A D_B f$
$\delta_\xi M = (f\partial_u + \mathcal{L}_Y + 3\dot{f}) M - \frac{1}{2} D_A N^{AB} D_B f + \frac{1}{4}\partial_u (C^{AB} D_A D_B f)$
$-I_\xi \Omega_\Sigma \hat{=} \delta Q_\xi - \mathcal{F}_\xi$ ; $Q_\xi = \frac{1}{8\pi}\oint (2f M_p + Y^A J_A)\epsilon_S$ ; $\mathcal{F}_\xi = \frac{1}{32\pi}\oint f N_{AB}\delta C^{AB} \epsilon_S$
$n = \partial_u$
$\mathscr{I}$
$u$
$\theta, \varphi$
$\Omega = \frac{1}{r}$
$u =$ const.
$r, x^A = (\theta, \varphi)$
$r \to \infty$ : $\mathscr{I}$
covariant description of $\mathscr{I}$.
Bondi
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ ; $\hat{g}_{\mu\nu} = \hat{g}_{ab} =: q_{ab} = \delta_a^A \delta_b^B q_{AB}$ ; $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
$\Omega = 0$ identifies $\mathscr{I}$
Ass: $\mathscr{I} = \mathbb{R} \times S^2$
smooth $\hat{g}_{\mu\nu}$
null hypersurface
$n_\mu := \partial_\mu \Omega$ ; $n^\mu = \hat{g}^{\mu\nu} n_\nu \in T\mathscr{I} \to n^a$ ; $q_{ab} n^b = 0$
$\mu = 0,1,2,3$
$a = 1,2,3$ . $u, x^A$
$A = 2,3$
background
universal quantity
$\Omega \to \Omega' = \omega\Omega$ : $q_{ab} \to \omega^2 q_{ab}$ ; $n^a \to \omega^{-1} n^a$
Diffeos which isometries of the background
structure are symmetries : BMS $\subset$ Diff($\mathscr{I}$)
BMS: $SL(2,\mathbb{C}) \ltimes ST$
contains a unique subgroup of global translations
2 cut sol , $(K=2)$
$\Delta(J_i)$


covariant description of $\mathcal{I}$.
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ ; $\underleftarrow{\hat{g}_{\mu\nu}} = \hat{g}_{ab} =: q_{ab} \overset{\text{Bondi}}{=} \delta^A_a \delta^B_b q_{AB}$ ; $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
$(\mathcal{L}_n \omega = 0$ Bondi condition$)$
$\Omega = 0$ identifies $\mathcal{I}$
Ass: $\mathcal{I} = \mathbb{R} \times S^2$
smooth $\hat{g}_{\mu\nu}$
null hypersurface
$n_\mu := \partial_\mu \Omega$ ; $n^\mu = \hat{g}^{\mu\nu} n_\nu \in T\mathcal{I} \to n^a$ ; $q_{ab} n^b = 0$
$\mu = 0,1,2,3$
$a = 1,2,3$ . $u, x^A$
$A = 2,3$
background
$\Omega \to \Omega' = \omega\Omega$ : $q_{ab} \to \omega^2 q_{ab}$ ; $n^a \to \omega^{-1} n^a$
BMS: $SL(2,\mathbb{C}) \ltimes ST$
BMS $\subset$ Diff(M)
Remark: vector fields $\xi$
are defined intrinsically to $\mathcal{I}$
Huge freedom in choosing $\Omega$ that can be tamed:
• div.-free frames: $\hat{\nabla}_\mu n^\mu = 0 \xrightarrow{\text{EES}} \mathcal{I}$ is NEH ; $\mathcal{L}_n q_{ab} = 0$ Bondi condition
$\underleftarrow{\hat{\nabla}_\mu} = \hat{\nabla}_a = D_a$
• use round spheres: $q_{AB} = \mathring{q}_{AB}$ ; $\mathring{R} = 2$. Bondi frames
Can always be done but not always convenient:
1. inv. becomes harder
2. FB


Covariant description of ℐ.
(£_n ω = 0 Bondi condition)
Bondi
ĝ_μν = Ω² g_μν ; ĝ_μν = ĝ_ab =: q_ab = δ_a^A δ_b^B q_AB ; (q_ab, n^a) ~ (ω² q_ab, ω⁻¹ n^a)
Ω = 0 identifies ℐ
Differs which isometries of the background structure are gauge: BMS ⊂ Diff(M)
Ass: ℐ = ℝ × S²
smooth ĝ_μν
null hypersurface
BMS: SL(2,ℂ) ⋉ ST
Contains a unique subgroup of global translations
n_μ := ∂_μ Ω ; n^μ = ĝ^μν n_ν ∈ Tℐ → n^a ; q_ab n^b = 0
Remark: gauge vector fields ξ are defined intrinsically to ℐ
μ = 0,1,2,3
a = 1,2,3 : u, x^A
A = 2,3
background
intrinsic quantity
Ω → Ω' = ωΩ : q_ab → ω² q_ab ; n^a → ω⁻¹ n^a
Radiation at ℐ
R̂_μν(ĝ)
Geroch '77
look at Weyl component at ℐ → Ŝ_μν = R̂_μν − 1/6 R̂ ĝ_μν not conf. inv.
There is ! and universal tensor ρ_ab st. N_ab := Ŝ_ab − ρ_ab is conf. inv.
and also traceless and transverse. N_ab n^b = 0
2 components.
In Bondi: N_AB = −Ċ_AB − ρ_⟨AB⟩
0 on round spheres
n = ∂_u N_ab → N_AB


covariant description of ℐ
Bondi
(q_ab, n^a) ~ (ω^2 q_ab, ω^-1 n^a)
Ω = 0 identifies ℐ
Ass: ℐ = ℝ × S^2
smooth
null hypersurface
Differs which isometries of the background
structure are symmetries : BMS ⊂ Diff(M)
BMS SL(2,ℂ) ⋉ ST
Contains a unique subgroup of global translations
Remark: symmetry vector fields ξ
are defined intrinsically to ℐ
μ = 0,1,2,3
a = 1,2,3
A = 2,3
Questions
factorization space
Radiation at ℐ
Geroch '77
not conf. inv
is conf. inv.
2 components
In Bondi: N_AB = − Ċ_AB −
n.l = −1
CAUTION


covariant description of I.
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ ; $\hat{g}_{\mu\nu} = \hat{g}_{ab} =: q_{ab} = \delta_a^A \delta_b^B q_{AB}$ ; $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
$(\mathcal{L}_n \omega = 0$ Bondi condition$)$
$\Omega = 0$ identifies I
Ass: $I = \mathbb{R} \times S^2$
smooth $\hat{g}_{\mu\nu}$
null hypersurface
$n_\mu = \partial_\mu \Omega$ ; $n^\mu = \hat{g}^{\mu\nu} n_\nu \in TI \to n^a$ ; $q_{ab} n^b = 0$
$\mu = 0,1,2,3$
$a = 1,2,3$ : $u, x^A$
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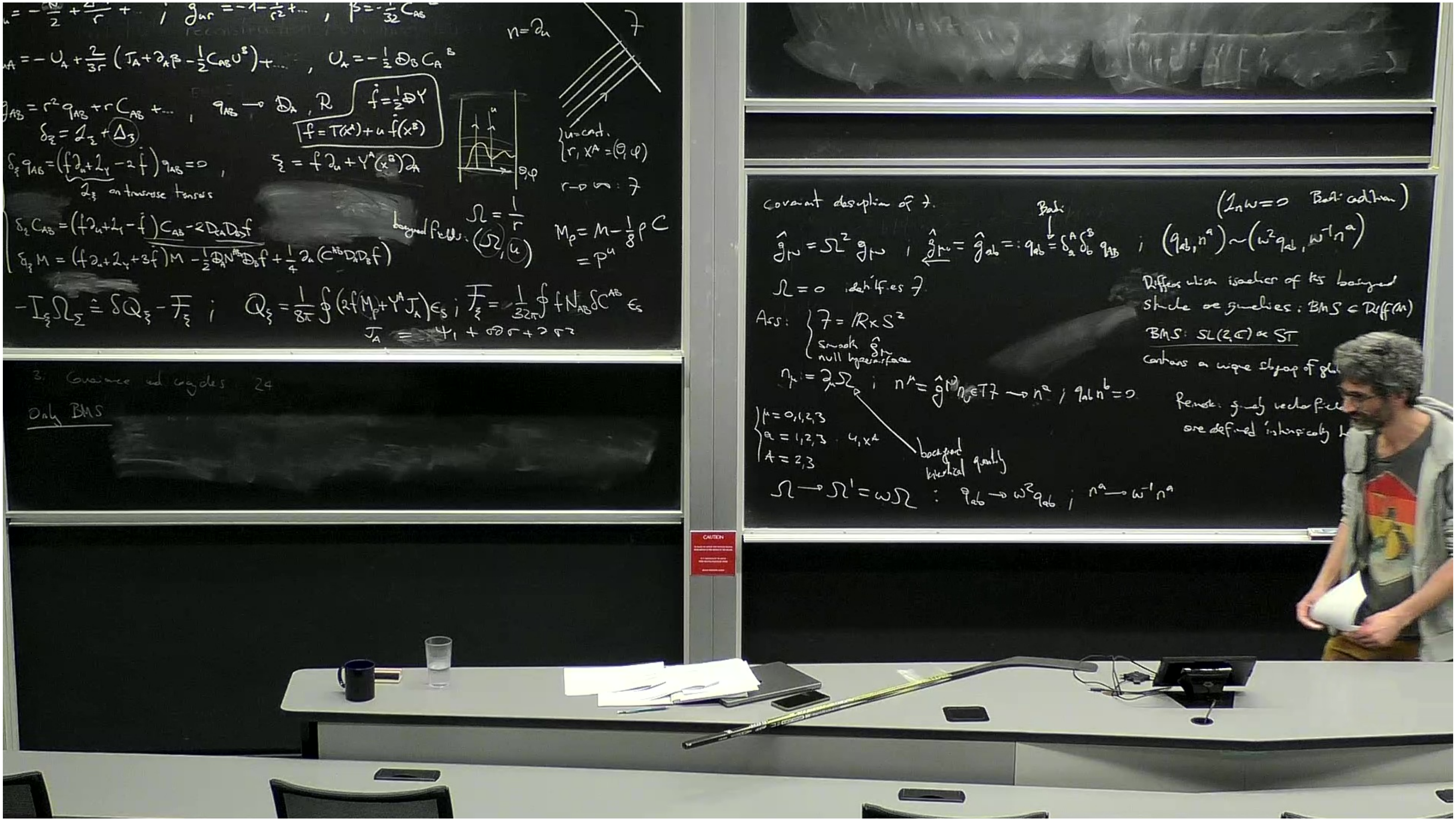
background
universal quantity
$\Omega \to \Omega' = \omega \Omega$ : $q_{ab} \to \omega^2 q_{ab}$ ; $n^a \to \omega^{-1} n^a$
Diffeos which isometries of this background structure are symmetries: BMS ⊂ Diff(M)
BMS: $SL(2,\mathbb{C}) \ltimes ST$
Contains a unique subgroup of global translations
Remark: given vector fields $\xi$ are defined intrinsically to I
$n \cdot l = -1$
$\hat{g}_{\mu\nu} = -2 l_{(\mu} n_{\nu)} + 2 m_{(\mu} \bar{m}_{\nu)} \to \gamma_{\mu\nu}$ ; $\gamma_{AB} = q_{AB}$
$= e_\mu^I e_\nu^J \eta_{IJ}$ ; $l = -du$
$\sigma_{ab} = \gamma_{\langle a}^c \gamma_{b\rangle}^d D_c l_d = \frac{1}{2} \delta_a^A \delta_b^B C_{AB}$
$\mathcal{L}_n l = 0$ ; $\mathcal{L}_n l = \tau$
$N_{ab} = 2 \mathcal{L}_n \sigma_{ab} - \rho_{\langle ab \rangle}$
DS '84
$N_{ab} = 2\mathcal{L}_n \sigma_{ab} + (D_a + \tau_a) \tau_b + l_{\langle a} \mathcal{L}_n \tau_{b\rangle} - \rho_{\langle ab\rangle}$
foliation
conf. inv.
$\sigma \to \omega \sigma$
$l \to \omega l$ ; $\tau = 0$ can be preserved requiring $l \to \omega l + \dots$



Covariant description of ℐ.
Bondi
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ ; $\hat{g}_{\mu\nu} = \hat{g}_{ab} =: q_{ab} \stackrel{\text{Bondi}}{=} \delta^A_a \delta^B_b q_{AB}$ ; $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
($\mathcal{L}_n \omega = 0$ Bondi condition)
$\Omega = 0$ identifies ℐ
Ass: ℐ = $\mathbb{R} \times S^2$, smooth $\hat{g}_{\mu\nu}$, null hypersurface
$n_\mu = \partial_\mu \Omega$ ; $n^\mu = \hat{g}^{\mu\nu} n_\nu \in T$ℐ $\to n^a$ ; $q_{ab} n^b = 0$
$\mu = 0,1,2,3$
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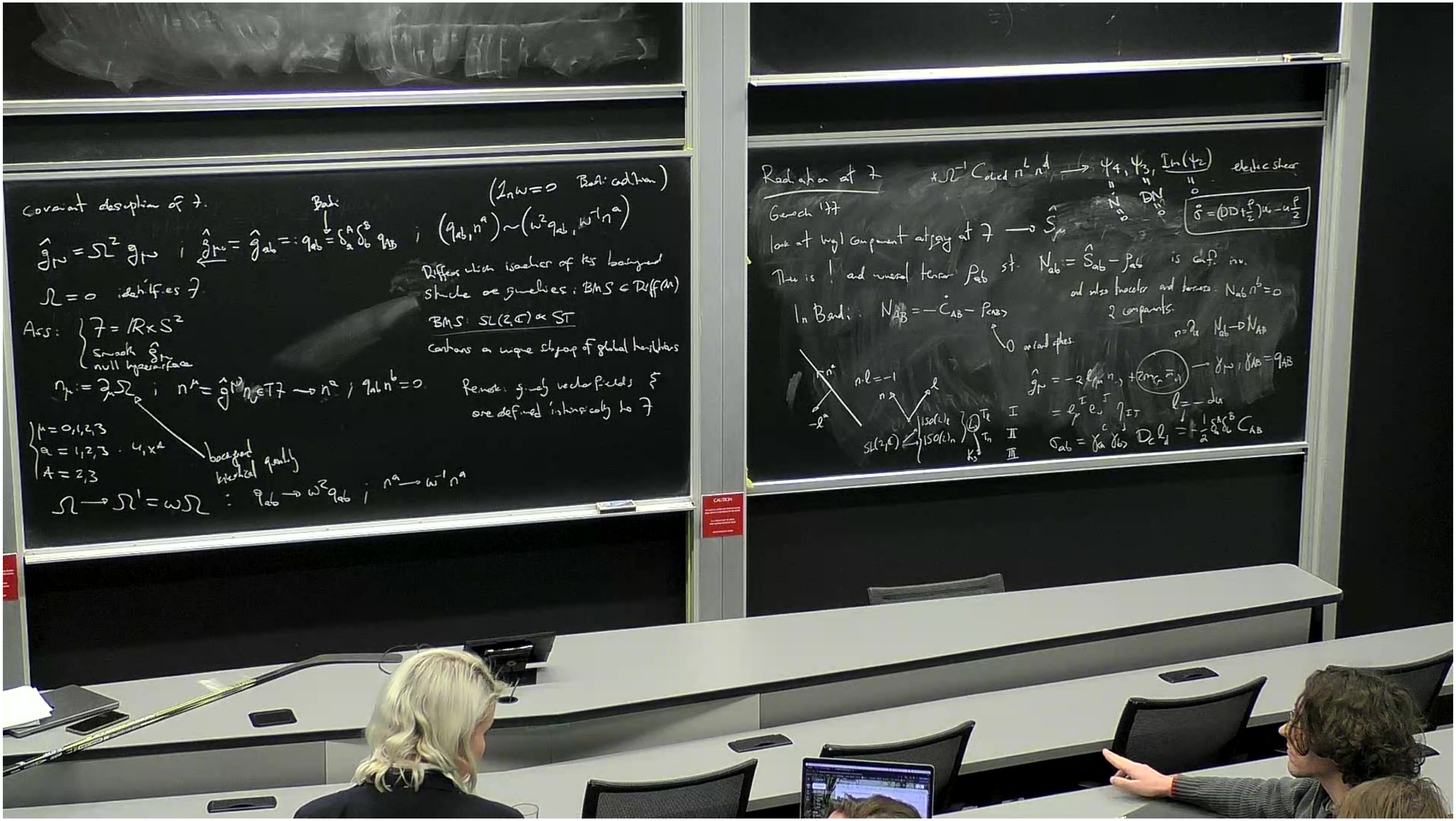
Diffeos which isometries of the background
structure are symmetries: BMS ⊂ Diff(ℐ)
BMS: $SL(2,\mathbb{C}) \ltimes ST$
Contains a unique subgroup of
Remark: generally vector fields
are defined intrinsically
$\delta_\xi q_{AB} = (f\partial_u + \mathcal{L}_Y - 2\dot{f}) q_{AB} = 0$ , $\xi = f\partial_u + Y^A(x^B)\partial_A$
$\boxed{f = T(x^A) + u \dot{f}(x^B)}$
$\dot{f} = \frac{1}{2}\mathcal{D}Y$
$\delta_\xi C_{AB} = (f\partial_u + \mathcal{L}_Y - \dot{f}) C_{AB} - 2 D_A D_B f$
$\delta_\xi M = (f\partial_u + \mathcal{L}_Y + 3\dot{f}) M - \frac{1}{4} \partial_u (C^{AB} D_A D_B f)$
background fields: $(\Omega, u)$
$\Omega = \frac{1}{r}$
$M_p = M - \frac{1}{8} \rho C = P^u$
$-I_\xi \Omega_\Sigma \doteq \delta Q_\xi - F_\xi$ ; $Q_\xi = \frac{1}{8\pi} \oint (2f M_p + Y^A J_A) \epsilon_S$ ; $F_\xi = \frac{1}{32\pi} \oint f N_{AB} \delta C^{AB} \epsilon_S$
$r \to \infty$ : ℐ
$n = \partial_u$
Only BMS


Covariant description of ℐ
Radiation at ℐ
Geroch '77
look at Weyl component at ℐ
BMS: SL(2,ℂ) ⋉ ST
In Bondi:
electric shear