

Title: Type I von Neumann algebras from gravitational path integrals: Ryu-Takayanagi as entropy without holography

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Series: Quantum Gravity

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Abstract: We show that the Ryu-Takayanagi (RT) formula, originally introduced to compute the entropy of a holographic boundary CFT, can be interpreted as entropy of an algebra of bulk gravitational observables. In particular, we show that any Euclidean gravitational path integral satisfying a simple and familiar set of axioms defines type I von Neumann algebras of bulk observables acting on closed codimension-2 asymptotic boundaries. The entropies associated to these algebras, defined via the gravitational path integral, can be written in terms of standard density matrices and standard Hilbert space traces, and in appropriate semiclassical limits are computed by the RT formula with quantum corrections. Our work thus provides a bulk state-counting interpretation of the Ryu-Takayanagi entropy. Since our axioms do not severely constrain UV bulk structures, they may be expected to hold equally well for successful formulations of string field theory, spin-foam models, or any other approach to constructing a UV-complete theory of gravity.

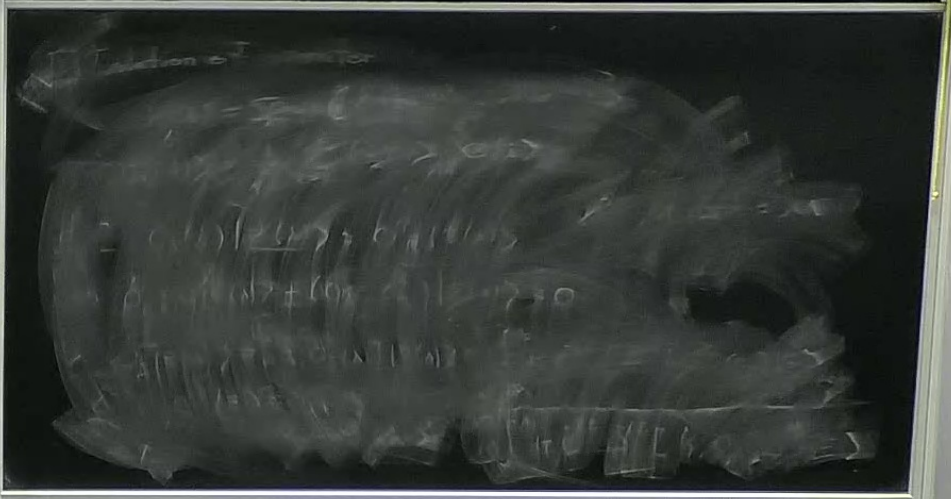
Zoom link TBA

$$\delta_2 C_{AB} = (\dots) C_{AB} - 2D_A D_B f$$

$$\delta_2 M = (\dots) M - \frac{1}{2} \mathbb{D}_A^B \mathbb{D}_B f + \frac{1}{4} 2 \dots$$

$$-I_{\xi} \Omega_{\Sigma} \doteq \delta Q_{\xi} - \mathbb{F}_{\xi} ; \quad Q_{\xi} = \frac{1}{8\pi} \oint (2f M_p + Y^A J_A) \epsilon_{\Sigma} ; \quad \mathbb{F}_{\xi} = \frac{1}{32\pi} \oint N_{AB} \delta C^{AB} \epsilon_{\Sigma}$$

- Scri: as WIH II: covariant charges and fluxes
- with Abhy Ashtekar "Sci as WIH"
 - with Antoine Rippen-Briet "Covariance and gravity algebras"
1. Divergence of charges à la Abhy (AS '81, AS '24)
 2. Compare with the WZ procedure '99
 3. Covariance and cycles '24



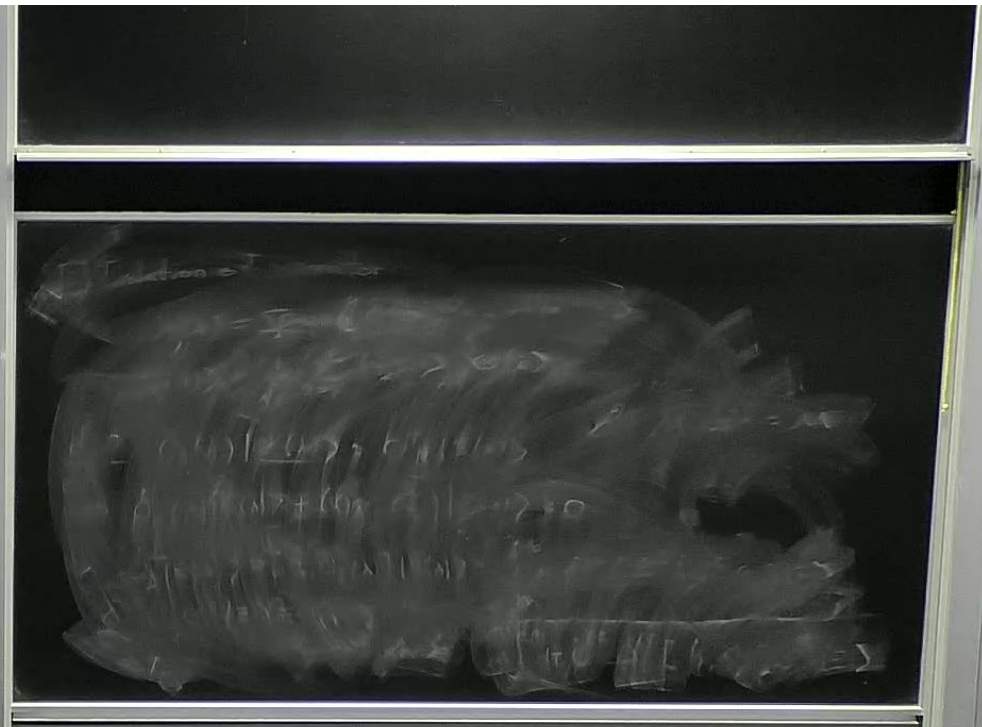
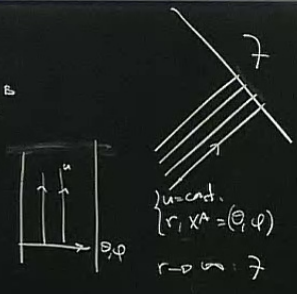
$$g_{tt} = -\frac{R_0}{2} + \frac{2M}{r} + \dots ; g_{rr} = -1 - \frac{2\beta}{r^2} + \dots, \beta = -\frac{1}{32} C_{AB}^2$$

$$g_{tA} = -U_A + \frac{2}{3r} (J_A + \partial_A \beta - \frac{1}{2} C_{AB} U^B) + \dots, U_A = -\frac{1}{2} \partial_0 C_A^B$$

$$g_{AB} = r^2 q_{AB} + r C_{AB} + \dots, q_{AB} \rightarrow \mathcal{D}_A, \mathcal{R}$$

$$\left\{ \begin{aligned} \delta_\xi q_{AB} &= \left(\frac{1}{2} \partial_u + 2\gamma - 2f \right) q_{AB} = 0, & \xi = f \partial_u + \gamma^A \partial_A \\ \delta_\xi C_{AB} &= \left(\frac{1}{2} \partial_u + 2\gamma - f \right) C_{AB} - 2 \mathcal{D}_A \mathcal{D}_B f \\ \delta_\xi M &= \left(\frac{1}{2} \partial_u + 2\gamma + 3f \right) M - \frac{1}{2} \partial_A N^{AB} \mathcal{D}_B f + \frac{1}{4} \partial_u (C^{AB} \mathcal{D}_A \mathcal{D}_B f) \end{aligned} \right.$$

$$I_{\xi} \Omega_{\Sigma} \doteq \delta Q_{\xi} - \mathcal{F}_{\xi}; \quad Q_{\xi} = \frac{1}{8\pi} \oint (2f M_p + \gamma^A J_A) \epsilon_S; \quad \mathcal{F}_{\xi} = -\frac{1}{32\pi} \oint N_{AB} \delta C^{AB} \epsilon_S$$



$$\delta_2 C_{AB} = (\frac{1}{2} \partial_a + \frac{1}{2} \partial_b - f) C_{AB} - 2 D_a D_b f$$

$$\delta_2 M = (\frac{1}{2} \partial_a + \frac{1}{2} \partial_b + 3f) M - \frac{1}{2} \partial_a^b M + \frac{1}{4} \partial_a (C^{AB} D_b f)$$

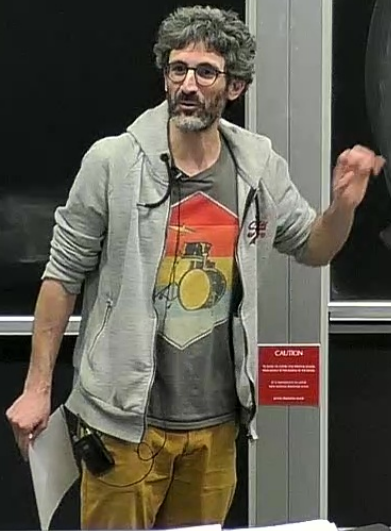
$$- \int_{\Sigma} \Omega_{\Sigma} \doteq \delta Q_{\Sigma} - \mathcal{F}_{\Sigma} ; \quad Q_{\Sigma} = \frac{1}{8\pi} \oint (2f M_p + Y^A J_A) \epsilon_{\Sigma} ; \quad \mathcal{F}_{\Sigma} = \frac{1}{32\pi} \oint f N_{AB} \delta C^{AB} \epsilon_{\Sigma}$$

Scrivasi WIIH II: covariant charges and fluxes

- with Abhay Ashtekar "Sci as WIIH"
 - with Antoine Ripstein "Covariance and gravity algebras"
1. Divergence and charges à la Abhay (AS '81, AS '24)
 2. Compare with the WZ procedure '99
 3. Covariance and charges '24
- Only BMS

CPS: $L^{EH} = \frac{1}{16\pi} R \epsilon$, $\delta L \doteq d\Theta \rightarrow$ symplectic potential current

$\Theta \xrightarrow{?} \text{flux}$ $\delta^{EH} = -\frac{1}{32\pi} \delta C^{AB} \delta C^{AB} + \delta b$; $\omega = \delta\Theta$



$$g_{AB} = r^2 q_{AB} + r C_{AB} + \dots, \quad q_{AB} \rightarrow \mathbb{D}_A, \mathbb{R}$$

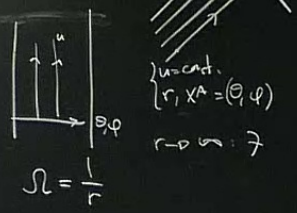
$$\delta_\xi q_{AB} = (f \partial_u + 2 \gamma - 2 \dot{f}) q_{AB} = 0, \quad \xi = f \partial_u + \gamma^A \partial_A$$

δ_ξ on transverse tensors

$$\delta_\xi C_{AB} = (f \partial_u + 2 \gamma - \dot{f}) C_{AB} - 2 D_{[A} D_{B]} f$$

$$\delta_\xi M = (f \partial_u + 2 \gamma + 3 \dot{f}) M - \frac{1}{2} \mathbb{D}_A^M \mathbb{D}_B f + \frac{1}{4} \partial_u (C^{AB} D_B f)$$

$$-I_\xi \Omega_\Sigma \doteq \delta Q_\xi - \mathbb{F}_\xi; \quad Q_\xi = \frac{1}{8\pi} \oint (2f M_\mu + \gamma^A J_A) \epsilon_S; \quad \mathbb{F}_\xi = \frac{1}{32\pi} \oint f N_{AB} \delta C^{AB} \epsilon_S$$



1. Derive the fluxes and charges
 2. Compare with the WZ procedure '99
 3. Covariance and cycles '24
- Only BMS

[Faded handwritten notes]

Covariant description of $\mathbb{7}$.

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \hat{g}_{ab} = \hat{g}_{ab} = q_{ab} = \delta_a^c \delta_b^d q_{cd}, \quad (q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$$

$\Omega = 0$ defines $\mathbb{7}$

Ass: $\mathbb{7} = \mathbb{R} \times \mathbb{S}^2$

smooth $\hat{g}_{\mu\nu}$
null hypersurface

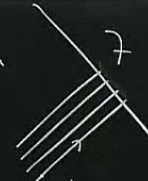
$$n_\mu = \partial_\mu \Omega, \quad n^a = \hat{g}^{ab} n_b \in T\mathbb{7} \rightarrow n^a, \quad q_{ab} n^b = 0$$

$\mu = 0, 1, 2, 3$
 $a = 1, 2, 3, 4, x^A$
 $A = 2, 3$

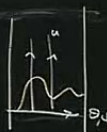
background
hatched quality

$$\Omega \rightarrow \Omega' = \omega \Omega : \quad q_{ab} \rightarrow \omega^2 q_{ab}; \quad n^a \rightarrow \omega^{-1} n^a$$

$g_{uu} = -\frac{R_u}{2} + \frac{2M}{r} + \dots$; $g_{ur} = -1 - \frac{2\beta}{r^2} + \dots$, $\beta = -\frac{1}{32} C_{AB}^2$
 $g_{AA} = -U_A + \frac{2}{3r} (T_A + \partial_A \beta - \frac{1}{2} C_{AB} U^B) + \dots$, $U_A = -\frac{1}{2} \partial_0 C_A^B$
 $g_{AB} = r^2 q_{AB} + r C_{AB} + \dots$, $q_{AB} \rightarrow \partial_A R \sqrt{f = \frac{1}{2} \partial Y}$
 $f = T(x^A) + u f(x^B)$
 $\delta_f q_{AB} = \frac{(f \partial_u + 2\partial_u - 2f) q_{AB}}{2_f \text{ on transverse tensors}} = 0$, $\xi = f \partial_u + Y^A(x^B) \partial_A$
 $\delta_f C_{AB} = (f \partial_u + 2\partial_u - f) C_{AB} - 2 \partial_u \partial_{AB} f$
 $\delta_f M = (f \partial_u + 2\partial_u + 3f) M - \frac{1}{2} \partial_u \partial_{AB} f + \frac{1}{4} \partial_u (C_{AB} \partial_{AB} f)$
 $-I_{\xi} \Omega_{\Sigma} \hat{=} \delta Q_{\xi} - \mathcal{F}_{\xi}$; $Q_{\xi} = \frac{1}{8\pi} \oint (2f M_p + Y^A J_A) \epsilon_{\Sigma}$; $\mathcal{F}_{\xi} = -\frac{1}{32\pi} \oint f N_{AB} S^{AB} \epsilon_{\Sigma}$



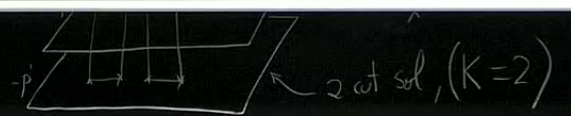
$n = \partial_u$



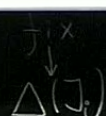
$u = \text{const.}$
 $(r, x^A) = (\theta, \varphi)$
 $r \rightarrow \infty : \mathcal{I}^+$

$\Omega = \frac{1}{r}$

covariant description of \mathcal{I} .
 $\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$; $\hat{g}_{ab} = \hat{g}_{ab} = q_{ab} = \delta_a^A \delta_b^B q_{AB}$; $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
 $\Omega = 0$ identifies \mathcal{I}
 Ass: $\mathcal{I} = \mathbb{R} \times S^2$
 smooth \mathcal{I} null hypersurface
 $n^a = \partial_a \Omega$; $n^a = \hat{g}^{ab} n_b \in T\mathcal{I} \rightarrow n^a$; $q_{ab} n^b = 0$
 $\mu = 0, 1, 2, 3$
 $a = 1, 2, 3$; u, x^A
 $A = 2, 3$
 $\Omega \rightarrow \Omega' = \omega \Omega$: $q_{ab} \rightarrow \omega^2 q_{ab}$; $n^a \rightarrow \omega^{-1} n^a$
 Differs from isotherm of \mathcal{I} by gauge choice or gauges: $BMS \subset \text{Diff}(\mathcal{I})$
 $BMS: SL(2, \mathbb{C}) \ltimes \mathcal{ST}$
 contains a wide class of global horizons
 back-pod vertical quality



\mathcal{I} at \mathcal{I}^+ ($K=2$)



$\Delta(J_i)$

Covariant description of \mathcal{I} .

$$\hat{g}_M = \Omega^2 g_M \quad ; \quad \hat{g}_{ab} = \hat{g}_{ab} =: q_{ab} = \delta_a^A \delta_b^B q_{AB}$$

$\Omega = 0$ identifies \mathcal{I} .

Ass: $\mathcal{I} = \mathbb{R} \times S^2$
 smooth \hat{g}_M
 null hypersurface

$$\eta_{\mu\nu} = \Omega^2 \eta_{\mu\nu} \quad ; \quad n^A = \hat{g}^{AB} \eta_B \in T \rightarrow n^a \quad ; \quad q_{ab} n^b = 0$$

$\mu = 0, 1, 2, 3$
 $a = 1, 2, 3 \quad ; \quad 4, \infty$
 $A = 2, 3$

$$\Omega \rightarrow \Omega' = \omega \Omega \quad ; \quad q_{ab} \rightarrow \omega^2 q_{ab} \quad ; \quad n^a \rightarrow \omega^{-1} n^a$$

($\Omega = 0$ Bondi condition)

$$(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$$

Differs from isotherm of Kerr based
 structure or geometries: BMS \subset Diff(M)

BMS: $SL(2, \mathbb{C}) \ltimes \mathcal{ST}$

Contains a unique subgroup of global translations

Remark: globally vector fields ξ
 are defined intrinsically to \mathcal{I}

Huge freedom in choosing Ω that can be tamed:

• div-free frames: $\hat{\nabla}_\mu n^a = 0 \xrightarrow{\text{ECS}} \mathcal{I}$ is NEH; $\int n^a q_{ab} = 0$ Bondi condition

$$\hat{\nabla}_\mu = \hat{\nabla}_a = D_a$$

• use round spheres: $q_{AB} = \hat{q}_{AB}$; $\hat{Q} = 2$. Bondi frames

can always be done but not always convenient:

1. Choice of inv. becomes harder
2. Choice of FB eq. results in far away cross-sections

Covariant description of \mathbb{F} .

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad ; \quad \hat{g}_{ab} = \hat{g}_{ab} =: q_{ab} = \sum_a^A \sum_b^B q_{AB} \quad ; \quad (q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$$

$\Omega = 0$ identifies \mathbb{F}

Ass: $\mathbb{F} = \mathbb{R} \times S^2$
 smooth $\hat{g}_{\mu\nu}$
 null hypersurface

$n_\mu = 2\Omega \partial_\mu$; $n^a = \hat{g}^{ab} n_b \in T\mathbb{F} \rightarrow n^a / q_{ab} n^b = 0$ Remark: global vector fields E are defined intrinsically to \mathbb{F}

- $\mu = 0, 1, 2, 3$
- $a = 1, 2, 3 \quad ; \quad u, v, \lambda$
- $A = 2, 3$

$\Omega \rightarrow \Omega' = \omega \Omega \quad ; \quad q_{ab} \rightarrow \omega^2 q_{ab} \quad ; \quad n^a \rightarrow \omega^{-1} n^a$

background
 hierarchical
 quality

($2n\omega = 0$ Bondi condition)
 Differential isomorphism of \mathbb{F} s based
 structure or geometries: $\text{BMS} \subset \text{Diff}(M)$
 $\text{BMS} = \text{SL}(2, \mathbb{C}) \ltimes \text{ST}$
 contains a wide class of global horizons

Radiation at \mathbb{F}

Geodesic \mathbb{F}

low at wgl component at \mathbb{F} $\rightarrow \hat{S}_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{1}{2} \hat{R} \hat{g}_{\mu\nu}$ not conf. inv

This is ! and universal tensor P_{ab} st. $N_{ab} := \hat{S}_{ab} - P_{ab}$ is conf. inv.
 and also traceless and hermitian. $N_{ab} n^b = 0$
 2 components.

In Bondi: $N_{AB} = -\dot{C}_{AB} - P_{AB}$

or (and) eqs.

$n = \partial_u \quad N_{ab} = \dot{N}_{AB}$

Covariant description of \mathcal{I} .

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad ; \quad \hat{g}_{\mu\nu} = \hat{g}_{ab} =: q_{ab} = \Omega^2 g_{ab}$$

$\Omega = 0$ identifies \mathcal{I}

Ass: $\mathcal{I} = \mathbb{R} \times S^2$
 smooth $\hat{g}_{\mu\nu}$
 null hypersurface

$$\eta_{\mu\nu} = 2\Omega \delta_{\mu\nu}$$

$$\begin{cases} \mu = 0, 1, 2, 3 \\ a = 1, 2, 3, 4, x^A \\ A = 2, 3 \end{cases}$$

$$\Omega \rightarrow \Omega' = \omega \Omega$$

Bondi $(\Omega, \omega = 0$ Bondi condition)

$$(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$$

Differences with isotherm of KS: bounded structure on geometries: BMS \subset Diff(M)

BMS: $SL(2, \mathbb{C}) \ltimes \mathcal{ST}$

contains a wide class of global horizons

Remark: global vector fields X are defined intrinsically to \mathcal{I}

$$x \rightarrow n^a \quad ; \quad q_{ab} n^b = 0$$

$$\omega^2 q_{ab} \quad ; \quad n^a \rightarrow \omega^{-1} n^a$$

Questions: $V_1(g) \rightsquigarrow$ factorization space
 B.I. $V \rightsquigarrow V_{\mathcal{I}^{-1}}$

Radiation at \mathcal{I}

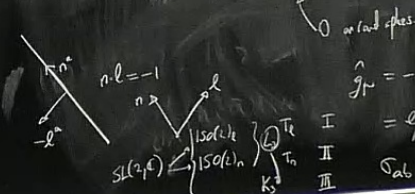
$$\hat{R}_{\mu\nu\rho\sigma}(\hat{g})$$

Guess \mathcal{I}^{\pm}

$$\text{look at large component outgoing at } \mathcal{I} \rightarrow \hat{S}_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{1}{6} \hat{R} \hat{g}_{\mu\nu} \quad \text{not def. in}$$

This is! add universal tensor P_{ab} st. $N_{ab} = \hat{S}_{ab} - P_{ab}$ is def. in
 and also traceless and torsion. $N_{ab} n^b = 0$
 2 components.

$$\text{In Bondi: } N_{AB} = -\dot{C}_{AB} - P_{AB}$$



$$\hat{g}_{\mu\nu} = -2\ell_{\mu} n_{\nu} + 2\omega^2 \gamma_{AB} \hat{g}_{AB}$$

$$= \ell_{\mu}^I e_{\nu}^J \gamma_{IJ} \quad ; \quad \ell = -du$$

$$S_{ab} = \gamma_{ab} \hat{g}_{ab} \quad ; \quad D_c \ell_d = \frac{1}{2} \hat{g}_{ab}^c \hat{g}_{ab}^d C_{AB}$$

Covariant description of \mathcal{F} .

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad ; \quad \hat{g}_{ab} = \hat{g}_{ab} = q_{ab} = \sum_a^A \sum_b^B q_{AB} \quad ; \quad (q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$$

$\Omega = 0$ identifies \mathcal{F} .

Ass: $\mathcal{F} = \mathbb{R} \times S^2$
smooth $\hat{g}_{\mu\nu}$
null hypersurface

$$\eta_\mu = 2\Omega \partial_\mu \quad ; \quad n^\mu = \hat{g}^{\mu\nu} \eta_\nu \in T\mathcal{F} \rightarrow n^\mu \quad ; \quad q_{ab} n^b = 0 \quad ; \quad \text{Remark: } g_{\mu\nu} \text{ vector fields } \mathcal{F} \text{ are defined intrinsically to } \mathcal{F}$$

$\mu = 0, 1, 2, 3$
 $a = 1, 2, 3 \quad ; \quad 4, x^4$
 $A = 2, 3$

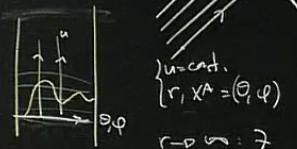
$$\Omega \rightarrow \Omega' = \omega \Omega \quad ; \quad q_{ab} \rightarrow \omega^2 q_{ab} \quad ; \quad n^a \rightarrow \omega^{-1} n^a$$

($\Omega = 0$ Best. action)
Differentiation is aether of \mathcal{F} 's boundary structure or geometries: $\text{BMS} \subset \text{Diff}(M)$
 $\text{BMS} = \text{SL}(2, \mathbb{C}) \ltimes \text{ST}$
contains a wide class of global transformations

$\gamma_{\mu\nu} = -2\ell_{\mu\nu} n_\nu + 2m_{\mu\nu} \tilde{n}_\nu \rightarrow \gamma_{\mu\nu} = q_{AB} = q_{AB}$
 $= \gamma^I e_{\mu I} \gamma^{JT} = \gamma^I e_{\mu I} \gamma^{JT} \quad ; \quad \ell = -du$
 $\sigma_{ab} = \gamma_{ab} \delta_{ab} \quad ; \quad D_C \ell_A = \frac{1}{2} \delta_{AB} C_{AB}$

$\mathcal{L}_n \ell = 0 \quad ; \quad \mathcal{L}_n \sigma = \epsilon$
 $N_{ab} = 2\mathcal{L}_n \sigma_{ab} + (D_a + \gamma_a) \epsilon_{ab} + \epsilon_{ca} \mathcal{L}_n \epsilon_{cb} - \epsilon_{cab}$
foliation
conf. inv.
 $\mathcal{L}_n \omega = \epsilon$
 $\epsilon = 0$ can be pres. req. $\mathcal{L}_n \omega = \epsilon + \dots$ class II

$u = -\frac{M}{2} + \frac{L}{r} + \dots$; $g_{ur} = -1 - \frac{2M}{r} + \dots$, $\beta = -\frac{1}{32} C_{AB}$
 $U_A = -U_A + \frac{2}{3r} (J_A + \partial_A \beta - \frac{1}{2} C_{AB} U^B) + \dots$, $U_A = -\frac{1}{2} \partial_B C_A^B$
 $g_{AB} = r^2 q_{AB} + r C_{AB} + \dots$, $q_{AB} \rightarrow \mathcal{D}_A \mathcal{R} \left(f = \frac{1}{2} \mathcal{D}^2 Y \right)$
 $\delta_{\xi}^2 = \delta_{\xi}^2 + \delta_{\xi}^3$ $\left(f = T(x^A) + u f(x^B) \right)$
 $\delta_{\xi}^2 q_{AB} = (f \partial_A + 2 \partial_A f - 2 f) q_{AB} = 0$, $\xi = f \partial_u + Y^A(x^B) \partial_A$
 $\delta_{\xi}^2 C_{AB} = (f \partial_u + 2 \partial_u f - f) C_{AB} - 2 \mathcal{D}_A \mathcal{D}_B f$
 $\delta_{\xi}^2 M = (f \partial_u + 2 \partial_u f + 3f) M - \frac{1}{2} \mathcal{D}_A \mathcal{D}^A \mathcal{D}_B f + \frac{1}{4} \partial_A (C^{AB} \mathcal{D}_B f)$
 $-\mathcal{I}_{\xi} \Omega_{\Sigma} \hat{=} \delta Q_{\xi} - \mathcal{F}_{\xi}$; $Q_{\xi} = \frac{1}{8\pi} \oint (2f M_A + Y^A J_A) \epsilon_S$; $\mathcal{F}_{\xi} = \frac{1}{32\pi} \oint f N_{AB} \delta C^{AB} \epsilon_S$
 $J_A = \psi_A + \partial \sigma + \dots$



3. Covariance and charges 24
Only BMS

Covariant description of \mathcal{I} . Back: $(\mathcal{I}, \omega = 0$ Bondi addition)
 $\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, $\hat{g}_{ab} = \hat{g}_{ab} = q_{ab} = \delta_a^A \delta_b^B q_{AB}$; $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$
 $\Omega = 0$ identifies \mathcal{I}
 Differs when isoscher of its background structure or geometries: BMS \subset Diff(M)
 BMS: $SL(2, \mathbb{C}) \ltimes \mathcal{ST}$
 Contains a wave shape of light.
 Rem: g_{ab} vector fields are defined intrinsically to \mathcal{I} .
 As: $\mathcal{I} = \mathbb{R} \times S^2$
 $\left\{ \begin{array}{l} \text{smooth } \hat{g}_{ab} \\ \text{null hypersurface} \end{array} \right.$
 $n^a = \partial_u \Omega$; $n^a = \hat{g}^{ab} \partial_b \Omega \in T\mathcal{I} \rightarrow n^a$; $q_{ab} n^b = 0$
 $\left\{ \begin{array}{l} \mu = 0, 1, 2, 3 \\ a = 1, 2, 3 \end{array} \right.$; u, x^A
 $A = 2, 3$
 background geometrical quality
 $\Omega \rightarrow \Omega' = \omega \Omega$: $q_{ab} \rightarrow \omega^2 q_{ab}$; $n^a \rightarrow \omega^{-1} n^a$



Covariant description of \mathcal{I} .

$\hat{g}_M = \Omega^2 g_M$, $\hat{g}_{ab} = \hat{g}_{ab} = q_{ab} = \delta_a^A \delta_b^B q_{AB}$ (Bard) $(q_{ab}, n^a) \sim (\omega^2 q_{ab}, \omega^{-1} n^a)$ ($\int n^a \omega = 0$ Bard action)

$\Omega = 0$ identifies \mathcal{I}

Ass: $\mathcal{I} = \mathbb{R} \times S^2$
 smooth \hat{g}_M
 null hypersurface

$n_a = \partial_a \Omega$, $n^a = \hat{g}^{ab} \partial_b \Omega \in T\mathcal{I} \rightarrow n^a, q_{ab} n^b = 0$

$\mu = 0, 1, 2, 3$
 $a = 1, 2, 3, 4, x^a$
 $A = 2, 3$

background
 hierarchical family

$\Omega \rightarrow \Omega' = \omega \Omega$: $q_{ab} \rightarrow \omega^2 q_{ab}$; $n^a \rightarrow \omega^{-1} n^a$

Differentiation is a her of KS background
 structure or geometries: BMS $\in \text{Diff}(M)$
 BMS: $SL(2, \mathbb{C}) \ltimes \mathbb{R}^4$
 contains a unique subgroup of global translations

Remove: g. only vector fields ξ
 are defined intrinsically to \mathcal{I}

Radiation at \mathcal{I} $\star \Omega^{-1} \text{Covet } n^L n^d \rightarrow \psi_4, \psi_3, \dots, \frac{\text{In}(\psi_2)}{\Omega}$ electric shear

Geodesic \mathcal{I} $\rightarrow \hat{S}_M$

late at wgl component at \mathcal{I} $\rightarrow \hat{S}_M$

This is 1 ad universal tensor P_{ab} st $N_{ab} = \hat{S}_{ab} - P_{ab}$ is def. inv. and also traceless and torsion $N_{ab} n^b = 0$
 2 components $n^a n^b N_{ab} = 0$

In Bard: $N_{AB} = -\dot{C}_{AB} - P_{AB}$

$\hat{g}_M = -2 \ell^a \ell^b \eta_{ab} + 2 \text{Im}(\sigma) \bar{\sigma} \rightarrow \gamma_{AB}, \gamma_{AB} = q_{AB}$
 $\ell^a = \partial^a \Omega$
 $\sigma = \frac{1}{2} \partial_a \ell^b \ell^c \eta_{bc}$
 $\sigma_{ab} = \gamma_{ab} \ell^c \ell^d = \frac{1}{2} \partial_c \ell^d \ell^e \eta_{de} C_{AB}$

$SL(2, \mathbb{C}) \leftarrow \text{isot}(U_1) \leftarrow \text{isot}(U_1) \leftarrow \text{isot}(U_1)$
 $\left. \begin{matrix} T_1 \\ T_2 \\ T_3 \end{matrix} \right\} \text{III}$