Title: Constraining CFTs with moduli spaces

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Abstract: Moduli spaces of vacua are an intriguing property of certain supersymmetric QFTs which have been widely explored. However, a first-principles approach to moduli spaces and how they constrain observables is still lacking. This question is even more pressing due to recent interest in moduli spaces in theories with only two supercharges, where supersymmetry is extremely weak and does not allow for exact computations. In this talk we attempt to bootstrap conformal field theories with moduli spaces. First we assume an additional global symmetry which is spontaneously broken along the moduli space, and use techniques from the large charge expansion to show that the existence of a moduli space directly constrains CFT data of charged operators. We then study the generic case by using a "moduli space bootstrap equation" to write down perturbative sum rules on observables of CFTs order-by-order in a small coupling. We discuss several examples and applications of our results.

Zoom link

Constraining CFTs w/ moduli spaces

w/ L Rastelli and G Cuomo, to appear

"Bootstrap" moduli spaces

Q How does the existence of a moduli space constrain CFT data?

1) assume $U(1)$ sym which is spontaneously broken on moduli space

2) generic

Constraining CFTs w/ moduli spaces

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"Bootstrap" moduli spaces

Q: How does the existence of a moduli space constrain CFT data?

$\boxed{1)}$ assume $U(1)$ sym which is spontaneously broken on moduli space

2) generic

SUSY

3d $\mathcal{N}=2$ SUSY
4d $\mathcal{N}=1$ SUSY

$\# Q = 4$

mod space
$\# Q = 4 \implies \Delta_{X^n} = n \Delta_X = q_R$

conjecture moduli space $\cong$ spectrum of chiral ring

$W = X^k$
$X^{k-1} = 0$

1) SCFT always have $U(1)_R$ sym $\Rightarrow$ broken on moduli space

2) directions on moduli space $\longleftrightarrow$ chiral operators

Example

3d $\mathcal{N}=2$ XYZ model $(XY)^2 + (YZ)^2 + (XZ)^2$

$W = XYZ \Rightarrow V = \sum_\phi (\partial_\phi W)^2$

F-term eqs $\frac{\partial W}{\partial X} = \frac{\partial W}{\partial Y} = \frac{\partial W}{\partial Z} = 0$ $+ \partial^2_\phi W \psi^2_\phi$

$XY = YZ = XZ = 0$

$X \neq 0$ or $Y \neq 0$ or $Z \neq 0$

$\Delta_{X^n} = n \Delta_X = \frac{2}{3} n$

Constraining CFTs w/ moduli spaces

moduli space $\Rightarrow$ $\boxed{\Delta_{min}(Q) = \alpha Q\ ?}$

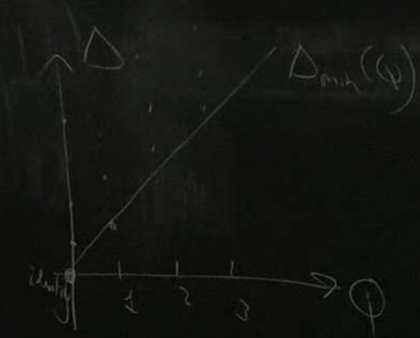
define $\Delta_{min}(Q) \equiv$ min dim of all ops of charge Q

Q charge of $U(1)$

large charge expansion $Q \gg 1$

1) "generic" $\Delta_{min}(Q) \sim Q^{d/d-1}$

2) free/BPS $\Delta_{min}(Q) = Q$



3d $\mathcal{N}=1$ theories

$$\int d^2\theta \, P(A, B, C)$$

$$\downarrow$$

$$\left[\int d^2\theta \, ABC \, P(A^2, B^2, C^2)\right]$$

$A \neq 0$

$\mathbb{Z}_2^R$ sym $\Rightarrow$ exact moduli space

$$\mathcal{L} = \int d^2\theta \, K + \int d^2\theta \, W$$

$$\int d^2\theta \, \overbrace{\Phi D^2 \Phi}^{K} + \int d^2\theta \, \overbrace{m\Phi^2 + \lambda\Phi^3}^{W}$$

$$\mathcal{L} \ni \int \underline{d^2\theta} \, \underline{A}BC$$

$\mathbb{Z}_2^R$-sym

$d^2\theta \to -d^2\theta$, $A \to -A$

$d^2\theta \to -d^2\theta$, $B \to -B$

$C \to -C$

F-term eqs

$AB = BC = AC = 0$

$\Rightarrow$ moduli space $A \neq 0$ or $B \neq 0$ or $C \neq 0$

Constraining CFTs w/ moduli spaces

[Hellerman, Watanabe, Maeda]

$\Delta_{mn}(Q)$ assuming moduli space

method: 1) effective action on moduli space

2) put on cylinder, and compute gr energy at large φ

3) state-op correspondence $gs = \Delta_{mn}(\varphi)$

1) a) expansion in derivatives

b) syms:

1) dilation Φ

2) NGB π from u(1) $g_{\mu\nu}$

$\pi \to \pi + \alpha \longrightarrow \partial\pi$

trick:

$\Phi, \pi, g_{\mu\nu} \to \pi, \hat{g}_{\mu\nu}$

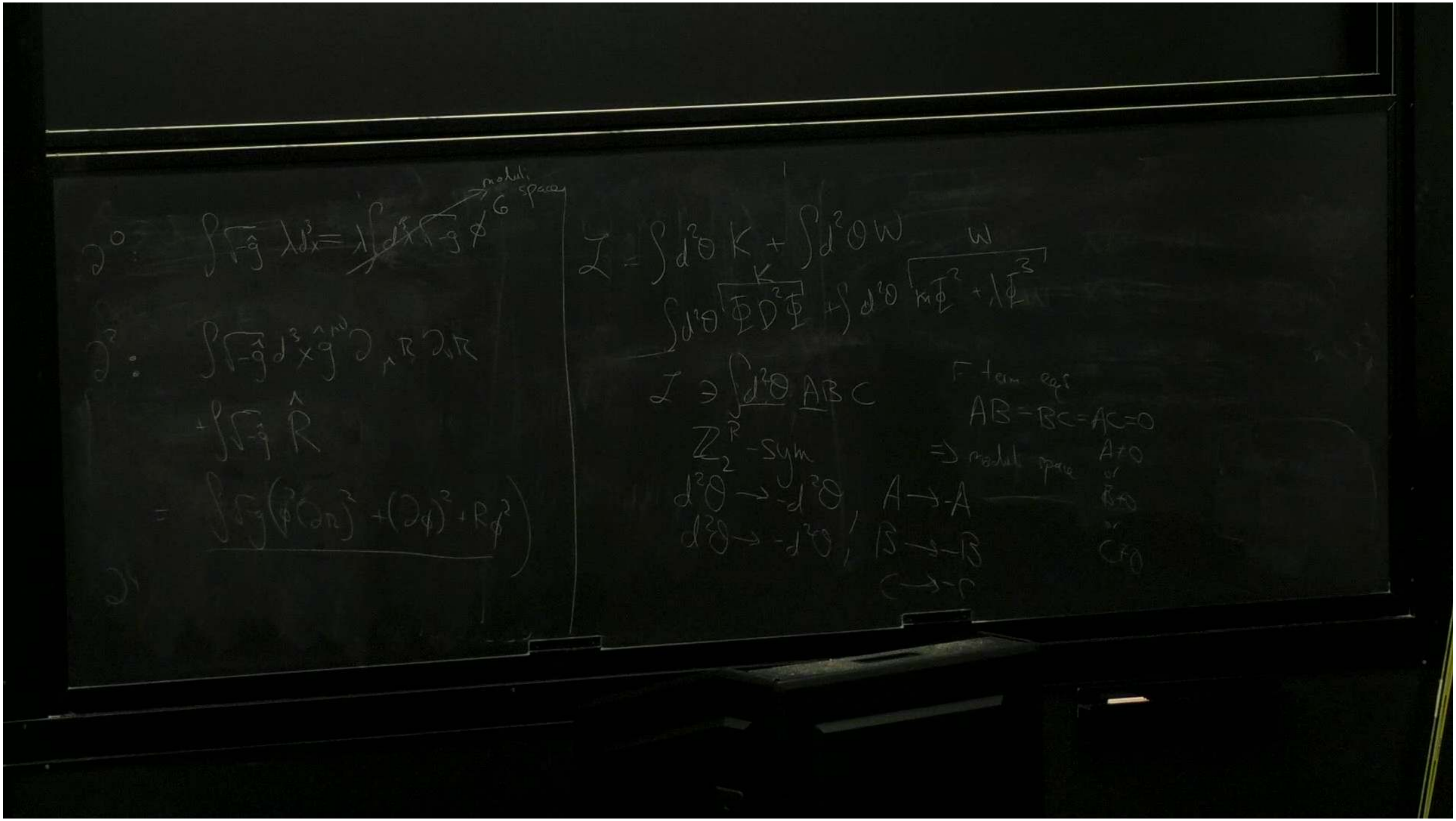
1) $\Phi \to \underbrace{e^{-\frac{1}{2}\sigma}\Phi}_{=1}$

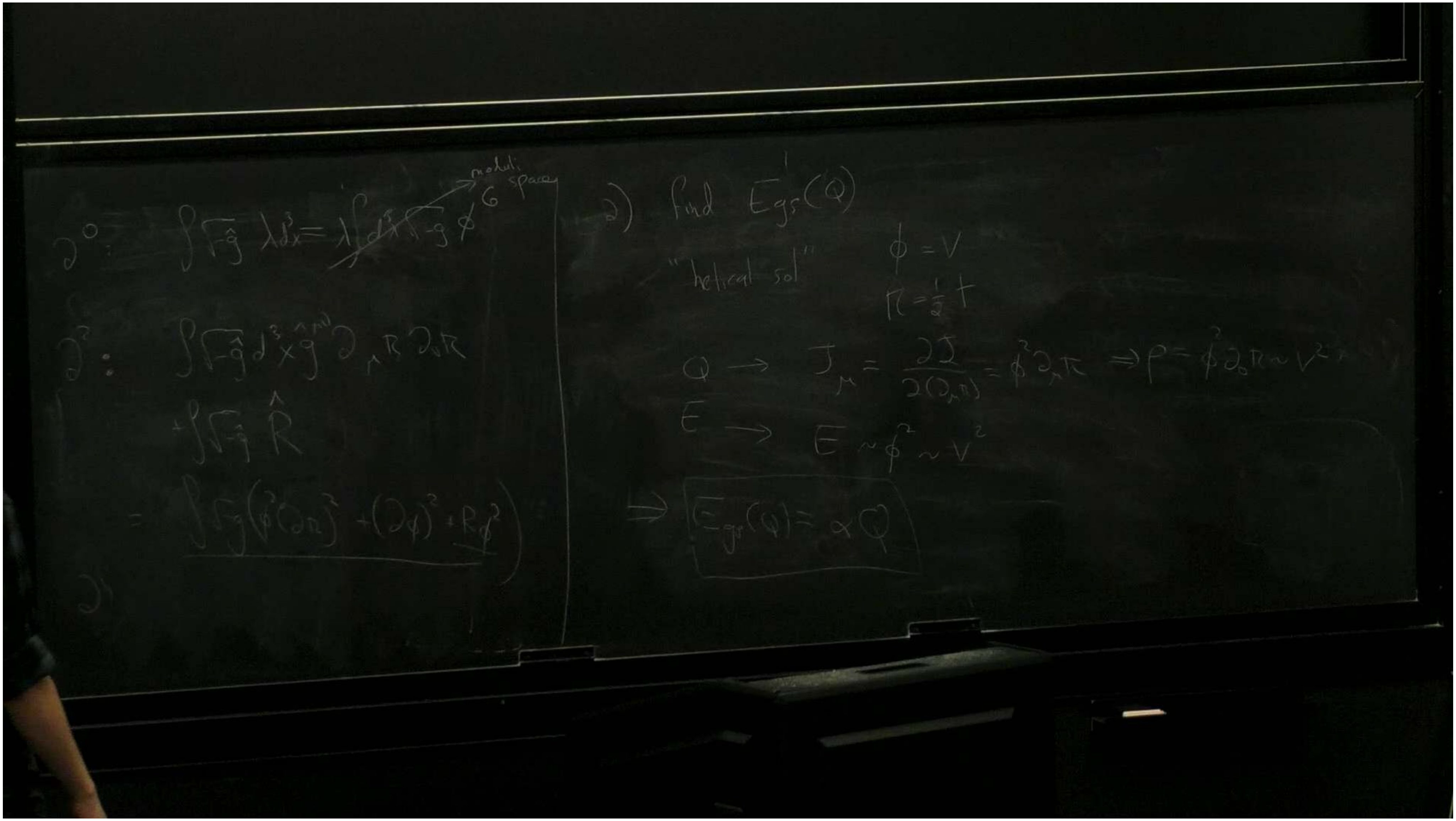
$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$

$\Phi \to 1$

$g_{\mu\nu} \to \hat{g}_{\mu\nu} = g_{\mu\nu}\varphi^{4}$

2) Weyl transformations


moduli space
∂^0: $\int \sqrt{-\hat g}\,\lambda d^3x = \lambda \int d^3x \sqrt{-g}\,\phi^6$
∂^2: $\int \sqrt{-\hat g}\, d^3x\, \hat g^{\mu\nu} \partial_\mu R\, \partial_\nu R$
$+\int \sqrt{-\hat g}\, \hat R$
$= \int \sqrt{g}\left(\phi^2 (\partial R)^2 + (\partial\phi)^2 + R\phi^2\right)$
∂^4
$\mathcal{L} = \int d^2\theta\, K + \int d^2\theta\, W$
$\int d^2\theta\, \bar\Phi D^2 \Phi + \int d^2\theta\, m\Phi^2 + \lambda\Phi^3$
$\mathcal{L} \ni \int d^2\theta\, ABC$
$\mathbb{Z}_2^R$-sym
$d^2\theta \to -d^2\theta$, $A \to -A$
$d^2\theta \to -d^2\theta$, $B \to -B$
$C \to -C$
F term eqs
$AB = BC = AC = 0$
$\Rightarrow$ moduli space $A \neq 0$ or $B \neq 0$ or $C \neq 0$


moduli space
∂^0: $\int \sqrt{\hat{g}}\, \lambda d^3x = \lambda \int d^3x \sqrt{-g}\, \phi^6$
∂^2: $\int \sqrt{-\hat{g}}\, d^3x\, \hat{g}^{\mu\nu} \partial_\mu \pi \partial_\nu \pi$
$+ \int \sqrt{-\hat{g}}\, \hat{R}$
$= \int \sqrt{-g} \left(\phi^2 (\partial \pi)^2 + (\partial \phi)^2 + R\phi^2 \right)$
2) find $E'_{gs}(Q)$
"helical sol"
$\phi = V$
$\pi = \frac{1}{2} t$
$Q \longrightarrow J_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} = \phi^2 \partial_\mu \pi \Rightarrow \rho = \phi^2 \partial_0 \pi \sim V^2$
$E \longrightarrow E \sim \phi^2 \sim V^2$
$\Rightarrow \boxed{E_{gs}(Q) = \alpha Q}$

Constraining CFTs w/ moduli spaces

$\partial^2 \quad \Delta_{min}(Q) = \boxed{\alpha Q + Q^0} + 1/Q + 1/Q^2 + \dots$

$\partial^4 : \quad \phi^{-2}\partial^4 \sim v^{-2} \sim 1/Q$

$\partial^6 \quad 1/Q^2$

quantum corrections

$\underbrace{Q^0}_{\text{1-loop}} + 1/Q + 1/Q^2 \dots$

1-loop Casimir energy

4d

$\Delta_{min}(Q) = \boxed{\alpha Q + \log Q} + Q^0 + \dots$

Constraining CFTs w/ moduli spaces

∂^2 $\quad \Delta_{min}(Q) = \alpha Q + Q^0 + 1/Q + 1/Q^2 + \dots$

∂^4: $\quad \phi^{-2}\partial^4 \sim v^{-2} \sim 1/Q$

∂^6 $\quad 1/Q^2$

quantum corrections

$\underbrace{Q^0}_{\text{1-loop}} + 1/Q + 1/Q^2 \dots$

Casimir energy

Examples

1) $3d$ $\mathcal{N}=1$ SQED

A_μ, λ

$+ N_f \; \phi_i, \psi_i \quad i=1, \dots, N_f$

complex Dirac

$\mathcal{L} = (\text{kinetic}) + \bar{\psi}_i \phi_i \lambda + c.c.$

$\langle \phi_i \rangle \neq 0$

$W(|\Phi_i|^2)$

$\underbrace{\int d^2\theta_i \, W(|\Phi|^2)}_{\sim \partial^2 \theta}$

$\Rightarrow \langle \phi_i \rangle \neq 0$

dim $2N_f - 1$

massless dof: 1 dilaton Φ

NGBs π

$SU(N_f) \to S[U(N_f-1)\times U(1)]$

$\mathbb{C}P^{N_f-1} = \dfrac{SU(N_f)}{S[U(N_f-1)\times U(1)]}$ $2(N_f-1)$

dilaton + NGBs $= 2N_f - 1$

$$(\partial\Phi)^2 + R\Phi^2 + 2c\,\Phi^2 \underbrace{\gamma_{ij}\,\partial z_i \partial \bar{z}_j}_{\mathbb{C}P^{N_f-1}\ \text{NLSM}}$$

z_i complex $i = 1, \ldots, N_f - 1$

γ_{ij} Fubini–Study metric

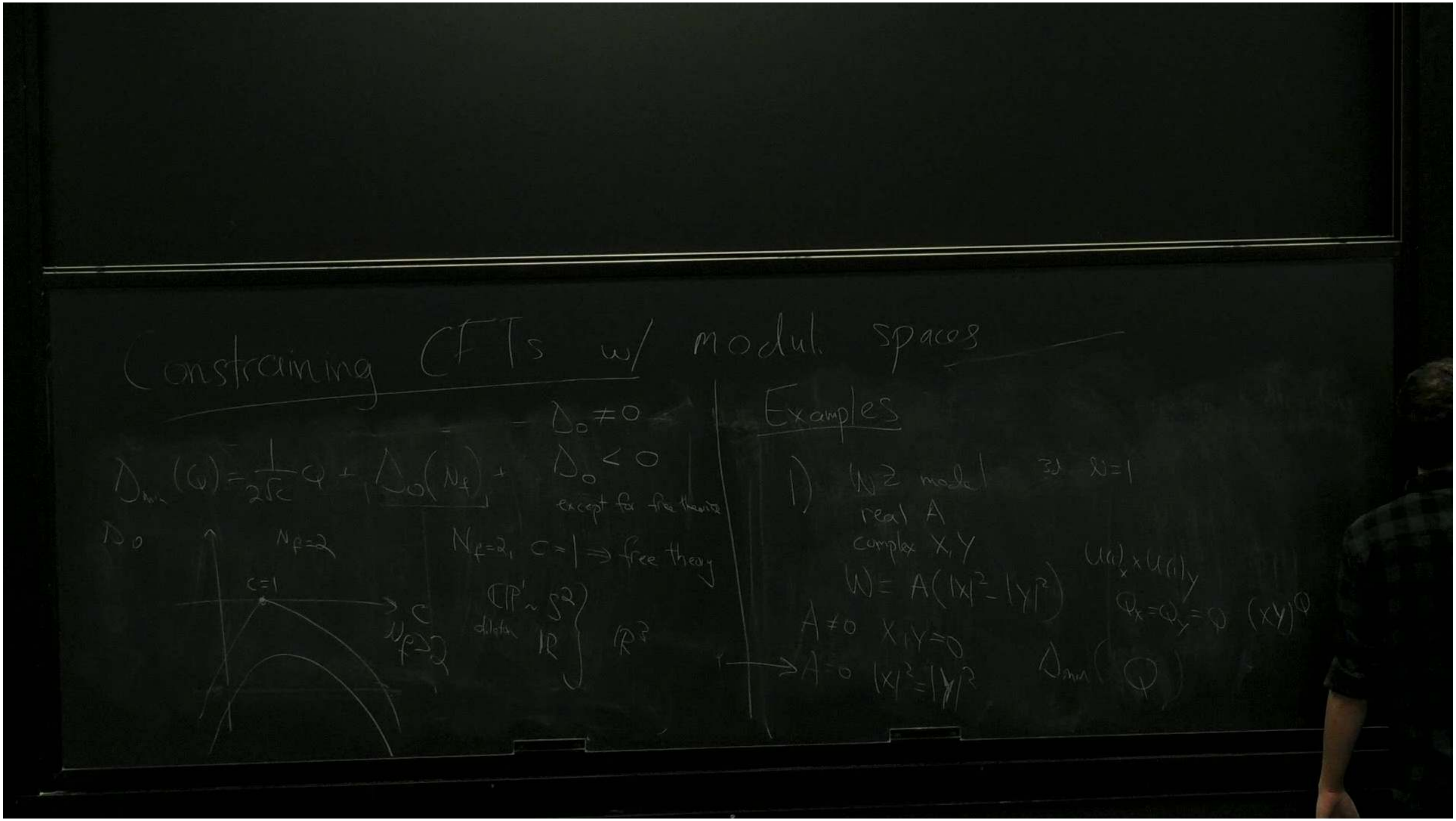
$E_{gr}(Q)$

$\Phi = v + \phi$ $\quad z_1 \to e^{i\theta} z_1$

$z_1 = e^{i\frac{1}{2c}t} + \tilde{z}_1$ $\quad Q_{2v^2} \Rightarrow$

$z_{D_i} = 0$

$E_{gr}(Q) = \frac{1}{2\sqrt{c}} Q + \ldots$


Constraining CFTs w/ moduli spaces
$\Delta_0 \neq 0$
$\Delta_0 < 0$
$\Delta_{min}(Q) = \frac{1}{2\sqrt{c}} Q + \Delta_0(N_f) + \ldots$
except for free theories
Δ_0
$N_f = 2$
$c=1$
c
$N_f > 2$
$N_f = 2,\ c = 1 \Rightarrow$ free theory
$\mathbb{CP}^1 \sim S^2$
dilaton $\mathbb{R}$
$\mathbb{R}^3$
Examples
1) $\mathcal{N}=2$ model
real A
complex X, Y
$W = A(|X|^2 - |Y|^2)$
$A \neq 0 \quad X, Y = 0$
$A = 0 \quad |X|^2 = |Y|^2$
$U(1)_X \times U(1)_Y$
$Q_X = Q_Y = Q \quad (XY)^Q$
$\Delta_{min}(Q)$

ε - expansion

$d = 4 - \varepsilon$

$g^2 \sim \varepsilon$

obstruction to 3d $\mathcal{N}=1$ in ε-expansion

3d

M_{a_j} #dof = 2

4d

M_{a_j} #dof = 4

trick

[Klebanov, Fei, Tarnopolsky, Giombi]

$$\boxed{\# \psi = N_f \longrightarrow N_f = 1/2} \quad \checkmark$$

$$\Delta_{min}(Q) = \frac{1}{\varepsilon} \Delta_{-1}(\varepsilon Q) + \varepsilon^0 \Delta_0(\varepsilon Q) + \varepsilon \Delta_1(\varepsilon Q) + \ldots$$

[Rattazzi, (Cuomo, Magan, Badel)]

$$\Delta_{min}(Q) = \underline{\left(2 - \frac{11\varepsilon}{Q} + O(\varepsilon^2)\right) Q} + \left(-\frac{1}{3d} + O(\varepsilon) \ldots\right) \log Q + \ldots$$

Constraining CFTs w/ modul. spaces

Application

Charge convexity conjecture [Aharony, Palti]

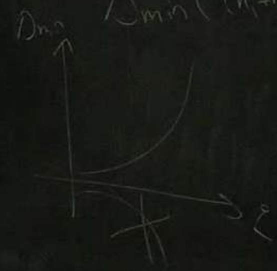
WGC $\exists$ particle $m < q$ ← U(1) gauge sym

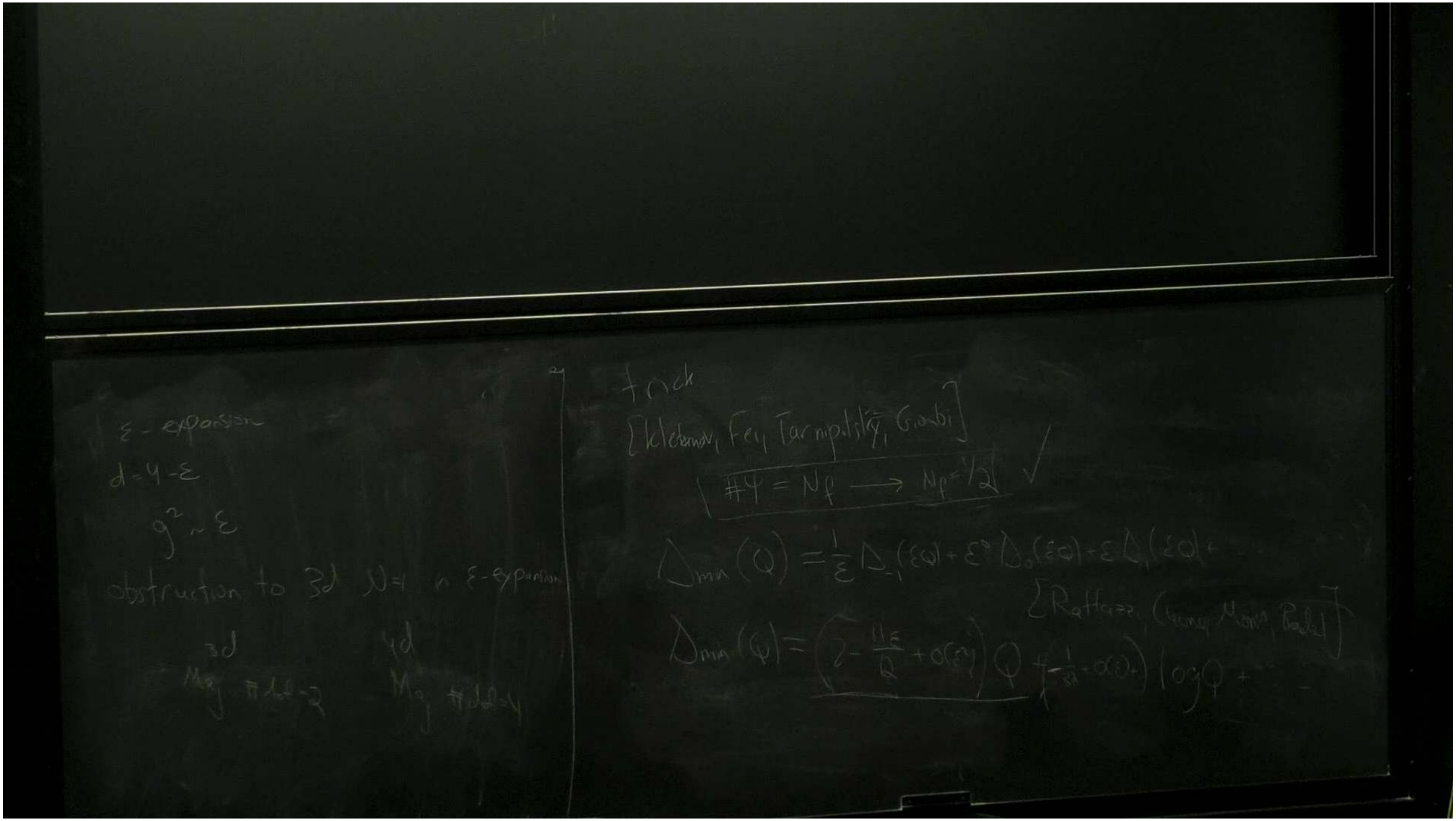
AdS WGC $\Rightarrow$ CFT?

constraint:

AdS WGC $\Longrightarrow$ $\exists\, q_0 \sim O(1)$ s.t.

$$\Delta_{min}\left((n_1+n_2)q_0\right) \geq \Delta_{min}(n_1 q_0) + \Delta_{min}(n_2 q_0)$$




ε - expansion
d = 4 - ε
g² ~ ε
obstruction to 3d N=1 in ε-expansion
3d
4d
trick
#ψ = Nf ⟶ Nf = 1/2
Δmin (Q) = 1/ε Δ-1 (εQ) + ε⁰ Δ0 (εQ) + ε Δ1 (εQ) + ...
Δmin (Q) = (2 - 11ε/Q + O(ε²)) Q + (- 1/3... + O(ε) +) log Q + ...

Constraining CFTs w/ modul. spaces

Application

Charge convexity conjecture [Aharony, Palti]

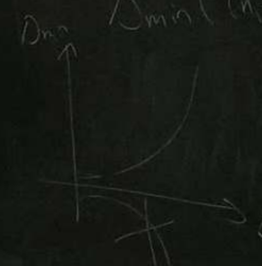
WGC: $\exists$ particle $m < q$ — U(1) gauge sym.

AdS WGC $\Rightarrow$ CFT?

AdS WGC $\Longrightarrow$ $\exists\, q_0$ s.t.

constraint:

$$\Delta_{min}((n_1+n_2)q_0) \geq \Delta_{min}(n_1 q_0) + \Delta_{min}(n_2 q_0)$$

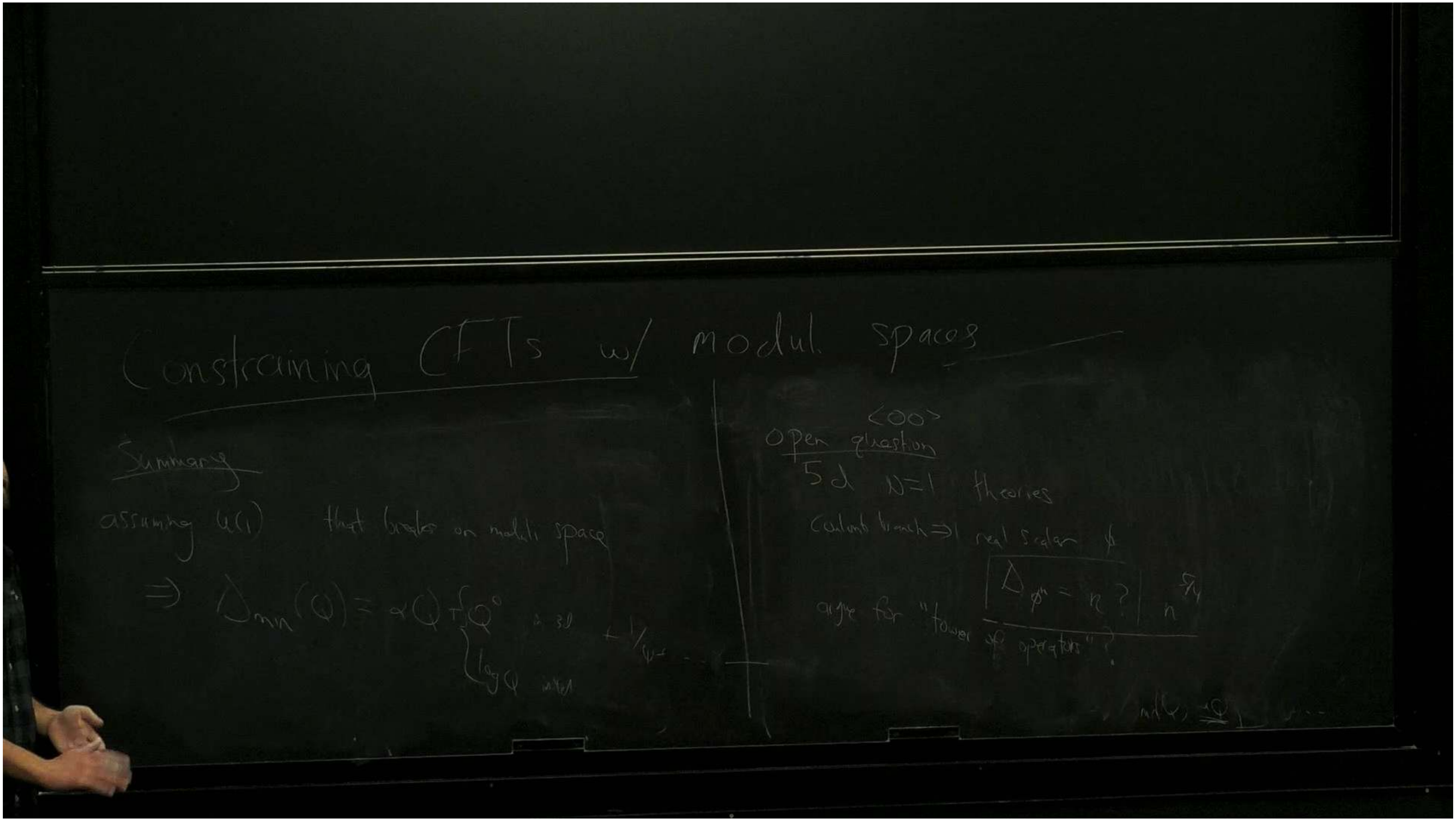


large charge

1) $\Delta_{min} \sim Q^{d/d-1}$

2) $\Delta_{min} = Q$

3) $\Delta_{min}(Q) = Q + \boxed{Q^{?}} + \ldots$


Constraining CFTs w/ moduli spaces
Summary
assuming U(1) that breaks on moduli space
$\Rightarrow \Delta_{min}(Q) = \alpha Q + \beta Q^0$
$\log Q$
$\langle OO \rangle$
Open question
5d N=1 theories
Coulomb branch ⇒ 1 real scalar ϕ
$\boxed{\Delta_{\phi^n} = n\,?}$ $n^{5/4}$
argue for "tower of operators"?

Constraining CFTs w/ moduli spaces

Summary

assuming $U(1)$ that breaks on moduli space

$\Rightarrow \Delta_{min}(Q) = \alpha Q + \begin{cases} Q^0 & \text{in } 3d \\ \log Q & \text{in } 4d \end{cases} + 1/Q + \ldots$

$\langle O \rangle_{\ldots} \sim v^{\Delta_0} \sim M^Q$

$\langle OO \rangle$

Open question

5d $N=1$ theories

Coulomb branch $\Rightarrow$ 1 real scalar ϕ

$\boxed{\Delta_{\phi^n} = n\,?}$ $\quad n^{5/4}$

argue for "tower of operators"?