

Title: Constraining CFTs with moduli spaces

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Abstract: Moduli spaces of vacua are an intriguing property of certain supersymmetric QFTs which have been widely explored. However, a first-principles approach to moduli spaces and how they constrain observables is still lacking. This question is even more pressing due to recent interest in moduli spaces in theories with only two supercharges, where supersymmetry is extremely weak and does not allow for exact computations. In this talk we attempt to bootstrap conformal field theories with moduli spaces. First we assume an additional global symmetry which is spontaneously broken along the moduli space, and use techniques from the large charge expansion to show that the existence of a moduli space directly constrains CFT data of charged operators. We then study the generic case by using a "moduli space bootstrap equation" to write down perturbative sum rules on observables of CFTs order-by-order in a small coupling. We discuss several examples and applications of our results.

Zoom link

Constraining CFTs w/ modul. spaces

w/ L. Rastelli and G. Cuomo, to appear

"Bootstrap" moduli spaces

Q How does the existence of a moduli space constrain CFT data?

- 1) assume $U(1)$ sym which is spontaneously broken on moduli space
- 2) generic

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"Bootstrap" moduli spaces

Q How does the existence of a moduli space constrain CFT data?

- 1) assume $U(1)$ sym which is spontaneously broken on moduli space
- 2) generic

Susy

mod space $\#Q=4 \Rightarrow \Delta_{X^n} = n \Delta_X$

3d N=2 susy }
4d N=1 susy }

$\#Q=4$

conjecture moduli space \cong chiral ring
SFT always have $U(1)_R$ sym \Rightarrow broken on moduli space

directions on moduli space \rightarrow chiral operators

$W = X^k$
 $X^{k+1} = 0$

Example

3d N=2 XYZ model $(XY)^2 + (YZ)^2 + (XZ)^2 = 0$

$W = XYZ \Rightarrow V = \sum \frac{1}{2} (\partial W)^2$
F-term eqs: $\frac{\partial W}{\partial X} = \frac{\partial W}{\partial Y} = \frac{\partial W}{\partial Z} = 0$
 $XY = YZ = XZ = 0$

$X \neq 0$
or $Y \neq 0$ or $Z \neq 0$

$\Delta_{X^n} = n \Delta_X$
 $\Delta_{X^k} = k \Delta_X$

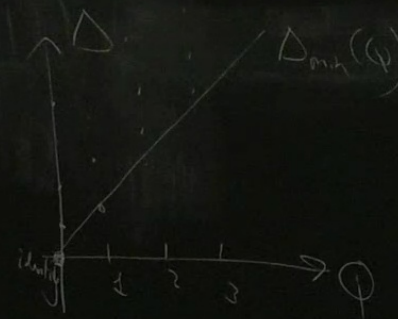
Constraining CFTs w/ modul. spaces

moduli space \Rightarrow $\Delta_{\text{min}}(\varphi) = \alpha \varphi$?

define $\Delta_{\text{min}}(\varphi) \equiv$ min dim of all
 φ charge of $U(1)$ ops of charge Q

large charge expansion $\varphi \gg 1$

- 1) "generic" $\Delta_{\text{min}}(\varphi) \sim \varphi^{d/d-1}$
- 2) free/BPS $\Delta_{\text{min}}(\varphi) = \varphi$



3 d N=1 theories

$$\int d^2\theta P(A, B, C)$$

$$\left[\int d^2\theta ABC P(A^2, B^2, C^2) \right]$$

$A \neq 0$
 \mathbb{Z}_2^R sym \Rightarrow exact moduli space

$$\mathcal{L} = \int d^2\theta K + \int d^2\theta W$$

$$\int d^2\theta \sqrt{\frac{K}{D^2}} + \int d^2\theta \sqrt{M\Phi^2 + \lambda\Phi^3}$$

$$\mathcal{L} \ni \int d^2\theta ABC$$

\mathbb{Z}_2^R -sym
 $d^2\theta \rightarrow -d^2\theta$
 $d^2\theta \rightarrow -d^2\theta$

$A \rightarrow -A$
 $B \rightarrow -B$
 $C \rightarrow -C$

F-term eqs
 $AB = BC = AC = 0$
 \Rightarrow moduli space
 $A=0$
 or
 $B=0$
 or
 $C=0$

Constraining CFTs w/ modul. spaces

[Hullman, Watanabe, Mochizuki]

$\Delta_{mn}(Q)$ assuming moduli space

method:

- 1) effective action on moduli space
- 2) put on cylinder, and compute gr energy at large Q
- 3) state-op correspondence $g_s = \Delta_{mn}(Q)$

trick:

$$\begin{aligned} & \Phi, \pi, g_{\mu\nu} \rightarrow \pi, \hat{g}_{\mu\nu} \\ & \Phi \rightarrow e^{i\sigma} \Phi \\ & g_{\mu\nu} \rightarrow e^{2\sigma} g_{\mu\nu} \end{aligned}$$

1) $\int \Phi = 1$

2) $\Phi \rightarrow 1$

$g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = g_{\mu\nu} e^{2\sigma}$

a) expansion in derivatives

b) syms:

1) dilation Φ

2) NGB π from $U(1)_{g_m}$

$\pi \rightarrow \pi + \alpha \rightarrow \partial\pi$

Weyl transformations

$$d^0: \int \sqrt{g} \lambda dx^3 = \int d^3x \sqrt{g} \lambda \xrightarrow{\text{moduli space}} \mathbb{C}^6$$

$$d^2: \int \sqrt{-g} d^3x g^{\mu\nu} \partial_\mu R \partial_\nu R + \int \sqrt{-g} \hat{R} = \int \sqrt{g} (p^2 \phi^2 + (\partial\phi)^2 + R\phi^2)$$

$$\mathcal{L} = \int d^2\theta K + \int d^2\theta W$$

$$\int d^2\theta \sqrt{\frac{K}{2}} \Phi D^2 \Phi + \int d^3\theta \sqrt{m\Phi^2 + \lambda\Phi^3}$$

$$\mathcal{L} \ni \int d^3\theta ABC$$

\mathbb{Z}_2^R -sym

$$d^2\theta \rightarrow -d^2\theta, A \rightarrow A$$

$$d^3\theta \rightarrow -d^3\theta, B \rightarrow -B$$

$$C \rightarrow -C$$

F-term eqs
 $AB = BC = AC = 0$

\Rightarrow moduli space
 or
 $A=0$
 or
 $B=0$
 or
 $C=0$

$$\partial^0: \int \sqrt{g} \lambda^3 = \int d^3x \sqrt{g} \phi \xrightarrow{\text{moduli space}} G$$

$$\partial^2: \int \sqrt{-g} d^3x g^{mn} \partial_m R \partial_n R + \int \sqrt{g} \hat{R} = \int \sqrt{g} (\dot{\phi}^2 (\partial \alpha)^2 + (\partial \phi)^2 + R \phi^2)$$

2) find $E_{gr}(Q)$

"helical sol" $\phi = v$
 $\pi = \frac{1}{2} t$

$$Q \rightarrow J_m = \frac{2J}{2(\partial_m \pi)} = \phi^2 \partial_m \pi \Rightarrow P = \phi^2 \partial_t \pi \sim v^2$$

$$E \rightarrow E \sim \dot{\phi}^2 \sim v^2$$

$$\Rightarrow E_{gr}(Q) = \alpha Q$$

Constraining CFTs w/ modul. spaces

$$\mathcal{D}^2 \quad \Delta_{\text{min}}(\phi) = \alpha Q + Q^0 + 1/Q + 1/Q^2 + \dots$$

$$\mathcal{D}^4: \quad \phi^2 \mathcal{D}^4 \sim v^{-2} \sim 1/Q$$

$$\mathcal{D}^6$$

$$1/Q^2$$

quantum corrections

$$Q^0 = 1/Q + 1/Q^2 + \dots$$

Hoop Casimir energy

4d

$$\Delta_{\text{min}}(\phi) = \alpha Q + \log Q + Q^0 + \dots$$

Constraining CFTs w/ modul. spaces

$$d^2 \quad \Delta_{\text{min}}(\phi) = \alpha Q + Q^0 + 1/Q + 1/Q^2 + \dots$$

$$d^4: \quad \phi^2 \sim v^2 \sim 1/Q$$

$$d^6: \quad 1/Q^2$$

quantum corrections

$$Q^0 = 1/Q + 1/Q^2 + \dots$$

Hoop Casimir energy

Examples

1) 3d N=1 SQED

A_μ, λ

+ N_f ϕ_i, ψ_i $i=1, \dots, N_f$
 complex Dirac

$$\mathcal{L} = (\text{kinetic}) + \bar{\psi}_i \phi_i \lambda + \text{c.c.}$$

$\langle \phi_i \rangle \neq 0$

$W(\Phi_i)$

$$\int d^3 \mathcal{D} \Phi_i W(\Phi_i^2)$$

$\rightarrow \text{JTB}$
 $\Rightarrow \langle \phi_i \rangle \neq 0$

dim $2N_F - 1$

massless def:

$SU(N_F) \rightarrow S[U(N_F + 1) \times U(1)]$
 dilaton Φ
 NGBs π

$$[IP^{N_F-1}] = \frac{SU(N_F)}{S[U(N_F+1) \times U(1)]} \quad 2(N_F-1)$$

dilaton + NGBs = $2N_F - 1$

$$(\partial\Phi)^2 + R\Phi^2 + 2c\Phi^2 \sum_j \partial z_j \partial \bar{z}_j$$

$[IP^{N_F-1}]$ NLSM

z_j : complex $j=1, \dots, N_F-1$
 r_j : Fubini-Study metric

$E_{gr}(\Phi)$

$$\Phi = v - \phi \rightarrow e^{i\theta} z_1$$

$$z_1 = e^{i\theta} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

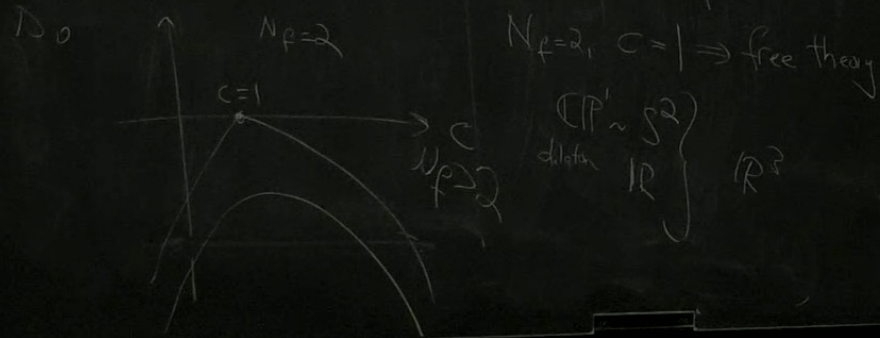
$$z_{N_F} = 0$$

$$E_{gr}(\Phi) = \frac{1}{2f^2} \Phi^2$$

Constraining CFTs w/ modul. spaces

$$D_{\text{min}}(\mathcal{G}) = \frac{1}{2R} Q + \Delta_0(N_f) + \begin{cases} \Delta_0 \neq 0 \\ \Delta_0 < 0 \end{cases}$$

except for free theories



Examples

- 1) $N \geq 2$ model $\Rightarrow N=1$
 real A
 complex X, Y

$$W = A(|X|^2 - |Y|^2)$$

$$A \neq 0 \quad X=0, Y=0$$

$$\rightarrow A=0 \quad |X|^2 = |Y|^2$$

$$U(1)_X \times U(1)_Y$$

$$Q_X = Q_Y = 0 \quad (XY)^0$$

$$D_{\text{min}}(\mathcal{G})$$

ϵ -expansion

$$d = 4 - \epsilon$$

$$g^2 \sim \epsilon$$

obstruction to 3d $N=1$ in ϵ -expansion

3d

$$M_g \neq 1, 2$$

4d

$$M_g \neq 1, 2, 4$$

trick

[Klebanov, Fei, Tarapitskiy, Gruber]

$$\#4 = N_f \rightarrow N_f = 1/2 \quad \checkmark$$

$$\Delta_{\text{min}}(Q) = \frac{1}{\epsilon} \Delta_{-1}(\epsilon Q) + \epsilon^0 \Delta_0(\epsilon Q) + \epsilon^1 \Delta_1(\epsilon Q) + \dots$$

[Rattazzi, (Luna, Marzoni, Padoa-Schioppa)]

$$\Delta_{\text{min}}(Q) = \left(2 - \frac{11\epsilon}{2} + o(\epsilon^2) \right) Q + \left(\frac{1}{2} + o(\epsilon) \right) \log Q + \dots$$

Constraining CFTs w/ modul. spaces

Application

Charge convexity conjecture [Aharony, Palti]

WGC

\exists particle $m < q$ with gauge sym.

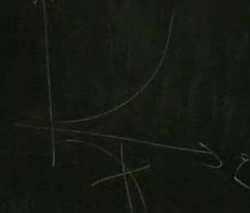
AdS WGC \Rightarrow CFT?

AdS
WGC

$\Rightarrow \exists q_0 \sim O(1)$ st.

constraint:

$$\Delta_{\min}(n_1 + n_2 q_0) \geq \Delta_{\min}(n_1, q_0) + \Delta_{\min}(n_2, q_0)$$



ϵ -expansion

$$d = 4 - \epsilon$$

$$g^2 \sim \epsilon$$

obstruction to 3d $N=1$ in ϵ -expansion

3d

$$M_g \neq 1/2 - 2$$

4d

$$M_g \neq 1/2 - 4$$

trick

[Klebanov, Fer, Tarapitskiy, Gruber]

$$\# \Psi = N_f \rightarrow N_f = 1/2 \quad \checkmark$$

$$\Delta_{\text{min}}(\phi) = \frac{1}{\epsilon} \Delta_1(\epsilon\phi) + \epsilon^0 \Delta_0(\epsilon\phi) + \epsilon \Delta_1(\epsilon\phi) + \dots$$

[Rattazzi, (Konec, Mores, Peneder)]

$$\Delta_{\text{min}}(\phi) = \left(2 - \frac{11\epsilon}{2} + o(\epsilon^2) \right) \phi + \left(\frac{1}{2} + o(\epsilon) \right) \log \phi + \dots$$

Constraining CFTs w/ modul. spaces

Application

Charge convexity conjecture [Aharony, Palti]

WGC

\exists particle $m < q$ (w/ gauge sym)

AdS WGC \Rightarrow CFT?

AdS
WGC

$$\Rightarrow \exists q_0$$

constraint:

s.t.

$$\Delta_{\min}(n, n_1 q_0) \geq \Delta_{\min}(n_1, q_0) + \Delta_{\min}(n_2, q_0)$$



large charge

1) $\Delta_{\min} \sim Q^{d/d-1}$

2) $\Delta_{\min} = Q$

3) $\Delta_{\min}(Q) \sim Q^{d/d-1}$

Constraining CFTs w/ modul. spaces

Summary

assuming $U(1)$ that breaks on moduli space

$$\Rightarrow \Delta_{\text{min}}(Q) = \alpha(Q) + \log Q \quad \text{in 3d} \quad + \frac{1}{d-2}$$

$\left\{ \begin{array}{l} \log Q \text{ term} \end{array} \right.$

Open $\langle 00 \rangle$
gluon

5d $N=1$ theories

Coulomb branch \Rightarrow real 5d dim ϕ

$$\Delta_{\phi^n} = n? \quad n \in \mathbb{Z}_4$$

argue for "tower of operators"?

with ϕ

Constraining CFTs w/ modul. spaces

Summary

assuming $u(1)$ that breaks on moduli space

$$\Rightarrow \langle \mathcal{O} \rangle_{\text{min}} = \alpha Q + \beta Q^0 \quad \text{in 3d} \quad + \frac{1}{\nu} \dots$$

$\left\{ \begin{array}{l} \log Q \text{ and} \\ \dots \end{array} \right.$

Open questions

5d $\mathcal{N}=1$ theories

Coulomb branch \Rightarrow real S-dual ϕ

$$\Delta_{\phi^n} = n? \quad \text{in } \mathcal{F}_4$$

argue for "tower of operators"?