

Title: An Efficient Quantum Algorithm for Port-based Teleportation

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Abstract: In this talk, we will outline an efficient algorithm for port-based teleportation, a unitarily equivariant version of teleportation useful for constructing programmable quantum processors and performing instantaneous nonlocal computation (NLQC). The latter connection is important in AdS/CFT, where bulk computations are realized as boundary NLQC. Our algorithm yields an exponential improvement to the known relationship between the amount of entanglement available and the complexity of the nonlocal part of any unitary that can be implemented using NLQC. Similarly, our algorithm provides the first nontrivial efficient algorithm for an approximate universal programmable quantum processor.

The key to our approach is a general quantum algorithm we develop for block diagonalizing so-called generalized induced representations, a novel type of representation that arises from lifting a representation of a subgroup to one for the whole group while relaxing a linear independence condition from the standard definition. Generalized induced representations appear naturally in quantum information, notably in generalizations of Schur-Weyl duality. For the case of port-based teleportation, we apply this framework to develop an efficient twisted Schur transform for transforming to a subgroup-reduced irrep basis of the partially transposed permutation algebra, whose dual is the $U^{n-k} \otimes (U^*)^k$ representation of the unitary group.

Zoom link

An Efficient Quantum Algorithm for Port-based Teleportation

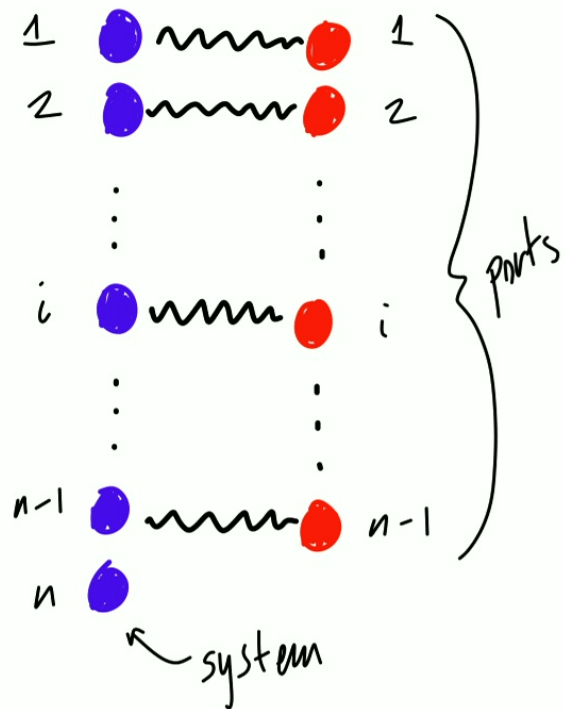
Jiani Fei and Sydney Timmerman, joint with Patrick Hayden
Stanford University

April 3, 2024

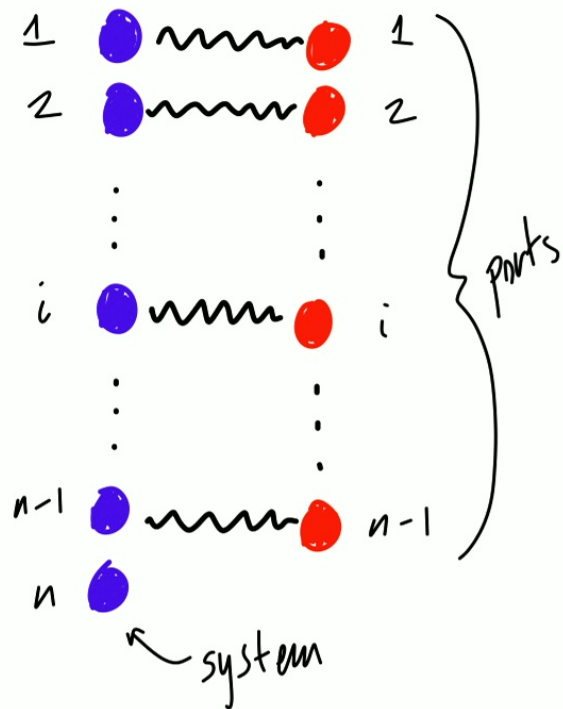
Outline

- Port-based teleportation: what it is and why we care
- Symmetries of PBT with the pretty good measurement
- Connections to more general rep-theoretic tasks
 - Schur-Weyl duality
 - generalized induced reps
- Block-diagonalizing homomorphic images of reps
- Efficient algorithm for PBT
- Further applications

Port-based teleportation (PBT)



Port-based teleportation (PBT)



Protocol:

1. Alice performs a joint POVM $\{\Pi_i\}_{i=1}^{n-1}$ on the system and her ports and receives outcome i
2. Alice sends i to Bob
3. Bob throws away all of his ports except i , which now holds the approximately teleported state

Why is PBT interesting?

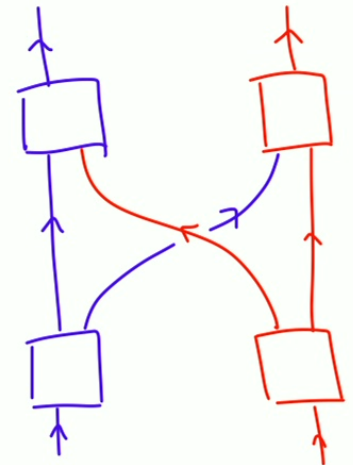
- Why use so much entanglement to achieve only approximate teleportation?
- PBT achieves **unitary equivariance**

Bob can perform a unitary operation U on the teleported state before or after he learns which port it ends up on



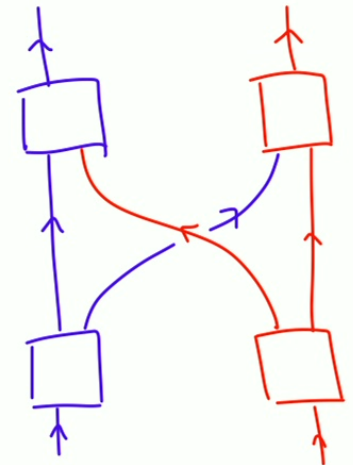
PBT and non-local quantum computation

- The unitary equivariance of PBT makes it a key subroutine for **NLQC**
- **NLQC**: spacelike-separated Alice and Bob perform a joint unitary on their quantum systems using local unitaries and a single round of communication
- [May '19, '21]: in AdS/CFT, local quantum computations in the bulk correspond to NLQCs on the boundary
- [May '22] bounds the entanglement consumed in an NLQC protocol above and below by the complexity of the unitary accomplished



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- ▶ Efficient PBT improves the gap from triply to doubly exponential



Symmetries of the PGM

$$\{\Pi_i = \rho^{-1/2} \rho_i \rho^{-1/2} + \Delta\}_{i=1}^{n-1} \quad \rho_i = |\phi_+\rangle\langle\phi_+|_{in} \otimes \frac{I_{\bar{i}}}{d^{n-2}} \quad \rho = \sum_{i=1}^{n-1} \rho_i$$

$$|\phi_+\rangle\langle\phi_+|_{in} = \frac{1}{d} \begin{array}{c} i \quad n \\ \frown \\ \smile \end{array}$$

$$(U_i \otimes U_n^*) |\phi_+\rangle\langle\phi_+|_{in} (U_i \otimes U_n)^\dagger = \frac{1}{d} \begin{array}{c} i \quad n \\ | \quad | \\ \mathcal{U} \quad \mathcal{U}^\dagger \\ \smile \\ \mathcal{U}^\dagger \quad \mathcal{U} \\ | \quad | \end{array} = \frac{1}{d} \begin{array}{c} i \quad n \\ | \quad | \\ \mathcal{U} \quad \mathcal{U}^\dagger \\ \frown \\ \mathcal{U}^\dagger \quad \mathcal{U} \\ | \quad | \end{array} = \frac{1}{d} \begin{array}{c} i \quad n \\ \frown \\ \smile \end{array}$$

$\implies \rho$ and ρ_i have a partially conjugated unitary symmetry

$$[\rho_i, U^{\otimes n-1} \otimes U^*] = 0, \quad [\rho, U^{\otimes n-1} \otimes U^*] = 0$$

Symmetries of the PGM

$$\{\Pi_i = \rho^{-1/2} \rho_i \rho^{-1/2} + \Delta\}_{i=1}^{n-1} \quad \rho_i = |\phi_+\rangle\langle\phi_+|_{in} \otimes \frac{I_i}{d^{n-2}} \quad \rho = \sum_{i=1}^{n-1} \rho_i$$

$$|\phi_+\rangle\langle\phi_+|_{in} = \frac{1}{d} \begin{matrix} i & n \\ \frown & \smile \\ & \end{matrix} = \frac{1}{d} \begin{matrix} i & n \\ \frown & \smile \\ \smile & \frown \\ & \end{matrix} = V[(i \ n)]^{t_n}$$

$\implies \rho, \rho_i$ are elements of the algebra of partially transposed permutation operators $A_n^{t_n}(d) \equiv \text{span}_{\mathbb{C}}\{V[\sigma]^{t_n} : \sigma \in S_n\}$

Actually, this statement is equivalent to the partially conjugated unitary symmetry!

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Standard Schur-Weyl duality

- ▶ The algebra of permutation operators $\mathbb{C}V[S(n)]$ and the algebra $\mathbb{C}[U(d)^{\otimes n}]$

$$V(\sigma)|i_1\rangle \otimes \dots \otimes |i_n\rangle = |i_{\sigma^{-1}(1)}\rangle \otimes \dots \otimes |i_{\sigma^{-1}(n)}\rangle \quad U^{\otimes n}|i_1\rangle \otimes \dots \otimes |i_n\rangle = U|i_1\rangle \otimes \dots \otimes U|i_n\rangle$$

are each other's **commutants** in $(\mathbb{C}^d)^{\otimes n}$


- ▶ As a consequence, the Hilbert space enjoys a multiplicity-free decomposition into irreps of $S(n)$ and $U(d)$

$$(\mathbb{C}^d)^{\otimes n} \cong \bigoplus_{\lambda \vdash n, h(\lambda) \leq d} \mathcal{P}_\lambda \otimes \mathcal{Q}_\lambda^d$$

Twisted Schur-Weyl duality

- ▶ The algebra of partially transported permutation operators $A_n^{t_n}(d)$ and the algebra $\mathbb{C}[U(d)^{\otimes n-1} \otimes U(d)^*]$ are each other's **commutants** in $(\mathbb{C}^d)^{\otimes n}$
- ▶ As a result, the Hilbert space enjoys an analogous multiplicity-free decomposition into irreps of $A_n^{t_n}(d)$ and $U(d)$

$$(\mathbb{C}^d)^{\otimes n} \cong \left(\bigoplus_{\alpha \vdash n-2, h(\alpha) \leq d} \mathcal{H}_\alpha \otimes \mathcal{Q}_\alpha^d \right) \oplus \mathcal{H}_S$$


 irreps of $A_n^{t_n}(d)$

Irreps of $A_n^{t_n}(d)$

- Elements of $A_n^{t_n}(d)$ that permute and transpose the last qudit are only represented nontrivially on

$$\text{span}_{\mathbb{C}}\{ |i\rangle_{\overline{kn}} |\phi_+\rangle_{kn} : k \in [1, n), i \in [1, d^{n-2}] \}$$

computational basis state

$$= \text{span}_{\mathbb{C}}\{ V[(k \ n - 1)] |i\rangle |\phi_+\rangle_{n-1n} : k, i \}$$

- The irreps are labeled by $\alpha \vdash n - 2$

$$\mathcal{H}_\alpha = \text{span}_{\mathbb{C}}\{ \underbrace{V[(k \ n - 1)]}_{\text{transversal of } S(n-1)/S(n-2)} \underbrace{|\alpha, r, k_\alpha\rangle^{n-2}}_{\text{irrep basis of } S(n-2)} |\phi_+\rangle : k \in [1, n), k_\alpha \in [1, d_\alpha] \}$$

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Generalized induced rep

- Suppose we have
- a rep (ρ, U) of the group G that we call the **parent rep**
 - a subgroup $H \subset G$
 - the restriction $(\rho|_H, V) \subseteq (\rho, U)$ that we call the **base rep**

When acting by $h \in H$, $\rho_h : V \rightarrow V$
by $g \in G$ $\rho_g : V \rightarrow U$

\implies we can define a new, interesting representation of G

$$V \uparrow_H^\rho G = \text{span}_{\mathbb{C}} \{ \rho_g(v) : g \in G, v \in V \}$$

Big idea: we want to lift the base rep to a rep of G ; the parent rep dictates how G acts on the base rep

Generalized induced rep

- ▶ **parent rep:** (ρ, U) of G
 - ▶ **base rep:** $(\rho|_H, V) \subseteq (\rho, U)$ of $H \subset G$
- $$V \uparrow_H^\rho G = \text{span}_{\mathbb{C}}\{\rho_g(v) : g \in G, v \in V\}$$

Note, for generic $g \in G$, $g = \tau h$ for some transversal $\tau \in G/H$, $h \in H$

$$\implies \rho_g(v) = \rho_\tau(\rho_h(v)) = \rho_\tau(v') \quad \text{where } v, v' \in V$$

Rewriting as a sum of subspaces,
$$V \uparrow_H^\rho G = \sum_{\tau \in G/H} \rho_\tau(V)$$

When this sum is direct, we have a true induced rep!

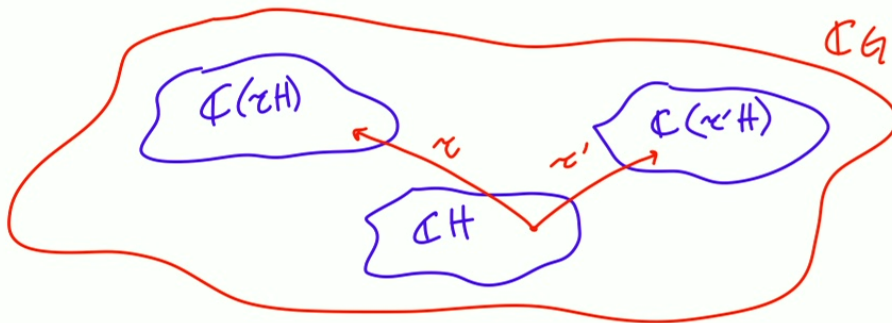
Example (regular representation)

Parent rep: left regular representation of a group G ($\rho, U \cong \mathbb{C}G$)

Base rep: for a subgroup $H \subset G$, choose an irrep $V \subset \mathbb{C}H$

What is the action of G on V ? $v \in V = \sum_{h \in H} c_h h \implies \rho_\tau(v) = \sum_{h \in H} c_h \tau h \in \mathbb{C}(\tau H)$

For $\tau \neq \tau'$, $\rho_\tau(V) \subset \mathbb{C}(\tau H)$ and $\rho_{\tau'}(V) \subset \mathbb{C}(\tau' H)$ will be disjoint in $\mathbb{C}G$!



$$\implies V \uparrow_H^\rho G = \sum_{\tau \in G/H} \rho_\tau(V) = \bigoplus_{\tau \in G/H} \rho_\tau(V)$$

Example (three qubits)

Parent rep: Hilbert space of three qubits $U \cong (\mathbb{C})^{\otimes 3}$

$G \cong S(3)$ acts by permuting the qubits

Base rep: $H = S(2)$ permutes the first two qubits, V is the sign rep

$$V \cong \text{span}_{\mathbb{C}}\{v = |010\rangle - |100\rangle\}$$

$$\rho_e(v) = |010\rangle - |100\rangle$$

$$\rho_{(23)}(v) = |001\rangle - |100\rangle$$

$$\rho_{(13)}(v) = |010\rangle - |001\rangle$$

$$\implies \rho_{(13)} = \rho_e - \rho_{(23)}$$

$$\implies V \uparrow_H^\rho G = \rho_e V + \rho_{(23)} V + \rho_{(13)} V$$

is 2d instead of 3d!

The action of the transversals on the third qubit can't be fully captured!

Example (irreps of $A_n^{t_n}(d)$)

Parent rep: Hilbert space of n qudits $U \cong (\mathbb{C}^d)^{\otimes n}$

$G \cong S(n-1)$ acts by permuting the first $n-1$ qudits

Base rep: $H = S(n-2)$ permutes the first $n-2$ qubits, $V_\alpha = \mathcal{P}_\alpha \otimes |\phi_+\rangle$

$$V_\alpha = \text{span}_{\mathbb{C}}\{|\alpha, r, k_\alpha\rangle^{n-2} |\phi_+\rangle : k_\alpha \in [1, d_\alpha]\}$$

The generalized induced rep is then

$$V_\alpha \uparrow_{S(n-2)}^\rho S(n-1) = \text{span}_{\mathbb{C}}\{V[(k \ n-1)] |\alpha, r, k_\alpha\rangle^{n-2} |\phi_+\rangle : k \in [1, n), k_\alpha \in [1, d_\alpha]\}$$

Recap so far

- We want to find an efficient algorithm for PBT with the PGM

$$\{\Pi_i = \rho^{-1/2} \rho_i \rho^{-1/2} + \Delta\}_{i=1}^{n-1} \quad \rho_i = |\phi_+\rangle\langle\phi_+|_{in} \otimes \frac{\mathbb{1}_i}{d^{n-2}} \quad \rho = \sum_{i=1}^{n-1} \rho_i$$

- The PBT operator ρ and states ρ_i are elements of $A_n^{t_n}(d)$
- The irreps of $A_n^{t_n}(d)$ where ρ, ρ_i are represented nontrivially are generalized induced reps

$$\mathcal{H}_\alpha = V_\alpha \uparrow_{S(n-2)}^\rho S(n-1)$$

Algorithmic strategy

Because each \mathcal{H}_α is also a rep of $S(n-1)$, it can be reduced into irreps of $S(n-1)$

$$\mathcal{H}_\alpha \cong \bigoplus_{\mu=\alpha+\square, h(\mu)\leq d} \mathcal{P}_\mu$$

this is a multiplicity-free decomposition!

\implies operators with an $S(n-1)$ symmetry, like ρ are diagonal in this basis

Strategy:

1. transform to an irrep basis of $A_n^t(d)$
2. further decompose each generalized induced rep \mathcal{H}_α into irreps of $S(n-1)$

i.e. perform a subgroup-reduced **twisted Schur transform!**

Efficient port-based teleportation

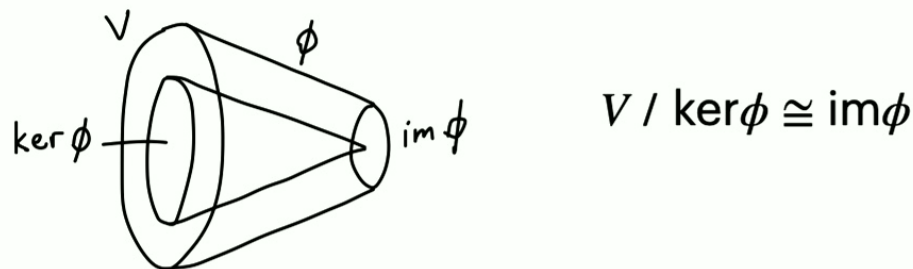
- Performing the twisted Schur transform diagonalizes ρ and simplifies ρ_i , making it possible to perform PBT in $\text{poly}(n, d)$ time
- **Independent work:** [Ngyuen '23] (efficient mixed Schur transform)
[Grinko, Burchardt, Ozols '23] (efficient PBT)
- **Our perspective:** performing the twisted Schur transform from the $A_n^{t_n}(d)$ side of the duality is a *special case* of the *general problem* of block-diagonalizing generalized induced representations

Generalized induced reps are homomorphic images of induced reps!

Linear representation of a group: $(\rho : G \rightarrow GL(U), U) \quad g \cdot u \equiv \rho_g(u)$

Module (representation) homomorphism

linear map $\phi : V \rightarrow W$ such that $v \mapsto w \implies g \cdot v \mapsto g \cdot w$



e.g. $\phi : \bigoplus_{\tau} \tau \cdot V \rightarrow V \uparrow_H^{\rho} G = \sum_{\tau} \rho_{\tau}(V) \quad \forall \rho \quad \phi \text{ surjective homomorphism!}$

$\tau \cdot v \mapsto \rho_{\tau}(v) \implies g_1(\tau \cdot v) = (g_1\tau) \cdot v \mapsto \rho_{g_1\tau}(v) = \rho_{g_1}(\rho_{\tau}(v))$

A general procedure for reducing homomorphic reps into irreps

$\phi : V \rightarrow W = \text{im } \phi$, basis element $v_i \mapsto w_i$

1. **Once** we have an **irrep-decomposing** basis transformation matrix X for V

$$v'_i = \sum_j X_{ji} v_j \text{ such that } V \cong \bigoplus_{\mu} \bigoplus_{i=1}^{n_{\mu}} \mathcal{P}_{\mu}^i \text{ and } X\Phi(g)X^{-1} = \bigoplus_{\mu} \mathbb{1}_{n_{\mu}} \otimes \psi^{\mu}(g)$$

Idea is $\phi(\mathcal{P}_{\mu}^i)$ would either $\cong \mathcal{P}_{\mu}$ or $= 0$ because $\ker \phi|_{\mathcal{P}_{\mu}^i} = 0$ or \mathcal{P}_{μ}^i

This means $\{w'_i = \phi(v'_i)\}$ would be **irrep-decomposing** spanning vectors of W

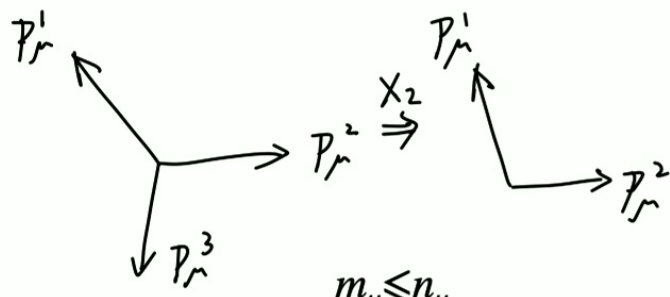
A general procedure for reducing homomorphic reps into irreps

2. **But** $\phi(\mathcal{P}_\mu^i) \neq 0$ and $\phi(\mathcal{P}_\mu^j) \neq 0$ may be linearly dependent

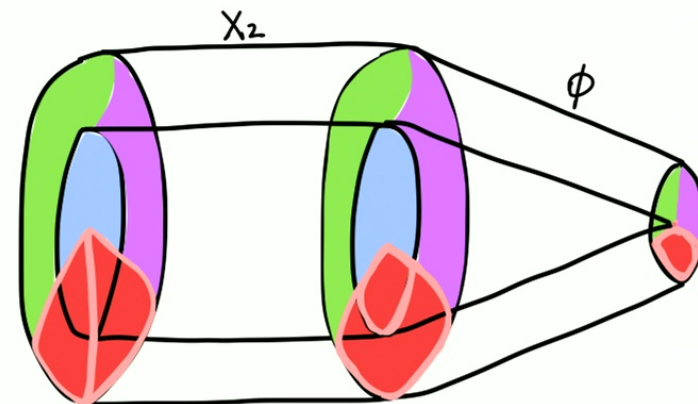
(i.e. $W \cong \bigoplus_{\mu} \sum_i \mathcal{P}_\mu^i$)

So do

Gaussian elimination



We have $W \cong \bigoplus_{\mu} \bigoplus_{i=1}^{m_{\mu} \leq n_{\mu}} \mathcal{P}_\mu^i$



A general procedure for reducing homomorphic reps into irreps

3. Quantum subroutine – orthonormal irrep-decomposing basis of W ?

Achievable if measured by a G -invariant inner product $\langle \cdot, \cdot \rangle_W$

Block-diagonalize into unitary matrix irreps ψ^μ as $\Phi'(g) = \bigoplus_{\mu} \mathbb{1}_{n_{\mu}} \otimes \psi^\mu(g)$

track the Gram matrix $Q_{ij} = \langle w_i, w_j \rangle \xrightarrow{X} Q'_{ij} = \langle w'_i, w'_j \rangle$ will find that

$$[\Phi'(g) Q' \Phi'(g)^{-1}]_{kl} = \sum_{ij} \Phi'(g)_{ki} \langle w'_i, w'_j \rangle_W \Phi'(g^{-1})_{jl}$$

$$= \overset{\psi^\mu \text{ unitary}}{\langle \sum_i \Phi'(g^{-1})_{ik} w'_i, \sum_j \Phi'(g^{-1})_{jl} w'_j \rangle_W} = \overset{\text{homomorphism}}{\langle \rho_{g^{-1}}(w'_k), \rho_{g^{-1}}(w'_l) \rangle_W} = \overset{\langle \cdot, \cdot \rangle_W \text{ } G\text{-invariant}}{Q'_{kl}}$$

and so $Q' = \bigoplus_{\mu} q^{\mu} \otimes \mathbb{1}_{d_{\mu}}$

A general procedure for reducing homomorphic reps into irreps

Recap : Measure orthogonality using a G -invariant inner product $\langle \cdot, \cdot \rangle_W$

1. Find an X to irrep-decompose the group action in $\{v_i\}$ as $X\Phi(g)X^{-1} = \bigoplus_{\mu} \mathbb{1}_{n_{\mu}} \otimes \psi^{\mu}(g)$ with ψ^{μ} unitary **for preimage**
2. Orthonormalize the irrep-decomposing $\{w'_i\}$ in W by diagonalizing the Gram matrix $Q' = \bigoplus_{\mu} q^{\mu} \otimes \mathbb{1}_{d_{\mu}}$ **for image**

Block-diagonalizing generalized induced reps

Step 1: block-diagonalize a true induced rep (preimage)

- We explicitly derived a basis transformation matrix that block-diagonalizes an arbitrary induced rep $\bigoplus_{\tau} V_{\alpha}$ from an irrep V_{α} of $H \leq G$ (base case)

$$X = \left[U_{QFT,G}(\mathbb{1}_A \otimes U_{QFT,H}^{\tau}) \right]_{|\cdot\rangle_A | \alpha, 1, \cdot \rangle_B}$$

$U_{QFT,G}$ **subgroup-reduced** down $G \supset H$, i.e. obtained irreps of G are block-diagonalized when restricting to H

- Proof idea: Frobenius reciprocity

$$\text{Hom}^H(V_{\alpha}, \text{Res}_H^G E_{\nu}) \xrightarrow{\sim} \text{Hom}^G(\text{Ind}_H^G V_{\alpha}, E_{\nu})$$

Block-diagonalizing generalized induced reps

Step 2: multiplicity separation (image)

▸ Diagonalize $Q' = \bigoplus_{\mu} q^{\mu} \otimes \mathbb{1}_{d_{\mu}} \rightarrow Q'' = \mathbb{1}$ to orthonormalize $\{w'_i\}$

▸ In general inefficient

▸ Cases that exponentially speedup the procedure:

the original $\{w_i\}$ orthonormal; degenerate eigenvalues in q^{μ} ;

an orthogonal compositional structure:

$$V \uparrow_H^{\rho} G = (V \uparrow_H^{\rho} H_1) \uparrow_{H_1}^{\rho} H_2 \uparrow_{H_2}^{\rho} \cdots \uparrow_{H_{p-1}}^{\rho} G$$

Running examples of the algorithm

1. Schur transform [Harrow '05]

$$(\mathbb{C}^d)^{\otimes n} = \bigoplus_{\sum_i n_i = n} Y(n_1, \dots, n_d)$$

where $Y(n_1, \dots, n_d)$ is spanned by standard basis elements with $\#n_i$ $|i\rangle$

$$Y(n_1, \dots, n_d) = \mathbb{C}v \uparrow_{S_{n_1} \times \dots \times S_{n_d}}^{\rho} S_n$$

where $v = |1\rangle^{\otimes n_1} |2\rangle^{\otimes n_2} \dots |d\rangle^{\otimes n_d}$ is a trivial rep of $S_{n_1} \times \dots \times S_{n_d}$

▸ **Complexity:**

▸ X_2 omitted because $\{\rho_{\tau}(v)\}$ orthonormal

▸ U_{QFT, S_n} subgroup-reduced down $S_n \supset S_{n_1} \times \dots \times S_{n_d} \implies$ inefficient X

Running examples of the algorithm

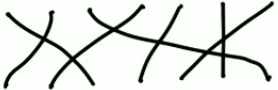
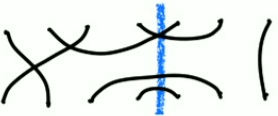

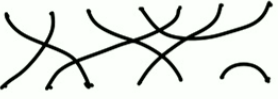
2. \mathcal{H}_α for (k-)port-based teleportation

$$\mathcal{H}_\alpha = V_{\alpha \vdash_d n-2k} \uparrow_{S_{n-2k}}^\rho S_{n-k}$$

- ▶ **Complexity:**
- ▶ q^μ all have degenerate eigenvalues
 $\implies X_2$ involves an amplitude amplification for each μ
- ▶ $U_{QFT, S_{n-k}}$ reduced down the standard tower $S_{n-k} \supset S_{n-2k}$
 $\implies X$ efficient
- ▶ For $k = 1$, X_2 incorporated efficiently into the whole circuit for PBT
 \implies efficient circuit for PBT!!!

Generalized induced reps in diagram algebras

Commutants in $(\mathbb{C}^d)^{\otimes n}$

Commutant	Algebra	Diagrams	(Generalized) induced reps
$\mathbb{C}[U^{\otimes n}]$	$\rho_1(\mathbb{C}S_n)$		$Y(n_1, \dots, n_d)$
$\mathbb{C}[U^{\otimes p} \otimes U^{*\otimes q}]$	$\rho_2(\mathcal{B}_{p,q}^d)$		$\mathcal{P}_{\alpha_1} \otimes \sum \text{diagram} \otimes \mathcal{P}_{\alpha_2} \uparrow_{S_{p-3} \times S_3 \times S_3 \times S_{q-3}}^{\rho_2} S_p \times S_q$
$\mathbb{C}[O(\mathbb{C})^{\otimes n}]$	$\rho_3(\mathcal{B}_n^{+d})$		$\mathcal{P}_{\alpha} \otimes \sum \text{diagram} \uparrow_{S_{n-6} \times S_6}^{\rho_3} S_n$
$\mathbb{C}[Sp(\mathbb{C})^{\otimes n}]$	$\rho_4(\mathcal{B}_n^{-d})$		$\mathcal{P}_{\alpha} \otimes \sum \text{diagram} \uparrow_{S_{n-6} \times S_6}^{\rho_4} S_n$

Results in our papers (arXiv 2310.01637 & ...)

- Give a $\text{poly}(n, d)$ quantum algorithm for port-based teleportation
Motivated by this problem, we (from general to specific)...
- Create a procedure of block-diagonalizing homomorphic images of representations
- Give a general quantum algorithm for block-diagonalizing generalized induced representations
- Pedagogical intro to diagram algebras and show that their irreps are generalized induced representations

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- Create a procedure of block-diagonalizing homomorphic images of representations
- Give a general quantum algorithm for block-diagonalizing generalized induced representations
- Pedagogical intro to diagram algebras and show that their irreps are generalized induced representations
- Applications of our general algorithm in diagram algebras: Schur transform, twisted Schur transform

Discussions

- Bridging the two sides of the (twisted) Schur transforms to give alternative algorithms for block-diagonalizing (generalized) induced representations
- Making non-orthogonality / non-unitarity useful in quantum computing?
- Indication of irrep / spectral distributions of generalized induced reps?