Title: Mathematical Physics Lecture

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Collection: Mathematical Physics 2023/24

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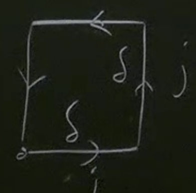
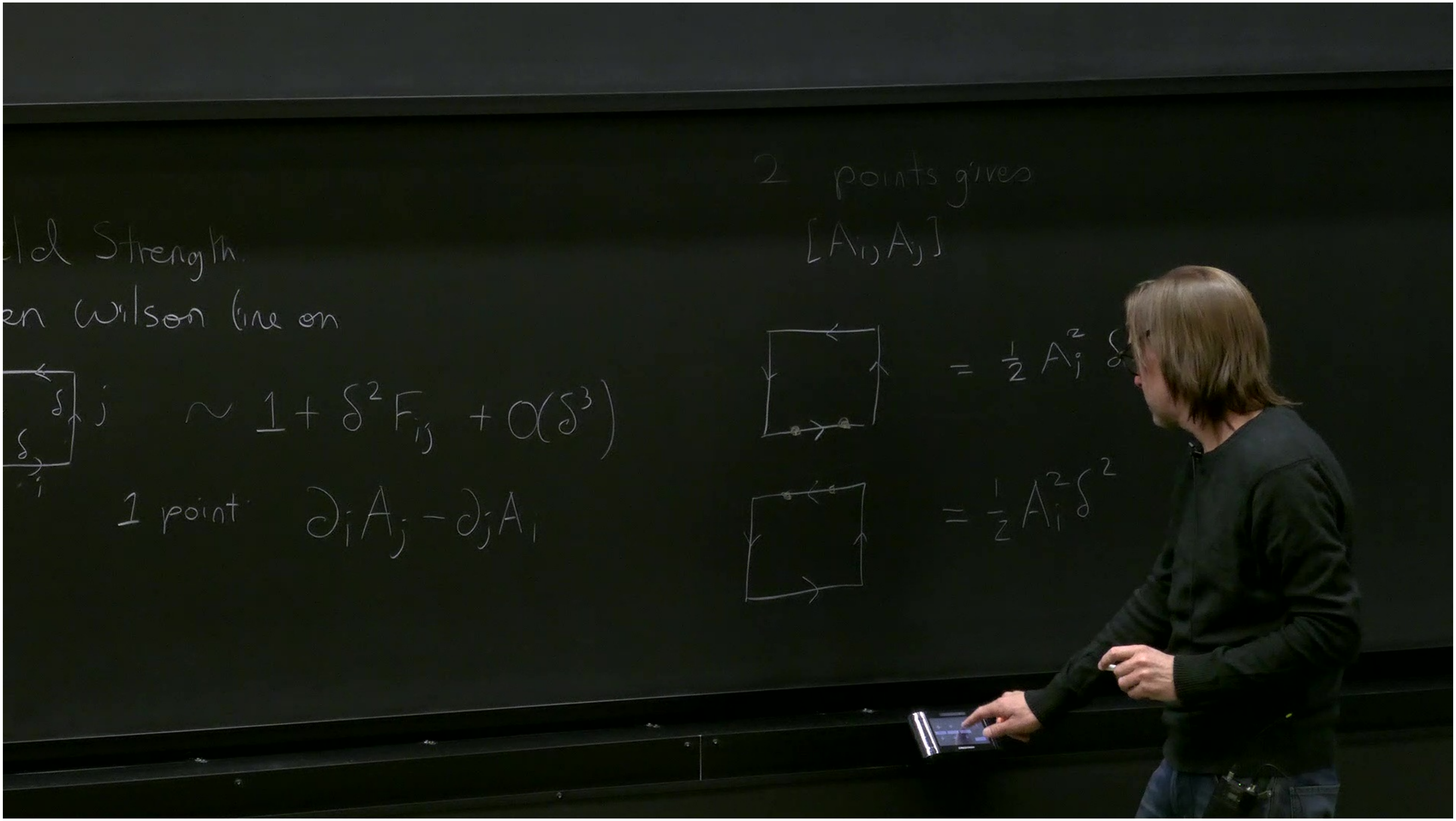
Field Strength.

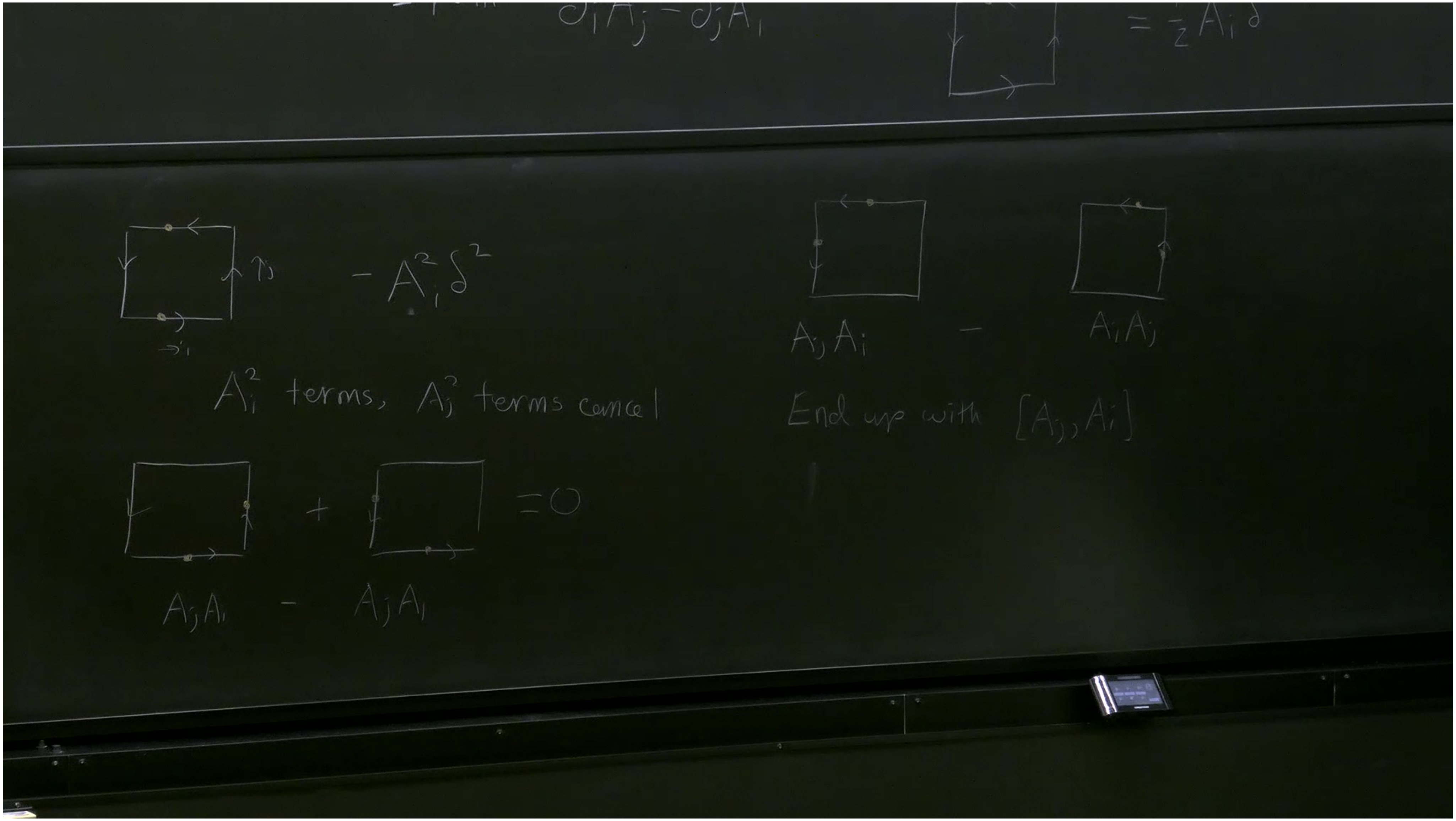
Open Wilson line on



$$\sim 1 + \delta^2 F_{ij} + O(\delta^3)$$

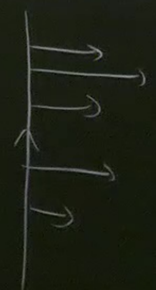
1 point: $\partial_i A_j - \partial_j A_i$


ld Strength.
en Wilson line on
$\sim \underline{1} + \delta^2 F_{ij} + O(\delta^3)$
1 point $\partial_i A_j - \partial_j A_i$
2 points gives
$[A_i, A_j]$
$= \frac{1}{2} A_j^2$
$= -\frac{1}{2} A_i^2 \delta^2$


$\partial_i A_j - \partial_j A_i$
$-A_i^2 \delta^2$
$A_j A_i$ $-$ $A_i A_j$
A_i^2 terms, A_j^2 terms cancel
End up with $[A_j, A_i]$
$+$ $=0$
$A_j A_i$ $-$ $A_j A_i$

Variation of a Wilson line

$\gamma(\theta) = \{\gamma_i(\theta)\}$

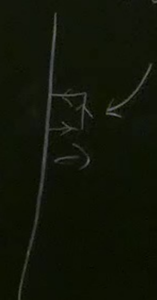


Vary by a vector $V_i(\theta)$

How does Wilson line change?

If V = a δ fn at some point

then variation of path is



Wilson line for a square with 1 side on original path, 2nd in direction V

e change?

at some point

path is

r a square

n original

irection V

If $W_\gamma(A)$, then

$\delta_V W_\gamma(A)$ is obtained by

inserting $\gamma^*(V^j F_{ij}\, dx^i)$ at some point

$$\delta_V W_\gamma(A) = W_\gamma(A + V \lrcorner F) - W_\gamma(A)$$

where $V \lrcorner F$ is one-form

$V^j F_{ij}\, dx^i$

Eom of CS theory are

$$F(A) = 0$$

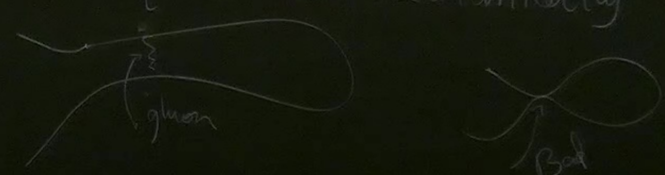
So, at least classically

$W_\gamma(A)$ in CS

does not change if

we vary γ continuously

(Introducing self intersections will cause problems quantum mechanically

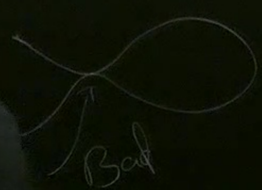


$U(N)$ $N=8$ SYM

continuously.

self intersections will cause problems

anically



$\Longrightarrow$ If $\gamma \subseteq S^3$ a knot

$$\int e^{k\int CS(A)} \quad W_\gamma(A)$$ should be a knot

invariant. Witten. $G = SU(2)$, Jones polynomial.

not change it

What is Phase space of CS?

Σ a surface (eg $\Sigma = T^2$)

Let $\gamma : S^1 \longrightarrow \Sigma$ a loop

Phase space = Sol^{n}s to EOM on $\Sigma \times \mathbb{R}$

invariant. Witten. $G = SU(2)$, Jones polynomial.

$A \longmapsto W_\gamma(A)$ is a function on phase space

If we change γ continuously (allowing self-intersections) then $W_\gamma(A)$ is unchanged.

$U(N)$ gauge theory, $N \longrightarrow \infty$, these W_γ are independent coordinates.

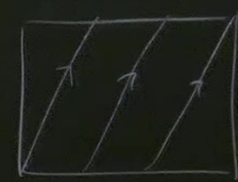
Eg

$\Sigma = T^2$

any loop can be straightened
so it winds around 1 cycle
m times, the other n times

Eg

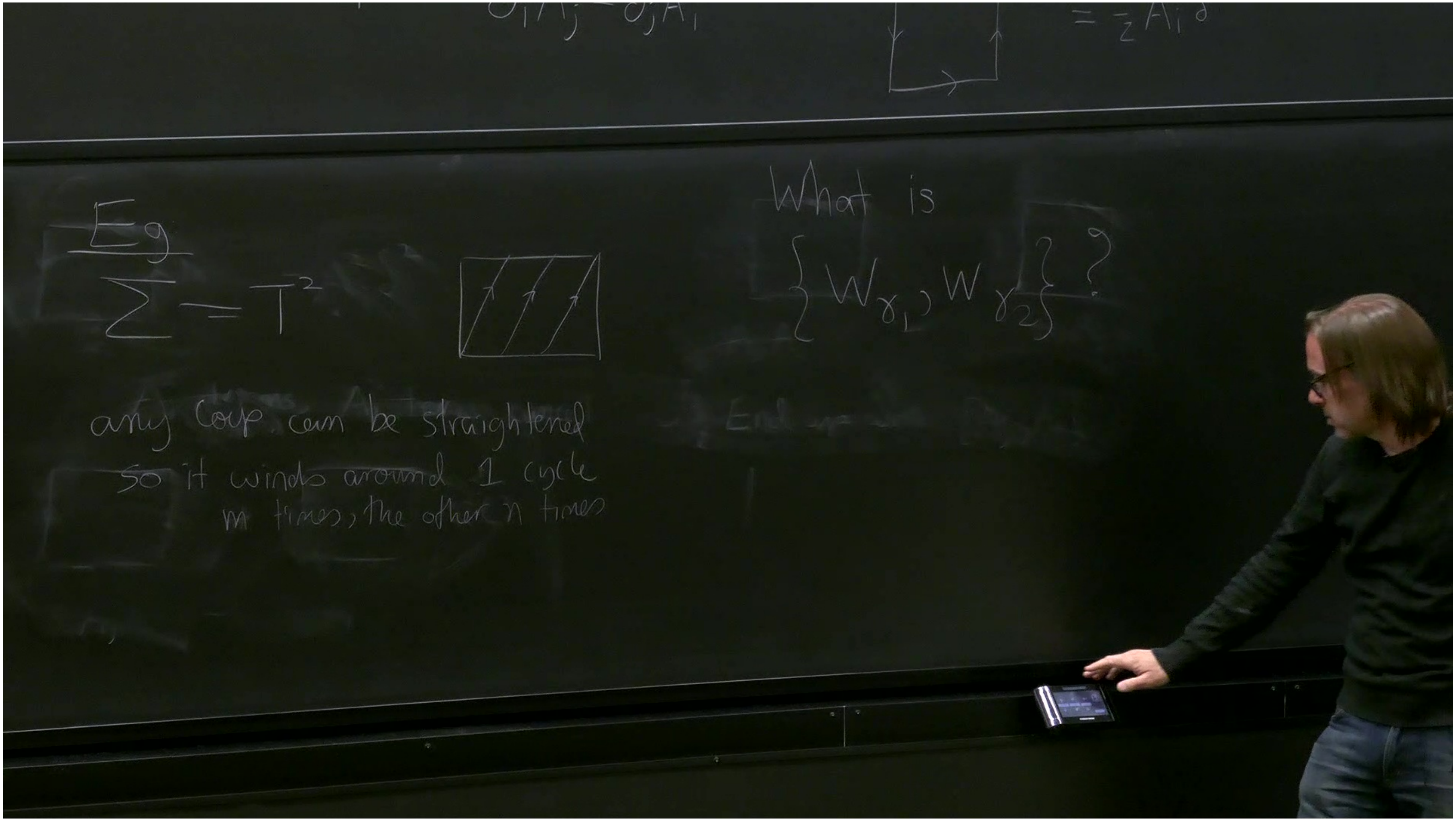
$\Sigma = T^2$



What is

$\{W_{\gamma_1}, W_{\gamma_2}\}$

any loop can be straightened
so it winds around 1 cycle
m times, the other n times

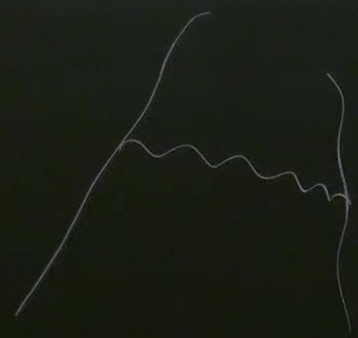

Eg
$\Sigma = T^2$
any loop can be straightened
so it winds around 1 cycle
m times, the other n times
What is
$\{W_{\gamma_1}, W_{\gamma_2}\}$?

To compute this, consider

$$W_{\gamma_1}(t=0)$$

$$W_{\gamma_2}(t_0>0)$$

Considering exchange of a single gluon.

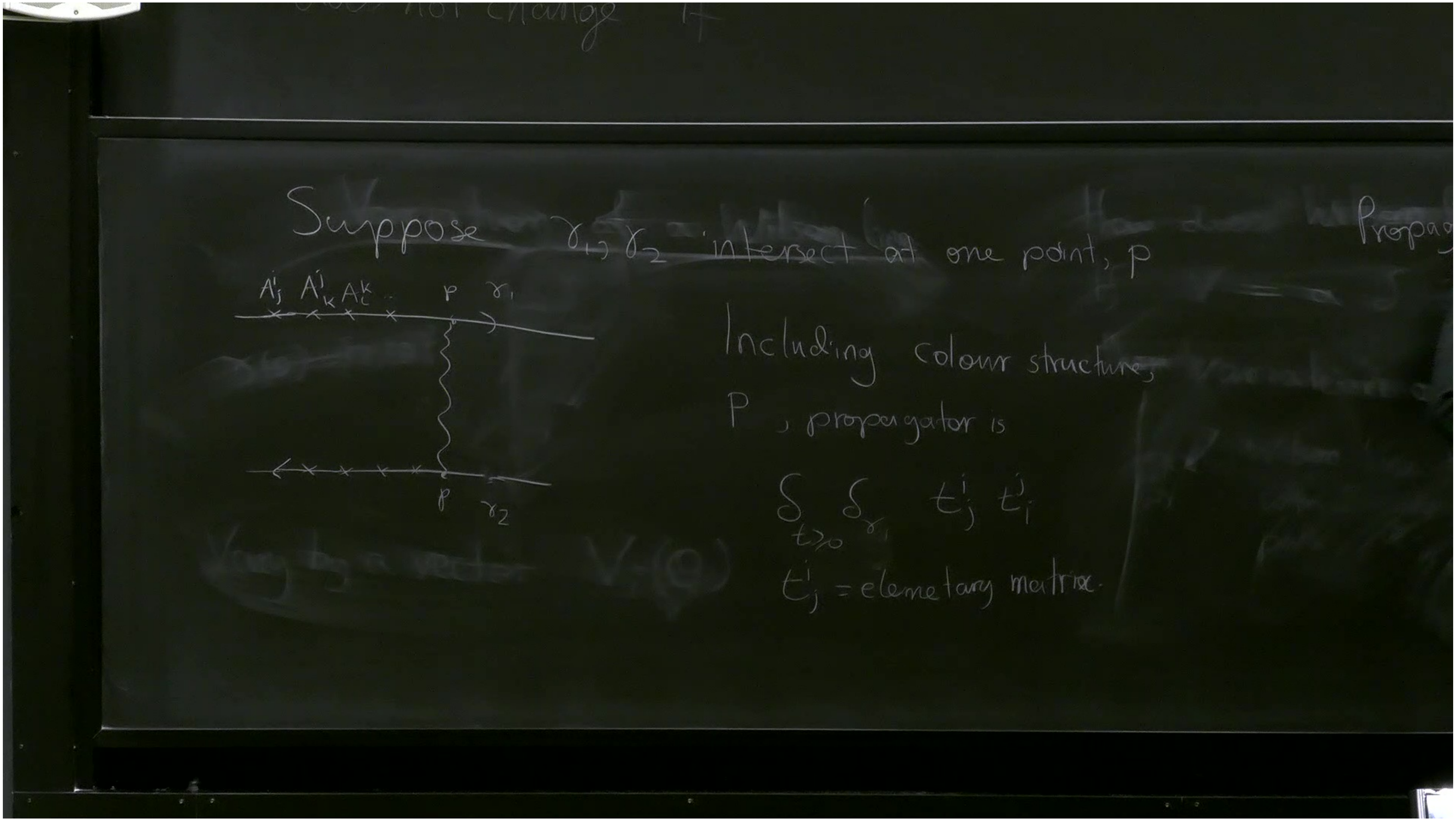


Field sourced by A on γ_1 is

$$\delta_{\gamma_1}\delta_{t\geq 0}$$

Measure Wilson line W_{γ_2} in the presence of this field: get

$$\delta_{t\geq 0}\left(\sum_{p\in\gamma_1\cap\gamma_2}(\text{something})\right)$$


Suppose γ1, γ2 intersect at one point, p
Aij Ajk Akl ..
p γ1
p γ2
Including colour structure,
P , propagator is
δt≥0 δγ1 tij tji
tij = elemetary matrix.
Propag

invariant W

γ_1, γ_2 intersect at one point, p

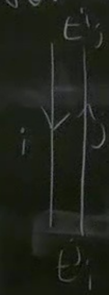
Including colour structure,

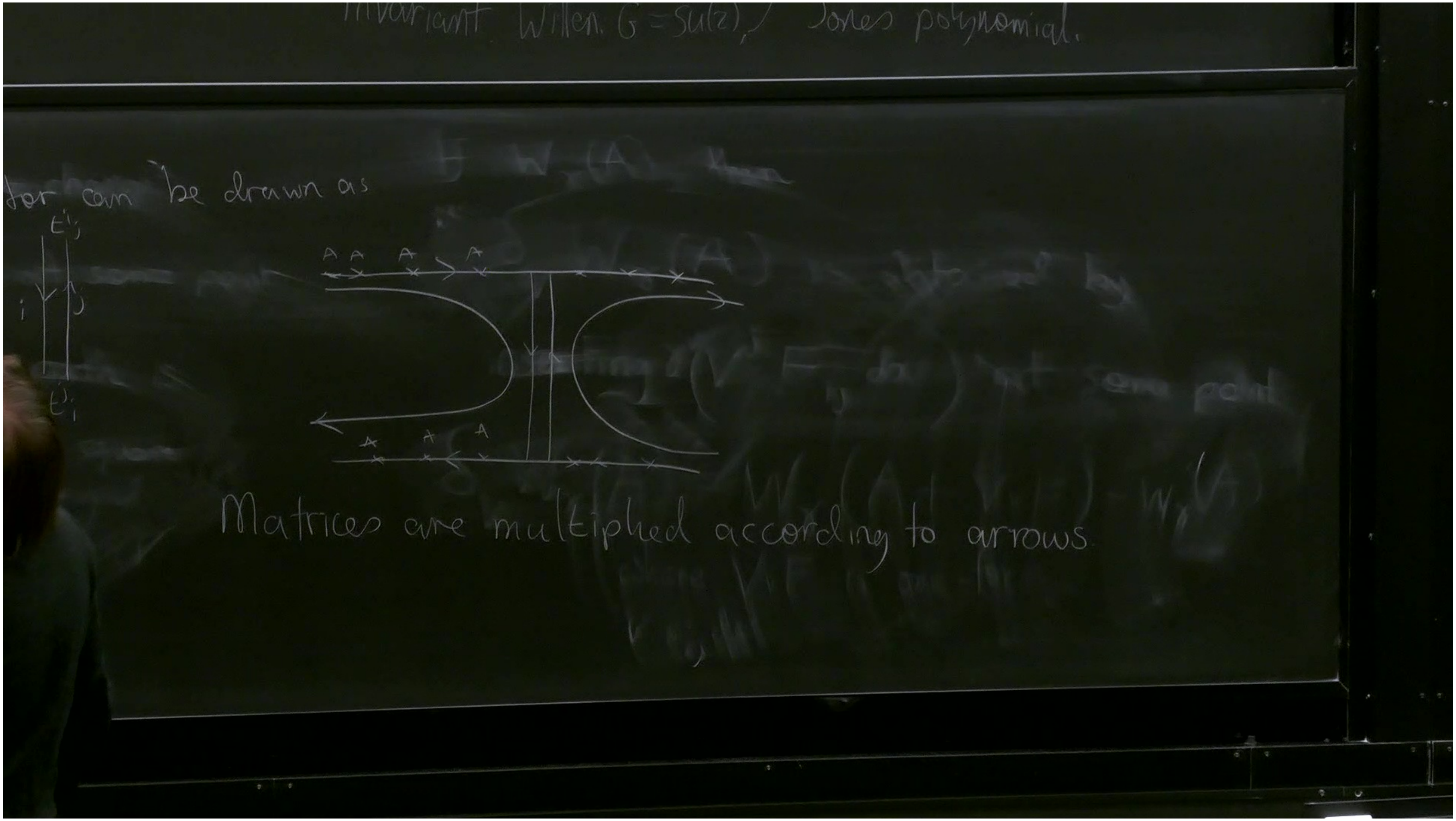
P , propagator is

$$\delta_{t>0}\, \delta_{\gamma_1}\ t^i_j\, t^j_i$$

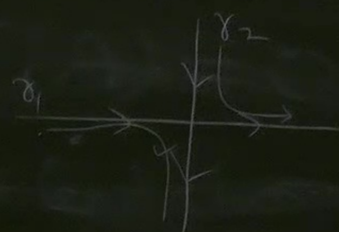
t^i_j = elementary matrix.

Propagator can be drawn as

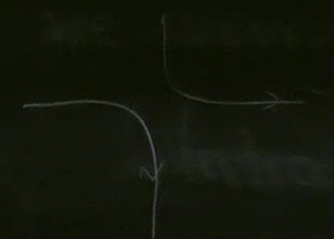



invariant Witten. G = SU(2), Jones polynomial.
tor can be drawn as
Matrices are multiplied according to arrows.

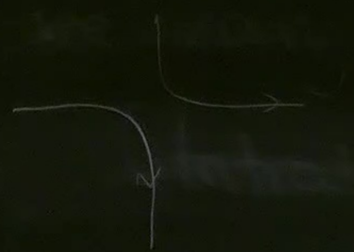
Draw γ_1, γ_2 in 2d plane



Let $\gamma_1 \pitchfork \gamma_2$ be curve
(or union of curves) which
looks like



Then $\{W_{\gamma_1}, W_{\gamma_2}$

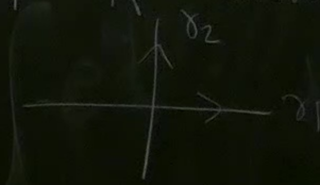


Then $\{W_{\gamma_1}, W_{\gamma_2}\} = W_{\gamma_1 *_p \gamma_2}$,

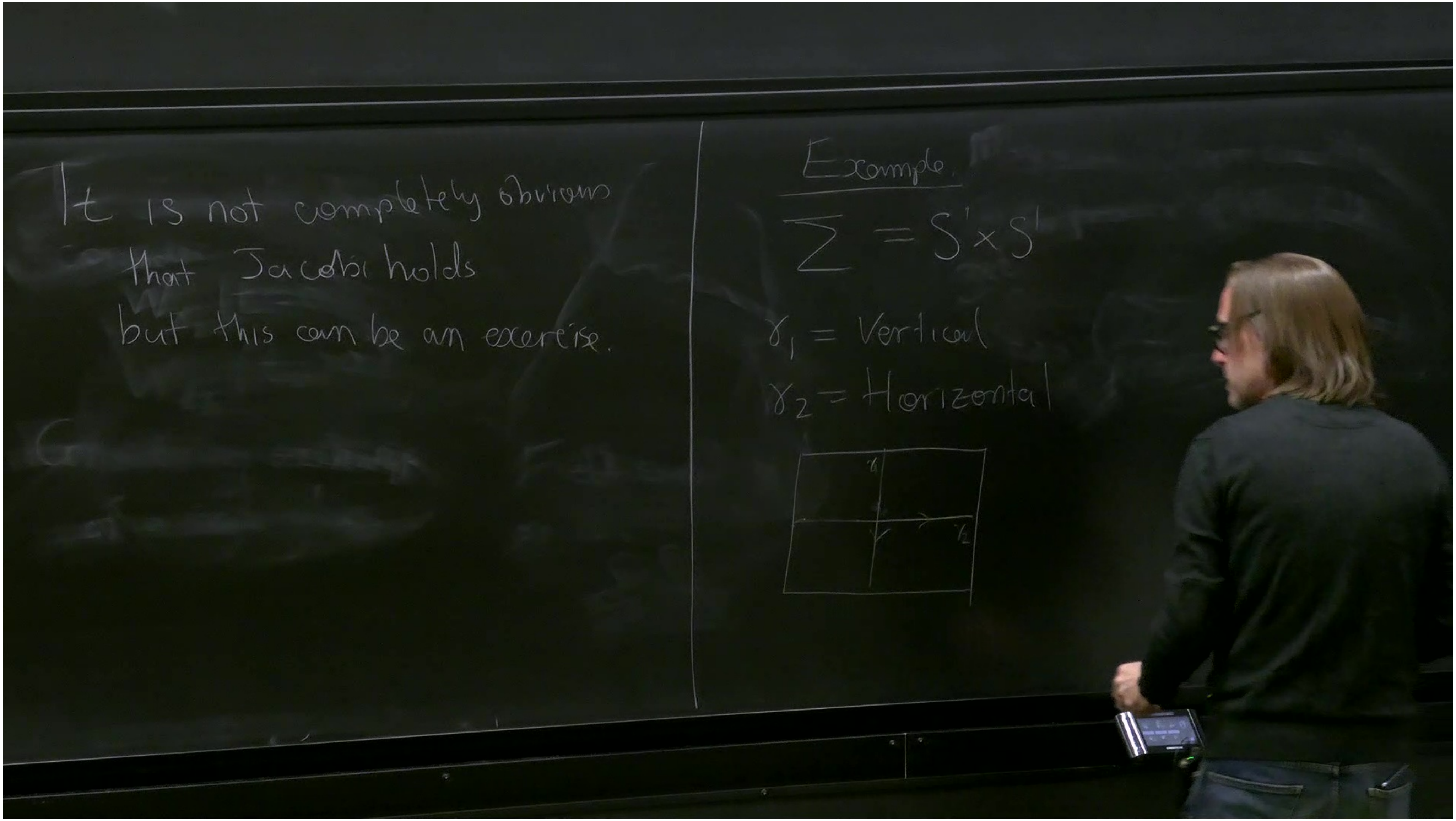
If there are many intersection points, we find

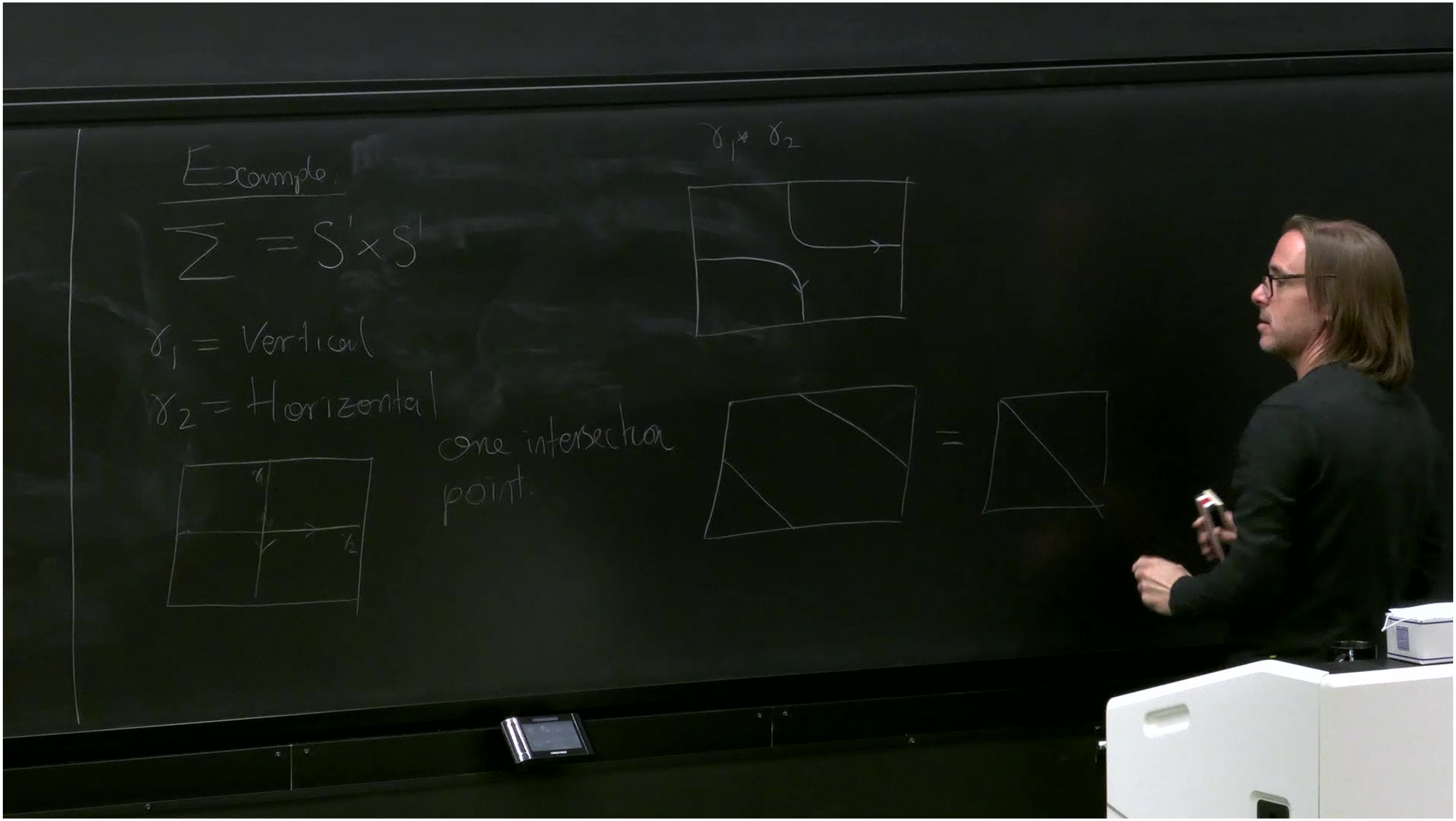
$$\{W_{\gamma_1}, W_{\gamma_2}\} = \sum_{p \in \gamma_1 \cap \gamma_2} \pm W_{\gamma_1 * \gamma_2}$$

$+$ if intersection is like



$-$ other possibility


It is not completely obvious
that Jacobi holds
but this can be an exercise.
Example.
$\Sigma = S^1 \times S^1$
γ_1 = Vertical
γ_2 = Horizontal
γ_1
γ_2


Example:
Σ = S¹ × S¹
γ₁ = Vertical
γ₂ = Horizontal
one intersection point.
γ₁ * γ₂
=

If we set $\gamma_{m,n}$ = path which

winds around vertical m times

horizontal n times

Then we've shown

$$\{\gamma_{-1,0}, \gamma_{0,1}\} = \gamma_{-1,1}$$

we set $\gamma_{m,n}$ = path which
winds around vertical m times
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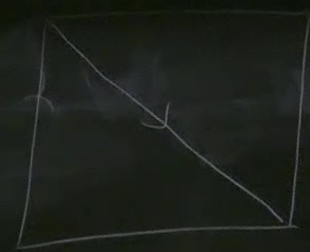
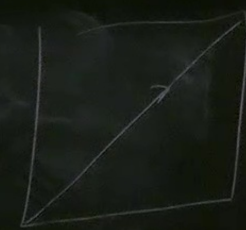
Then we've shown

$$\{\gamma_{-1,0}, \gamma_{0,1}\} = \gamma_{-1,1}$$

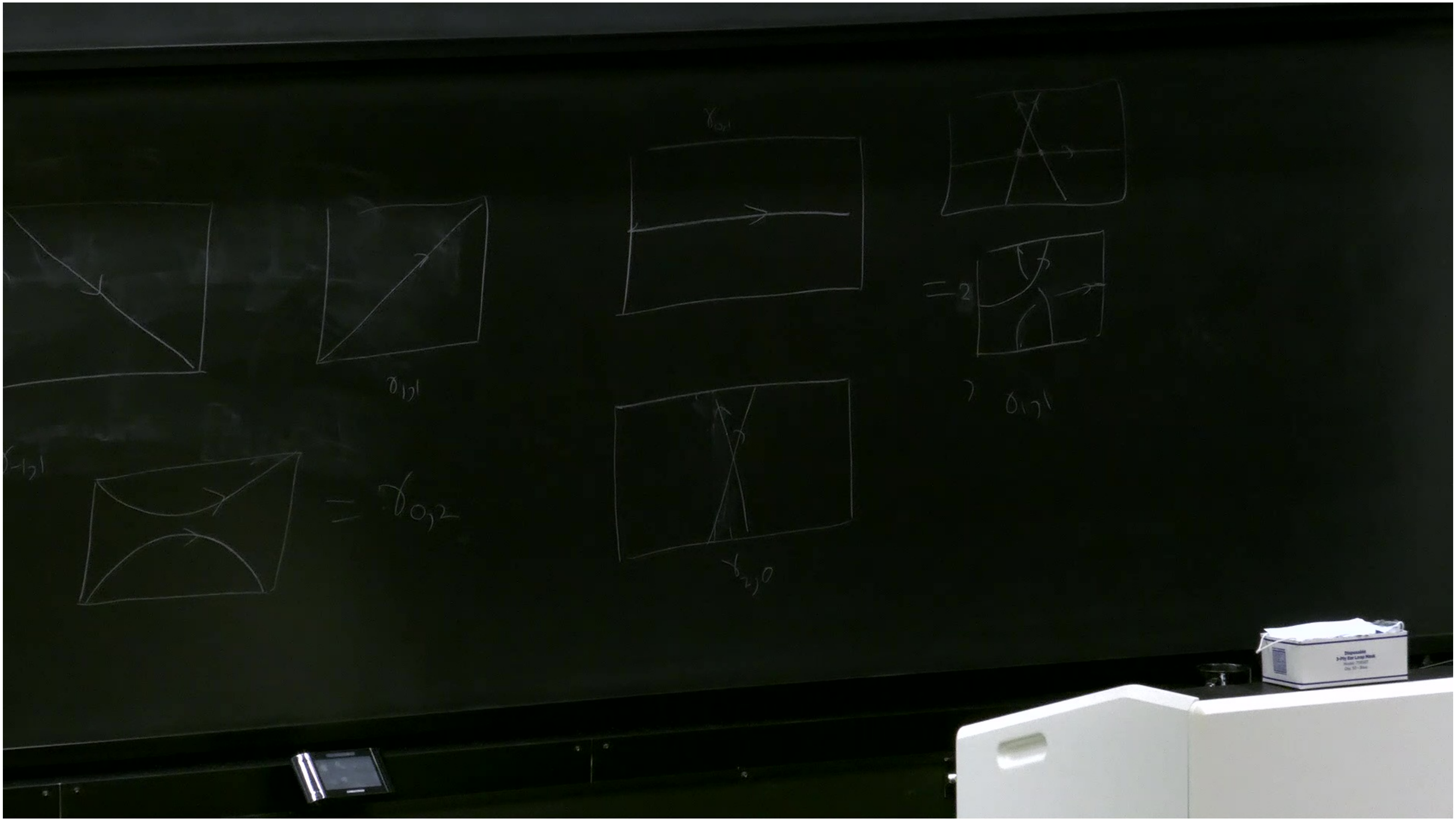


$\gamma_{-1,1}$



$\gamma_{1,1}$

$= \gamma_{0,2}$


$\gamma_{1,1}$
$\gamma_{-1,1}$
$= \gamma_{0,2}$
$\gamma_{0,1}$
$\gamma_{2,0}$
$= 2$
$\gamma_{1,1}$