

Title: String Theory Lecture

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GRAVITY ON

KALUZA-KLEIN

$$S^1_R \times \mathbb{R}^{D-1}$$

$$e^{ipx} \rightarrow e^{in\theta}$$

$$p_z = \frac{n}{R}$$

$$g_{\mu\nu} \rightarrow \begin{matrix} g_{\mu\nu}^{D-1} \\ g_{\mu\theta} \\ g_{\theta\theta} \end{matrix}$$

$$\mathbb{R}^1 \times S^1$$

$$X \rightarrow X + 2\pi R$$

GRA  
KAL



$$X \mapsto X + z \in R \quad (\partial_\sigma^2 + \partial_\tau^2) X = 0$$

$$X(\sigma, \tau) = wR\sigma$$

$$X(\sigma, \tau) = x - z i p \tau + wR\sigma + \sum_{n \neq 0} \frac{i}{n} a_n e^{in(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - i\tau)}$$

$$z = R\theta$$

$$p = \frac{n}{R}$$

$$|n, w\rangle$$

$$a_n | \rangle$$

$$\bar{a}_n | \rangle$$

$$L_0 = \frac{1}{2} \left( \frac{n}{R} - \frac{Rw}{2} \right)^2$$

$$\bar{L}_0 = \frac{1}{2} \left( \frac{n}{R} + \frac{Rw}{2} \right)^2$$

$$L_0 - \bar{L}_0 = nw$$

$$p^2 + \frac{m^2}{2R^2} = 2$$

KK TACHYON

$$|n, 0^-\rangle \otimes |p\rangle$$

$$|0, w\rangle \otimes |p\rangle$$

$$a_{-1} |n, w\rangle \otimes |p\rangle$$

$$p^2 + \frac{m^2}{8} R^2 = 2$$

$$\bar{a}_{-1} |n, w\rangle \otimes |p\rangle$$

$$a_{-1} |n, w\rangle \otimes \bar{a}_{-1}^n |p\rangle$$

$$e^{-i\alpha(\sigma - \sigma)}$$

$$\frac{i}{\kappa} a_n e^{-i\kappa(x-ct)}$$

$$p^2 + \frac{m^2}{2\kappa^2} = 2$$

KK TACHYON

$$p^2 + \frac{1}{2} \left( \frac{1-\kappa}{\kappa} \right)^2 = 2 \quad a_{-1} |1, 1\rangle \otimes |p\rangle$$

$$\bar{a}_{-1} |n, w\rangle \otimes |p\rangle$$

$$a_{-1} |n, w\rangle \otimes \bar{a}_{-1}^n |p\rangle$$

$$a_{-1}$$

$$\vdots$$

$$p^2 + \frac{m^2}{8} \kappa^2 = 2$$

$$|n, 0\rangle \otimes |p\rangle \quad |0, w\rangle \otimes |p\rangle$$

GRAVITY ON  
KALUZA-KLEIN

$$S^1 \times \mathbb{R}^{D-1}$$

$$e^{ipx} \rightarrow e^{im\theta}$$
$$p_i = \frac{n}{R}$$

$$g_{\mu\nu} \rightarrow \begin{cases} g_{\mu\nu}^{D-1} \\ g_{\mu 0} \\ g_{00} \end{cases}$$

$$B_{\mu\nu} \rightarrow B_{\mu 0}$$

$$\int_{\Sigma} B$$

$$C_0 = \frac{1}{2} \left( \frac{r}{R} + \frac{Rw}{z} \right)^2$$

$$|n, w\rangle \rightarrow O_{nm}$$

$$X(z) = X(\sigma) + 2\pi w R$$

$$|n, m\rangle$$

$$X(e^{2\pi i} z) = X(z) + 2\pi w R$$

$\circ O_{n,w}$



$z(k+1)$

$\mathcal{O}_{n,m}$

$$X(e^{zic} z) = X(z) + zcRw$$

$\mathcal{O}_{n,w}$

$$e^{i\frac{m}{2}X(z)}$$

$$\mathcal{O}_{m,w}(0)$$

$$C_0 = \frac{1}{z} \left( \frac{1}{R} + \frac{Rz}{z} \right)$$

→  $\mathcal{O}_{n,m}$

$$\mathcal{O}_{m,w}(z) \mathcal{O}_{m',w'}(z) \sim (z-\bar{z}') \in \mathcal{O}_{m+m',w+w'}$$

$\partial X(z)$   
 $\bar{\partial} X(\bar{z})$

$$X(e^{2\pi i} z) = X(z) + 2\pi i R w$$

$$\mathcal{O}_{n,w} = e^{i \left( \frac{n}{R} - \frac{nw}{z} \right) X(z) + i \left( \frac{n}{R} + \frac{nw}{z} \right) \bar{X}(\bar{z})}$$

$$\mathcal{O}_{m,w}(0)$$

$$X_{cl} = \sum (-1) \ln |z - z_i| + \sum (-1)$$

$$\langle \prod_i \mathcal{O}_{m_i, w_i}(z_i, \bar{z}_i) \rangle = \prod_i (z - z_i)^{\left( \frac{m_i}{R} - \frac{m_i w_i}{z} \right)} (\bar{z} - \bar{z}_i)^{\left( \frac{m_i}{R} + \frac{m_i w_i}{z} \right)}$$

Sol  $\langle \dots \rangle$   
 $e^{(+)(+)}$   
 $(\bar{z}_i - \bar{z}_j)$

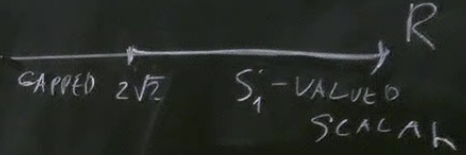
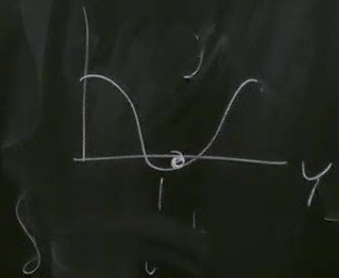
$$\int_{\text{MULTI-DIM}} DX e^{S(X)} = \int_{\text{SINGLE-DIM}} DX e^{S(X_{cl} + X)} = S(X_{cl}) + S(X)$$

$z = R\theta$   $P = \frac{m}{R}$   $z \neq 0$   
 $|n, w\rangle$   $L_0 = \frac{1}{2} \left( \frac{m}{R} - \frac{Rw}{z} \right)^2$   $L_0 - \bar{L}_0 = nw$   
 $L_{-1}$   $\bar{L}_0 = \frac{1}{2} \left( \frac{m}{R} + \frac{Rw}{z} \right)^2$   $\begin{pmatrix} R^2 & R^2 \\ \delta & \delta \end{pmatrix}$   
 $\bar{L}_{-1}$   
 $\vdots$

$\rightarrow \mathcal{O}_{n,m}$   $S(x) + g \mathcal{O}_{0,1} \mathcal{O}_{m,w}(z) \mathcal{O}_{m,w}(\bar{z}) \sim (z-\bar{z})^n \in \mathcal{O}_{n+1, w+w}$   
 $\mathcal{O}_{0,0} = e^{i\phi}$   
 $X(e^{2\pi i} z) = X(z) + 2\pi R w$   
 $\mathcal{O}_{n,w} = e^{i \left( \frac{m}{R} - \frac{mz}{z} \right) X(z) + i \left( \frac{m}{R} + \frac{m\bar{z}}{z} \right) \bar{X}(\bar{z})}$   $\mathcal{O}_{n,w}(0)$   
 $+ 2\pi n R$   $X_\alpha = \sum (-) \ln z - z_i$   
 $\langle \prod_i \mathcal{O}_{m_i, w_i}(z_i, \bar{z}_i) \rangle = \prod_i (z - z_i)^{\left( \frac{m_i}{R} - \frac{m_i}{z} \right) \left( \frac{m_i}{R} - \frac{m_i R}{z} \right)}$   $\int D\alpha e^{S(\alpha)} = \int D\alpha e^{S(\alpha) + S(\phi)}$   
 $\int D\alpha e^{S(\alpha)}$   $S(\alpha + \phi) = S(\alpha) + S(\phi)$   
 $(\bar{z}_i - \bar{z}_j)^{(+)(+)}$

$$R \stackrel{24,1}{\times} S_R^1$$

$$X \rightarrow X + 2\pi R$$



GRA  
KAL

SPECIAL

KT

IN  $2d$

