Title: String Theory Lecture

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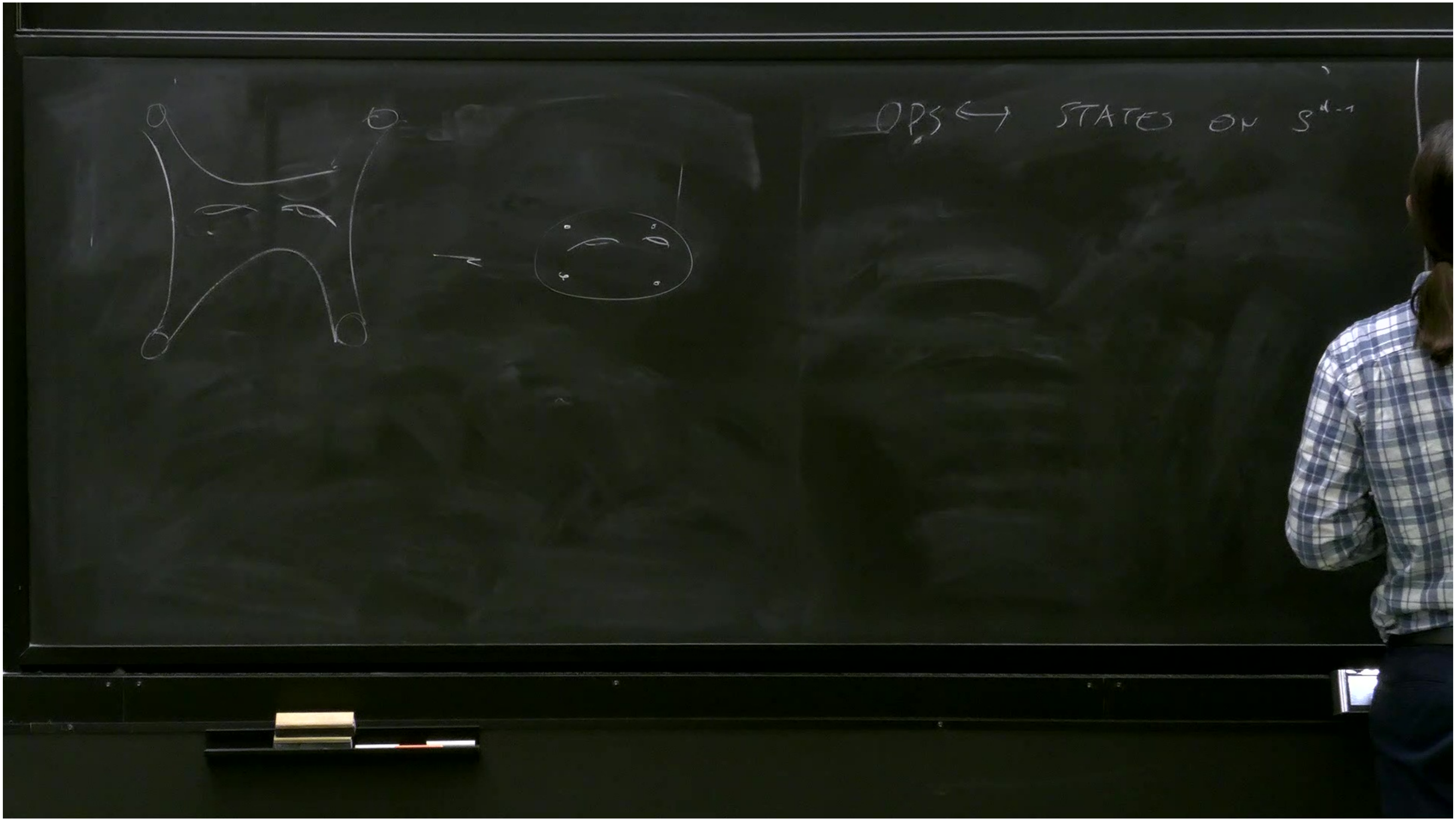
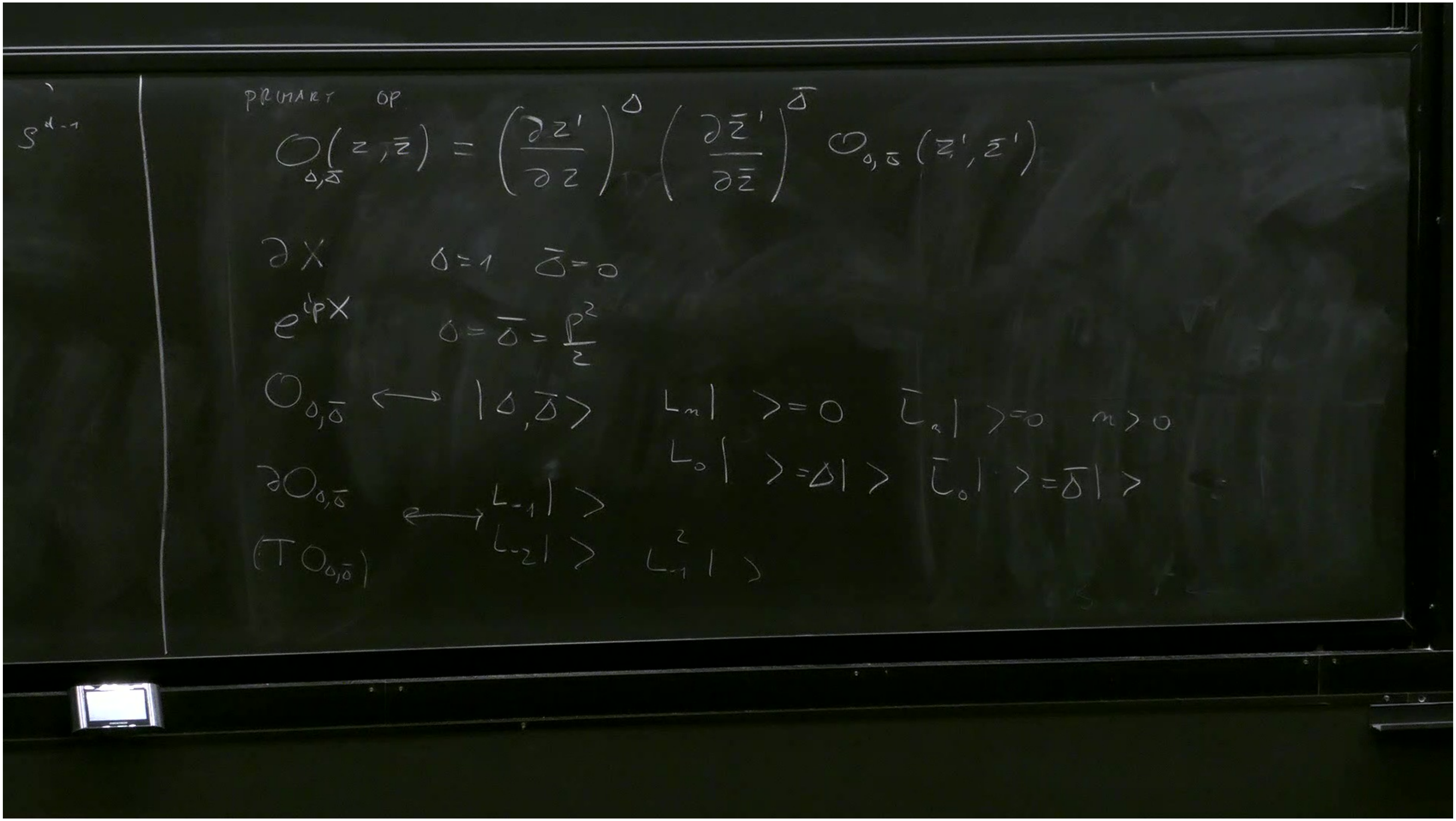
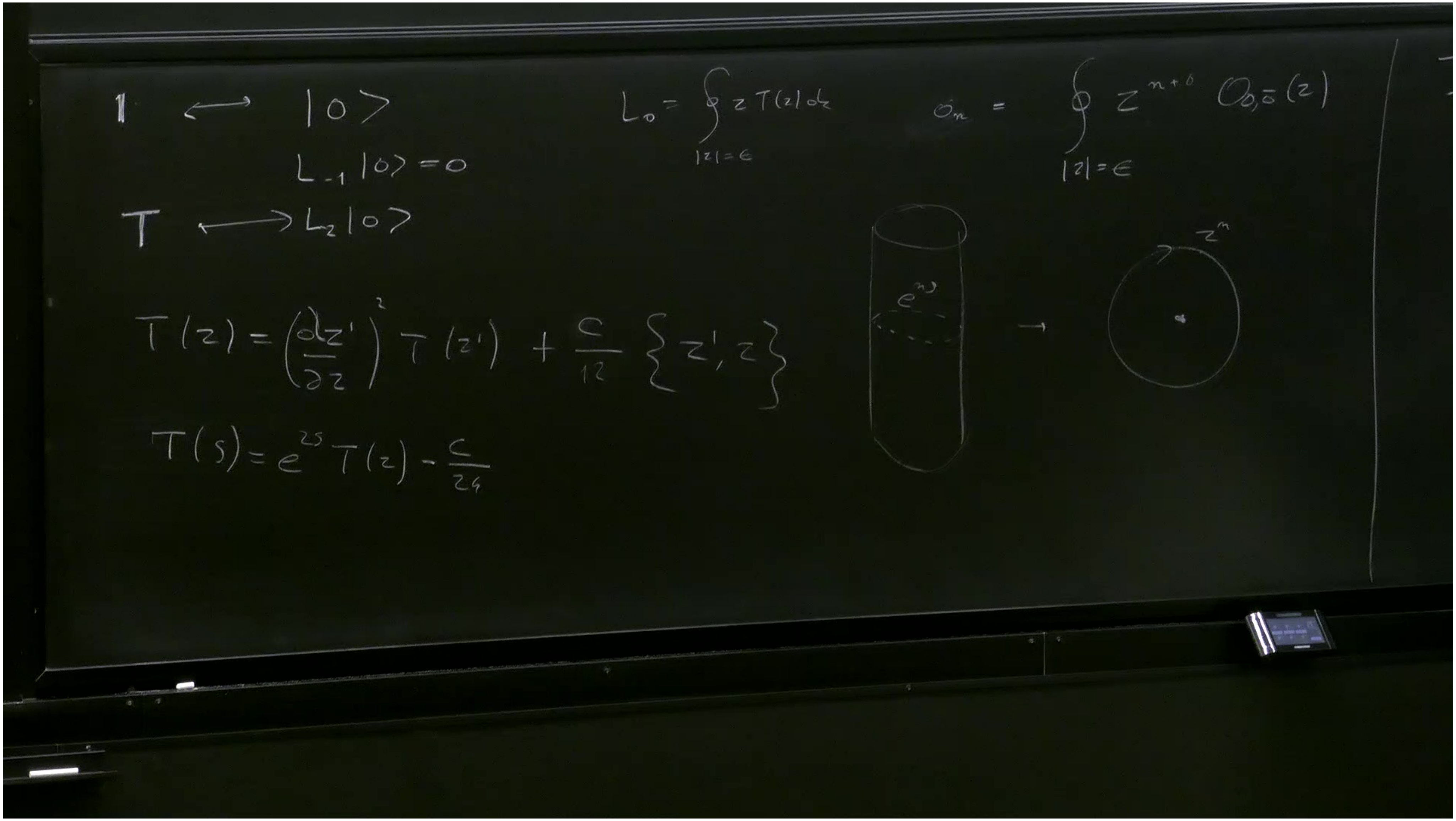
Collection: String Theory 2023/24

Date: April 19, 2024 - 10:15 AM

URL: https://pirsa.org/24040040


OPS ↔ STATES ON S^{d-1}


S^{d-1}
PRIMARY OP
$\mathcal{O}_{\Delta,\bar\Delta}(z,\bar z) = \left(\frac{\partial z'}{\partial z}\right)^{\Delta}\left(\frac{\partial \bar z'}{\partial \bar z}\right)^{\bar\Delta}\mathcal{O}_{\Delta,\bar\Delta}(z',\bar z')$
∂X $\Delta=1$ $\bar\Delta=0$
e^{ipX} $\Delta=\bar\Delta=\frac{p^2}{2}$
$\mathcal{O}_{\Delta,\bar\Delta} \longleftrightarrow |\Delta,\bar\Delta\rangle$ $L_n|\ \rangle=0$ $\bar L_n|\ \rangle=0$ $n>0$
$L_0|\ \rangle=\Delta|\ \rangle$ $\bar L_0|\ \rangle=\bar\Delta|\ \rangle$
$\partial\mathcal{O}_{\Delta,\bar\Delta} \longleftrightarrow L_{-1}|\ \rangle$
$(T\mathcal{O}_{\Delta,\bar\Delta})$ $L_{-2}|\ \rangle$ $L_{-1}^2|\ \rangle$


$\mathbb{1} \longleftrightarrow |0\rangle$
$L_{-1}|0\rangle = 0$
$T \longleftrightarrow L_2|0\rangle$
$L_0 = \oint_{|z|=\epsilon} z\, T(z)\, dz$
$O_n = \oint_{|z|=\epsilon} z^{n+\delta}\, O_{\delta,\bar{\delta}}(z)$
$T(z) = \left(\frac{dz'}{dz}\right)^2 T(z') + \frac{c}{12}\{z', z\}$
$T(s) = e^{2s}\, T(z) - \frac{c}{24}$
z^n

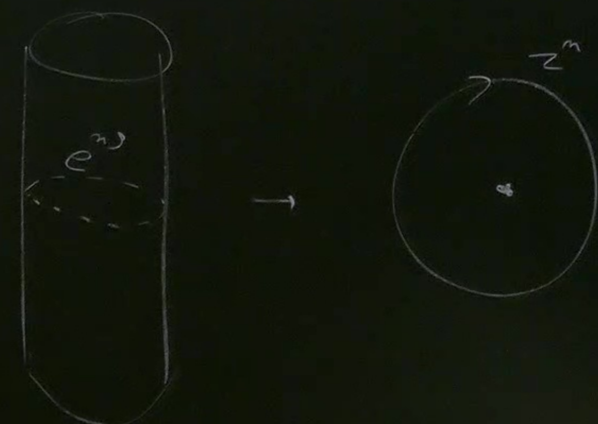
$\mathbb{1} \longleftrightarrow |0\rangle$

$L_{-1}|0\rangle = 0$

$T \longleftrightarrow L_2|0\rangle$

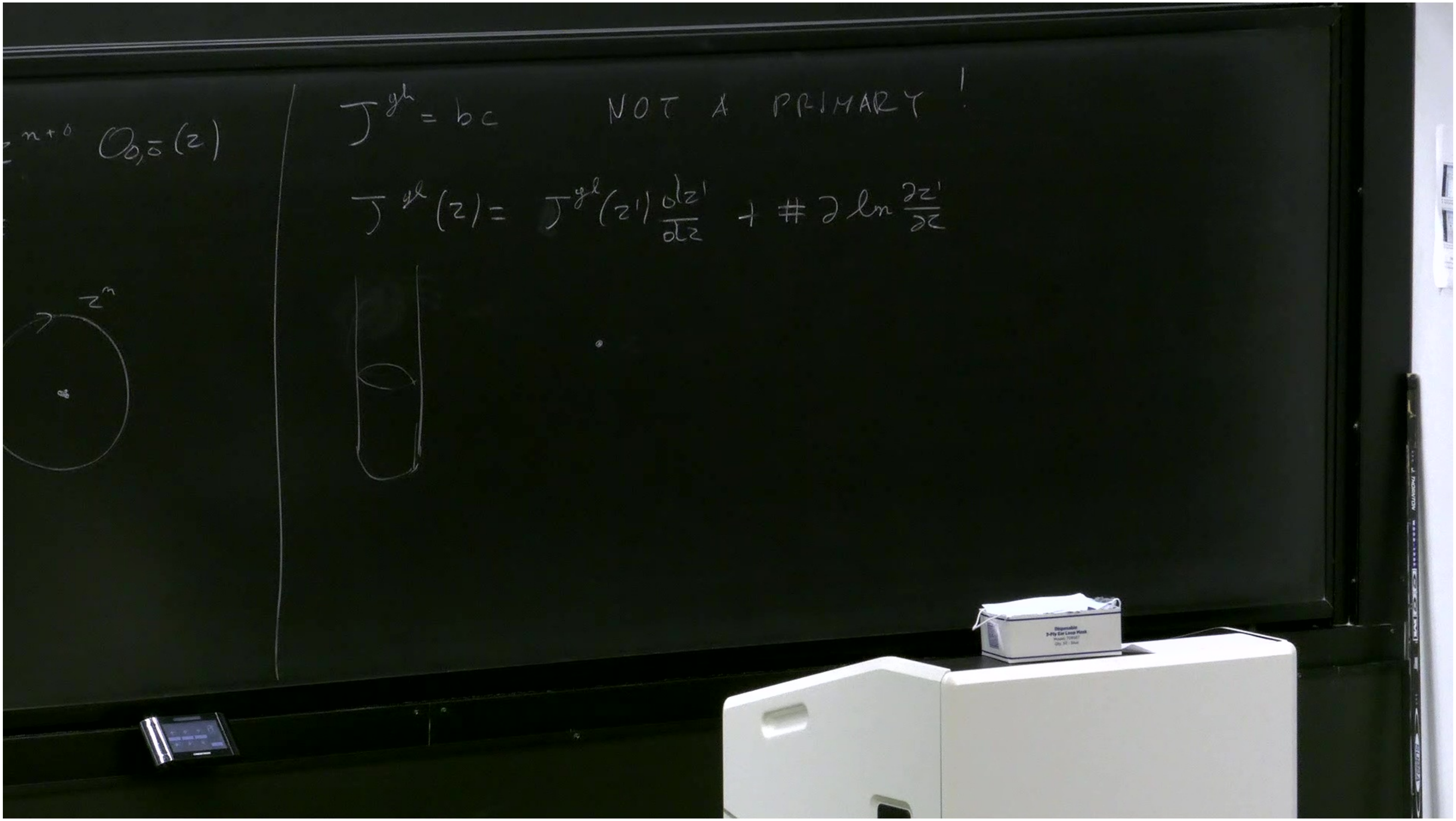
$$L_0 = \oint_{|z|=\epsilon} z\, T(z)\, dz$$

$$\mathcal{O}_n = \oint_{|z|=\epsilon} z^{n+0}\, \mathcal{O}_{0,\bar{0}}(z)$$



$$T(z) = \left(\frac{\partial z'}{\partial z}\right)^2 T(z') + \frac{c}{12}\left\{z', z\right\}$$

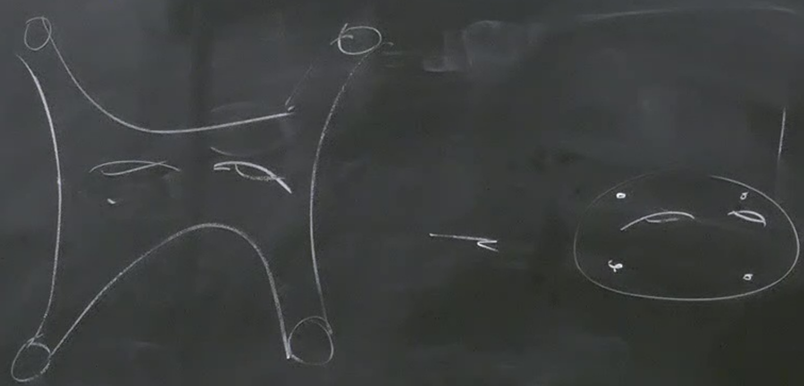
$$T(s) = e^{2s}\, T(z) - \frac{c}{24}$$


$J^{gh} = bc$
NOT A PRIMARY !
$J^{gh}(z) = J^{gh}(z') \frac{dz'}{dz} + \# \partial \ln \frac{\partial z'}{\partial z}$
$O_{0,\bar{0}}(z)$
z^n

$|g\rangle \qquad c_0|g\rangle \qquad\qquad\qquad |g\rangle \quad \longleftrightarrow$

$c_{-1}|g\rangle \qquad b_{-1}|g\rangle \qquad c_{-1}c_0|g\rangle \qquad b_{-1}c_0|g\rangle$

$\vdots$



OPS $\leftrightarrow$ STATES ON S^{d-1}

$$|g\rangle \otimes |\mathrm{PHYS}\rangle \longleftrightarrow c(z)\,\bar{c}(\bar{z})\,\mathcal{O}_{\mathrm{PHYS}}$$

$\to \quad C$

$$b_{-1}|g\rangle \longleftrightarrow 1$$

$$\|$$

$$|0\rangle$$

$$L_{-1}|0\rangle = 0 \quad , \ldots$$

$$\langle g | c_0 \bar{c}_0 | g \rangle = 1$$

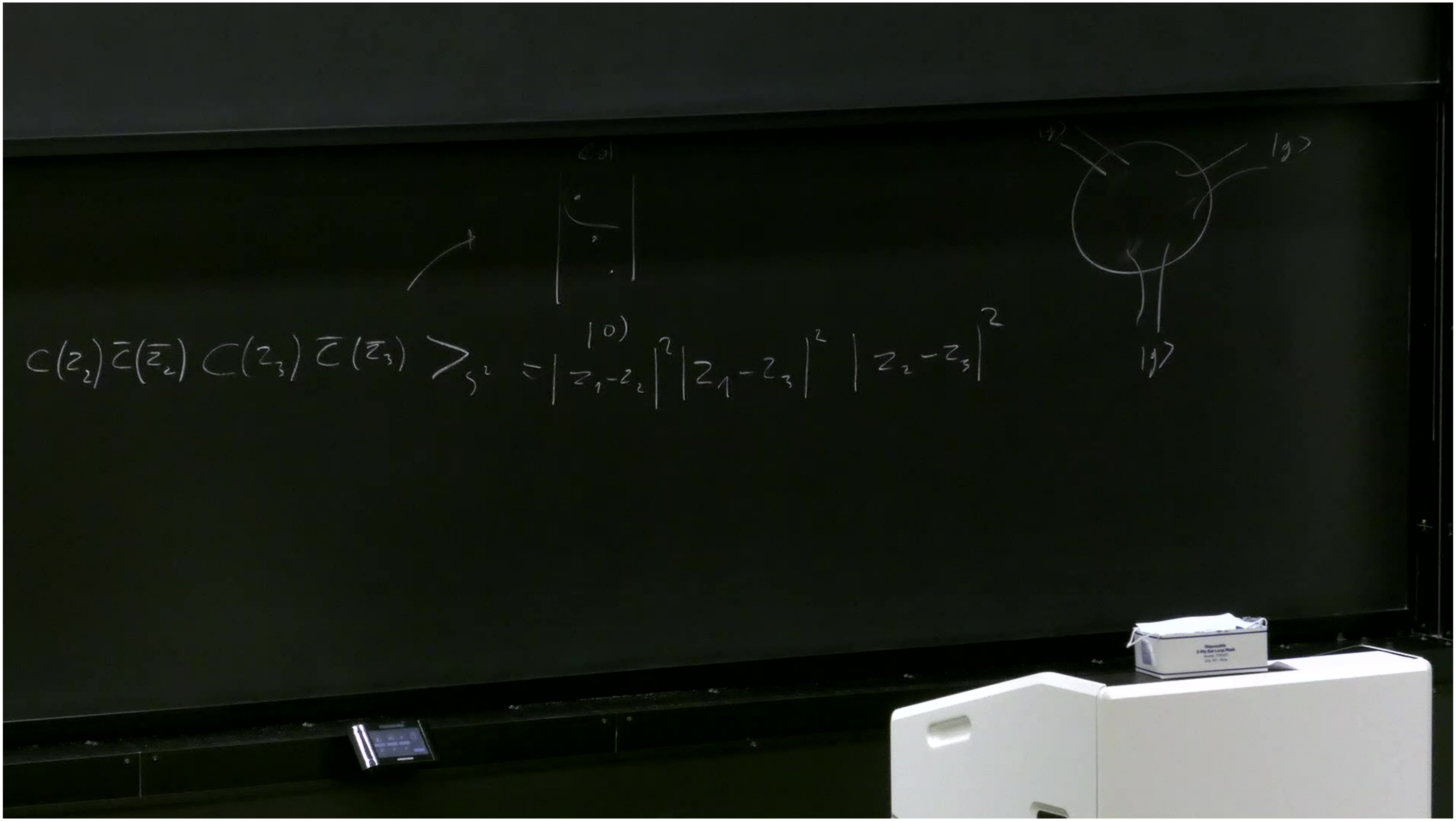
$$\Downarrow$$

$$\langle c(\infty)\bar{c}(\infty)\, c(1)\bar{c}(1)\, c(0)\bar{c}(0) \rangle_{S^2} = 1$$

$$\langle 0 | c_{-1}\bar{c}_{-1}\, c_0\bar{c}_0\, c_1\bar{c}_1 | 0 \rangle = 1$$

$$c(1) = \sum c_n$$

$$\langle c(z_1)\bar{c}(\bar{z}_1)\; c(z_2)\bar{c}(\bar{z}_2)\; c(z_3)\bar{c}(\bar{z}_3) \rangle$$



$\langle 0|$
$|0\rangle$
$|g\rangle$
$|g\rangle$
$|g\rangle$
$C(z_2)\bar{C}(\bar{z}_2)\, C(z_3)\bar{C}(\bar{z}_3) \rangle_{S^2} = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$

$$\ldots(z_1)\; e^{ip_2 X(z_2)}\; e^{ip_3 X(z_3)} \rangle_{S^2} = |z_1 - z_2|^{2p_1p_2}\, |z_1 - z_3|^{2p_1p_3}\, |z_2 - z_3|^{2p_2p_3}$$

$$c(1) = \sum c_n$$

c_0

$|0)$

$$\langle\, c(z_1)\bar{c}(\bar{z}_1)\; c(z_2)\bar{c}(\bar{z}_2)\; c(z_3)\bar{c}(\bar{z}_3) \,\rangle_{S^2} = |z_1 - z_2|^2\, |z_1 - z_3|^2\, |z_2 - z_3|^2$$

$$\langle e \quad e \quad e \rangle_{S^2} = \ldots$$

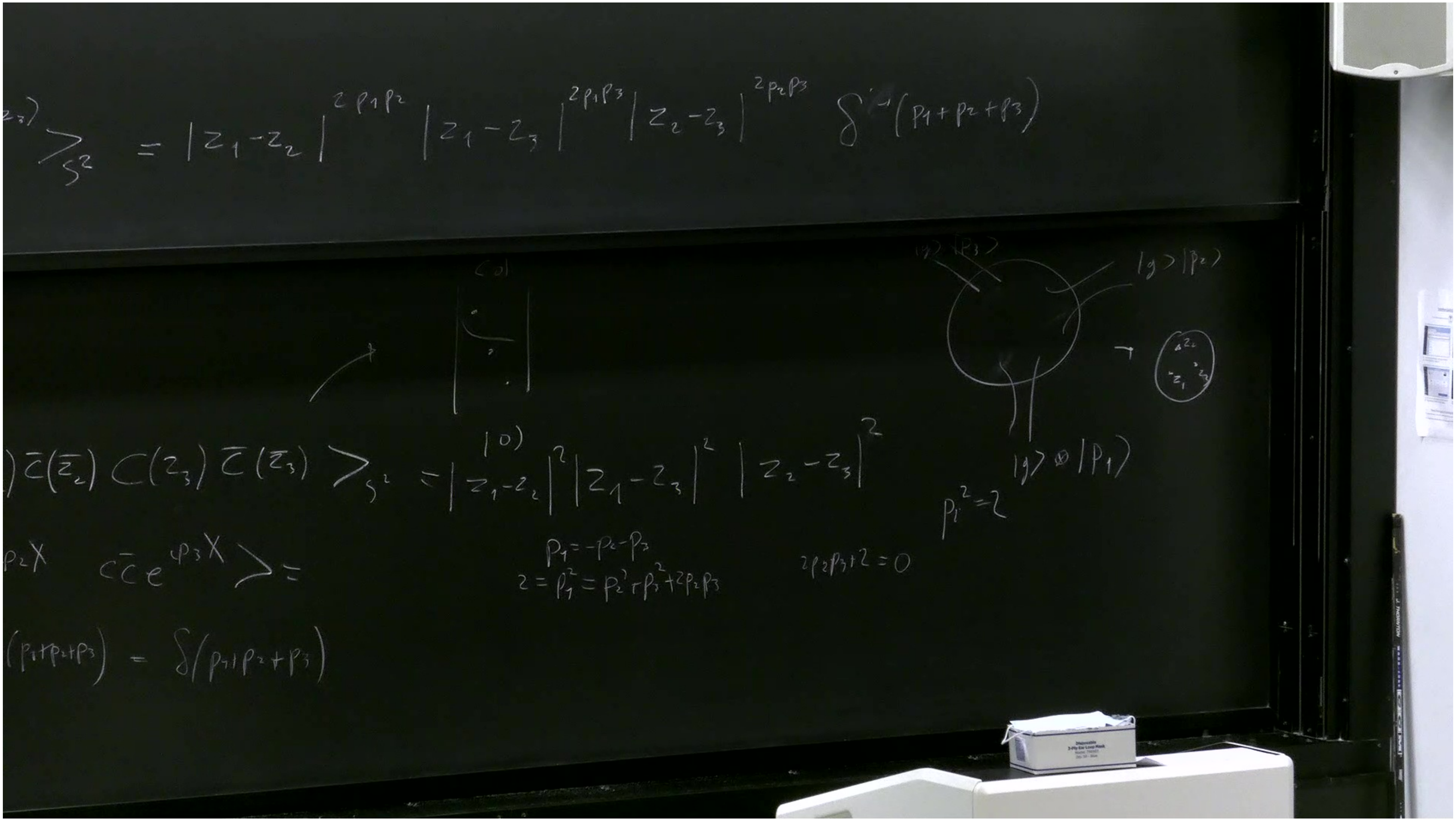
$$\langle g | c_0 \bar{c}_0 | g \rangle = 1 \qquad c(1) = \sum c_n$$

$$\Downarrow$$

$$\langle c(\infty)\bar{c}(\infty)\, c(1)\bar{c}(1)\, c(0)\bar{c}(0) \rangle_{S^2} = 1 \qquad \langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3) \rangle_{S^2} = |$$

$$\langle 0 | c_{-1}\bar{c}_{-1}\, c_0\bar{c}_0\, c_1\bar{c}_1 | 0 \rangle = 1 \qquad \langle c\bar{c}e^{ip_1X}\; c\bar{c}e^{ip_2X}\; c\bar{c}e^{ip_3X} \rangle =$$

$$|z_1 - z_2|^{2p_1\cdot p_2+2}\, |z_1 - z_3|^{2p_1\cdot p_3+2}\, |z_2 - z_3|^{2p_2\cdot p_3+2}$$



$\rangle_{S^2} = |z_1 - z_2|^{2p_1p_2} |z_1 - z_3|^{2p_1p_3} |z_2 - z_3|^{2p_2p_3} \delta(p_1+p_2+p_3)$
$|g\rangle|p_3\rangle$
$|g\rangle|p_2\rangle$
$|g\rangle \otimes |p_1\rangle$
$\bar{c}(\bar{z}_2)\, c(z_3)\, \bar{c}(\bar{z}_3) \rangle_{S^2} = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$
$p_1^2 = 2$
$\bar{c} c\, e^{ip_3 X} \rangle =$
$p_1 = -p_2 - p_3$
$2 = p_1^2 = p_2^2 + p_3^2 + 2p_2p_3$
$2p_2p_3 + 2 = 0$
$(p_1+p_2+p_3) = \delta(p_1+p_2+p_3)$

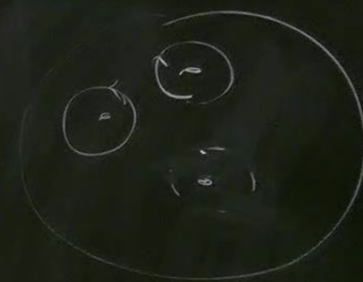
$$\langle O_1(z_1)\, O_2(z_2)\, \partial_3 O_3(z_3) \rangle = \langle O_1\, O_2\, Q O^b \rangle = -\langle O$$

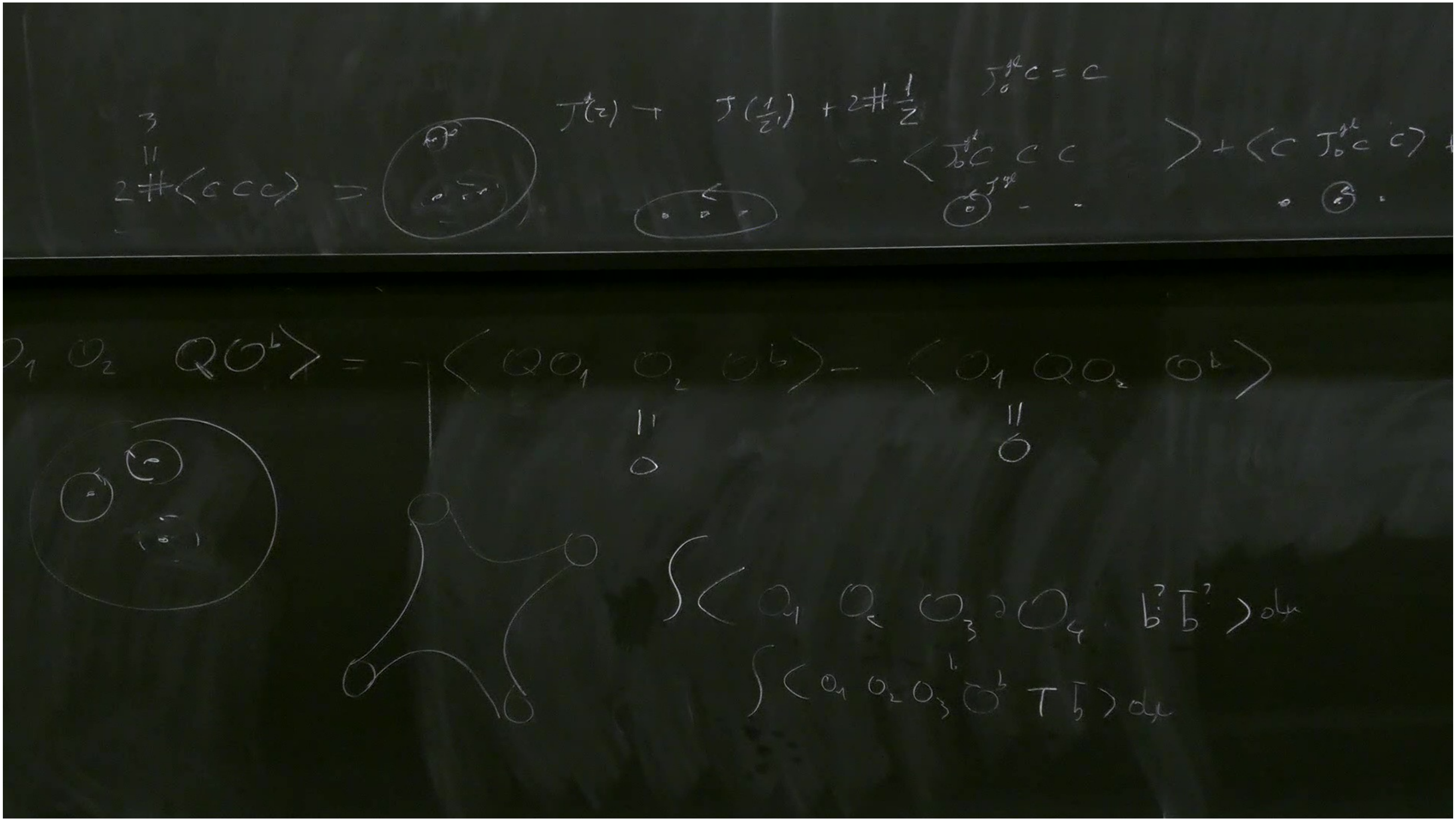
$Q O_i(z_i)$

$$Q\, \partial O(z) = 0$$

$$\partial O(z) = Q\, O^b$$

$$L_{-1}|O\rangle = Q\, b_{-1} O$$




J₀^{gh} c = c
J(z) + J(1/z) + 2# 1/2
2#⟨ccc⟩ =
− ⟨J₀^{gh}c c c⟩ + ⟨c J₀^{gh}c c⟩ +
O₁ O₂ QO^b⟩ = −⟨QO₁ O₂ O^b⟩ − ⟨O₁ QO₂ O^b⟩
∫⟨O₁ O₂ O₃ ∂O₄ b b̄⟩ dμ
∫⟨O₁ O₂ O₃ O^b T b̄⟩ dμ

$$\langle e \quad e \quad e \quad \rangle_{S^2}$$

$$\rightarrow \begin{cases} c\bar{c}\,\mathcal{O}_{PHYS}(z_i) & \text{IF } z_i \text{ IS FIXED} \\ \mathcal{O}_{PHYS}(z_i)\,dz_i\,d\bar{z}_i & \text{IF } z_i \text{ IS INTEGRATED OVER} \end{cases}$$

$$Q\,\mathcal{O}_{PHYS}(z) = \partial\left(c(z)\,\mathcal{O}_{PHYS}(z)\right)$$

$$= \begin{cases} c\bar{c}\, \mathcal{O}_{PHYS}(z_i) & \text{IF } z_i \text{ IS FIXED} \\ \int \mathcal{O}_{PHYS}(z_i)\, dz_i\, d\bar{z}_i & \text{IF } z_i \text{ IS INTEGRATED OVER} \end{cases}$$

$$Q\, \mathcal{O}_{PHYS}(z) = \partial\big(c(z)\, \mathcal{O}_{PHYS}(z)\big)$$

$$b_{-1}(c\,\mathcal{O}_{PHYS}) = \mathcal{O}_{PHYS}$$

$$Q\,\mathcal{O}^{(0)} = 0$$

$$d\,\mathcal{O}^{(0)} = Q\,\mathcal{O}^{(1)}$$

$$d\,\mathcal{O}^{(1)} = Q\,\mathcal{O}^{(2)}$$

$$\mathcal{O}^{(0)} + \mathcal{O}^{(1)} + \mathcal{O}^{(2)}$$

$$\langle e \quad e \quad e \quad \rangle_{S^2}$$

$$= \begin{cases} c\bar{c}\,\mathcal{O}_{PHYS}(z_i) & \text{IF } z_i \text{ IS FIXED} \\ \int \mathcal{O}_{PHYS}(z_i)\,dz_i d\bar{z}_i & \text{IF } z_i \text{ IS INTEGRATED OVER} \end{cases}$$

$$Q\,\mathcal{O}_{PHYS}(z) = \partial\left(c(z)\,\mathcal{O}_{PHYS}(z)\right)$$

$$b_{-1}\left(c\mathcal{O}_{PHYS}\right) = \mathcal{O}_{PHYS}$$

$$\mathcal{O}^{(0)} + \mathcal{O}^{(1)} + \mathcal{O}^{(2)}$$

$$Q\,\mathcal{O}^{(0)} = 0$$

$$d\mathcal{O}^{(0)} = Q\mathcal{O}^{(1)}$$

$$d\mathcal{O}^{(1)} = Q\mathcal{O}^{(2)}$$

$$\langle Q O_1 \;\; O_2 \;\; O^b \rangle - \langle O_1 \;\; Q O_2 \;\; O^b \rangle$$

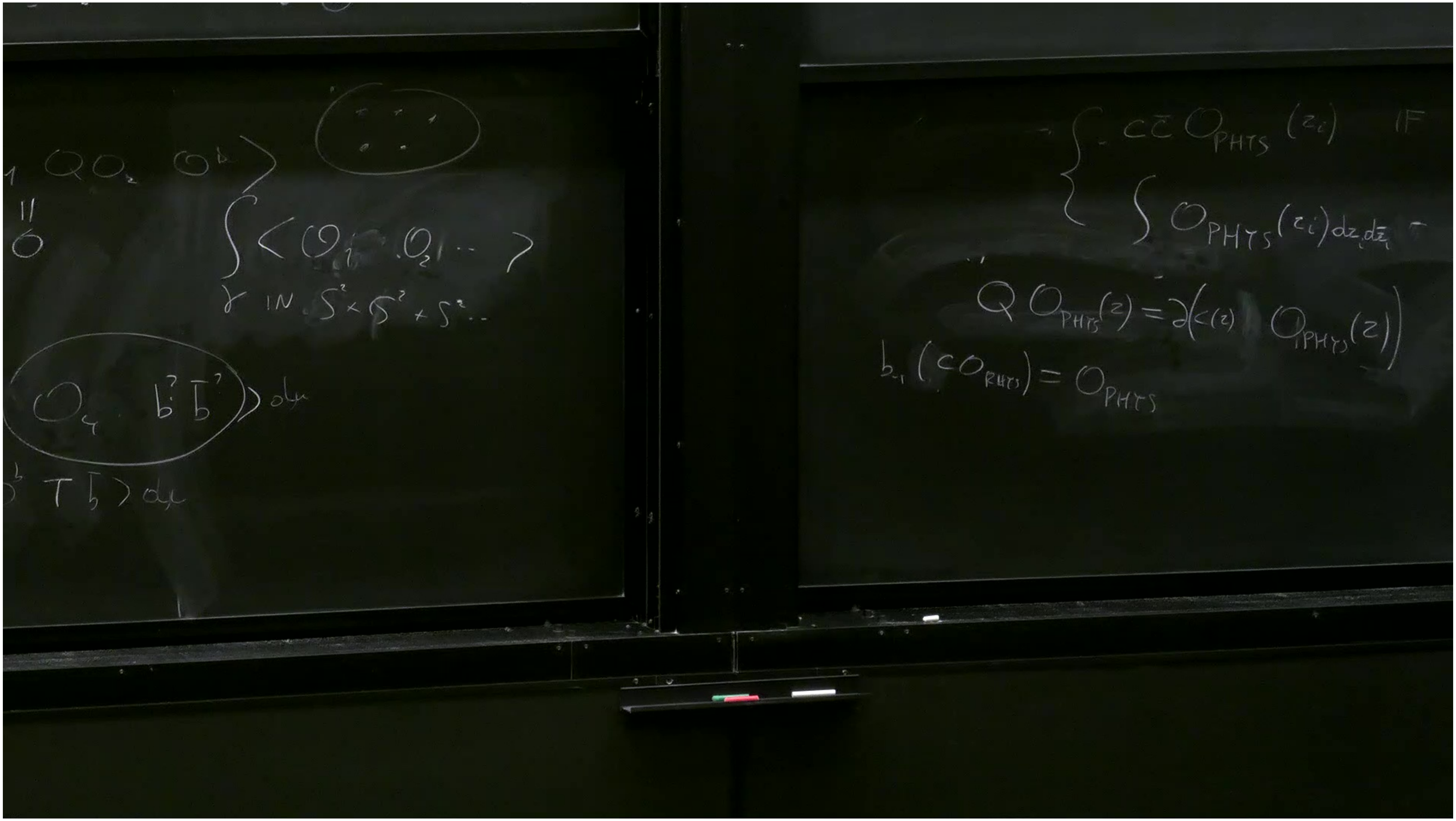
$$\underset{0}{\parallel} \qquad\qquad \underset{0}{\parallel}$$

$$\int \langle O_1 \; O_2 \cdots \rangle$$

γ IN $S^2 \times S^2 \times S^2 \cdots$

$$\int \langle O_1 \;\; O_2 \;\; O_3 \;\; O_4 \;\; b^? \bar{b}^? \rangle \, d\mu$$

$$\int \langle O_1 \;\; O_2 \; O_3 \; O^b \;\; T \bar{b} \rangle \, d\mu$$


$\int c\bar{c}\,\mathcal{O}_{PHYS}(z_i)$ IF
$\int \mathcal{O}_{PHYS}(z_i)\,dz_i\,d\bar{z}_i$
$Q\,\mathcal{O}_{PHYS}(z) = \partial(c(z)\,\mathcal{O}_{PHYS}(z))$
$b_{-1}(c\mathcal{O}_{PHYS}) = \mathcal{O}_{PHYS}$
γ IN $S^2 \times S^2 \times S^2 \ldots$